Brick problems in McGill-AMY

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Plan

- AMY Method Overview
- Brick results
- Discussions

Big Picture

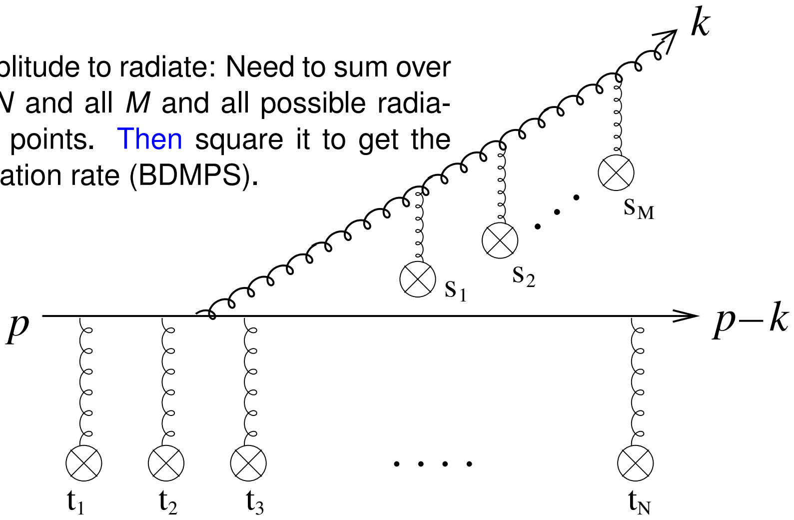
- $K \sum \left(f_{a/A} \otimes f_{b/A} \otimes \frac{d\sigma_{ab\to cd}}{dx} \right) \otimes \text{(E-loss module)} \otimes D_{\mathrm{frag}}$
- Parton-parton scattering: $\left(f_{a/A} \otimes f_{b/A} \otimes \frac{d\sigma_{ab\to cd}}{dx} \right)$
- K: By comparing the LO and Aurenche et al.'s NLO.
- D_{frag}: As in vacuum but with reduced energy.
- Energy loss module – Three separate pieces
  - Energy change rate: $\frac{d\Gamma}{dtdk}(\epsilon, k; T)$
  - Evolution:

$$\frac{dP(\epsilon,t)}{dt} = \int dk \frac{d\Gamma}{dtdk} P(\epsilon+k,t) - \int dk \frac{d\Gamma}{dtdk} P(\epsilon,t)$$

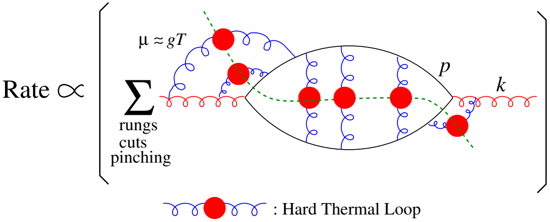
  - Space-time dependence thru $T(\mathbf{x},t), u^{\mu}(\mathbf{x},t)$: Must be obtained independently.

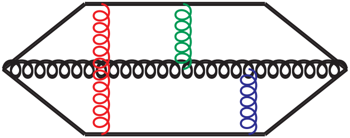
Gluon Radiation Calculation

Amplitude to radiate: Need to sum over all *N* and all *M* and all possible radiation points. Then square it to get the radiation rate (BDMPS).



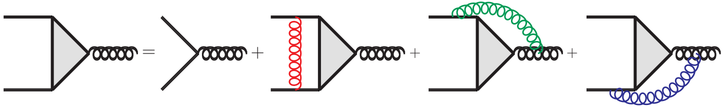
Diagrams





Pinching poles give the leading order result.

SD equation for the vertex



$$2\mathbf{h} = i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \times$$
$$\times \Big\{ (C_s - C_{\rm A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{-}k\,\mathbf{q}_\perp)]$$
$$+(C_{\rm A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{+}p\,\mathbf{q}_\perp)]$$
$$+(C_{\rm A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{-}(p{-}k)\,\mathbf{q}_\perp)] \Big\},$$

$$\delta E(\mathbf{h},p,k) = \frac{\mathbf{h}^2}{2pk(p{-}k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p{-}k)} - \frac{m_p^2}{2p}\,.$$

- m^2: Medium induced thermal masses.
- $\mathbf{h} = (\mathbf{p} \times \mathbf{k}) \times \mathbf{e}_{||}$ — Must keep track of both $\mathbf{p}_\perp$ and $\mathbf{k}_\perp$ now.

Gluon Radiation Rate

Rate using **F**

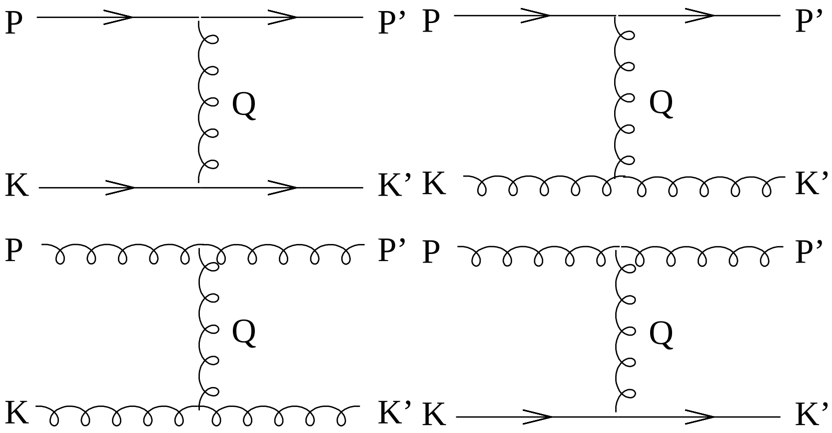
$$\frac{d\Gamma_g(p,k)}{dkdt} = \frac{C_s g_s^2}{16\pi p^7} \frac{1}{1 \pm e^{-k/T}} \frac{1}{1 \pm e^{-(p-k)/T}} \times$$
$$\times \left\{ \begin{array}{cc} \frac{1+(1-x)^2}{x^3(1-x)^2} & q \to qg \\ N_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \to qq \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \to gg \end{array} \right\}$$
$$\times \int \frac{d^2\mathbf{h}}{(2\pi)^2} 2\mathbf{h} \cdot \mathrm{Re}\, \mathbf{F}(\mathbf{h},p,k)\,,$$

where $x \equiv k/p$ is the momentum fraction in the gluon (or either quark, for the case $g \to qq$). $\mathbf{h} \equiv \mathbf{p} \times \mathbf{k}$: 2-D vector. $O(gT^2)$

- Correctly incorporates *both* the BH limit and the LPM limit.

Elastic scattering rate

Coulombic *t*-channel dominates



Elastic scattering rate

We need

$$\frac{dE}{dt} = \frac{1}{2E}\int_{k,k',p'} \delta^4(p+k-p'-k')\,(E-E')|M|^2\, f(E_k)[1 \pm f(E'_k)]$$
$$= C_r \pi\, \alpha_s^2 T^2 \left[\ln(ET/m_g^2) + D_r\right]$$

where C_r and D_r are channel dependent $O(1)$ constants.

Putting them together

- Fokker-Planck Eqn.

$$\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} + 2P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt},$$

$$\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dkdt} + P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} - P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt} + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(k-p/2)\right)$$

- Inelastic part is solved as it is.
- Elastic part – Soft exchange dominated.
  Implement it as either
  (i) drag + diffusion or
  (ii) $\Gamma = \Gamma_{\rm el} + \Gamma_{\rm inel}$

Included Physics

- Parton splitting due to scattering in the medium; the treatment includes
  - treatment of the medium as dynamical scattering, with HTL screening
  - solution of integral equation which treats the LPM effect as an $O(1)$ effect, smoothly interpolating between Bethe-Heitler for low emission energy and strong LPM effect for high emission energy
  - Bose stimulation factors at low energies and inverse Bremsstrahlung absorption from the medium
  - medium induced dispersion corrections
- inclusion of all subsequent evolution of all daughters of a splitting process (above a cutoff energy)
- Compton-type QCD scattering, qq to gg and qg to gq
- Elastic energy loss
- All processes explicitly obey detailed balance.

McGill-AMY Brick Results

10 GeV

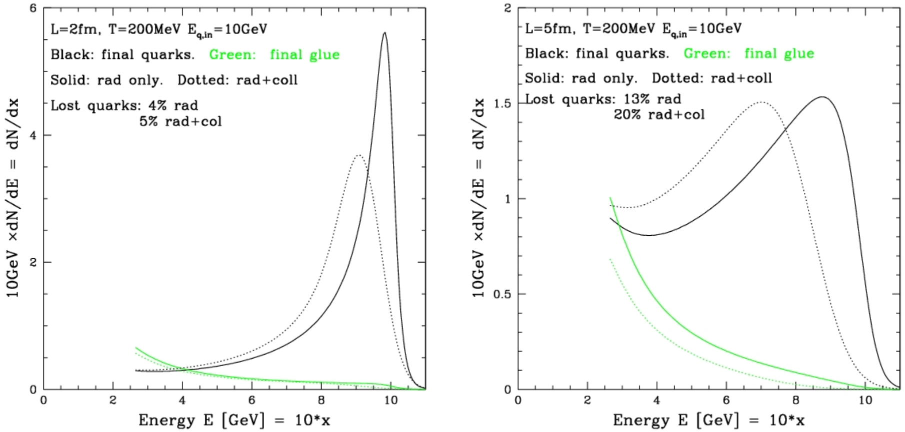
NOTE: Our

$$P(\epsilon) = \frac{dN}{d\epsilon}$$

It is NOT the probability density.

Original Brick

A 10 GeV quark propagating. Δx dependence.



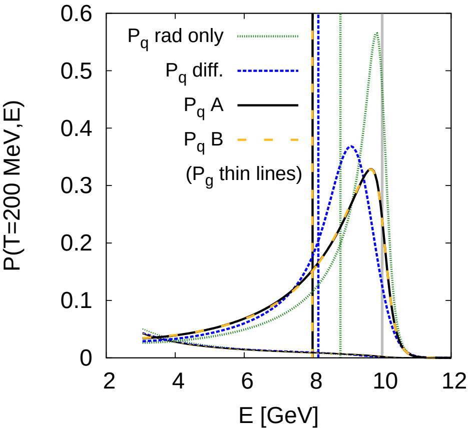
2 fm

5 fm

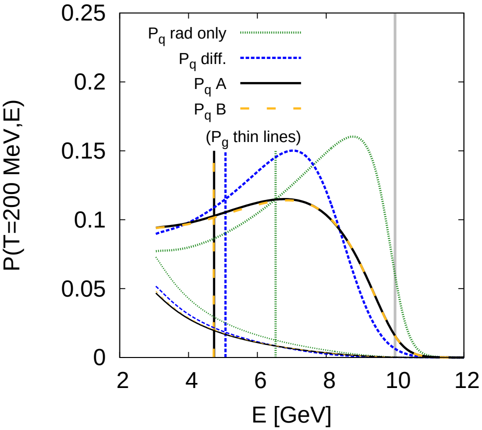
Full implementation of elastic scattering Γ (labeled A, B)

arXiv:0901.3498, Schenke, Gale, Qin

A 10 GeV quark propagating. Δx dependence. (Again, $P = dN/dE$)



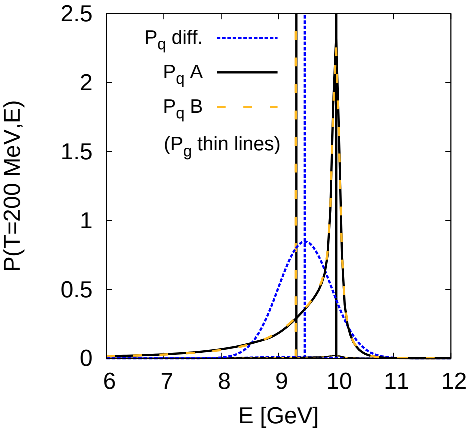
2 fm



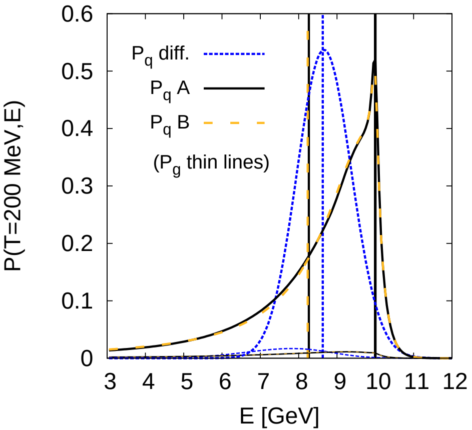
5 fm

A, B: Full implementation of elastic scattering Γ
arXiv:0901.3498, Schenke, Gale, Qin

Elastic only



2 fm



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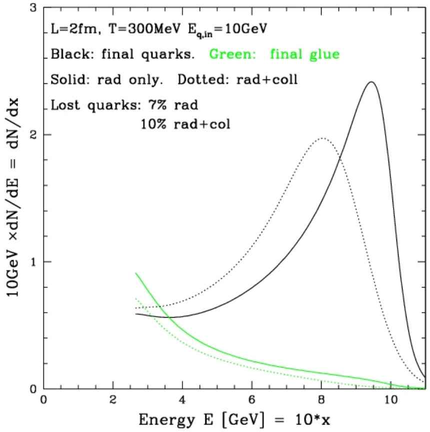
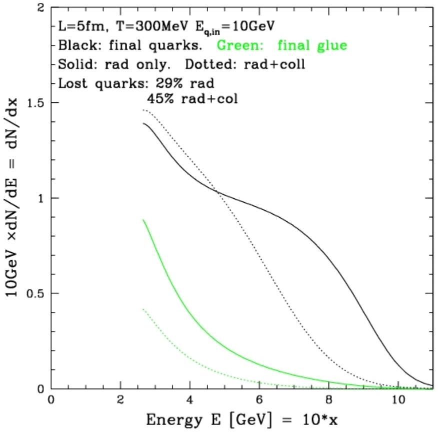
Original Brick

A 10 GeV quark propagating. Δx dependence.





Original Brick

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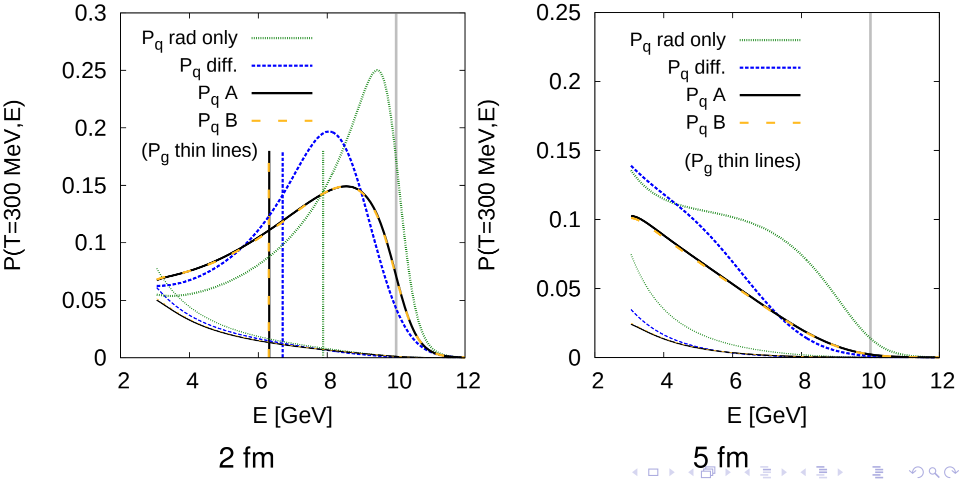
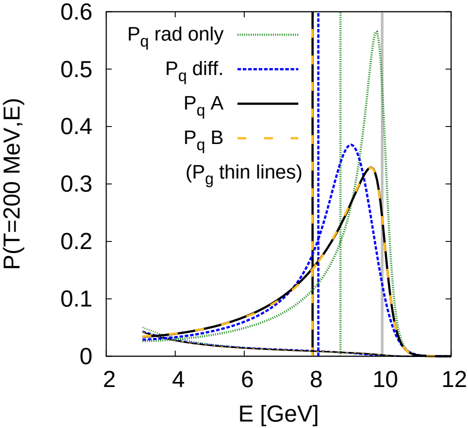
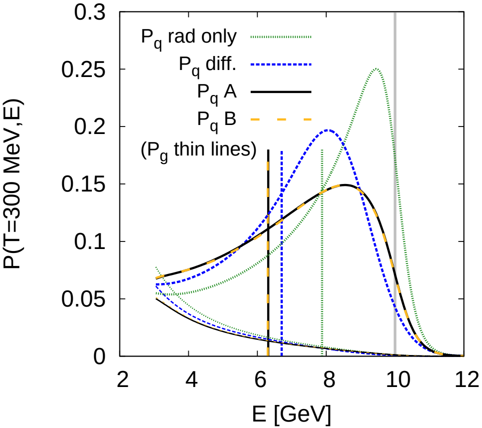


A 10 GeV quark propagating. T dependence. $\Delta x = 2\,\mathrm{fm}$.





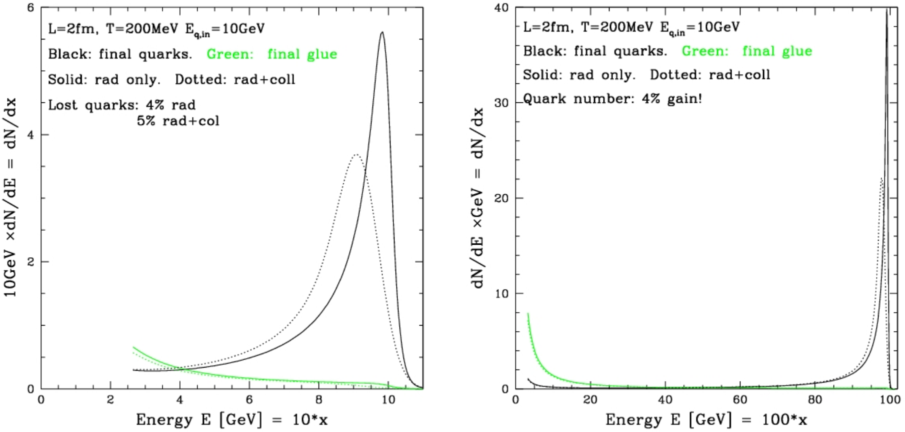
10 GeV Summary

- Higher T more effective (of course)
- Elastic contribution significant
- Full implementation of elastic scattering is different than diffusion approx.
  - Peak does not shift away from the original energy
  - Power-law tail for small E
- Around $\Delta x = 5\,\mathrm{fm}$, the memory of the peak is almost gone for both $T = 200\,\mathrm{MeV}$ and $T = 300\,\mathrm{MeV}$.

100 GeV Results

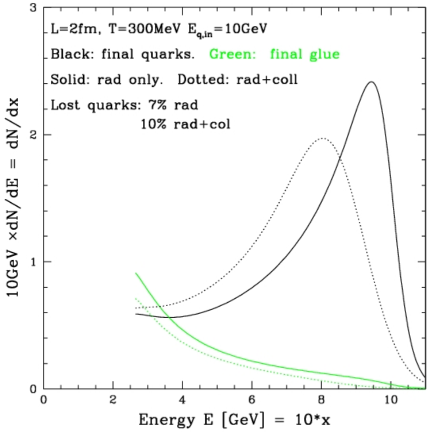
Original Brick

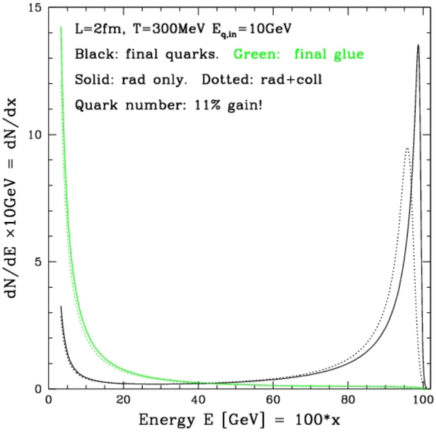
Energy dependence. 10 GeV vs. 100 GeV. $T = 200\,\text{MeV}$.



Original Brick

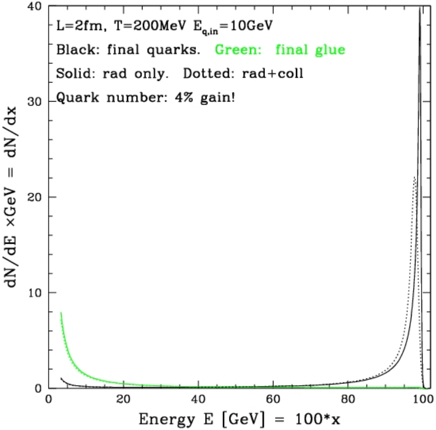
Energy dependence. 10 GeV vs. 100 GeV. $T = 300\,\text{MeV}$.



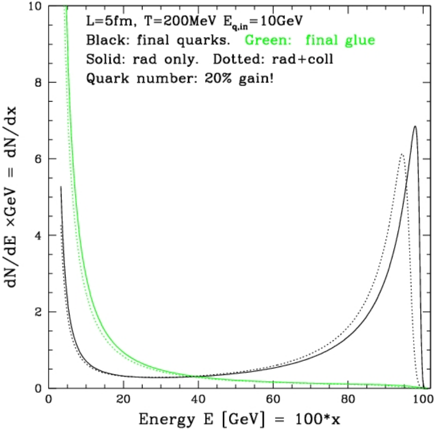


Original Brick

100 GeV. Δx dependence

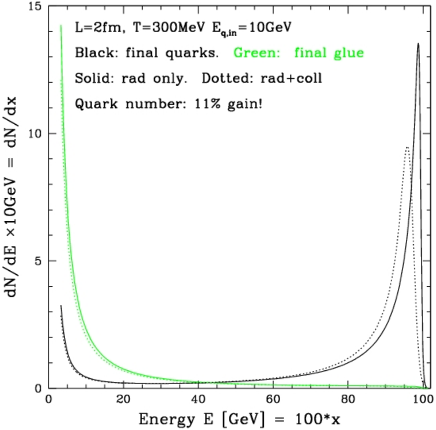


2 fm

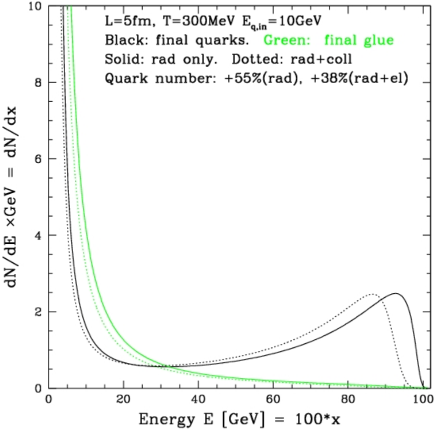


5 fm

100 GeV. Δx dependence



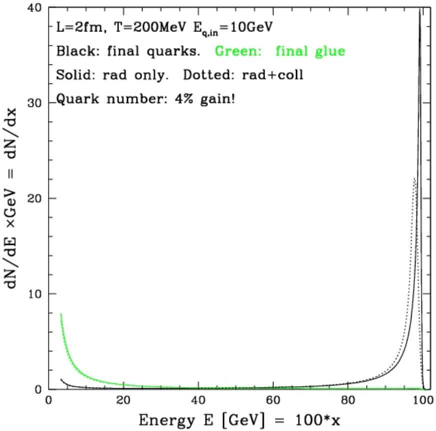
2 fm



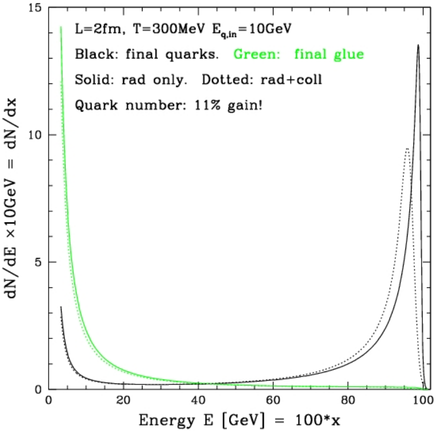
5 fm

Original Brick

100 GeV. T dependence. $\Delta x = 2\,\mathrm{fm}$



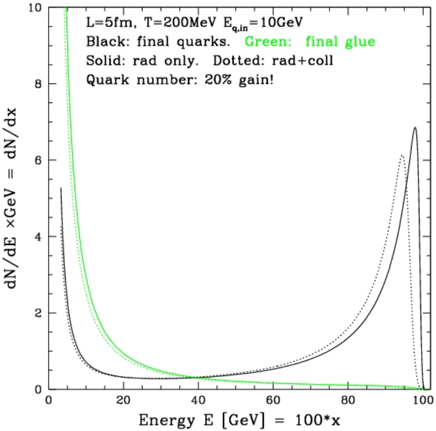
200 MeV



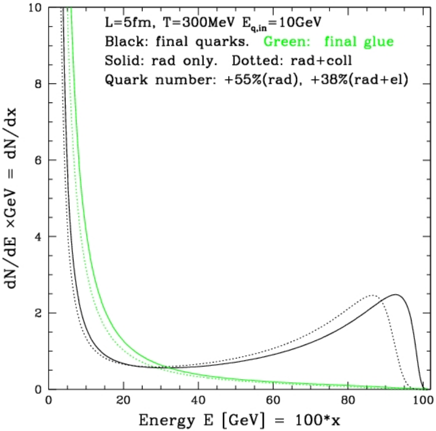
300 MeV

Original Brick

100 GeV. T dependence. $\Delta x = 5\,\mathrm{fm}$



200 MeV



300 MeV

100 GeV Summary

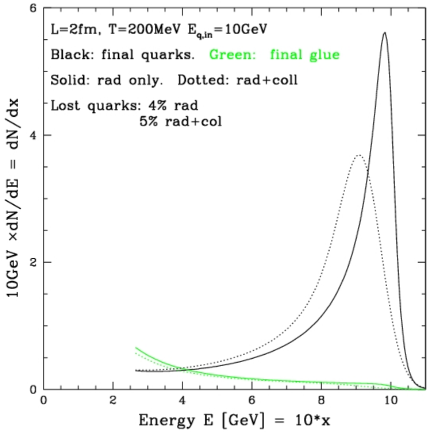
- Net gain in $q\bar{q}$ population
  - Radiated g is hard enough for $g \to q\bar{q}$ to contribute significantly
  - May influence in-jet multiplicity distribution at LHC – both $\langle N \rangle$ and $\langle \Delta N^2 \rangle$ may become larger.
- Strong low energy tail for $T = 300\,\text{MeV}$
  - Strong presence of $20 - 60\,\text{GeV}$ secondary partons – Phenomenology implications? v_2? Photons? γ-jet correlations?

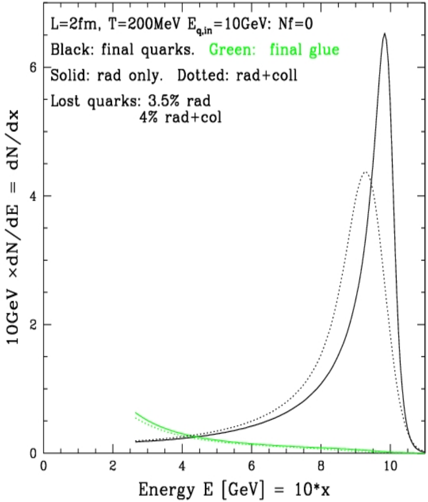
QGP vs. Gluon plasma

Original Brick

QGP vs. Gluon plasma – No medium quarks and no $g \to q\bar{q}$ splittings.

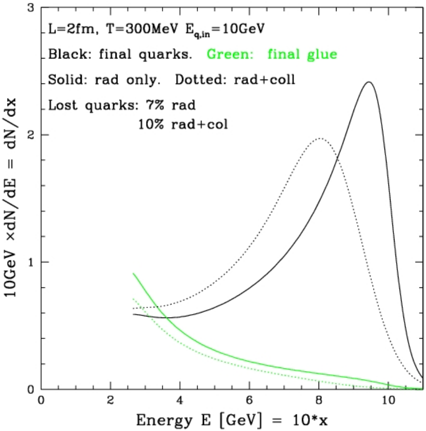
$\Delta x = 2\,\mathrm{fm}$, $T = 200\,\mathrm{MeV}$

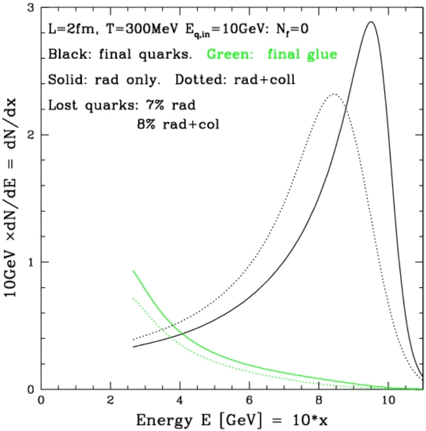




Original Brick

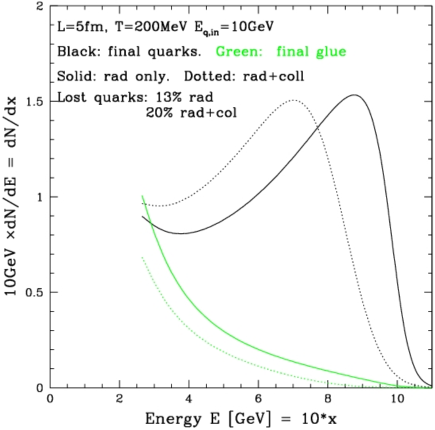
QGP vs. Gluon plasma. $\Delta x = 2\,\mathrm{fm}$, $T = 300\,\mathrm{MeV}$

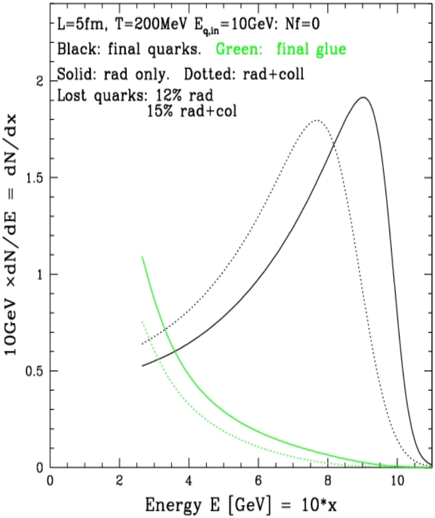




Original Brick

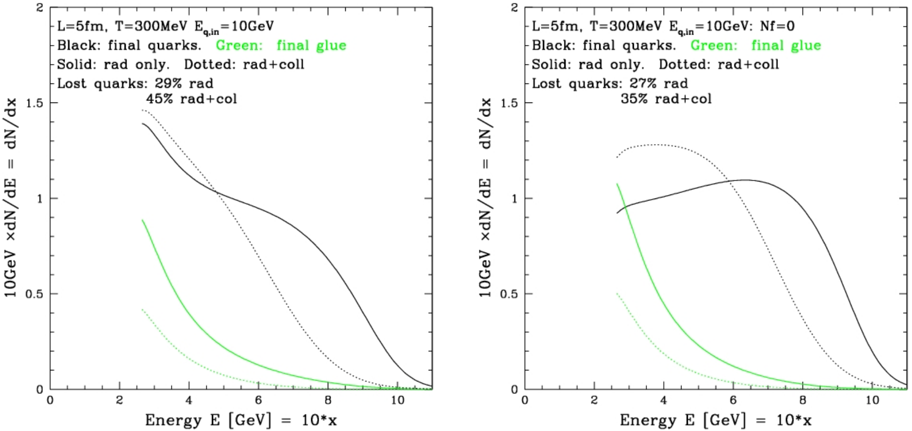
QGP vs. Gluon plasma. $\Delta x = 5\,\mathrm{fm}$, $T = 200\,\mathrm{MeV}$





Original Brick

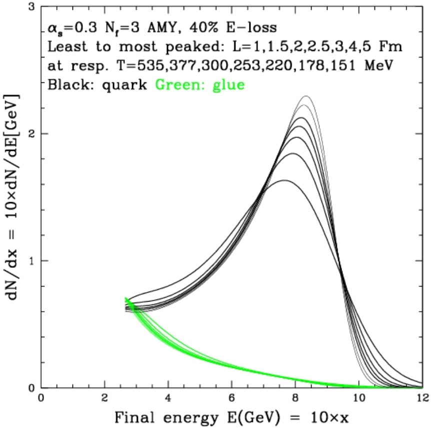
QGP vs. Gluon plasma. $\Delta x = 5\,\mathrm{fm}$, $T = 300\,\mathrm{MeV}$



Gluon Plasma Summary

- Gluon plasma:
  - Density of scatterer is smaller at the same T – reduces emission
  - No splitting – reduces emission
  - Screening mass is smaller – enhances emission
  - Thermal mass is smaller – enhances emission
  - No processes involving thermal quark – reduces loss
  - Overall one needs higher temperature to achieve the same amount of E-loss

Wiedemann's Brick – Fixed energy loss

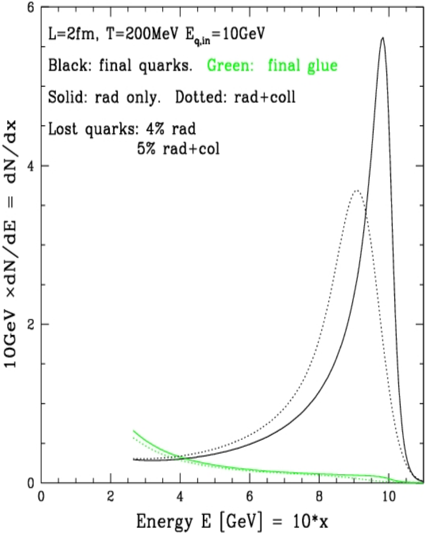


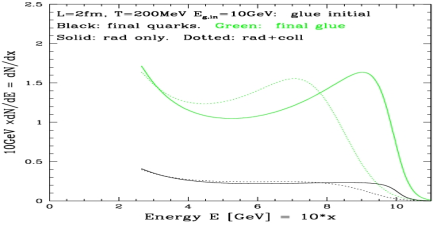
Gluon dN_g/dE is almost unchanging.

Quarark jet vs Gluon jet

Original Brick

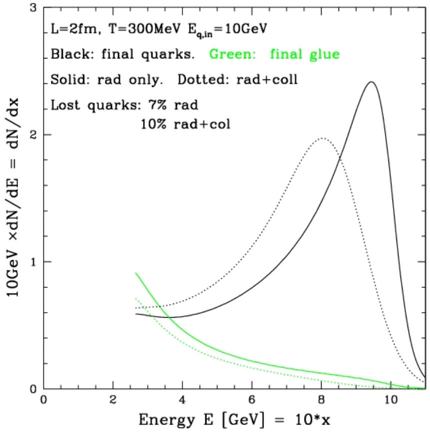
Quark jet vs. Gluon jet. $T = 200\,\mathrm{MeV}$, $\Delta x = 2\,\mathrm{fm}$

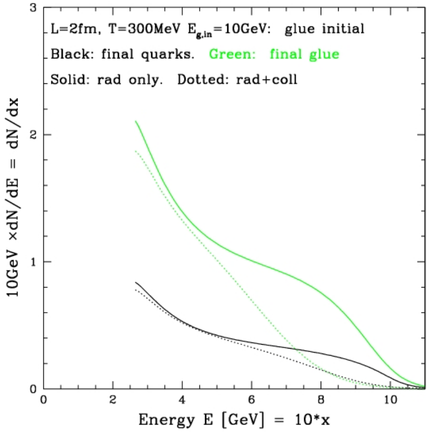




Original Brick

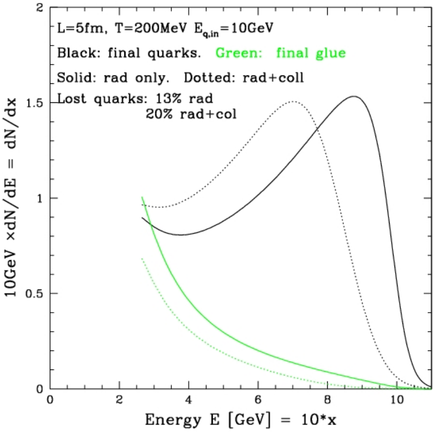
Quark jet vs. Gluon jet. $T = 300\,\text{MeV}$, $\Delta x = 2\,\text{fm}$

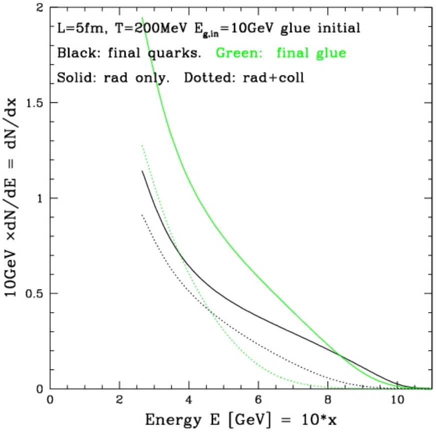




Original Brick

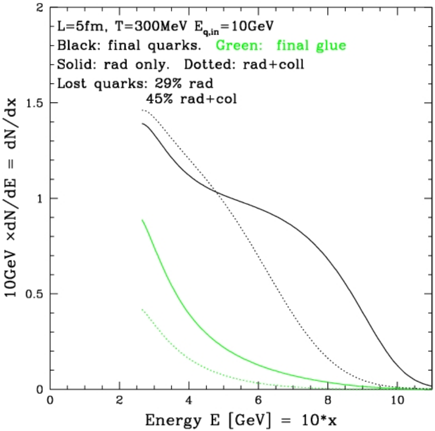
Quark jet vs. Gluon jet. $T = 200\,\text{MeV}$, $\Delta x = 5\,\text{fm}$

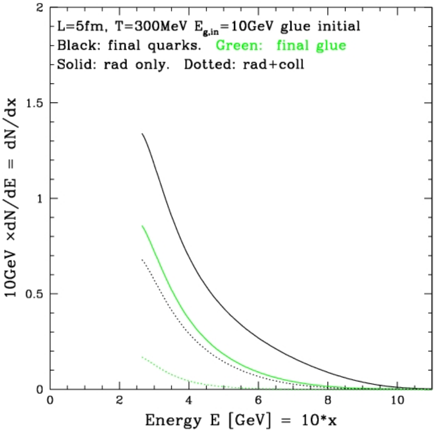




Original Brick

Quark jet vs. Gluon jet. $T = 300\,\mathrm{MeV}$, $\Delta x = 5\,\mathrm{fm}$



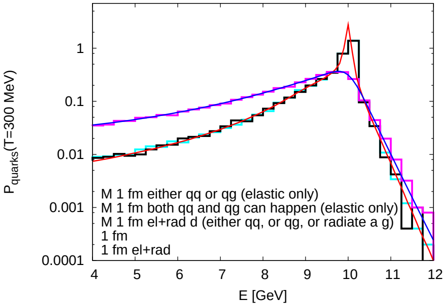


Gluon Jet Summary

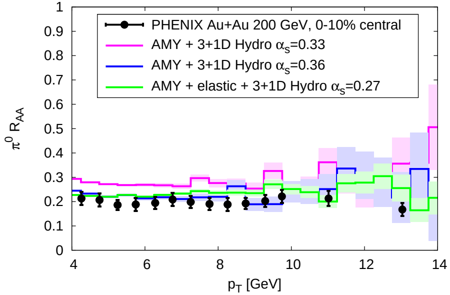
- As expected:
  Gluons lose energy much quicker than the quarks
  Leaves very strong $q\bar{q}$ remnant.

MARTINI

(Schenke, Gale, Jeon)
MC Event Generator based on PYTHIA 8.1 and McGill-AMY.
Preliminary, but getting there.



Brick problem verification.
Quark, 1 fm.



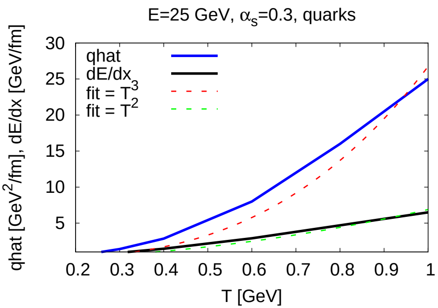
Pion R_{AA}

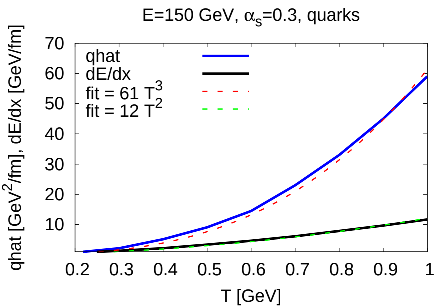
Summary for the Brick problem

McGill-AMY:

- Thermal QCD based approach.
- Solves PF-equation w/ detailed balance $\Longrightarrow$ No probability leakage
- Radiational + Collisional now on equal footings $\Longrightarrow$ Diffusion approx. is not adequate
- 100 GeV results: Many semi-hard (20 – 60 GeV) $q\bar{q}$ daughters
- Monte-Carlo event generator version (MARTINI) in prep.

$\hat{q}$ in MARTINI





$E = 25\,\text{GeV}$ $\qquad$ $E = 150\,\text{GeV}$

AMY Estimate: $\hat{q} \approx 0.3 \left(\frac{T}{0.2\,\text{GeV}}\right)^3 \text{GeV}^2/\text{fm}$