Recent advances in string model building

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Why F-theory

Aim of string phenomenology:

Exploration of landscape of string vacua from perspective of 4D particle phenomenology and cosmology

$\leftrightarrow$ string model building

Two traditional working horses:

1) heterotic string:

orbifolds, free fermionic, het. on CY with vector bundles

- exceptional gauge groups ✓
- gravity and gauge fields all in bulk

2) perturbative Type II orientifolds/brane worlds:

Type IIA with D6-branes

Type IIB with D3/D7-branes

- perturbative gauge groups: U(N), Sp(N), SO(N) , but no E-groups
- localised gauge degrees of freedom ✓

Why F-theory

F-theory combines both pros:

- model building based on exceptional groups
- includes "local features"

Besides:
Promising moduli stabilisation scenarios similar to Type IIB/M-Theory

But F-theory is fascinating beyond phenomenology:
truly non-perturbative and backreacted compactification framework
in some sense: THE way to think about 7-branes

Dynamics accessible from several different viewpoints:

- as strongly coupled Type IIB with 7-branes
- as dual to M-theory
- as dual to heterotic strings

Overview

Lecture 1: Introduction to F-theory

Lecture 2: Local gauge theory and global Weierstrass models

Lecture 3: Spectral covers for F-theory models

Lecture 4: Aspects of F-theory GUT phenomenology

Overview

Part I: Introduction to F-theory

- F-theory as Type IIB with backreaction
- Elliptic fibrations and their M-theory origin
- 7-branes and non-abelian singularities

Reminder: Type IIB orientifolds

Compactify 10 D Type IIB theory on Calabi-Yau 3-fold X

quotient by $\Omega(-1)^{F_L}\sigma$ σ: $\mathbb{Z}_2$ hol. involution of X

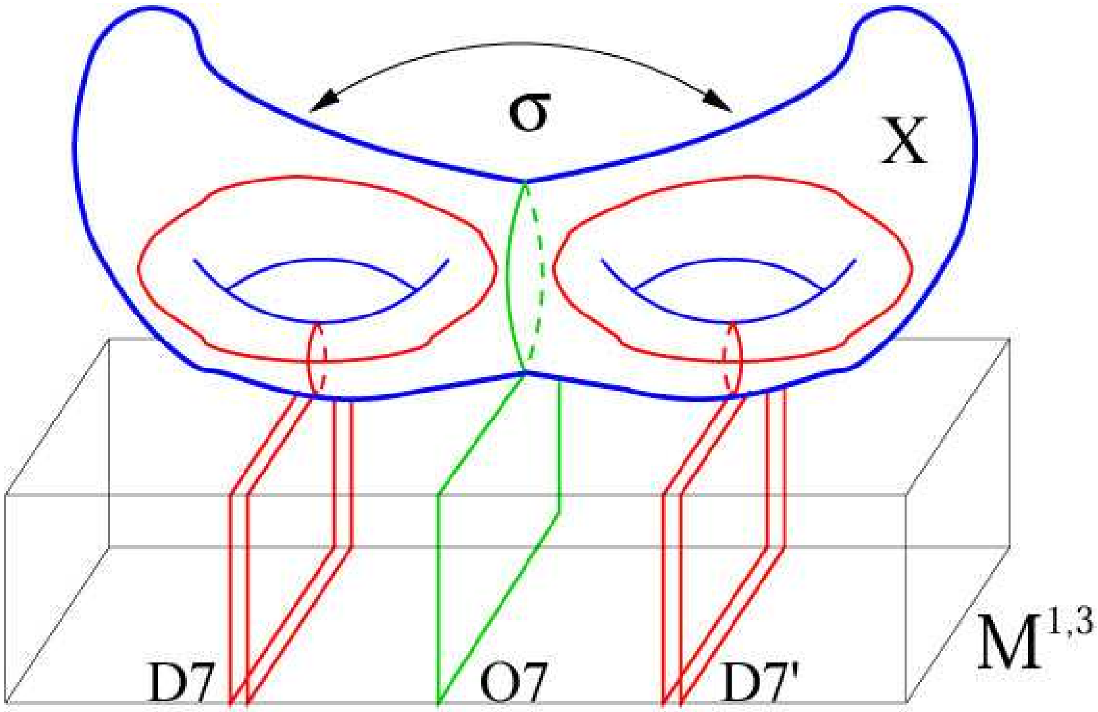
$\Rightarrow$ Fixpoint set: O7-planes and O3-planes

charge cancellation requires 7-branes on Γ_a + image on $\Gamma_{a'}$

$$\sum_a N_a(\Gamma_a + \Gamma_{a'}) = 8\Pi_{O7}$$

further constraints:

- induced D5/D3-tadpole cancellation
- D-term SUSY



Probe approximation

D-branes and O-planes treated in **probe approximation**

- neglect backreaction on metric and on sourced RR fields
- only ensure integrability conditions $=$ tadpole conditions

For generic Dp-brane $p \neq 7$ backreaction is asymptotically negligible

- Dp brane is pointlike source in $n = 9 - p$ normal directions
- Heuristically: need to solve Poincaré equation in n dimensions

  $n > 2$: $\Delta\Phi(r) = \delta(r) \rightarrow \Phi(r) = \frac{1}{r^{n-2}}$

more precisiely: BPS solution

$$ds^2 = H_p^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H_p^{1/2}dx^i dx^i$$

$$e^{2\phi} = e^{2\phi_0} H_p^{\frac{3-p}{2}} \qquad C_{p+1} = \frac{H_p^{-1} - 1}{e^{\phi_0}} dx^0 \wedge \ldots \wedge dx^p$$

$$H_p = 1 + \left(\frac{r_p}{r}\right)^{(7-p)}, \qquad r_p^{(7-p)} = \# e^{\phi_0} N$$

Note: $H_p \rightarrow 1$ as $r \rightarrow \infty$ ✓

Probe approximation

In general we wish to go beyond this probe approximation:

- While above is fine for Dp-branes, O-planes have negative tension $\Rightarrow$ SUGRA is ill-defined unless all tadpoles are cancelled **locally**
- we need control of the in general varying dilaton as this gives the string coupling
- for the interesting case of D7-branes in Type IIB orientifolds solution does not asymptote to flat space in above sense (see later)!

F-theory = SUSY Type IIB/$\Omega\sigma(-1)^{F_L}$ with backreaction of D7-branes and O7-planes on the geometry and on the varying dilaton taken into account

Type IIB and $SL(2,\mathbb{Z})$

Consider the IIB action in Einstein frame:

$$S_{IIB} = \frac{2\pi}{\ell_s^8}\Big(\int d^{10}x\sqrt{-g}R - \frac{1}{2}\frac{\partial\tau\overline{\partial}\overline{\tau}}{(\mathrm{Im}\tau)^2} + \frac{1}{\mathrm{Im}\tau}G_3\wedge *G_3 + \frac{1}{2}\tilde{F}_5\wedge *\tilde{F}_5 + $$

$$+C_4\wedge H_3\wedge F_3\Big)$$

$$\ell_s = 2\pi\sqrt{\alpha'}, \qquad \tau = C_0 + ie^{-\Phi}, \quad G_3 = F_3 - \tau H_3, \quad H_3 = dB_2,$$

$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3 \quad F_p = dC_{p-1}$$

Classical action is invariant under $SL(2,\mathbb{R})$:

$$\tau \to \frac{a\tau+b}{c\tau+d}, \qquad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} aC_2 + bB_2 \\ cC_2 + dB_2 \end{pmatrix} = M\begin{pmatrix} C_2 \\ B_2 \end{pmatrix}, \qquad \det M = 1$$

non-perturbatively $SL(2,\mathbb{R}) \to SL(2,\mathbb{Z})$ due to D(-1)-instanton

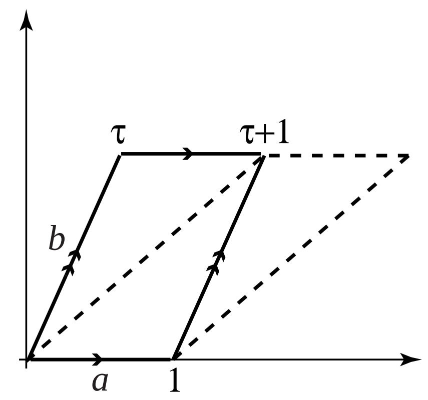
$$M = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}: \quad C_0 \to C_0 + b \quad \Rightarrow e^{2\pi i\int C_0} \quad \text{invariant for } b\in\mathbb{Z}$$

Type IIB and $SL(2,\mathbb{Z})$

$$SL(2,\mathbb{Z}):\ T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} : \tau \to \tau + 1 \qquad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} : \tau \to -\frac{1}{\tau}$$

same symmetry group acts on complex structure of a torus T^2

τ takes values in fundamental domain



T^2: lattice in $\mathbb{C}$:

$w \simeq w + 1, \qquad w \simeq w + \tau$

$\tau \in \mathbb{C} \quad \Re\tau > 0$: complex structure

S-duality is strong-weak duality: maps F1-string to D1-string

F1: (1,0) string, D1: (0,1) string $S \begin{pmatrix} 1 \\ 0 \end{pmatrix} \simeq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(p,q) string: BPS-bound state of p F-strings and q D-strings
stable for p,q relatively prime

7-brane backreaction

D7 action: $S = S_{DBI} + S_{CS}$ (string frame)

$$S_{DBI} = -\mu_7 e^{-\Phi} \int \sqrt{-det(G + B + 2\pi\alpha' F)}, \qquad S_{CS} = \mu_7 \int C_8 + \dots$$

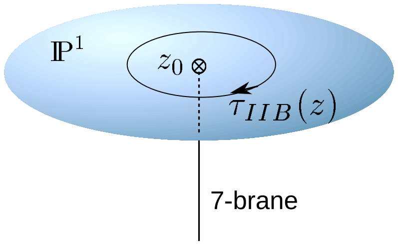
$$\mu_7 = \frac{2\pi}{l_s^{p+1}}|_{p=7} = \frac{2\pi}{l_s^8}$$

D7 brane is electric source for $C_8 \leftrightarrow$ magnetic source for $C_0 = \mathrm{Re}\tau$

Task: find the backreacted solution for τ

D7 brane is pointlike source in 2 normal directions $z = x + iy$

- SUSY requires: $\tau = \tau(z) \rightarrow F_1(z) = dC_0(z)$
- e.o.m. for $F_9 = dC_8$: $d * F_9 = \delta^{(2)}(z - z_i)$
- duality $*F_9 = F_1$
- Gauss law: $1 = \int_{\mathbb{C}} d * F_9 = \oint_{S^1} F_1 = \oint_{S^1} dC_0$



pic from: Lerche, 9910207

Solution for τ: $\tau(z) = \tau_0 + \frac{1}{2\pi i}\ln(z - z_i)$

Interpretation: non-trivial profile for τ, but still have unfixed offset

7-brane backreaction

This is a **severe backreaction on geometry**:

1) deficit angle $\leftrightarrow$ "long-range" effect
2) monodromies

ad 1) D7-brane looks like "cosmic string" in ambient space

metric ansatz: $ds^2 = -dt^2 + \sum_{i=1}^{7} dx_i^2 + H_7(z,\overline{z})dz d\overline{z}$

- pointlike source in ambient space
  $\Rightarrow$ Expect 2 dim. Poisson equation for $H_7(z,\overline{z})$
- Einstein equation: $\partial\overline{\partial}\log H_7 = \frac{\partial\tau\overline{\partial}\overline{\tau}}{\tau_2^2}$ `Greene,Shapere,Vafa,Yau 1989`
- at long distances away from brane, space has a deficit angle
  `see also Bergshoeff et al. '06, A. Braun et al. '08`
  $\Longrightarrow$ long-range effect that does not asymptote away
- for sufficiently small τ_0 still exists regime near brane where metric can be treated as in probe picture
  but for too strong coupling, this regime might become substringy!

7-brane backreaction

ad 2) near D7-brane: $\tau(z) \simeq \frac{1}{2\pi i}\ln(z - z_0)$ has branch cut

as we encircle z_0: $\tau \to \tau + 1$ "monodromy"

consistent because Type IIB theory is $SL(2,\mathbb{Z})$ invariant

More drastic monodromy for more general (p,q) branes

- D7-brane: electric charge under $C_8 \leftrightarrow$ magnetic charge under τ
- dyonic (p,q) branes $=$ branes on which (p,q) strings can end
  magnetic charge p and electric charge q under τ
- $SL(2,\mathbb{Z})$ transforms various (p,q) branes into one another
  We had: monodromy of (1,0) brane: $\tau \to \tau + 1 \leftrightarrow T_{1,0}$

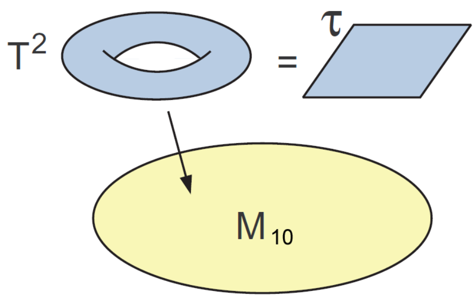
$$\text{for (p, q) brane}: \tau \to \frac{a\tau + b}{c\tau + d} \qquad T = \begin{pmatrix} 1 - pq & p^2 \\ -q^2 & 1 + pq \end{pmatrix}$$

- In general, different (p,q) type branes cannot be treated perturbatively

From branes to F-theory

F-theory geometrises the varying axio-dilaton field τ: Vafa 1996

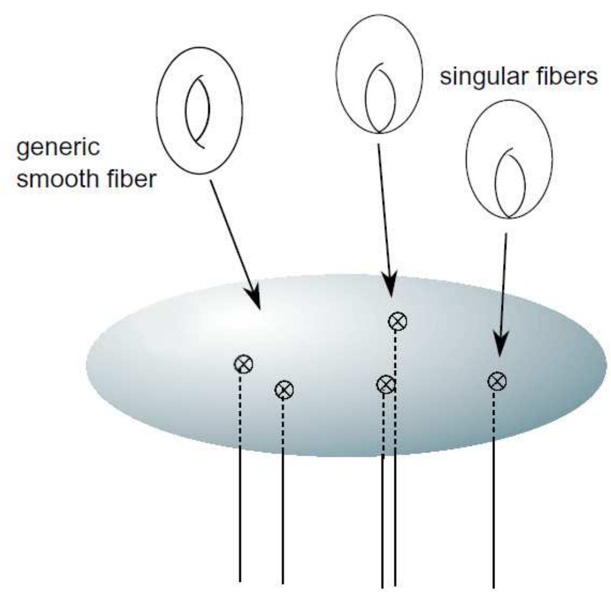
- Interpret τ as complex structure of auxiliary torus T^2
- Kähler structure has no physical meaning



pic adapted from: Denef, 0803.1194

τ varies $\leftrightarrow$ shape of T^2 varies

$\Longrightarrow$ fibration of $T^2 \to \mathcal{M}_{10}$



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locally near position of D-brane

$\tau = \frac{1}{2\pi i} \ln(z - z_0) + \ldots \to i\infty$

$\leftrightarrow$ T^2 fiber degenerates as cycle $a \to 0$

F from M

Precise definition of F-theory via duality with M-theory

Idea: Compactify 11-dim SUGRA on $M_9 \times T^2$

Take limit of vanishing volume of $T^2 = S^1_A \times S^1_B$ as follows:

- S^1_A: M-theory circle of radius R_{10}
  $R_{10} \to 0 \leftrightarrow$ weakly coupled IIA on $M_9 \times S^1_B$
- T-duality along S^1_B of radius $R_9 \to$ IIB on $M_9 \times \tilde{S}^1_B$
  $R_9 \to 0 \leftrightarrow$ decompactification of $\tilde{S}^1_B \Rightarrow$ IIB on M_9

Roughly: $g_s = \frac{R_{10}}{R_9}$ **is** $Im(\tau)$ **for original** T^2

More precisely:

- M-theory action:

$$S = \frac{2\pi}{\ell_M^9}\Big(\int d^{11}x\sqrt{-g}R - \frac{1}{2}\int G_4 \wedge *G_4 - \frac{1}{6}C_3 \wedge G_4 \wedge G_4 + \ell_M^6 \int C_3 \wedge I_8(R)\Big)$$

F from M

- Compactification ansatz:

$$ds_M^2 = \frac{v}{\tau_2}\left((dx + \tau_1 dy)^2 + \tau_2^2 dy^2\right) + ds_9^2$$

v: area of T^2, $\quad x \leftrightarrow S_A^1, \quad y \leftrightarrow S_B^1$

- relation to IIA:

$$ds_M^2 = L^2 e^{4\chi/3}(dx + C_1)^2 + e^{-2\chi/3} ds_{IIA}^2$$

$$C_1 = \tau_1 dy, \quad e^{4\chi/3} = \frac{v}{L^2\tau_2}, \quad ds_{IIA}^2 = \frac{\sqrt{v}}{L\sqrt{\tau_2}}(v\tau_2 dy^2 + ds_9^2)$$

- T-duality along y:

$$C_0 = (C_1)_y = \tau_1, \quad L_{IIB}^y = \frac{\ell_s^2}{L_{IIA}^y}, \quad g_{IIB} = \frac{\ell_s}{L_{IIA}^y} g_{IIA},$$

F from M

- comparison M_2 brane with F_1/D_2 brane

$$\frac{1}{\ell_s^2} = \frac{L}{\ell_M^3}, \qquad \frac{1}{g_{IIA}\ell_s^3} = \frac{1}{e^{\chi}\ell_M^3}$$

E.g: $M_2 \to F_1$: $\frac{2\pi}{\ell_M^3} \times Le^{2\chi/3} \times e^{-2\chi/3} Vol_2^{IIA} = \frac{2\pi}{\ell_s^2} Vol_2^{IIA}$

Result:

$$C_0 + \frac{i}{g_{IIB}} = \tau, \qquad ds^2_{IIB,E} = \frac{\sqrt{v}}{L}\left(\frac{L^2\ell_s^4}{v^2}dy^2 + ds_9^2\right)$$

Pick $L = \sqrt{v}$ (by redef. of χ)

F-theory limit is $v \to 0$ while ℓ_s finite

Newton's law of string theory:

$$F = M|_{A(T^2)\to 0}$$

Indeed: volume of T^2 has no physical meaning in IIB

Elliptic fibrations

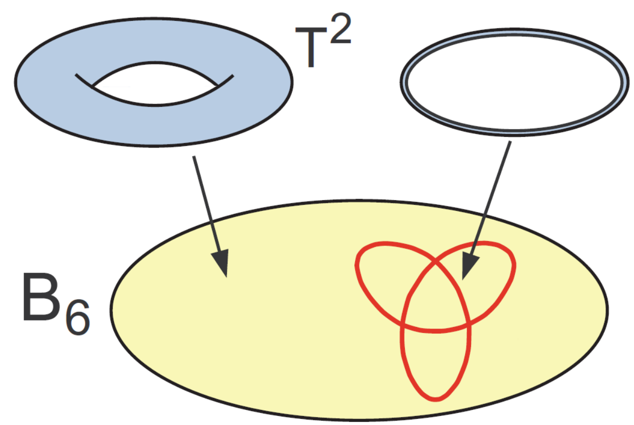
Case of phenomenological interest:

Type IIB orientifold on $\mathcal{M}_{10} = \mathbb{R}^{1,3} \times X_6/\sigma$ with $\mathbb{R}^{1,3}$ filling 7-branes

- physics encoded in geometry of $Y : T_2 \to B_6$ $\quad B_6 = X_6/\sigma$
- $\mathcal{N} = 1$ SUSY on M-theory side requires Y to be Calabi-Yau
- jargon: F-theory on elliptic fourfold Y = effective 4D theory obtained by compactification of Type IIB strings with D7-branes on X_6/σ

IIB language:

7-branes wrap 4-cycle $\Gamma_a \in X_6/\sigma$



pic adapted from: Denef, 0803.1194

F-theory language:

Γ_a = locus of fiber degeneration

Magic of F-theory:

✓all information of present D7-branes encoded in geometry of this fibration

✓backreaction fully taken into account

Elliptic fibrations

Understand possible degenerations of fiber via algebraic geometry :

- elliptic curve admits various representations as hypersurfaces/complete intersections in projective spaces
- most common: $T^2 = \mathbb{P}_{2,3,1}[6]$: $(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$ $(*)$

  $P \equiv y^2 - x^3 - fxz^4 - gz^6 = 0 \qquad f, g \in \mathbb{C} \qquad$ with $(*)$ set $z = 1$
- This is Calabi-Yau and thus T^2 ✓
- T^2 singular when $P = 0$ and $dP = 0$, i.e.

  1) $y = 0$

  2) $0 = x^3 - fx - g = (x - a_1)(x - a_2)(x - a_3)$

  3) $0 = (x - a_1)(x - a_2) + (x - a_2)(x - a_3) + (x - a_1)(x - a_3)$

$\Rightarrow$ only possible if two or more a_i coincide

Elliptic fibrations

Results from complex geometry:

- Coincidence of roots if **discriminant**

$$\Delta = 27g^2 + 4f^3 = 0$$

- complex structure τ encoded in f, g via

$$j(\tau) = \frac{4(24f)^3}{\Delta}$$

  $j(\tau)$: $SL(2,\mathbb{Z})$ invariant Jacobi function

  $j(\tau) = e^{-2\pi i\tau} + 744 + \mathcal{O}(e^{2\pi i\tau})$

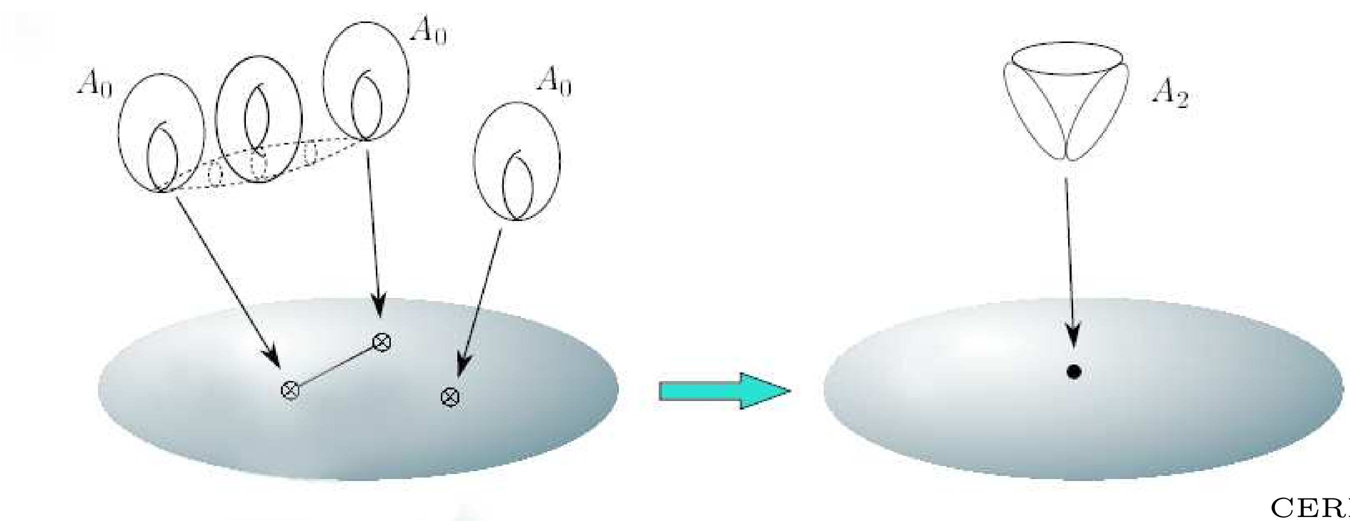
Elliptic fibrations

Now fiber T^2 over the base B_6 with coordinates u_i: $f, g \rightarrow f(u_i), g(u_i)$
$\rightarrow$ Weierstrass model $y^2 = x^3 + f(u_i)xz^4 + g(u_i)z^6$

(more details on properties of B and g, f later)

position of branes on B: $\Delta(u_i) = 0$ $\rightarrow$ holomorphic divisor ✓

- simple zero
  only T^2 degenerates, but fourfold Y remains smooth
  single 7-brane with U(1) gauge group
- multiple zero $\rightarrow$ genuine singularity of fourfold Y
  several coincident 7-branes with non-abelian gauge groups



ADE gauge groups

admissable singularities of Y analysed by Kodaira $\Rightarrow$ A-D-E classification

these are the singularities of Y whose resolution in fiber does not destroy the Calabi-Yau property of Y

ord(Δ)	ord(f)	ord(g)	sing.
n	0	0	$A_{n-1} \simeq SU(n)$
$n+6$	2	≥ 3	$D_{n+4} \simeq SO(2n+8)$
8	≥ 3	4	E_6
9	3	≥ 5	E_7
10	≥ 4	5	E_8

Physical interpretation:

gauge group of brane given by corresponding **A-D-E group**

`Bershadsky et al. 1996`

Note at this stage:

on general elliptic fibration mutually non-local monodromies are present

stacks of coincident (p,q) branes of different type $\rightarrow$ exceptional gauge symmetries

Gauge groups: M-theory viewpoint

Understand nonabelian gauge groups from M-theory:
Example: $SU(N)$ from N coincident branes of same (p,q) type
as N mutually local branes approach each other the fiber develops an A_{N-1} singularity

- obtained by shrinking N $\mathbb{P}^1$s in fiber
- if resolve the singularity, this gives N $\mathbb{P}^1$s, of which $N-1$ are homologically distinct: Γ_i, $i = 1, \ldots N-1$
- these Γ_i intersect as the nodes in Dynkin diagram of $SU(N)$

$\rightsquigarrow$ reduction of M-theory 3-form C_3 on Γ_i
  $\longrightarrow$ N-1 (independent) Cartan U(1) gauge potentials
$\rightsquigarrow$ extra states from light membranes wrapping chain $\Gamma_i \cup \ldots \cup \Gamma_j$, $i \leq j$
  2 orientations $\rightarrow$ $N(N-1)$ states

$\Longrightarrow$ $\mathbf{N^2 - 1}$ generators of $SU(N)$ ✓

Gauge groups: IIB viewpoint

Appearance of exceptional gauge symmetries also in strongly coupled IIB language well-known

- origin: coincident 7-branes of **mutually non-local (p,q) type**
- massless degrees of freedom from string junctions: string bound-states with multiple ends

$\longrightarrow$ graphical representation of adjoint of e.g. exceptional gauge groups

Example:

$A = (1,0), \quad B = (3,-1), \quad C = (1,-1)$ [Sen '96/97; Gaberdiel,Zwiebach'97]

$SU(N) : A^N, \qquad SO(2N) : A^N BC, \qquad E_8 : A^5 BC^2$

(More info, if desired, in discussion session)

F-Theory and IIB

Connection to IIB orientifold on CY 3-fold X by **Sen limit**: [Sen '96/97]

Weierstrass:

$$y^2 = x^3 - f(u_i)\,x + g(u_i), \qquad \Delta = 27g^2 + 4f^3$$

General parametrisation: $f = -3h^2 + \epsilon\eta, \qquad g = -2h^3 + \epsilon h\eta - \frac{\epsilon^2}{12}\chi$

Generically: Δ is single, connected I_1 object

IIB limit: $\epsilon \to 0 \Rightarrow \Delta = -9\epsilon^2 h^2(\eta^2 - h\chi) + \mathcal{O}(\epsilon^3)$

$$O7 : h = 0, \qquad D7 : \eta^2 - h\chi = 0$$

F-theory $\leftrightarrow$ non-perturbative recombination of O-plane and 7-branes!

X: double cover of base B branched over $h = 0$

Simplest case: X given by equation $h = \xi^2$, orientifold $\xi \to -\xi$

Uplift: Reversal of Sen limit

$\leftrightarrow$ Define $B = X/\sigma$ and consider Weierstrass model thereof

7-brane tadpole

all "extra" 7-brane consistency conditions of probe IIB picture **automatically** incorporated in geometry: Morrison, Vafa 1996

- Kodaira: first Chern class of tangent bundle of Y descends from tangent bundle of B modulo singular divisor loci

  $c_1(T_Y) \simeq \pi^*(c_1(T_B) - \sum_i \frac{a_i}{12}\delta(\Gamma_i)) \qquad a_i = \mathcal{O}(\Delta)|_{\Gamma_i}$

- Since $\mathcal{N} = 1$ SUSY requires $c_1(T_Y) = 0$

  $$\sum_i a_i \delta(\Gamma_i) = 12\, c_1(T_B)$$

  B is no longer Calabi-Yau

- cf. 7-brane tadpole: $\sum_i N_i \delta(\Gamma_i) = 4\delta(O7)$

  $\rightarrow$ curvature of base B has absorbed orientifold charge

elliptic Calabi-Yau fourfold $\leftrightarrow$ consistent, non-pert. 7-brane configuration

Example: 7-branes on S^2: $K_S = \mathcal{O}(-2) \rightarrow$ need 24 branes

Consistency: D3 tadpole

While D7-tadpole automatic in F-theory, D3-tadpole ensured extra:
Consider dual M-theory on $\mathbb{R}^{(1,2)} \times Y$ w/ M_2 branes along $\mathbb{R}^{(1,2)} \times \mathrm{point}_i$
integrate e.o.m. for G_4 field strength

$$0 = \int d * G_4 = \int \left(\frac{1}{2} G_4 \wedge G_4 - \ell_M^6 I_8(R) + \ell_M^6 \sum_i \delta(M_2^{(i)}) \right)$$

Key: $\int_Y I_8(R) = \frac{\chi(Y)}{24}$ $\qquad$ $\chi(Y) = \int_Y c_4(T_Y)$: Euler characteristic

$$\boxed{\frac{\chi}{24} = \frac{1}{2\ell_M^6} \int G_4 \wedge G_4 + N_{M_2}}$$

upon M/F-theory duality:

- $\frac{\chi}{24}$: curvature dependent D3-charge of O7-planes and 7-branes
- $N_{M_2} \longrightarrow N_{D_3}$
- $\frac{1}{2\ell_M^6} \int G_4 \wedge G_4$: bulk and brane flux induced D3-charge

Consistency: D3 tadpole

Reduction of M-theory 3-form:

$$C_3 = \tilde{C}_3 + B_2 \wedge L\,dx + C_2 \wedge L\,dy + B_1 \wedge L\,dx \wedge L\,dy$$

IIB: $C_4^y = \tilde{C}_3 \wedge dy$, $g_{iy} = (B_1)_i$, B_2: NSNS 2-form C_2: RR 2-form

Similarly: IIB 3-form flux from: $G_4 = H_3 \wedge L\,dx + F_3 \wedge L\,dy$

- IIB bulk flux: reduce G_4 on 4-cycle which is product of 1-cycle in fiber and non-trival three-cycle Σ_3 on B
  $\leftrightarrow H_3/F_3$ along Σ_3
- IIB gauge flux F_2 along the 7-brane on divisor S: $G_4 = F_2 \wedge \omega$
  ω anti-selfdual 2-form dual to 2-cycle from fibering pinched S^1 between 2 branes

Consistency: D3 tadpole

In absence of bulk flux:

$$\frac{\chi}{24} = N_{D3} \quad \underbrace{-\frac{1}{2\ell_s^4}\int_S F_2 \wedge F_2}_{\geq 0 \text{ due to } F_2 = -*F_2}$$

Constraint: $N_{D3} \geq 0$ to avoid uncontrolled SUSY breaking

Important: in presence of non-abelian gauge groups, Y is singular

$\chi(Y)$ refers to Calabi-Yau obtained after resolving these singularities

special case: no non-abelian gauge groups $\Rightarrow$ only I_1 degenerations

$\leftrightarrow$ Euler character: $\chi^*(Y) = 12 \int_B c_1(B)\, c_2(B) + 360 \int_B c_1^3(B)$

[Sethi,Vafa,Witten '96]

We will give a closed expression for certain more general configurations required for model building

Overview

Part II:

Local gauge theory and global Weierstrass model

- Twisted gauge theory and massless matter
- Weierstrass and Tate model
- Application: SU(5) gauge theory

Summary so far

F-theory on Calabi-Yau fourfold $Y \to B$

- $B \leftrightarrow$ backreacted compactification space of IIB orientifolds with 7-branes: $B = X/\sigma$
- T_2 fiber: varying dilaton
- ADE group G along divisor $S \subset B$

Idea of "local" analysis:
Focus on effective 4D $\mathcal{N} = 1$ Super-Yang-Mills theory on S

- Describe degrees of freedom $\leftrightarrow$ phenomenology
- dictionary geometry $\leftrightarrow$ gauge degrees of freedom

[Beasley,Heckman,Vafa 0802.3391, 0806.0102]

[Donagi, Wijnholt 0802.2969, 0904.1218]

[Hayashi, Kawano,Tatar, Watari 0901.4941]

Local gauge theory on S

$D = 4, \mathcal{N} = 1$ twisted gauge theory of group G on S

- Starting point: 8D SYM from 10 D SYM

  SYM on $\mathbb{R}^{1,7} \subset \mathbb{R}^{1,9}$: symmetry $SO(1,7) \otimes U(1)_R$

  Bosonic: gauge potential A_i^{8D}, scalar $\varphi = \phi_8 + i\phi_9$: $U(1)_R = -1$
  Fermionic: 10 D spinor $(\mathbf{16}_+) \to (S_+, \frac{1}{2}) + (S_-, -\frac{1}{2})$

- Compactification on $\mathbb{C}^2$: $SO(1,3) \otimes SO(4) \otimes U(1)_R$

$$\begin{aligned} \left(S_+, \frac{1}{2}\right) &\to \left((2,1),(2,1),\frac{1}{2}\right) + \left((1,2),(1,2),\frac{1}{2}\right) \\ \left(S_-, -\frac{1}{2}\right) &\to \left((2,1),(1,2),-\frac{1}{2}\right) + \left((1,2),(2,1),-\frac{1}{2}\right) \end{aligned}$$

- S is compact Kähler space: $SO(1,3) \otimes U(2) \otimes U(1)_R$:

$$(2,1) \to 2_0, \qquad (1,2) \to 1_1 + 1_{-1}$$

Local gauge theory on S

To keep $\mathcal{N}=1$ SUSY in we need supercharges $Q_\alpha, \overline{Q}_{\dot\alpha}$ which are scalars under internal group

Define twist by combining $U(1) \subset U(2)$ with $U(1)_R$ s.t. 2 scalars remain

solution: $J_{top} = J_{U(1)} + 2J_{U(1)_R}$:

$$\Big((2,1)\otimes 2_1\Big) + \Big((1,2)\otimes(1_2+1_0)\Big) \qquad \Big((1,2)\otimes 2_{-1}\Big) + \Big((2,1)\otimes(1_0+1_{-2})\Big)$$

- $\exists$ chiral and anti-chiral spinors which are internal scalars ✓ $\leftrightarrow Q_\alpha, \overline{Q}_{\dot\alpha}$
- Identify internal part of spinor with differential forms:
  $U(1)_{top}$: $\quad +/-p \leftrightarrow$ antiholomorphic/holomorphic p-form

$\rightarrow$ fermion = section of $\Omega^p \otimes ad(G)$ and $\overline{\Omega}^p \otimes ad(G)$

Local gauge theory on S

Transverse scalar $\varphi, \overline{\varphi}$: $\Big((1,1) \otimes 1_{\mp 2}\Big)$

$\Rightarrow \varphi \subset \Omega^2_S \otimes ad(G)$: $\qquad \varphi = \Phi_{mn} dz^m \wedge dz^n$

$\Leftrightarrow$ **deformations are given by section of** $K_S \otimes ad(G)$

Altogether:

$\mathcal{N} = 1$ chiral multiplets: (A_μ, λ_α)	$(A_{\overline{m}}, \psi_{\alpha\,\overline{m}})$	$(\Phi_{mn}, \chi_{\alpha mn})$

Local gauge theory on S

- **F-term**:
  Superpotential e.g. by reduction of CS potential on D9-branes:
  $$\int_{\mathbb{R}^{1,3}} d^4x\, d^2\theta\ W = \int_{\mathbb{R}^{1,3}\times S} d^4x\, d^2\theta\, \mathrm{Tr}\Big(\mathbb{F}_S^{(0,2)} \wedge \Phi\Big)$$
  BPS equation: $F^{(0,2)} = 0 = F^{(2,0)} \rightarrow$ holomorphic G-bundle V
  $$\overline{\partial}_A \varphi = 0 = \partial_A \overline{\varphi}$$
- **D-term:** $J \wedge F_S + \frac{i}{2}[\varphi, \varphi] = 0$

2 ways to break gauge symmetry G on S further:

- VEV for (some components of) field strength F
  $\leftrightarrow$ holomorphic bundle V with structure group $H \subset G$
  E.g.: $G = SU(5)$, line bundle $\mathcal{L}$ with generator $T_Y = (2, 2, 2, -3, -3)$
  $\Longrightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1)_Y$
- VEV for adjoint scalar field φ $\leftrightarrow$ local geometry around $S \subset B$

(F_S, φ) defines a holomorphic Higgs bundle over S

Matter from cohomologies

chiral	(A_μ, λ_α)	$(A_{\overline{m}}, \psi_{\alpha\,\overline{m}})$	$(\Phi_{mn}, \chi_{\alpha mn})$
anti-chiral	$(\overline{A}_\mu, \overline{\lambda}_{\dot\alpha})$	$(\overline{A}_m, \overline{\psi}_{\dot\alpha\,\overline{m}})$	$(\overline{\Phi}_{\overline{mn}}, \overline{\chi}_{\dot\alpha\overline{mn}})$

- trivial gauge bundle $\rightarrow$ valued in ad(G)
- in presence of H-bundle: $G \rightarrow G' \times H$: $ad(G) \rightarrow \bigoplus_i (R_i, U_i)$:

massless "bulk" matter on S:

- bundle-valued zero-modes of $\overline{\partial}_A$, $\overline{\partial}_A^\dagger \rightarrow \Delta = \overline{\partial}_A \overline{\partial}_A^\dagger + \overline{\partial}_A^\dagger \overline{\partial}_A$
- only equivalence classes modulo exact part count different degrees of freedom $\Rightarrow$ cohomology groups $H^i(S, V)$

massless matter in representation R_i:

$\overline{\eta}_{\dot\alpha} : H^0_{\overline{\partial}}(S, U_i)$
$\psi_\alpha : H^1_{\overline{\partial}}(S, U_i)$
$\overline{\chi}_{\dot\alpha} : H^2_{\overline{\partial}}(S, U_i)$

$\eta_\alpha : \overline{H^0_{\overline{\partial}}(S, U_i)} \simeq H^0_{\overline{\partial}}(S, U_i^*)^*$
$\overline{\psi}_{\dot\alpha} : \overline{H^1_{\overline{\partial}}(S, U_i)} \simeq H^1_{\overline{\partial}}(S, U_i^*)^*$
$\chi_\alpha : \overline{H^2_{\overline{\partial}}(S, U_i)} \simeq H^2_{\overline{\partial}}(S, U_i^*)^*$

Matter from cohomologies

$\eta_\alpha + c.c.$ would be partners of extra gauge multiplets $\leftrightarrow$ must be absent
Indeed: For a stable bundle $H^0_{\bar\partial}(S, U_i) = 0 = H^0_{\bar\partial}(S, U_i^*)$ ✓

chiral index:

$$\#(\text{chiral}, R_i) - \#(\text{anti} - \text{chiral}, R_i) =$$

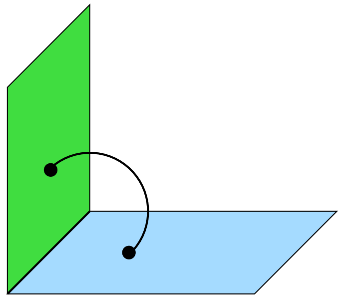
$$h^0(S, U_i^*) + h^1(S, U_i) + h^2(S, U_i^*) - (h^0(S, U_i) + h^1(S, U_i^*) + h^2(S, U_i)) =$$

$$\chi(S, U_i^*) - \chi(S, U_i) = -\int_S c_1(S) c_1(U_i)$$

$$\chi(S, U_i) = h^0(S, U_i) - h^1(S, U_i) + h^2(S, U_i) = \int_S ch(U_i) Td(S)$$

Brane intersections

gauge symmetry on compl. codimension-1 locus = divisor Γ_a on B



Γ_a, Γ_b intersect along curve $C_{ab} = \Gamma_a \cap \Gamma_b$
$\rightsquigarrow$ IIB picture: bifundamental matter
$\rightsquigarrow$ F-theory: singularity enhanced on C_{ab}

Picture: collision of zero-size $\mathbb{P}^1$ in fiber over Γ_a, Γ_b

Example:
Γ_a: SU(N) symmetry, Γ_b: U(1) symmetry $\Longrightarrow C_{ab}$: $SU(N+1)$

along C_{ab}: massless states from wrapped membranes fill $\mathrm{adj}_{(N+1)}$
extra states are localised as massless matter on C_{ab} `Katz, Vafa 1998`

group theoretic decomposition:
$SU(N+1) \to SU(N) \times U(1) \quad \mathrm{adj}_{N+1} \to (\mathrm{adj}_N)_0 + 1_0 + ((N)_1 + (\overline{N})_{-1})$

Matter at intersections

Define 6D "defect theory" on $\mathbb{R}^{1,3} \times C_{ab}$ of enhanced gauge group G_{ab}

G_{ab} higgsed down to G_a and G_b away from C_{ab} by $\langle \varphi \rangle \neq 0$

- 6D hypermultiplet along C_{ab} in $\mathrm{ad}(G_{ab}) \Longleftrightarrow (b, \chi_\alpha), \qquad (\overline{b}, \overline{\chi}_{\dot\alpha})$
- $\mathrm{ad}(G_{ab}) \to \mathrm{ad}(G_a) \oplus \mathrm{ad}(G_b) \oplus \bigoplus_i (R_a^i, R_b^i)$
- Topological twist similar to before:
  $SO(1,5) \times SU(2)_R \to SO(1,3) \times U(1) \times SU(2)_R$
  $J_{top} = J_{U(1)} - \frac{1}{2} U(1)_R$

$$(b^i, \chi_\alpha^i): \quad H^0(C_{ab}, K_C^{\frac{1}{2}} \otimes R_a^i \otimes R_b^i),$$

$$(\overline{b}^i, \overline{\chi}_{\dot\alpha}^i): \quad H^1(C_{ab}, K_C^{\frac{1}{2}} \otimes R_a^i \otimes R_b^i)$$

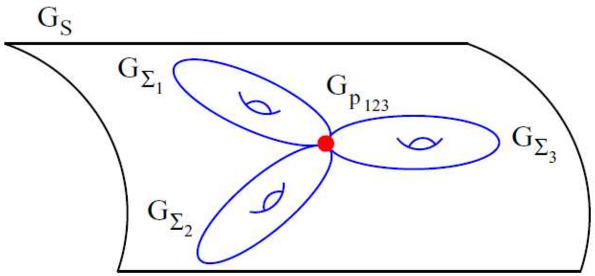
Chiral spectrum requires gauge flux F_a, F_b

(depending on embedding this will further decompose various reps.)

$$\boxed{\chi(R_a^i \otimes R_b^i) = \int_{C_{ab}} \left(c_1(R_a^i) + c_1(R_b^i) \right)}$$

Yukawas

at intersection of two or three matter curves: further rank enhancement



pic from: Beasley,Heckman,Vafa 0806.0102

e.g. $\Gamma_5 : SU(5), \quad \Gamma_a : U(1)_a, \quad \Gamma_b : U(1)_b$

$\mathbf{5}_{1_a,0} + c.c., \quad \mathbf{5}_{0,1_b} + c.c., \quad 1_{1_a,-1_b} + c.c.$

all states incorporated at point of $SU(7)$ enhancement

there: Yukawa couplings $\langle \mathbf{5}_{1_a,0}\ \overline{\mathbf{5}}_{0,-1_b}\ 1_{-1_a,1_b} \rangle$

- Yukawas descend from cubic Yang-Mills interaction for enhanced gauge group upon decomposition of adjoint
- Holomorphic piece given by overlap of wave functions at intersection point