Massive Antennae

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Outline

Infrared Singularities in NLO Cross Sections

Notation

n-jet cross section at NLO:

$$d\sigma = \underbrace{\left|\mathcal{M}^n_{tree}\left(p_1,\ldots,p_n\right)\right|^2 \mathrm{d}\Phi_n}_{\mathrm{d}\sigma_{\mathrm{LO}}} + \underbrace{\alpha_s \left|\mathcal{M}^n_{1-loop}\left(p_1,\ldots,p_n\right)\right|^2 \mathrm{d}\Phi_n}_{\mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{NLO}}} +$$

$$\underbrace{\alpha_s\, J_n^{(n+1)}\left(p_1,\ldots,p_{n+1}\right)\left|\mathcal{M}^{n+1}_{tree}\left(p_1,\ldots,p_{n+1}\right)\right|^2 \mathrm{d}\Phi_{n+1}}_{\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}}}$$

$\mathcal{M}^n$: n-particle matrix element
J: jet function
Φ_n: n-particle phase space
$\left|\mathcal{M}^n_{1-loop}\left(p_1,\ldots,p_n\right)\right|^2 := 2\Re\left(\mathcal{M}_n^{\text{1-loop}}\left(\mathcal{M}_n^{\text{tree}}\right)^*\right)$

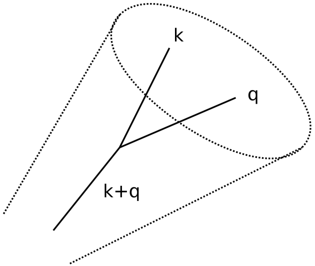
Infrared Singularities in NLO Cross Sections

Singularities

two kinds of IR (soft or collinear) singularities:

$$\left|\mathcal{M}^n_{1-loop}(p_1,\ldots,p_n)\right|^2 = \frac{+1}{\epsilon^2}c_{-2} + \frac{+1}{\epsilon}c_{-1} + c_0^{\mathrm{V}}$$

$$\left[\int_{\Phi_{\mathrm{rad}}} \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}}\right](p_1,\ldots,p_n) = \frac{-1}{\epsilon^2}c_{-2} + \frac{-1}{\epsilon}c_{-1} + c_0^{\mathrm{R}}$$



divergence:

$$\int_{\mathrm{cone}} \frac{1}{(k+q)^2}\cdots$$

Infrared Singularities in NLO Cross Sections

Cancellation

IR singularities cancel, but only after integration

problem

$\int_{\Phi_{\mathrm{rad}}} \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}}$ is

- difficult
- specific to jet definition

Subtraction Terms

make $n+1$ particles and n particles separately IR-finite

subtraction terms

(Z. Kunszt, D. Soper)

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \underbrace{\left(\mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{NLO}} + \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO;V}}\right)}_{\text{finite}} + \underbrace{\left(\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO;R}}\right)}_{\text{finite}}$$

$\mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO;V}}$: n-particle subtraction term

$\mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO;R}}$: $n+1$-particle subtraction term

$\mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO;V}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO;R}} = 0$ for n-jet observables

Subtraction Terms

Market

different methods to construct subtraction terms

- dipole subtraction (S. Catani, M. Seymour)
- $\mathcal{E}$-prescription (S. Frixione, Z. Kunszt, A. Signer)
- antenna subtraction (D. Kosower; J. Campbell, M. Cullen, N. Glover; A. Daleo, D. Maître, T. Gehrmann)

Subtraction Terms

Help from Physics - I

matrix element factorisation

for colour-ordered subamplitudes

$$\left|\mathcal{M}_{tree}^{n+1}(p_1,\dots,p_i,p_j,p_k,\dots,p_{n+1})\right|^2 \xrightarrow[j \text{ unresolved}]{} \mathcal{F}(p_i,p_j,p_k)\left|\mathcal{M}_{tree}^{n}(\dots,\tilde{p}_I,\tilde{p}_K,\dots)\right|^2 + \text{regular terms}$$

j unresolved: either $\vec{p_j} \parallel \vec{p_i}$ or $\vec{p_j} \parallel \vec{p_k}$ or $\vec{p_j} \to 0$

$\mathcal{F}(p_i,p_j,p_k)$: eikonal factor or splitting function

$\tilde{p}_I$, $\tilde{p}_K$: redefined on-shell momenta, $p_i + p_j + p_k = \tilde{p}_I + \tilde{p}_K$

Subtraction Terms

Help from Physics - II

phase space factorisation

$$\mathrm{d}\Phi_{n+1}(p_1, \ldots, p_i, p_j, p_k, \ldots, p_{n+1}; q) = \mathrm{d}\Phi_n(\ldots, \tilde{p}_I, \tilde{p}_K, \ldots ; q) \cdot \mathrm{d}\Phi_{X_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K)$$

q: total incoming four-momentum

$\Phi_{X_{ijk}}$: antenna phase space, $\mathrm{d}\Phi_{X_{ijk}} = \frac{1}{\int \mathrm{d}\Phi_2}\mathrm{d}\Phi_3$.

The Antenna Functions

Requirements

factorisation implies

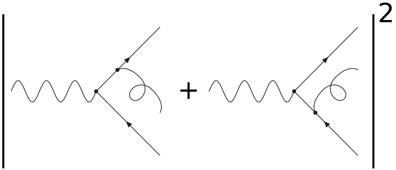
$$\begin{aligned} d\sigma^{\mathrm{S}}_{\mathrm{NLO;R}} =& X_{ijk}\left(p_i, p_j, p_k\right) d\Phi_{X_{ijk}} \\ & \left|\mathcal{M}^n_{tree}\left(\ldots, \tilde{p}_I, \tilde{p}_K, \ldots\right)\right|^2 d\Phi_n\left(\ldots, \tilde{p}_I, \tilde{p}_K, \ldots\right) \end{aligned}$$

criteria for X_{ijk}:

- $X_{ijk} \xrightarrow[j\,\text{unresolved}]{} \mathcal{F}(p_i, p_j, p_k)$
- $\int X_{ijk}\, d\Phi_{X_{ijk}}$ can be solved analytically to get $d\sigma^{\mathrm{S}}_{\mathrm{NLO;V}}$

The Antenna Functions

$$\text{duck test} \implies X_{ijk} \propto \left|\mathcal{M}\left(p_i, p_j, p_k\right)\right|^2, e.g.$$



- antennae are derived from physical matrix elements
- contain unresolved radiation between two hard particles

The Massive Case

masses shield soft/collinear singularities

but

$$\begin{aligned}\text{gluon massless} &\implies \text{some singularities stay}\\ \text{quark almost massless} &\implies \text{numerical instabilities}\end{aligned}$$

The Massive Case

Generalisations

factorisation with massive particles

adaptations:

- collinear limit $\longrightarrow$ quasi-collinear limit

$$2p_i p_j \longrightarrow 0 \qquad m_i^2 \longrightarrow 0 \qquad m_j^2 \longrightarrow 0 \qquad \frac{2p_i p_j}{m_{i/j}^2} \quad \text{fixed}$$

- $\mathcal{F}(p_i, p_j, p_k) \longrightarrow \mathcal{F}(p_i, p_j, p_k, m_i, m_j, m_k)$

The Massive Case

Steps

dipole subtraction extended to the massive case already (S. Catani, S. Dittmaier, M. Seymour, Z. Trócsányi; L. Phaf, S. Weinzierl)
massive antennae obtained recently (A. Gehrmann-De Ridder, MR in arXiv:0904.3297)

steps

- derive antenna functions
- check limits
- integrate over antenna phase space

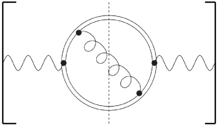
Reduction

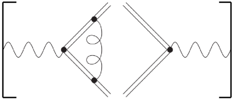
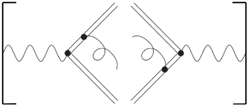
- loop integral reduction techniques extended to phase space integrals (C. Anastasiou, K. Melnikov) $\longrightarrow$ from ≈ 50 integrals to 5 master integrals
- 4 master integrals are hypergeometric functions
- 1 master integral calculated using a differential equation (T. Gehrmann, E. Remiddi)

Consistency Check

antennae derived from physical matrix elements $\Longrightarrow$ integrated antennae part of physical results

integrated massive quark-antiquark antenna checked using $\gamma^* \to Q\bar{Q} + X$ and correction to $\gamma Q\bar{Q}$-vertex (W. Bernreuther et al.; J. Jersak, E. Laermann, P. Zerwas; J. S. Schwinger; K. Chetyrkin, J. Kuhn, A. Kwiatkowsi)

$2\,\mathrm{Im}$  $=$

$\int \mathrm{d}\Phi_2\, 2\,\mathrm{Re}$  $+ \int \mathrm{d}\Phi_3$ 

End

Thanks for your attention