Overview

Part IV:

F-theory GUT phenomenology

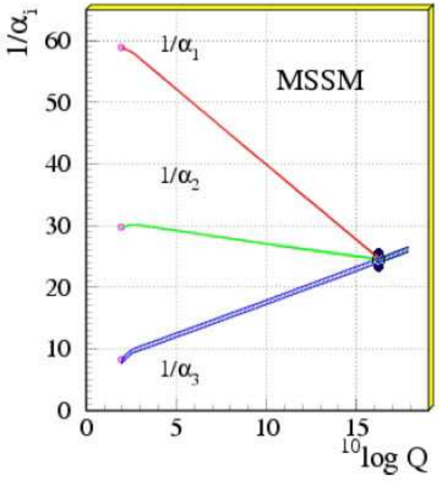
- SU(5) GUTS: basics
- GUT symmetry breaking
- Gauge coupling unification
- Proton decay
- Split spectral covers
- Summary

F-theory GUTs

Aim: construction of 4D models with realistic particle phenomenology

Guideline: unification of gauge couplings at $M_X \simeq 10^{16} GeV$

Strategy: Localisation of gauge sector to explain hierarchy $M_{GUT} = 10^{-3} M_{Pl.}$



pic: Kazakov, 0012288

- $S_{10D} = M_*^8 \int_{\mathbb{R}^{1,3}\times B_6} \sqrt{-g}R \quad \rightarrow M_{Pl.}^2 = M_*^8 \,\mathrm{Vol}(B_6) \qquad M_* = \ell_s^{-1}$
- $S_{YM} = M_*^4 \int_{\mathbb{R}^{1,3}\times \Gamma_4} F^2 \quad \rightarrow \alpha_{GUT}^{-1} = M_*^4 \,\mathrm{Vol}(\Gamma_4)$
- GUT breaking (see later) $\quad \rightarrow M_{GUT}^4 = \mathrm{Vol}(\Gamma_4)^{-1}$

3 different scales $\mathrm{Vol}(\Gamma_4)$, $\mathrm{Vol}(B_6)$, M_* to achieve both

- $M_{Pl.} = 10^3 M_{GUT} = 10^{19}\mathrm{GeV}$
- $\alpha_{GUT}^{-1} = 24$

F-theory GUTs

Achieving the hierarchy:

- In principle enough to ensure mild tuning $\underbrace{\ell_s}_{0.2x} < \underbrace{R_{\Gamma_4}}_{2.2x} < \underbrace{R_{B_6}}_{5.6x}$

  $x = 10^{-16}\text{GeV}^{-1}$
- Eventually this has to happen dynamically!
- Stronger requirement: decoupling $M_{Pl.} \to \infty$ while M_{GUT} finite
  While physically not absolutely necessary, appears as a sensible organising principle of GUT model building
- Mathematically one can show: decoupling forces S to be Fano: $\int_\Gamma K_S^{-1} \geq 0 \; \forall$ 2-cycles Γ
- 2 kompl.-dim. Fanos: $\mathbb{P}^1 \times \mathbb{P}^1$, $\mathbb{P}^2$, del Pezzo dP_r, $r = 1, \ldots, 8$
  $\longrightarrow$ severely constrains possible GUT surfaces
- for del Pezzo $H^{(0,2)}(S) = 0 = H^{(0,1)}(S)$
  $\Rightarrow$ no deformation modes or continuous Wilson lines
  Welcome feature: avoiding adjoint exotics would require stabilisation

F-theory GUTs

del Pezzo convenient choice, but - unfortunately - there is no general prediction that S must be of that type

Still: Assume now that S is del Pezzo

F-theory GUTs

- Simplest realisation: SU(5) GUTs BHV & DW '08
- SO(10) possible, but exotics arise in process of GUT breaking

Aim: SU(5) GUT group along divisor $S \subset B_6$ given by locus $w = 0$

$\rightarrow y^2 = x^3 + f(u_i, w)xz^4 + g(u_i, w)z^6$ with $f(u_i, w), g(u_i, w)$ s.t.

- $\Delta = w^5 P(u_i, w)$,
- $P(u_i, w) \neq w\, p(u_i, w)$

$$\Longrightarrow \underbrace{SU(5)}_{S:w=0} \times \underbrace{U(1)}_{D:P(u_i,w)=0}$$

a) matter:

enhancement of singularity type by rank one on intersection locus $S \cap D$

- $SU(5) \times U(1) \rightarrow SU(6)$
  $\mathbf{35} \rightarrow \mathbf{24} + 1 + \mathbf{5} + \overline{\mathbf{5}}$ $\Longrightarrow \overline{\mathbf{5}}_m = (d_R^c, L)$ or $\mathbf{5}_H + \overline{\mathbf{5}}_H$
- $SU(5) \times U(1) \rightarrow SO(10)$
  $\mathbf{45} \rightarrow \mathbf{24} + 1 + \mathbf{10} + \overline{\mathbf{10}}$ $\Longrightarrow \mathbf{10} = (Q_L, u_R^c, e_R^c)$

N_R^c: any SU(5) singlet with suitable couplings

F-theory GUTs

b) Yukawa couplings

further enhancements at mutual intersections of curves

- $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{H}}$ for masses of down quarks
- $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_{\mathbf{H}}$ for masses of up quarks
- $\mathbf{5}_{\mathbf{H}}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\mathbf{1}_{\mathbf{N}^c_{\mathbf{R}}}$ for Dirac neutrino masses

Group theory:

- $\langle \mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}} \rangle \subset \langle (\mathbf{66})^3 \rangle$ of SO(12) $\Longrightarrow$ enhancement $SU(5) \to SO(12)$
  possible also perturbativey in IIB theory
- $\langle \mathbf{10}\,\mathbf{10}\,\mathbf{5} \rangle \subset \langle (\mathbf{78})^3 \rangle$ of E_6 $\Longrightarrow$ requires exceptional enhancements
  not possible perturbatively in IIB theory

  (however: D-brane instantons can do it! `review Blumenhagen,Cvetič,Kachru,TW '09`)

  $\rightsquigarrow$**genuine F-theory effect**
- $\langle \mathbf{5}\,\overline{\mathbf{5}}\,\mathbf{1} \rangle$: SU(7) point sufficient

SU(5) GUT breaking

3 ways to break $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)_Y$

- via GUT Higgs in adjoint $\mathbf{24}$ of SU(5) $\leftrightarrow$ brane moduli $H^{(0,2)}(S)$
  problem: explicit GUT breaking potential from string theory $\leftrightarrow$ stabilisation of brane moduli?
  Note: Fluxes do stabilise moduli, but hard to arrange suitable potential
- Wilson lines $\leftrightarrow$ brane moduli $H^{(0,1)}(S)$
  again challenge of stabilising open moduli:
  now fluxes not even avaliable
  viable special case: discrete Wilson lines from discrete $\pi_1(S)$
- by line bundle $L_Y \leftrightarrow$ hypercharge $U(1)_Y$
  works for divisors with $H^{(0,1)}(S) = 0 = H^{(0,2)}(S)$
  $\leftrightarrow$ open moduli problem avoided $\rightarrow$ ✓del Pezzo S

We take $U(1)_Y$- approach:
first applied in F-theory context by [Beasley,Heckman,Vafa '08]
same as in heterotic: [Blumenhagen,Honecker,TW '06]

SU(5) GUT breaking (II)

Idea: VEV to F_Y on S with $T_Y = \mathrm{diag}(-2,-2,-2,3,3) \subset SU(5)$

$$SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)_Y$$

$$\begin{aligned}
\mathbf{24} &\rightarrow (\mathbf{8},\mathbf{1})_{0_Y} + (\mathbf{1},\mathbf{3})_{0_Y} + (\mathbf{1},\mathbf{1})_{0_Y} + (\mathbf{3},\mathbf{2})_{5_Y} + (\overline{\mathbf{3}},\mathbf{2})_{-5_Y} \\
\overline{\mathbf{5}} &\rightarrow (\overline{\mathbf{3}},\mathbf{1})_{2_Y} + (\mathbf{1},\mathbf{2})_{-3_Y} \\
\mathbf{10} &\rightarrow (\mathbf{3},\mathbf{2})_{1_Y} + (\overline{\mathbf{3}},\mathbf{1})_{-4_Y} + (\mathbf{1},\mathbf{1})_{6_Y}\,, \\
\mathbf{5}_H &\rightarrow (\mathbf{3},\mathbf{1})_{-2_Y} + (\mathbf{1},\mathbf{2})_{3_Y}\,, \quad \overline{\mathbf{5}}_H \rightarrow (\overline{\mathbf{3}},\mathbf{1})_{2_Y} + (\mathbf{1},\mathbf{2})_{-3_Y}
\end{aligned}$$

3 challenges:

1) $U(1)_Y$ must remain massless ✓

2) no exotic states $(\mathbf{3},\mathbf{2})_{5_Y} + (\overline{\mathbf{3}},\mathbf{2})_{-5_Y}$ from $\mathbf{24}$ ✓

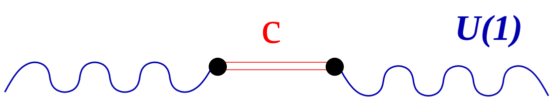
3) preservation of unification ???

SU(5) GUT breaking (III)

Massless $U(1)_Y$

Generically an abelian gauge factor acquires mass via Stückelberg mechanism:

$C^{(2)} \wedge F$-type coupling in 4D



Analyse in IIB 7-brane language:

$S_{CS} = \int_{\mathbb{R}_{1,3}\times S} \left(\iota^* C_8 + \mathrm{tr} F \wedge \iota^* C_6 + \mathrm{tr}(F^2) \wedge \iota^* C_4 + \ldots\right)$

Expand: $F = T_Y \left(F_Y^{4D} + c_1(\mathcal{L}_Y)\right) +$ (other SU(5) generators)

2 potential sources for mixing term:

- $\iota^* C_6 = \sum_a C_2^{(a)} \wedge \iota^* \alpha_a, \qquad \alpha_a \in H^4(Y)$
- $\iota^* C_4 = \sum_i C_2^{(i)} \wedge \iota^* \omega_i, \qquad \omega_i \in H^2(Y)$

since $\mathrm{tr}\, T_Y = 0$: only term involving $\mathrm{tr}(F^2)$ relevant

$\Rightarrow F_Y^{4D} \wedge C_2^{(i)}\, \mathrm{tr}\, T_Y^2 \int_S (c_1(\mathcal{L}_Y) \wedge \iota^* \omega_i)$

$\Rightarrow U(1)_Y$ massless iff $c_1(\mathcal{L}_Y)$ orthogonal to $\iota^* H^2(B, \mathbb{Z})$

SU(5) GUT breaking (IV)

Important concept: gauge flux in relative cohomology of $S \subset B$

gauge flux on divisor $S \Leftrightarrow H^2(S)$

2 types of 2-forms on divisor S: Lerche, Mayr, Warner '01/02;

pullbacks from B vs. 2-forms trivial on B Jockers, Louis '05

The latter 2-forms are dual to 2-cycles in $H_2(S)$ which are a boundary of a 3-chain in B

if $c_1(\mathcal{L}_Y)$ only through such "relative" 2-cycles on S, then
$\int_S (\mathcal{L}_Y \wedge \iota^* \omega_i) = 0$

Constraint: hypercharge flux must lie in relative cohomology of S in B

Non-trivial constraint on compactification geometry:

rigid GUT divisor must allow for such gauge flux which is not pull-back from ambient space

global feature - depends on compactification details, not on local data!

MSSM matter

Necessary condition: Recall: $l^2 = 1, \quad E_i \cdot E_j = -\delta_{ij}$

if S is del Pezzo, then $c_1(S) = 3l - \sum_i E_i$ non-trivial on $B \rightarrow$

$c_1(\mathcal{L}_Y) \cdot c_1(S) = 0 \leftrightarrow c_1(S)$ in Weyl lattice $\langle l - E_1 - E_2 - E_3, E_i - E_j \rangle$

Absence of exotics

$\mathbf{24} \rightarrow (\mathbf{8}, \mathbf{1})_{0_Y} + (\mathbf{1}, \mathbf{3})_{0_Y} + (\mathbf{1}, \mathbf{1})_{0_Y} + (\mathbf{3}, \mathbf{2})_{5_Y} + (\overline{\mathbf{3}}, \mathbf{2})_{-5_Y}$

- localised on entire divisor S
- counted by combinations of $H^i(S, \mathcal{L}_Y^5) = 0$

for $c_1(L_Y)$ in Weyl lattice of dP_r, this is not possible:
$H^i(E_i - E_i) = 0$, but not for 5th power!

But can redefine embedding of $\mathcal{L}_Y$ such that $H^i(S, \mathcal{L}_Y^{\pm 1})$ appears
In SCC we also have the $SU(5)_\perp \subset E_8$ bundle V on S
Define $S[U(5) \times U(1)_Y]$ bundle $V \oplus \mathcal{L}_Y$: $c_1(V) + c_1(\mathcal{L}_Y) = 0$
Cartan generators of the structure group of $V \oplus \mathcal{L}_Y$:

$$(1, 1, 1, 1, 1, -5) \subset SU(6) \subset E_8$$

MSSM matter

$$\mathbf{248} \quad \mapsto \left\{ \begin{array}{l} (\mathbf{24};\mathbf{1},\mathbf{1})_0 + (\mathbf{1};\mathbf{1},\mathbf{1})_0 + (\mathbf{1};\mathbf{8},\mathbf{1})_{\mathbf{0}} + (\mathbf{1};\mathbf{1},\mathbf{3})_{\mathbf{0}} \\ (\mathbf{5};\mathbf{3},\mathbf{2})_1 + (\mathbf{1};\mathbf{3},\mathbf{2})_5 + c.c. \\ (\mathbf{10};\overline{\mathbf{3}},\mathbf{1})_2 + (\mathbf{5};\overline{\mathbf{3}},\mathbf{1})_{-4} + c.c. \\ (\mathbf{10};\mathbf{1},\mathbf{2})_{-3} + (\mathbf{5};\mathbf{1},\mathbf{1})_6 + c.c. \end{array} \right\}$$

$SU(3)\times SU(2)\times U(1)_Y$	bundle	Standard Model particles	
$(\mathbf{3},\mathbf{2})_1$	V	q_L	L-handed quark
$(\mathbf{3},\mathbf{2})_5$	$\mathcal{L}_Y^{-1}$	$-$	(exotic matter)
$(\overline{\mathbf{3}},\mathbf{1})_2$	$\bigwedge^2 V$	$\bar{d}_L = d_R^c$	L-handed down antiquark
$(\overline{\mathbf{3}},\mathbf{1})_{-4}$	$V\otimes\mathcal{L}_Y$	$\bar{u}_L = u_R^c$	L-handed up antiquark
$(\mathbf{1},\mathbf{2})_{-3}$	$\bigwedge^2 V\otimes\mathcal{L}_Y$	l_L	L-handed lepton
$(\mathbf{1},\mathbf{1})_6$	$V\otimes\mathcal{L}_Y^{-1}$	$\bar{e}_L = e_R^c$	L-handed antielectron

MSSM matter - Higgs

appearance of full GUT matter multiplets:

$c_1(\mathcal{L}_Y) \cdot P_{\mathbf{10}} = 0 = c_1(\mathcal{L}_Y) \cdot P_{\overline{\mathbf{5}}_{\mathbf{m}}}$

Higgs sector:

$\mathbf{5}_H \;\; \rightarrow (\mathbf{3}, \mathbf{1})_{-2_Y} + (\mathbf{1}, \mathbf{2})_{3_Y}, \quad \overline{\mathbf{5}}_H \;\; \rightarrow (\overline{\mathbf{3}}, \mathbf{1})_{2_Y} + (\mathbf{1}, \mathbf{2})_{-3_Y}$

controlled mechanism for stringy doublet-triplet splitting via $U(1)_Y$

at first sight 2 possibilities arise [Beasley,Heckman,Vafa '08]

- $\mathbf{5}_H + \overline{\mathbf{5}}_H$ along 1 single elliptic curve P_H

  $(\mathbf{3}, \mathbf{1})_{-2_Y} + (\overline{\mathbf{3}}, \mathbf{1})_{2_Y} \quad : \quad H^*(P_H, \wedge V) = (0, 0)$

  $(\mathbf{1}, \mathbf{2})_{3_Y} + (\mathbf{1}, \mathbf{2})_{-3_Y} \quad : \quad H^*(P_H, \wedge V \otimes \mathcal{L}_Y) = (1, 1)$

  vanishing net flux through P_H for absence of chiral matter

  But: $\mathcal{L}_Y|_{P_H}$ can have non-trivial Wilson lines

  triplet mass of order $\mathcal{O}(R_{P_H}^{-1}) \leftrightarrow$ KK scale for excitations along P_H

Gauge coupling unification

- P_H splits into $P_u \cup P_d$ such that $\mathbf{5}_H$ along P_u, $\overline{\mathbf{5}}_H$ along P_d
  net $U(1)_Y$ flux through P_u and P_d such that:
  $\rightsquigarrow H^*(P_u, \wedge V \otimes \mathcal{L}_Y) = (1,0), \quad H^*(P_d, \wedge V \otimes \mathcal{L}_Y) = (0,1)$
  $\leftrightarrow$ one H_u and H_d

  $\rightsquigarrow H^*(P_u, \wedge V) = (0,0), \quad H^*(P_d, \wedge V) = (0,0)$
  $\leftrightarrow$ massless triplets projected out

  topological solution to doublet triplet splitting

  will see momentarily:

This secenario is preferred also due to proton decay

Gauge coupling unification

gauge kin. function f: $S_{4D} = \mathrm{Re}(f)\,\frac{1}{2}\mathrm{tr}F \wedge \star F + \mathrm{Im}(f)\,\frac{1}{2}\mathrm{tr}F \wedge F$

Consider again first Type IIB theory in string frame

tree-level expression from reduction of

$S_{\mathrm{DBI}} = -\mu_7 \int_{\mathbb{R}^{1,3}\times S} d^7\zeta\, e^{-\varphi} \sqrt{\det(g + \mathcal{F}/2\pi)}$

$S_{\mathrm{CS}} = -\mu_7 \int \int_{\mathbb{R}^{1,3}\times S} \mathrm{ch}(\mathcal{F}/2\pi) \sum_p C_{2p}$

- Leading order at string scale M_s:
  $f_S = \frac{1}{g_s}\frac{Vol_S}{\ell_s^4} + i\int_S C_4$
- Higher order terms:
  read off corrections to $\mathrm{Im}(f)$ from $S_{CS} \ni \mu^7 \int C_0 \wedge \mathrm{tr}F^4$
  from $\tau = e^{-\varphi} + iC_0$ is chiral superfield can complete this into full gauge kin. function

In F-theory g_s varies

better to go to Einstein frame in IIB by replacing $\frac{1}{g_s \ell_s^4} = M_*^4$

$M_* \leftrightarrow$ M-theory scale: constant over base B

Gauge coupling unification

$$F = \sum_{a=1}^{8} F^a_{SU(3)} \begin{pmatrix} \lambda_a/2 & 0 \\ 0 & 0 \end{pmatrix} + \sum_{i=1}^{3} F^i_{SU(2)} \begin{pmatrix} 0 & 0 \\ 0 & \sigma_i/2 \end{pmatrix} +$$

$$\tfrac{1}{6}\, F_Y \begin{pmatrix} -2_{3\times 3} & 0 \\ 0 & 3_{2\times 2} \end{pmatrix} + \tfrac{1}{5}\, \overline{f}_Y \begin{pmatrix} -2_{3\times 3} & 0 \\ 0 & 3_{2\times 2} \end{pmatrix},$$

plugging into expression for f and extracting real part (in Einstein frame)

$$\frac{1}{\alpha_s} = M_*^4 \, \mathrm{Vol}(S) - \frac{4}{50}\, M_*^4 \ell_s^4 \int_S c_1^2(\mathcal{L}_Y),$$

$$\frac{1}{\alpha_w} = M_*^4 \, \mathrm{Vol}(S) - \frac{9}{50}\, M_*^4 \ell_s^4 \int_S c_1^2(\mathcal{L}_Y),$$

$$\frac{3}{5}\frac{1}{\alpha_Y} = M_*^4 \, \mathrm{Vol}(S) - \frac{7}{50}\, M_*^4 \ell_s^4 \int_S c_1^2(\mathcal{L}_Y)$$

Internal $U(1)_Y$ flux induces corrections at $\mathrm{Vol_S}^{-1/4}$

[Donagi, Wijnholt], [Blumenhagen] '08

Gauge coupling unification

exact MSSM spectrum: unification at $M_{GUT} = 2.1 \times 10^{16} GeV$

ignoring higher order terms:

- $M_{GUT} \simeq M_{KK} = Vol_S^{-\frac{1}{4}}$ (KK scale for open string modes all over S)
- $\alpha_{\rm GUT}^{-1} = \frac{4\pi}{g^2_{SU(5)}} = M_*^4 \,{\rm Vol}_S = 24$

including higher order terms: at scale ${\rm Vol_S}^{-1/4}$

$$\frac{1}{\alpha_{QCD}} < \frac{3}{5}\frac{1}{\alpha_Y} < \frac{1}{\alpha_w} \qquad \frac{1}{\alpha_Y} = \frac{1}{\alpha_w} + \frac{2}{3}\frac{1}{\alpha_{QCD}} \;(\star)$$

$\Rightarrow$ **Minimal unification is lost!**

Note: This would not have happened with Wilson lines

Rescue: need new threshold below M_{GUT}

Minimal scenario: exploit heavy Higgs triplets at $M_{3\overline{3}} < M_{GUT}$ by heavy triplets $(\mathbf{3},\mathbf{1})_{\mathbf{-2_Y}}$

Note: $M_{3\overline{3}} = \mathcal{O}(R^{-1}_{P_u/d})$ need not coincide exactly with $Vol_S^{-\frac{1}{4}}$

Gauge coupling unification

Running of coupling:

below $M_{3\bar{3}}$: MSSM like: $(b_3, b_2, b_1) = (3, -1, -11)$

above $M_{3\bar{3}}$: $(\tilde{b}_3, \tilde{b}_2, \tilde{b}_1) = (2, -1, -\frac{35}{3})$

$$\frac{1}{\alpha_{QCD}(\mu)} = \frac{1}{\alpha_{QCD}(M_Z)} + \frac{b_3}{2\pi}\ln(\frac{\mu}{M_Z}) + \frac{\tilde{b}_3 - b_3}{2\pi}\ln(\frac{\mu}{M_{3\bar{3}}})$$

$$\frac{1}{\alpha_w(\mu)} = \frac{sin^2\theta}{\alpha_w(M_Z)} + \frac{b_2}{2\pi}\ln(\frac{\mu}{M_Z})$$

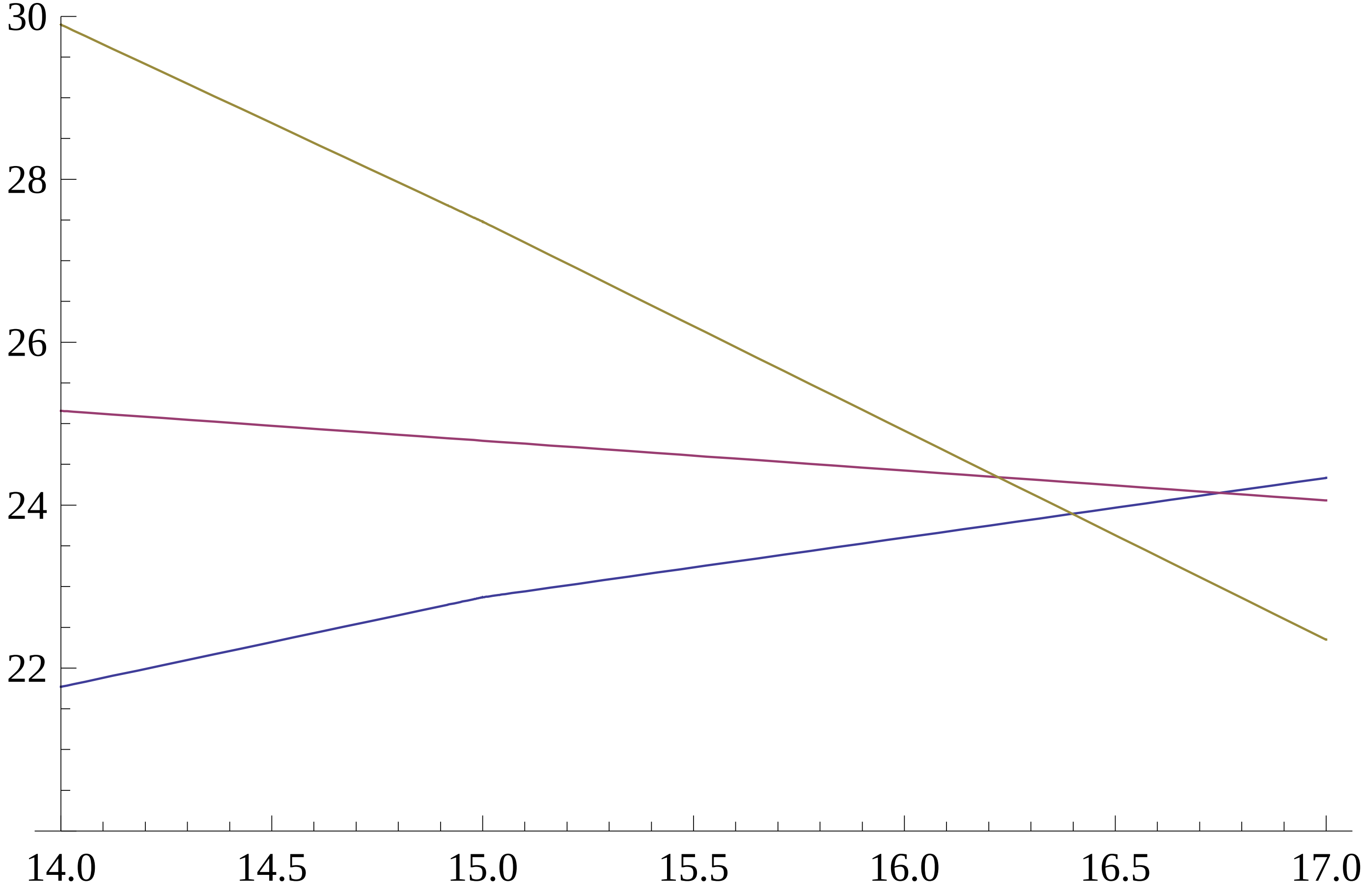
$$\frac{1}{\alpha_Y(\mu)} = \frac{cos^2\theta}{\alpha_Y(M_Z)} + \frac{b_1}{2\pi}\ln(\frac{\mu}{M_Z}) + \frac{\tilde{b}_1 - b_1}{2\pi}\ln(\frac{\mu}{M_{3\bar{3}}})$$

$(\star)$ at $M_{GUT} \Leftrightarrow \left((\tilde{b}_1 - b_1) - \frac{2}{3}(\tilde{b}_3 - b_3)\right)\log(\frac{M_{GUT}}{M_{3\bar{3}}}) = 0$ ✓

satisfied for arbitrary $M_{3\bar{3}} < M_{GUT}$

With this "universal" threshold: $\mathrm{Vol}_S^{-1/4} = 2.1 \times 10^{16} GeV$

Gauge coupling unification



from: Blumenhagen, 0812.0248

Proton Decay

Conventional SU(5) GUTs suffer from too large proton decay

Dimension 4:

- R-parity violating terms $u_R^c\, d_R^c\, d_R^c$, $L\, L\, e_R^c$, $Q\, L\, d_R^c \leftrightarrow \mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$
- must be absent due to experimental bound
- Necessary condition: P_m, P_H on different curves

  otherwise: $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{H}}$ implies $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$
- Note: **This is not sufficient** - see later

Dimension 6:

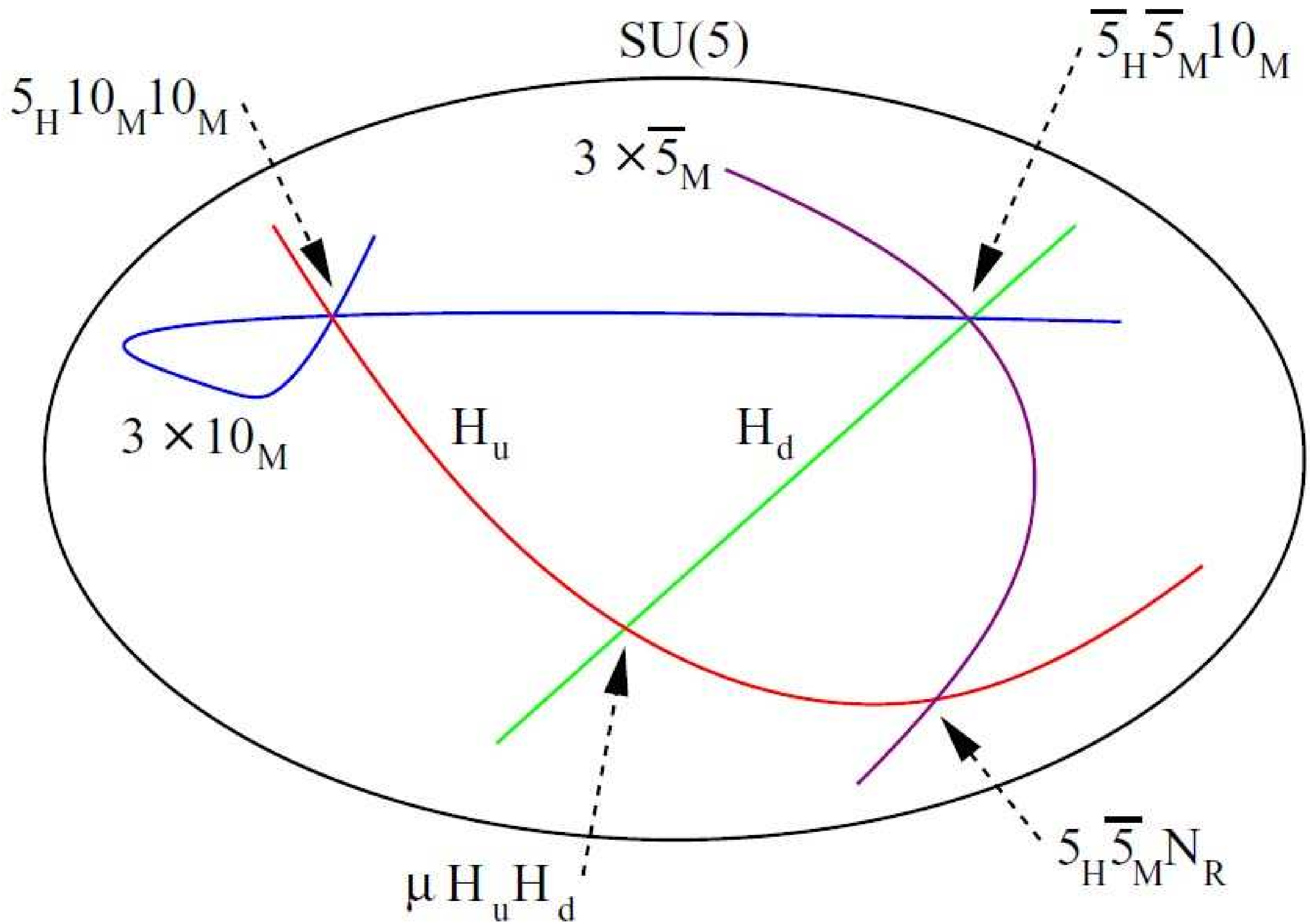
- From exchange of heavy gauge bosons in $(\mathbf{3}, \mathbf{2}) + (\overline{\mathbf{3}}, \mathbf{2})$
- leads to decay of type $p \longrightarrow \pi^0 + e^+$
- decay rate is well within experimental bounds $\longrightarrow$ no problem ✓

Proton decay

Dimension 5:

- effective terms of type $\frac{c^2}{M_{eff}}(QQQL + u^c_R u^c_R d^c_R e^c_R)$
- lead to decay of type $p \to K^+ \overline{\nu}$
- strongly constrained by proton lifetime
- Generation via triplet exchange: $\mathbf{5_H} = (T_u, H_u), \overline{\mathbf{5}}_\mathbf{H} = (T_d, H_d)$:
  $QQT_u + QLT_d + M_{KK} T_u T_d \to \frac{1}{M_{KK}} Q\,Q\,Q\,L$
- realised if triplets acquire mass by pairing up with each other
  $\leftrightarrow$ H_u, H_d on same curve P_H
- Remedy: Missing partner mechanism:
  $QQT_u + QLT_d + M_{KK} T_u \tilde{T}_d + M_{KK} \tilde{T}_u T_d$
  $\Longrightarrow$ no integration to $QQQL$ possible
- need H_u and H_d on two separate curves
- Note: In this case there is also no $\mathcal{O}(1)$ μ term $\mu H_u H_d$ ✓

SU(5) GUTs



pic from: Beasely,Heckman,Vafa, 0806.0102

Split Spectral Cover

Necessary for absence of $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$: split $P_5 \to P_m \cup P_H$

Complication:

If P_m, P_H meet in point without enhanced gauge symmetry, then wavefunctions are not independent due to same boundary conditions at intersection point

$\leftrightarrow P_m, P_H$ count as 1 component of pinched curve

$\Longrightarrow$ not sufficient to prevent couplings $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$

A safe way to prevent such intersection points is via spectral cover picture

There: wavefunctions live on curves $\mathcal{P}$ in auxiliary 3-fold X

curves P on S are projection of $\mathcal{P}$ onto S

Result of longer analysis:

absence of $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$ requires split of the SCC

Split Spectral Cover

$\mathcal{C}^{(5)}$ for G=SU(5) $\leftrightarrow (p, \lambda_i), i = 1, \ldots 5, \quad \lambda_i \leftrightarrow H = SU(5)_\perp$

10 curve $\mathcal{P}_{10}^{(i)}$: $\lambda_i = 0 \Rightarrow$ a priori: 5 distinct curves

$\leftrightarrow$ 10 from $(10, \overline{5}) \subset 248$

In this case: $\mathcal{C}^{(5)}$ factorises into 5 distinct components

merging of some components $\leftrightarrow$ curves form a single object of S

$\Leftrightarrow$ recombination of I_1 locus in the Tate formalism

Generic situation: single connected cover $\mathcal{C}^{(5)} \leftrightarrow$ 1 generic I_1

$\Longrightarrow$ one single curve $\mathcal{P}_{10}$ (but not necessarily single $\mathcal{P}_5$)

Simplest split example:

$C^{(5)} \to C^{(4)} \cup C^{(1)} \Rightarrow$ two **10** matter curves:

$10^{(1)} \leftrightarrow \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}, \quad 10^{(2)} \leftrightarrow \{\lambda_5\}$

Split can be formalised by notion of monodromy group G_5 that acts on λ^i

generic non-split case: G_5 acts transitively on all λ^i: $G = S_5$

split case: $G_5 = G_4 \times G_1$: $G_4 \leftrightarrow \lambda^i$, $i = 1, \ldots, 4 \qquad \lambda_5$ invariant

Split Spectral Cover

$\mathbf{5}$ curve $\mathcal{P}_5 : \lambda_i + \lambda_j = 0$

for most most general $G = S_4$:

Two 5-curves: $5^{(1)} \leftrightarrow \lambda_i + \lambda_j, i, j = 1, \ldots 4$ $5^{(2)} \leftrightarrow \lambda_i + \lambda_5$

$\Rightarrow$ enough at least for split $\mathcal{P}_5 \to \mathcal{P}_H \cup \mathcal{P}_m$ and absence of $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$ ✓

more refined monodromy action can lead also to further split of $\mathcal{P}_H \to \mathcal{P}_u \cup \mathcal{P}_d$

$\leftrightarrow$ required for absence of bare μ-term and dim. 5 proton decay

Note: Details more involved due to requirement that $\mathcal{L}_y \cdot P_{10} = 0$ for absence of exotic incomplete multiplets

more options are possible if this is relaxed

However:

requires lifting of exotics, e.g. via couplings to singlets Φ with $\langle \Phi \rangle \neq 0$

so far no working dynamical mechanism achieved

Summary of approach

Characteristic features of F-theory model building:

- localisation of gauge d.o.f.
- emergence of exceptional gauge groups

$\Longrightarrow$ good for GUTs

We have seen interesting mechanisms to accomodate:

- all required Yukawas - in principle ✓
- GUT breaking without GUT Higgs ✓
- suppress proton decay ✓
- achieve doublet-triplet splitting ✓

Price: non-minimal unification with extra threshold below M_{GUT}
But maybe this is a window of opportunity

Summary of approach

Many further phenomenological directions have been explored, inlcuding:

- hierarchy in flavour sector/structure of Yukawa couplings
- neutrino sectors

What about explicit constructions?

Many of the above realised in explicit compact models - thanks to

- spectral cover construction
- detailed understanding of Calabi-Yau fourfolds

Urgent directions:

- no stabilisation of moduli achieved
- no dynamical argument for specific choice of brane moduli
- coupling to, say, inflationary cosmology...