An MC algorithm implementing medium-induced gluon radiation

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Yesterday's talk = introduction
Motivation = 1. LHC phenomenology requires description of hard multi parton final states in heavy ion collisions. This motivates development of MC algorithm
2. exact E-p-conservation not yet implemented in previous calculations but straightforward to implement

Here: technical discussion, focussing on the guidance which we get for an MC algorithm from existing analytical calculations.

Report on work in progress with **K. Zapp and J. Stachel**

3rd TEC-HQM workshop,
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Medium-induced gluon radiation

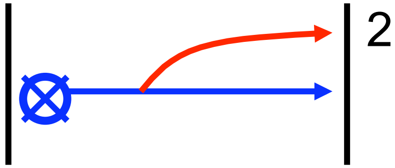
(expression with or without interference with vacuum term)

$$\frac{dI}{d\ln\omega\, dk_T} = \frac{\alpha_s C_R}{(2\pi)^2\omega^2} 2\,\mathrm{Re}\int_{\boxed{\xi_0}}^{\infty} dy \int_{y}^{\infty} d\bar{y} \int du\, e^{-ik_T u} e^{\left[-\frac{1}{2}\int_y^{\infty} d\xi\, n(\xi)\sigma(u)\right]}$$

$$\times \frac{\partial}{\partial u} . \frac{\partial}{\partial s} K(s=0, y; u, y \mid \omega)$$

If $\boxed{\xi_0 = 0}$ then projectile produced at time $\xi_0 = 0$
and this projectile radiates

$$\omega \frac{dI^{(0)}}{d\omega\, dk_T} = \frac{\alpha_s C_R}{\pi^2} H(k_T) =$$



$$, \qquad H(k_T) = \frac{1}{k_T^2}$$

If $\boxed{\xi_0 = -\infty}$ then projectile exists since tine $\xi_0 = -\infty$
and this projectile is “on-shell”

$$\omega \frac{dI^{(0)}}{d\omega\, dk_T} = 0$$

Consider here the simpler case $\xi_0 = -\infty$

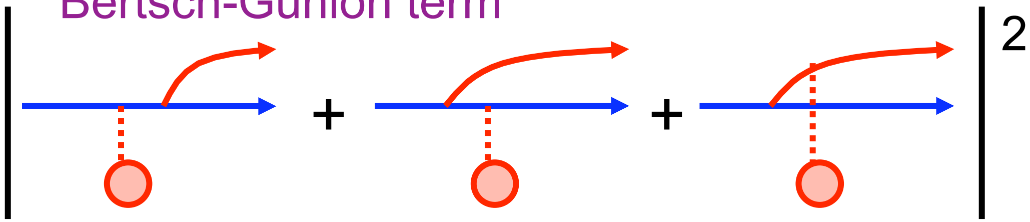
First order opacity for $\xi_0 = -\infty$

$$\omega \frac{dI^{(1)}}{d\omega\, dk_T\, dq_1} = \frac{\alpha_s C_R}{\pi^2}(n_0 L)\left(\left|A(q_1)\right|^2 - V_{tot}\delta(q_1)\right)\frac{q_1^2}{k^2\left(k+q_1\right)^2}$$

This describes radiation in a single scattering event, which occurs $(n_0 L V_{tot})$ times in a medium of length L.

$$V_{tot} = \int dq_1 \left|A(q_1)\right|^2$$

Bertsch-Gunion term



$$= \left|A(q_1)\right|^2 R(k;q_1) \equiv \left|A(q_1)\right|^2 \frac{q_1^2}{k^2\left(k+q_1\right)^2}$$

This is straightforward to implement in parton cascade.

Second order opacity for $\xi_0 = -\infty$

$$\omega \frac{dI^{(1)}}{d\omega\, dk_T\, dq_1\, dq_2} = \frac{\alpha_s C_R}{\pi^2}\left(\left|A(q_1)\right|^2 - V_{tot}\delta(q_1)\right)\left(\left|A(q_2)\right|^2 - V_{tot}\delta(q_2)\right)$$

$$\times\left[\frac{(n_0 L)^2}{2} R(k+q_{1;}q_2) - n_0^2 \frac{1-\cos LQ_1}{Q_1^2} R(k+q_{1;}q_2) + n_0^2 \frac{1-\cos LQ_1}{Q_1^2} R(k;q_1+q_2)\right]$$

The interference factors interpolate between totally coherent and totally incoherent limits

$$n_0^2 \frac{1-\cos LQ_1}{Q_1^2} = \begin{cases} n_0^2 L^2/2 & ,\tau_1 >> L \quad coherent \\ 0 & ,\tau_1 << L \quad incoherent \end{cases}$$

Where $\tau_1 = \dfrac{1}{Q_1} = \dfrac{2\omega}{\left(k+q_1\right)^2}$ is formation time before the 2nd scattering.

Incoherent production limit $L >> \tau_1$

If the gluon is fully formed before entering the 2nd interaction with momentum transfer q_1, then this momentum transfer simply shifts the transverse momentum distribution of the gluon

$$\omega \frac{dI^{(2)}}{d\omega\, dk_T dq_1\, dq_2} = \frac{\alpha_s C_R}{\pi^2}\left(\left|A(q_1)\right|^2 - V_{tot}\delta(q_1)\right)\left(\left|A(q_2)\right|^2 - V_{tot}\delta(q_2)\right)\frac{(n_0 L)^2}{2} R(k+q_1;q_2)$$

This is simply probabilistic rescattering of a fully formed gluon, Straightforward to implement in MC algorithm.

Coherent production limit $L << \tau_1$

If the gluon is not formed prior to the 2nd interaction, then both act coherently

$$\omega \frac{dI^{(2)}}{d\omega\, dk_T dq_1\, dq_2} = \frac{\alpha_s C_R}{\pi^2}\left(\left|A(q_1)\right|^2 - V_{tot}\delta(q_1)\right)\left(\left|A(q_2)\right|^2 - V_{tot}\delta(q_2)\right)\frac{(n_0 L)^2}{2} R(k;q_1+q_2)$$

This extends to all orders in opacity. Analytical results provide a lot of guidance, e.g.

$$\omega \frac{dI}{d\omega\, dk_T} = \exp\left[-n_0 L V_{tot}\right] \times \left(V_{tot}\, independent\ term\right)$$

This defines the **no-scattering probability** in JEWEL.

MC algorithm in a nutshell:

produce gluon in inelastic process, determine position of next scattering center. If formation time shorter than distance, then incoherent. Else, add momentum transfer from next scattering center to not fully formed gluon, reevaluate the formation time and proceed.

Consider back-of-the-envelope estimate for L²-dependence

Phase accumulated in medium: $\left\langle k_T^2 \, \Delta z \big/ 2\omega \right\rangle \approx \frac{\hat{q} L^2}{2\omega} = \frac{\omega_c}{\omega}$

Characteristic gluon energy

Number of coherent scatterings: $N_{coh} \approx \frac{t_{coh}}{\lambda}$, where $t_{coh} \approx \frac{2\omega}{k_T^2} \approx \sqrt{\omega/\hat{q}}$

$k_T^2 \approx \hat{q}\, t_{coh}$

Gluon energy distribution: $\omega \frac{dI_{med}}{d\omega\, dz} \approx \frac{1}{N_{coh}} \omega \frac{dI_1}{d\omega\, dz} \approx \alpha_s \sqrt{\frac{\hat{q}}{\omega}}$

Average energy loss $\Delta E = \int_0^L dz \int_0^{\omega_c} d\omega\, \omega \frac{dI_{med}}{d\omega\, dz} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2$

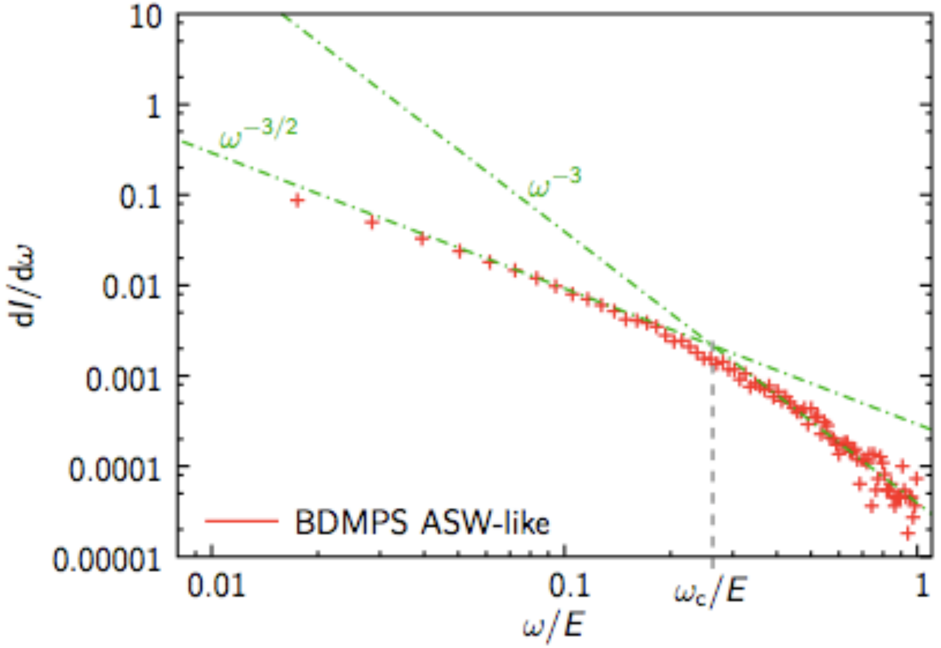
This indicates that accounting for interference in terms of **formation time** alone reproduces central features of medium-induced parton energy loss, namely at least sqrt-omega dependence and L²-dependence.

But MC implementation reveals significant quantitative differences to above estimate:
- it's numerical important that coherence time is not replaced by average coherence time
- L²-dependence arises only in BDMPS limit, where hard scattering tails are neglected.

If we implement approximations of BDMPS-ASW,

(i.e. no energy degradation of projectile, soft momentum transfer, matrix element w/o microscopic E-p-conservation at each vertex)

then MC algorithm returns BDMPS-ASW



BDMPS ASW results:

$$\frac{dI}{d\omega} \propto \omega^{-3/2} \quad \text{for} \quad \omega < \omega_c$$

$$\frac{dI}{d\omega} \propto \omega^{-3} \quad \text{for} \quad \omega > \omega_c$$

$$\Delta E \propto L^2 \quad \text{for} \quad L < L_c$$

$$\Delta E \propto L \quad \text{for} \quad L > L_c$$

But we can remove these approximations and keep interference terms

=> Some first results in talk by Korinna Zapp

Back-up:How to implement LPM-effect in MCs?

- General assumption about the medium:
  The medium gives the projectile the possibility to enter elastic or inelastic interactions, given by cross sections (scattering centers Q_T are distributed with density n

$$\sigma^{qQ_T \to qQ_T} \qquad \sigma^{qQ_T \to qQ_T g}$$

- General problem of putting LPM effect in Monte Carlos:
  How to decide whether different scattering centers act coherently?
  Answer: consider formation time for gluon produced in single scattering

$$t_F = 2\omega / k_T^2$$

If t_F < d (distance to next scattering center) then
-> gluon produced incoherently, probabilistic implementation trivial

If t_F > d then
-> add $q_{T,i}$ of next (ith) scattering center to get $q_{tot} = \sum_i q_{T,i}$
-> recalculate inelastic process under constraint that
q_{tot} is transferred from medium (i.e. assume coherent production)
-> determine new formation time $t'_F = 2\omega / (k_T + q_{tot})^2$
-> check whether **t'_F < d** , else repeat