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The Brick Problem in High-Twist Approximation

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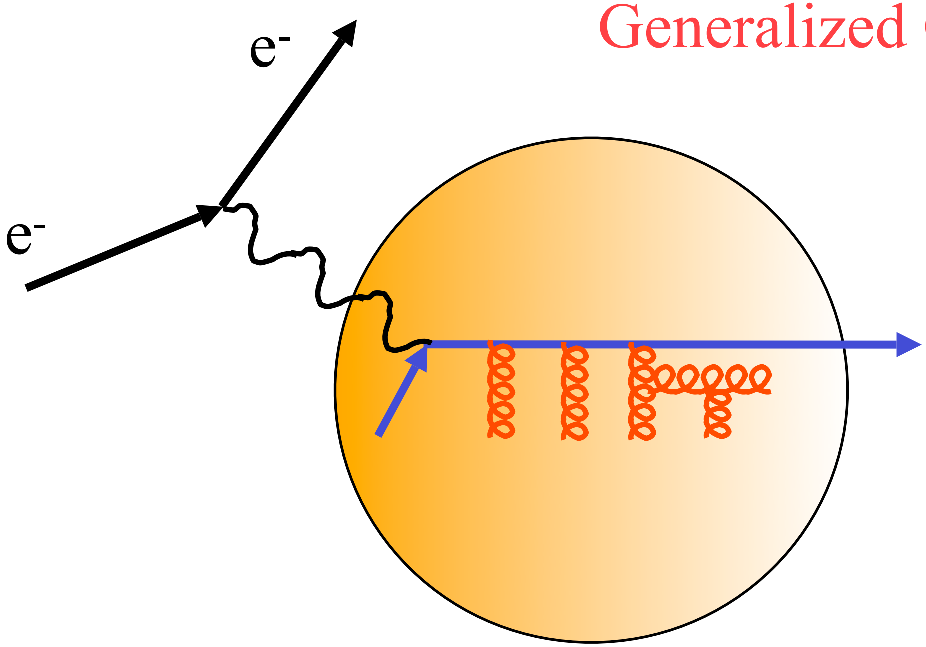
High-Twist Approach

Generalized Collinear Approx. in pQCD

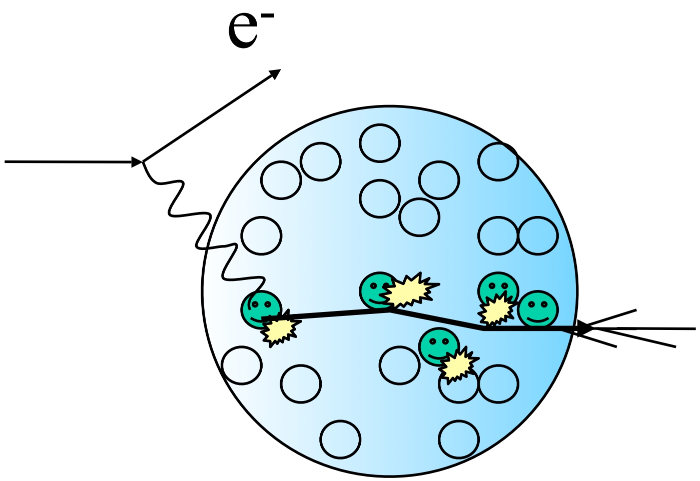


Production & propagation in the same framework

However, a general proof of factorization is not done

Extension to Hevay-ion collisions: $f_q^A(x_B) \leftrightarrow d\sigma/d^2p_T dx_1 dx_2$

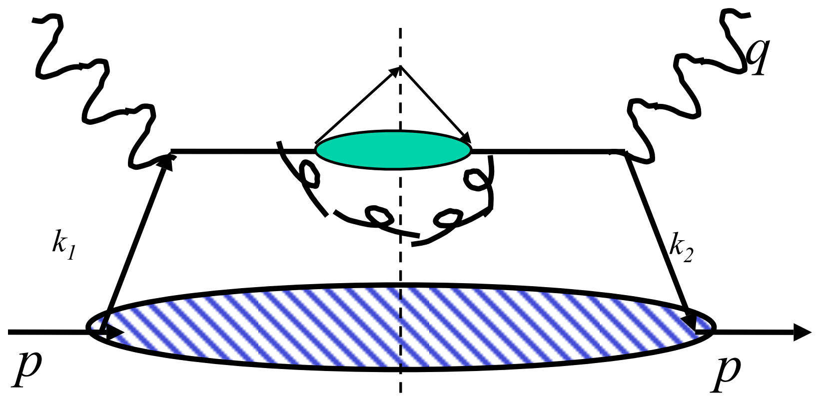
High-Twist approach to multiple scattering



Description of medium as collection of quasi-particle states

$$\langle\langle\cdots\rangle\rangle_A = \frac{1}{2p^+}\rho_A(\xi_N)\langle N(p)|\cdots|N(p)\rangle$$

DGLAP Evolution in vacuum



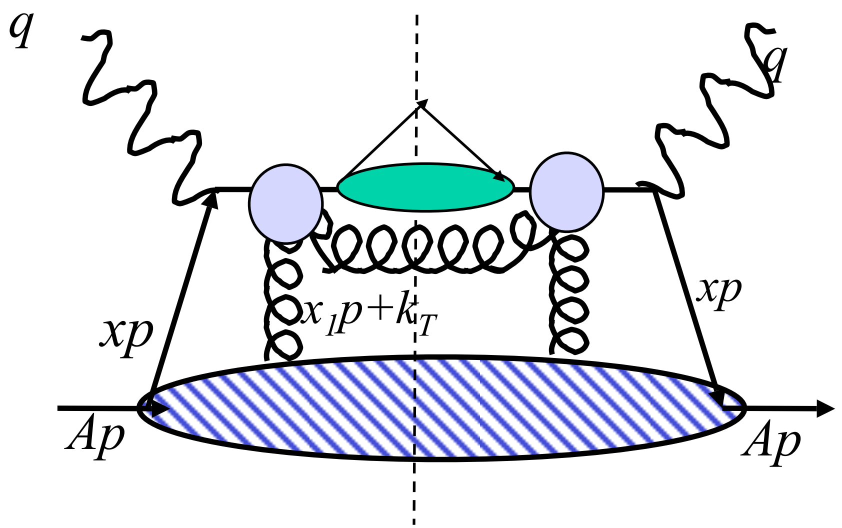
$$\Delta D_{q\to h}(z_h) = \frac{\alpha_s}{2\pi}\int^{\mu^2}\frac{d\ell_\perp^2}{\ell_\perp^2}\int_{z_h}^{1}\frac{dz}{z}\left[P_{q\to qg}(z)D_{q\to h}(\frac{z_h}{z}) + P_{q\to qg}(1-z)D_{g\to h}(\frac{z_h}{z})\right]$$

Splitting function $$P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

perturbative region: $$\ell_\perp^2 \geq Q_0^2$$

no matter how large is initial Q^2

Induced gluon emission in twist expansion

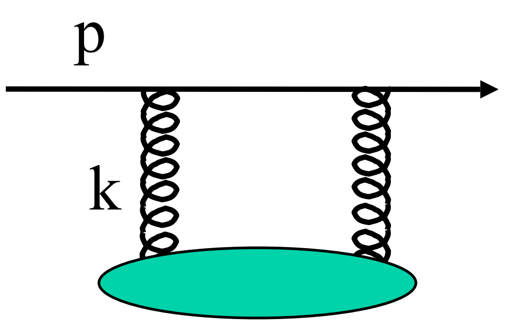


$$W^D_{\mu\nu} \propto \int d^2k_T e^{ik\cdot(y_1-y_2)} H^D_{\mu\nu}(p,q,k_T)\langle A|\bar{\psi}\gamma^+ A^+(y_1)A^+(y_2)\psi\,|A\rangle$$

k_T transverse momentum of medium gluon

Gauge Invariance

$$\int dk^+ \frac{e^{ik^+(y_1^- - y_2^-)}}{2k^+ p^- - k_\perp^2 + i\varepsilon} = -i\frac{2\pi}{2p^-}\theta(y_2^- - y_1^-)e^{i\frac{k_\perp^2}{2p^-}(y_1^- - y_2^-)}$$



Expansion in k_T $\qquad \vec{k}_\perp \Rightarrow i\vec{\partial}_\perp$

One should also consider $\vec{A}_\perp$ $\qquad i\vec{\partial}_\perp - g\vec{A}_\perp \equiv i\vec{D}_\perp$

(not all models have this)

Final matrix elements should contain: $\qquad \cdots D(y_1)\mathcal{L}(y_1, y_2)D(y_2)\mathcal{L}(y_2, y_3)\cdots$

Induced gluon emission in twist expansion

$$W_{\mu\nu}^{D} \propto \int d^2 k_T e^{ik\cdot(y_1-y_2)} H_{\mu\nu}^{D}(p,q,k_T)\langle A|\bar{\psi}\gamma^{+} A^{+}(y_1) A^{+}(y_2)\psi|A\rangle$$

Collinear expansion:

$$H_{\mu\nu}^{D}(p,q,k_T) = H_{\mu\nu}^{D}(p,q,0) + \partial_{k_T} H_{\mu\nu}^{D}(p,q,0) k_T + \partial_{k_T}^2 H_{\mu\nu}^{D}(p,q,0) k_T^2 + \cdots$$

$H_{\mu\nu}^{D}(p,q,0) \Rightarrow$ Eikonal contribution to vacuum brems.

Double scattering

$$W_{\mu\nu}^{D} \propto \partial_{k_T}^2 H_{\mu\nu}^{D}(p,q,k_T=0)\langle A|\bar{\psi}\gamma^{+} F^{+\sigma} F^{+}{}_{\sigma}\psi|A\rangle$$

Modified Fragmentation

$$\Delta D_{q\to h}(z_h,Q^2) = \frac{\alpha_s}{2\pi}\int_0^{Q^2}\frac{d\ell_\perp^2}{\ell_\perp^4}\int_{z_h}^1\frac{dz}{z}\left[\Delta\gamma(z,x_L)D_{q\to h}(\frac{z_h}{z})+\cdots\right]$$

Guo & XNW'00

Modified splitting functions

$$\Delta\gamma(z,x_L) = \frac{1+z^2}{(1-z)_+}\frac{T^A_{qg}(x,x_L)}{f_a^A(x)}\frac{C_A 2\pi\alpha_s}{N_c} - \delta(1-z)v(\ell_\perp^2)$$

Two-parton correlation:

$$T^A_{qg}(x,x_L) = \int\frac{dy^-}{2\pi}dy_1^- dy_2^- e^{-ix_B p^+ y^-}\langle A|\bar\psi(0)\frac{\gamma^+}{2}F_\sigma^+(y_1^-)F^{+\sigma}(y_2^-)\psi(y^-)|A\rangle$$
$$\times\left(1-e^{-ix_L p^+ y_2^-}\right)\left(1-e^{ix_L p^+(y_1^- - y^-)}\right)$$

Validity of collinear expansion

Collinear expansion:

$$H^D_{\mu\nu}(p,q,k_T) = H^D_{\mu\nu}(p,q,0) + \partial_{k_T} H^D_{\mu\nu}(p,q,0)k_T + \partial^2_{k_T} H^D_{\mu\nu}(p,q,0)k_T^2 + \cdots$$

Region of validity: $\ell_\perp^2 \gg k_\perp^2 \geq Q_0^2$ pQCD

If one is bold and goes beyond no one has gone before:

$\ell_\perp^2 \ll k_\perp^2 \geq Q_0^2$ One has to re-sum higher-twist terms (Or model the behavior of small l_T behavior)

Need to include all: $T^A_{qg}(x_B,x_L)$, $x_L \dfrac{\partial T^A_{qg}(x_B,x_L)}{\partial x_L}$, $x_L^2 \dfrac{\partial^2 T^A_{qg}(x_B,x_L)}{\partial^2 x_L}$

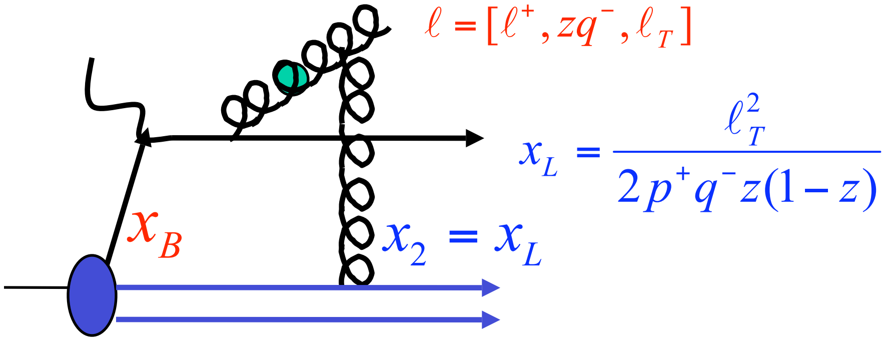
LPM limits $L_A \geq \dfrac{2z(1-z)E}{\ell_\perp^2}$

Comparison with GLV

$$\frac{dN_{\rm HT}}{dz} = \frac{N_c\alpha_s}{\pi}\frac{1+(1-z)^2}{z}\int\frac{d\ell_T^2}{\ell_T^4}\int d\xi[c(x_L)\hat{q}(\xi,0)+\hat{q}(\xi,x_L)] \left[1-\cos\frac{\ell_T^2\xi}{2q^-z(1-z)}\right].$$

$$\frac{dN_{\rm GLV}}{dz} = \frac{C_A\alpha_s}{\pi}\frac{1+(1-z)^2}{z}\int d\xi\,\rho(\xi)\,\sigma_{gN}\mu^2\int\frac{d\ell_T^2}{\ell_T^2(\ell_T^2+\mu^2)} \left[1-\cos\frac{\ell_T^2\xi}{2q^-z(1-z)}\right].$$

$\hat{q} \leftrightarrow \rho_A\sigma_g\mu^2$ ρ_A quasi-particle density



$$\frac{2\pi\alpha_s}{N_c}\frac{T^A_{qg}(x,x_L)}{f^A_q(x)} \approx \int d\xi^-[\hat{q}(\xi,x_T)+\hat{q}(\xi,x_L)][1-\cos(x_L p^+\xi^-)]$$

$$\hat{q}(\xi,x_L) = \frac{4\pi\alpha_s C_F}{N_c^2-1}\rho_A(\xi)x_L G(x_L)$$

$$x_L \leq 1$$

Modified DGLAP Evolution

$$\frac{\partial \tilde{D}_q^h(z_h,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\int_{z_h}^1 \frac{dz}{z}\Big[\tilde{\gamma}_{q\to qg}(z,\mu^2)\tilde{D}_q^h(\frac{z_h}{z},\mu^2) + \tilde{\gamma}_{q\to gq}(z,\mu^2)\tilde{D}_g^h(\frac{z_h}{z},\mu^2)\Big]$$

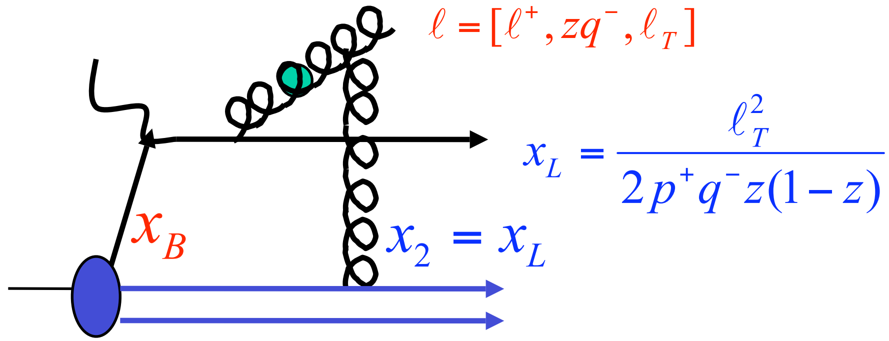
$$\frac{\partial \tilde{D}_g^h(z_h,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\int_{z_h}^1 \frac{dz}{z}\Bigg[\sum_{q=1}^{2n_f}\tilde{\gamma}_{g\to q\bar{q}}(z,\mu^2)\tilde{D}_q^h(\frac{z_h}{z},\mu^2) + \tilde{\gamma}_{g\to gg}(z,\mu^2)\tilde{D}_g^h(\frac{z_h}{z},\mu^2)\Bigg]$$

Modified splitting functions

$$\tilde{\gamma}_{a\to bc}(z,l_T^2) = \gamma_{a\to bc}(z) + \Delta\gamma_{a\to bc}(z,l_T^2)$$

Parton-gluon scattering

Schafer, XNW & Zhang 07



$$\partial_{k_T}^2 H_{\mu\nu}^D(p,q,k_\perp)\Big|_{k_\perp=0} \approx H_{\mu\nu}^{(0)} \frac{1+z^2}{(1-z)_+} \frac{\alpha_s^2}{\ell_T^4}\left(1-e^{-ix_L p^+ y_2^-}\right)\left(1-e^{ix_L p^+ (y_1^- - y^-}\right.$$

$$d\sigma_{qg}^N \simeq \frac{1+z^2}{(1-z)_+}\frac{\pi\alpha_s^2}{\ell_T^4} x_L G_N(x_L) dz d\ell_\perp^2$$

Quark-gluon Compton scattering

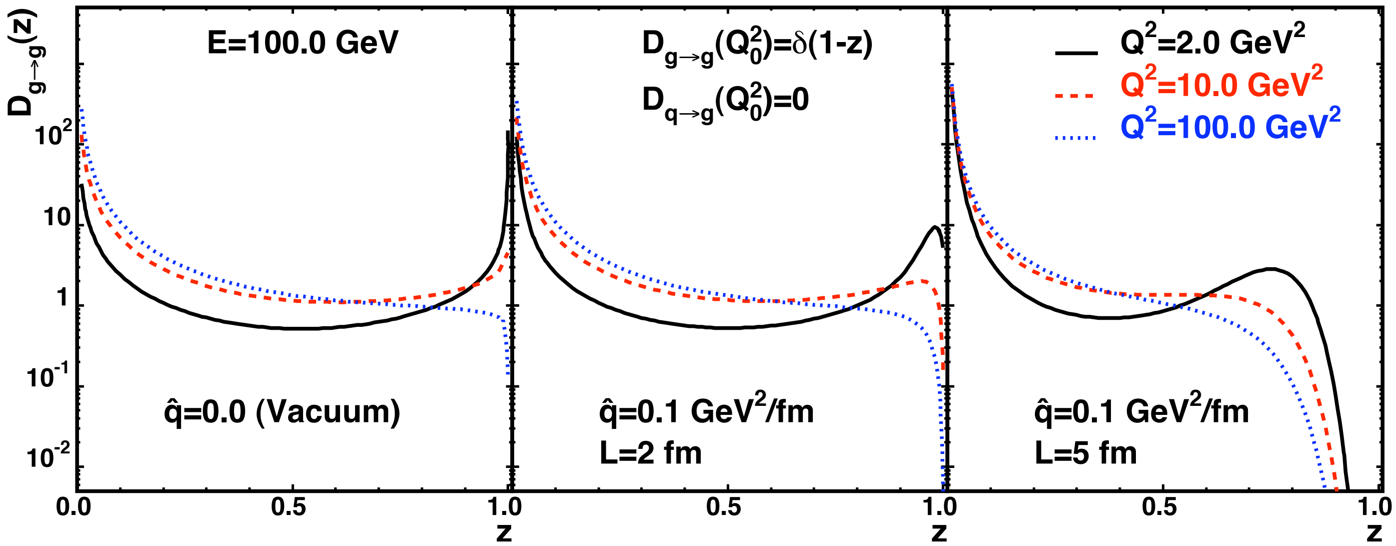
The Brick Problem

Initial conditions:

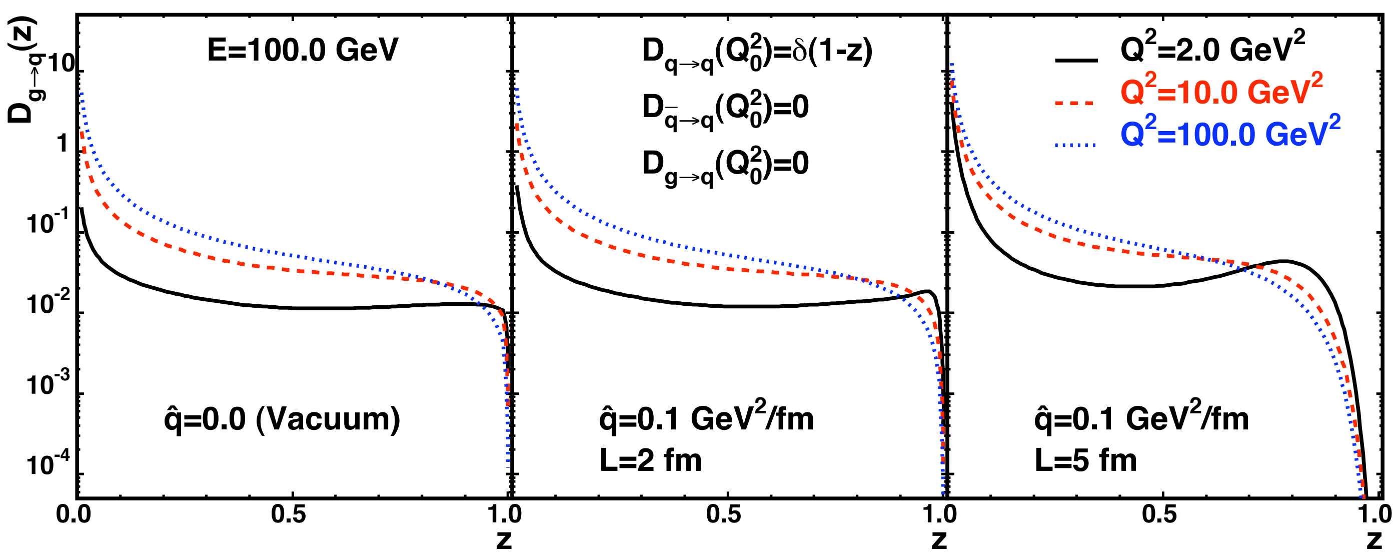
$$\tilde{D}_a(Q_0^2) = D_a(Q_0^2) + \Delta D_a(Q_0^2)\,, \quad a = g, q, \bar{q}.$$

$$\begin{aligned} D_a^a(z, Q_0^2) &= \delta(1-z); \\ D_a^{b\neq a}(z, Q_0^2) &= 0 \ \ (a, b = q, \bar{q}, g), \end{aligned}$$

Shower parton distr. in a gluon jet

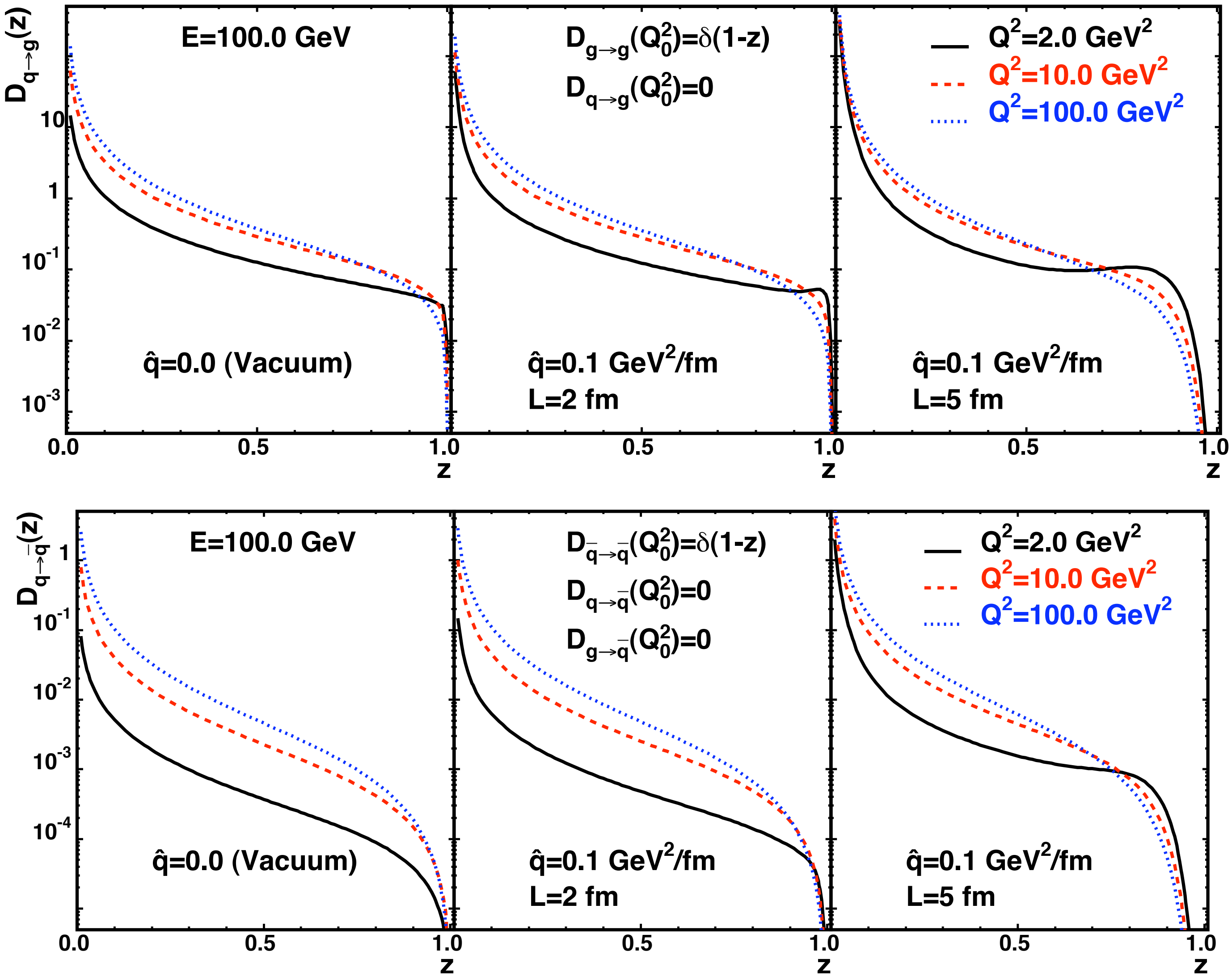


g--> g



g --> q

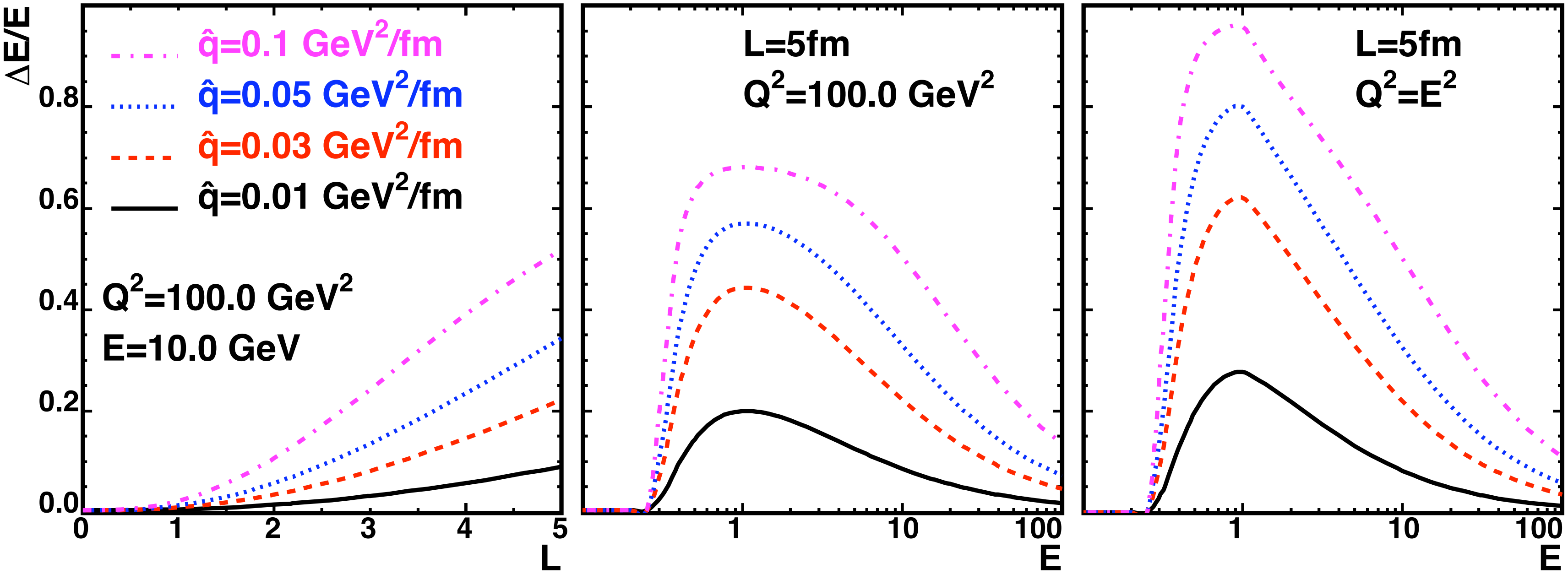
Shower parton distr. in a quark jet



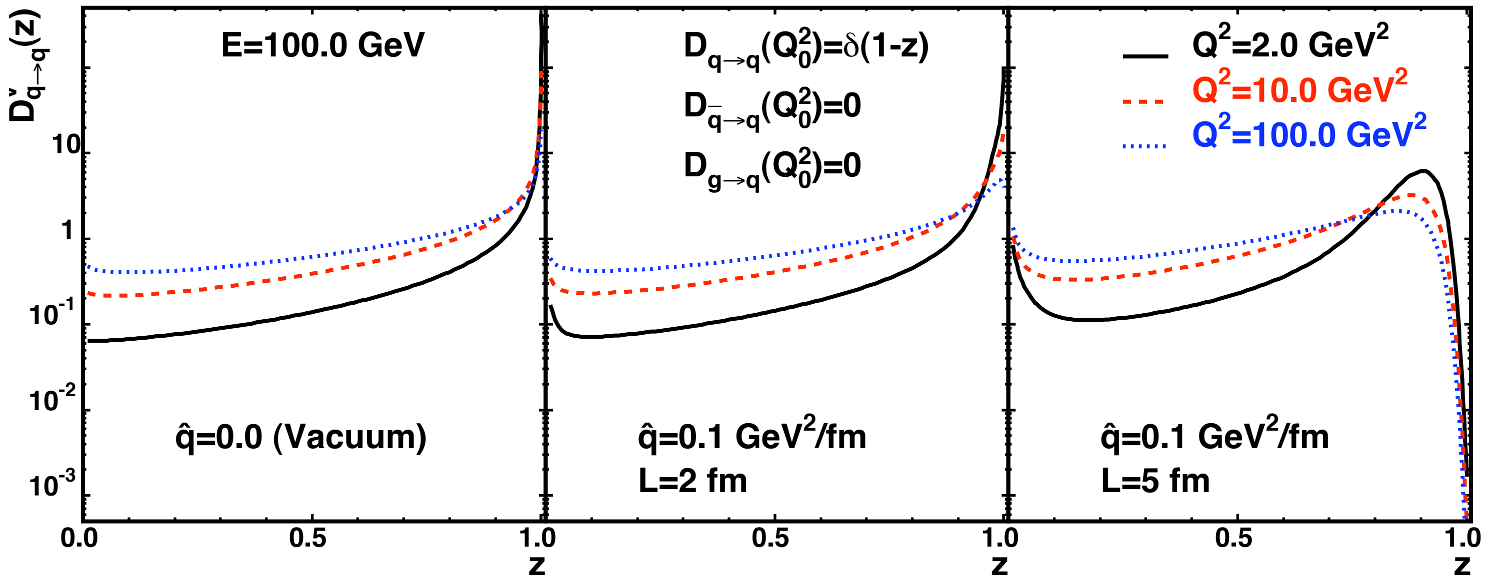
q --> g

q--> qbar

Valence quark energy loss



Valence quark distribution



$$\frac{\Delta E}{E} = \frac{\Delta E_m - \Delta E_v}{E} = \int_0^1 dz\, z \left[D_q^v(z,Q^2) - \tilde{D}_q^v(z,Q^2) \right]$$

Beyond the Brick Problem

Jet transport parameter & phases of dense matter

$$\hat{q}(\xi_N) \equiv \frac{4\pi^2 \alpha_s C_F}{N_c^2 - 1} \rho_A(\xi_N) x G_N(x)|_{x\approx 0}$$

Jet transport parameter

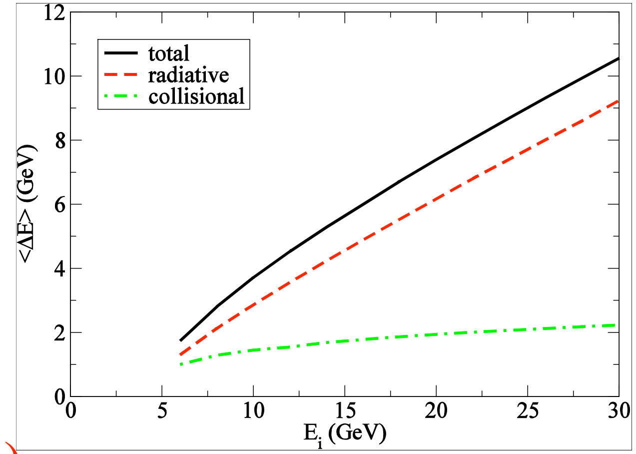
$$xG_N(x) = -\int \frac{d\xi^-}{2\pi p^+} e^{ixp^+\xi^-} \langle N | F_{+\sigma}(0) L_{\parallel}^A(0,\xi^-) F_+^\sigma(\xi^-) | N \rangle$$

Constrain “implementations” with DIS data (talk by Wei-tian Deng)

$$\langle \delta q_\perp^2 \rangle = 0.016 A^{1/3} GeV^2/fm \longleftrightarrow R_A(z)$$

Theoretical improvements

Qin, et al ''08

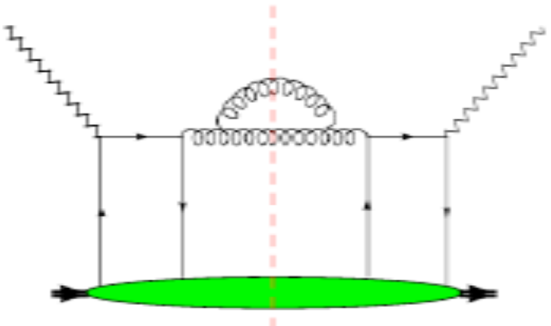


- Elastic vs radiative: for finite E & L

$$\frac{\Delta E_{rad}}{\Delta E_{el}} \simeq \frac{9\xi(3)N_c}{2\pi^2}\alpha_s LT \ln\frac{EL}{11}$$

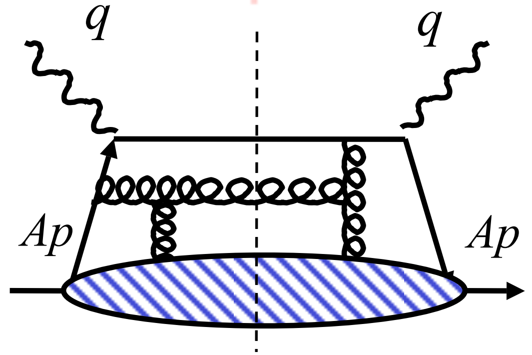
- Recoil in radiative process:

$$\frac{dN_g}{dz d\ell_\perp^2} \sim c(x_L)\hat{q}(\xi,0) + \hat{q}(\xi,x_L) \sim 2\hat{q}(\xi,0) + \mathcal{O}(\frac{\ell_\perp^2}{Q^2})$$



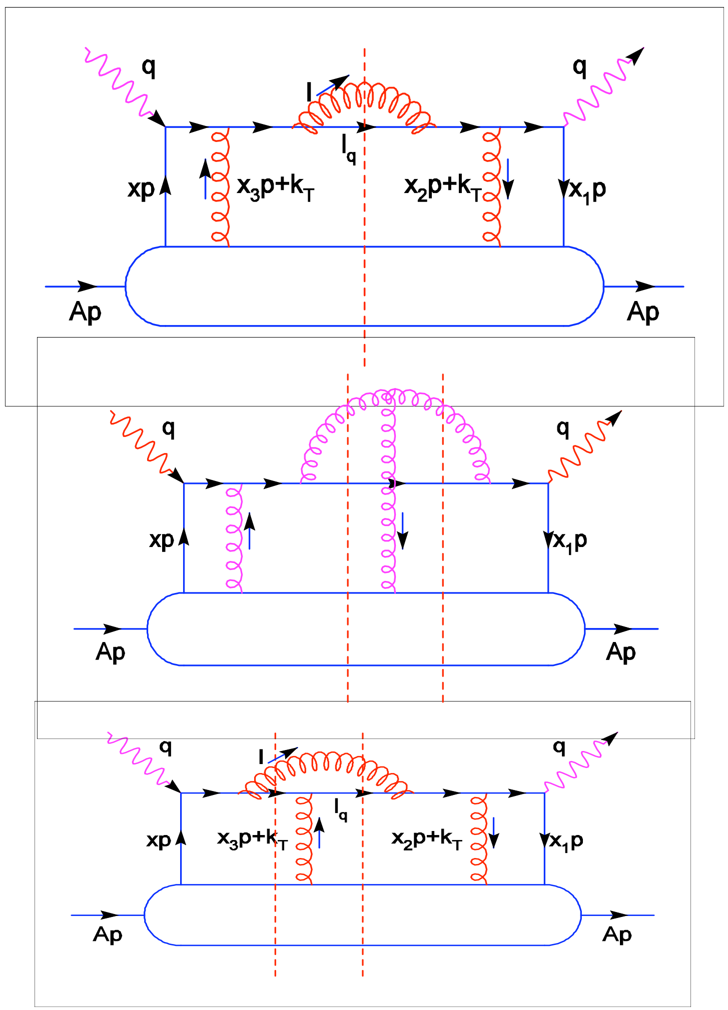
- Quark-annihilation
  Flavor changing process $\mathcal{O}(\frac{\ell_\perp^2}{Q^2})$

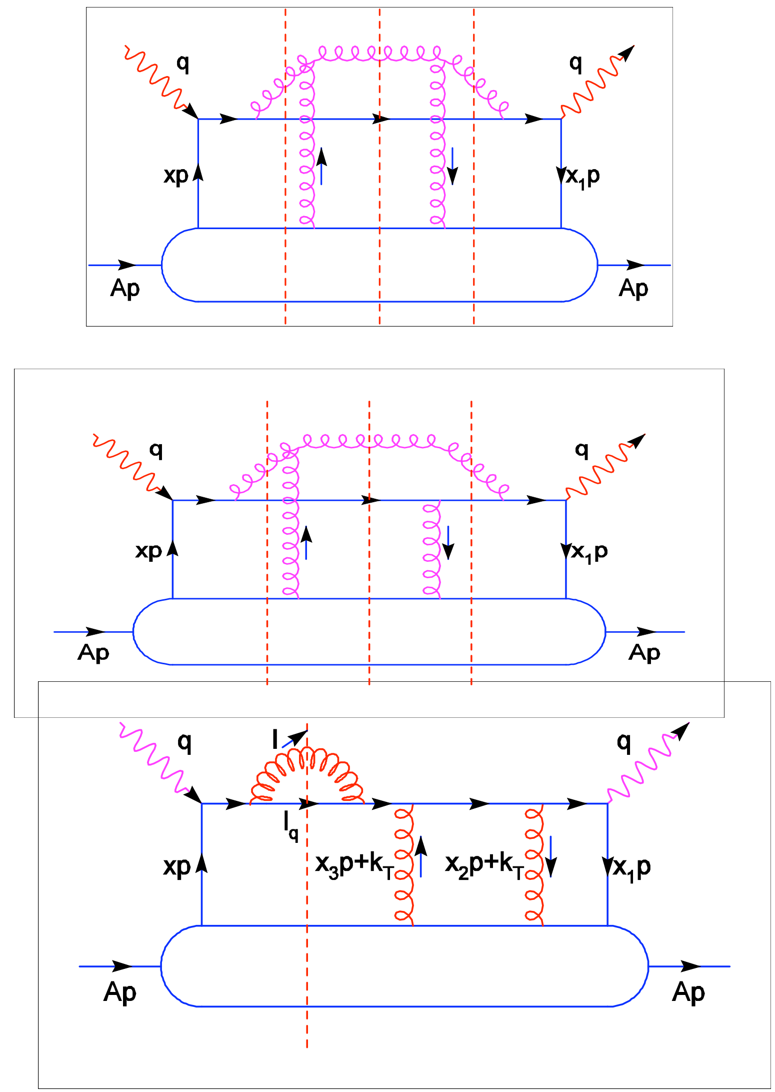
- NLO corrections to LO collinear factorized contribution



- Mass correction for heavy quarks $\mathcal{O}(\frac{M_Q^2}{Q^2})$

Different cut-diagrams





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