Implementation of Two Galactic Cosmic Rays Cutoff Rigidity Models

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New Opportunities in the AMS2 Era #2, Arlington Virginia

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Motivation and Outline

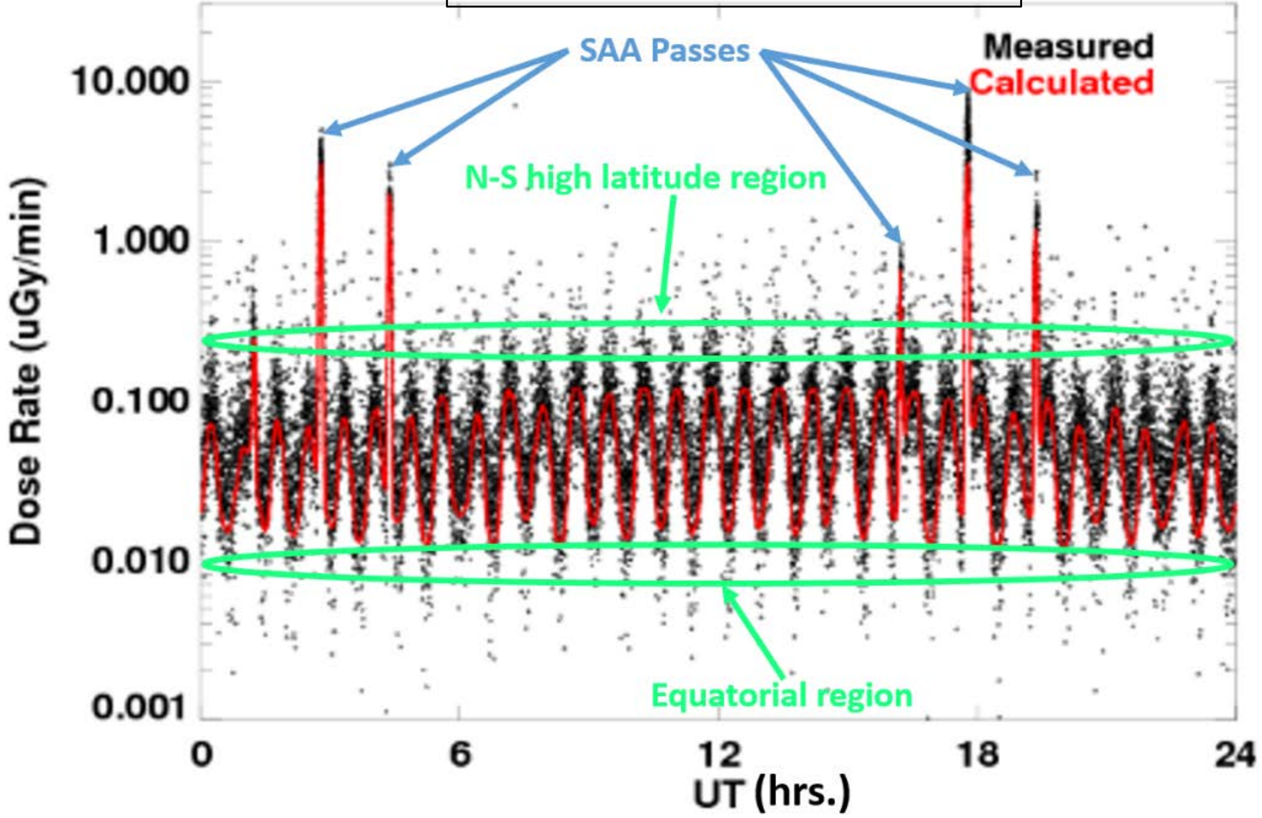
Motivation: To evaluate the accuracy of dose_eq prediction using the **Stormer** and the **trajectory-tracing** cutoff models at **Low Earth Orbit** (**LEO**)

Outline:

- Review of current status of **underestimation** in the **ISS** dosimetric validation
- Description of the ISS US-Lab Radiation Environment Monitor (**REM**) target point for dose_eq comparison
- Brief description of the **Stormer** and the **trajectory-tracing** methodologies
- Presentation of ISS dose_eq comparison results using **operational** and **simplified** ISS trajectories
- Discussion of how to speed up the trajectory-tracing computation using a hybrid method
- Summary

Validation Underestimation in Dose Rate at US-Lab REM - I

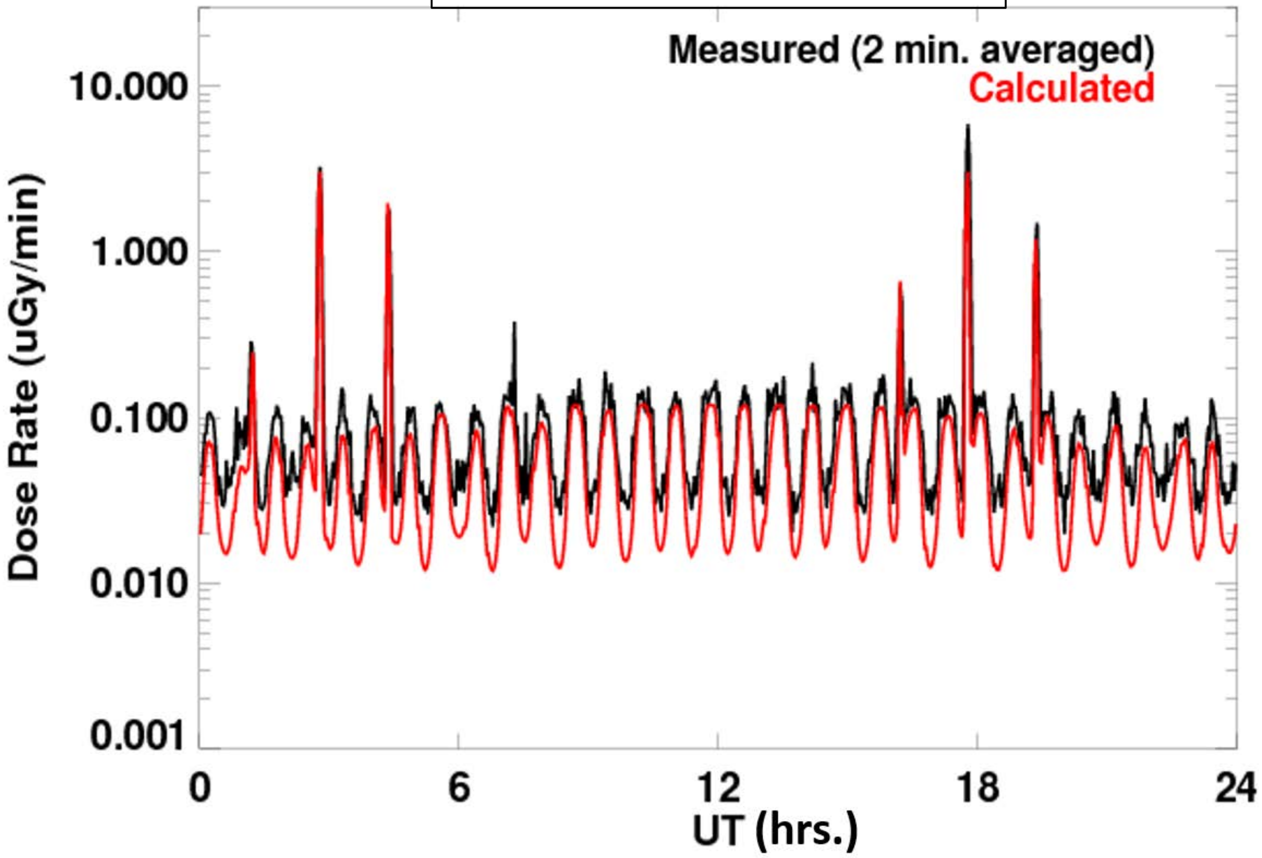
Coverage: Nov. 16, 2013



21600 data pts. (4 s. intervals)
720 data pts. (2 m. interval)
720 transport runs

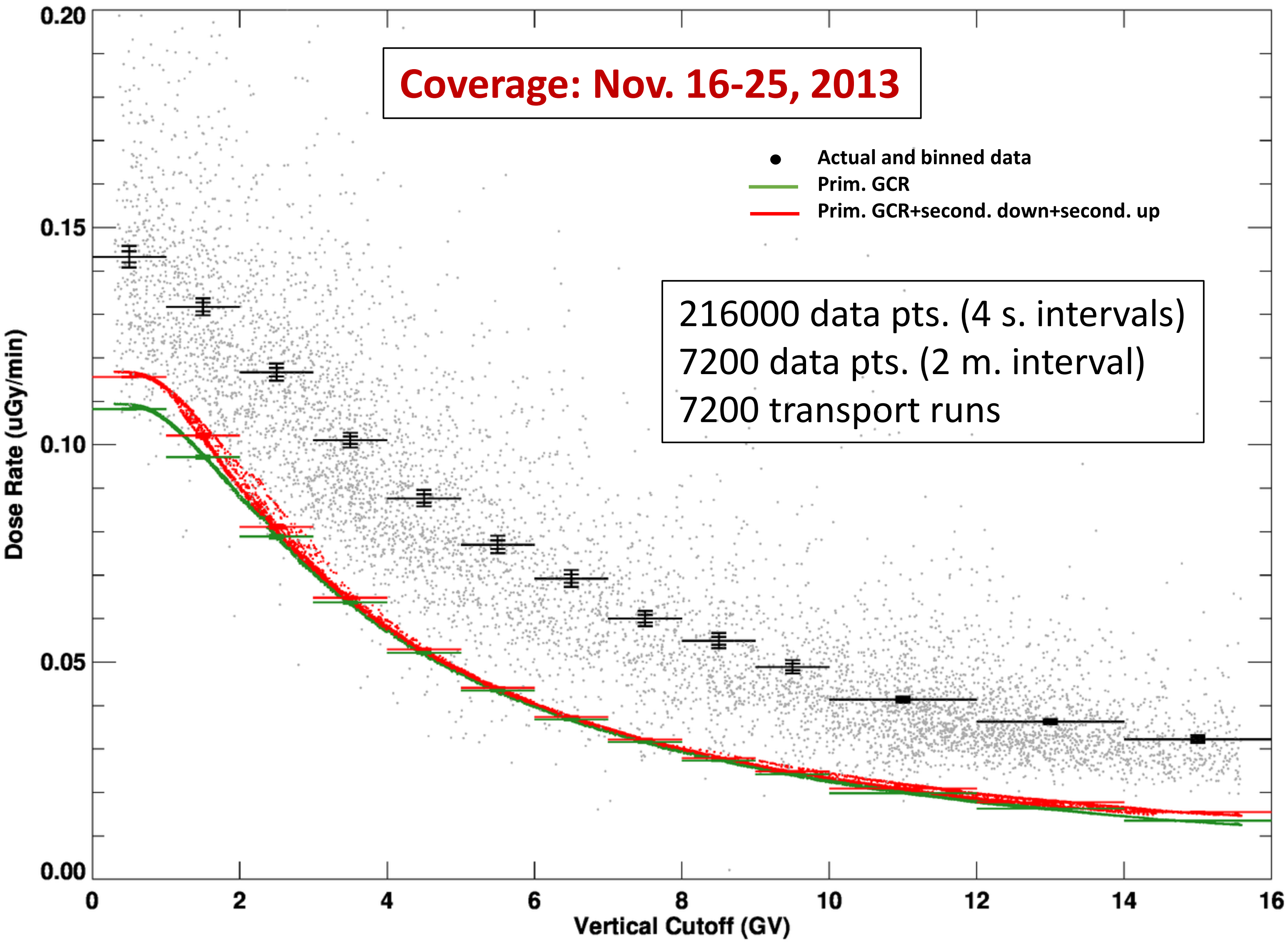
Validation Underestimation in Dose Rate at US-Lab REM - II

Coverage: Nov. 16, 2013

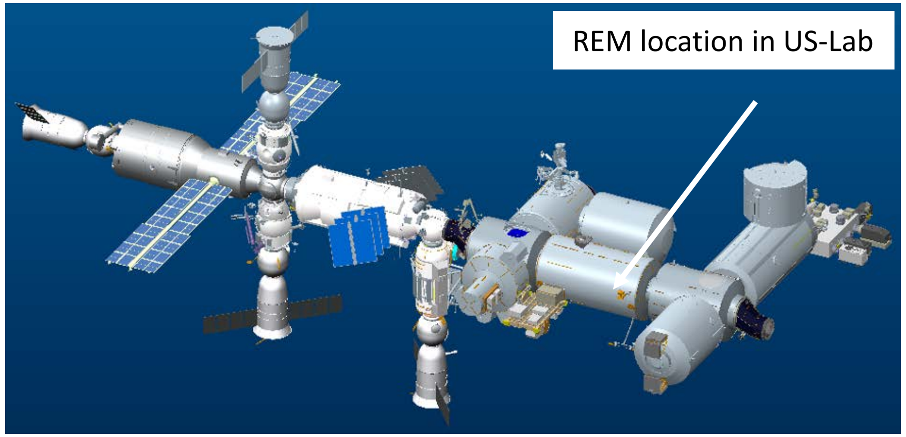


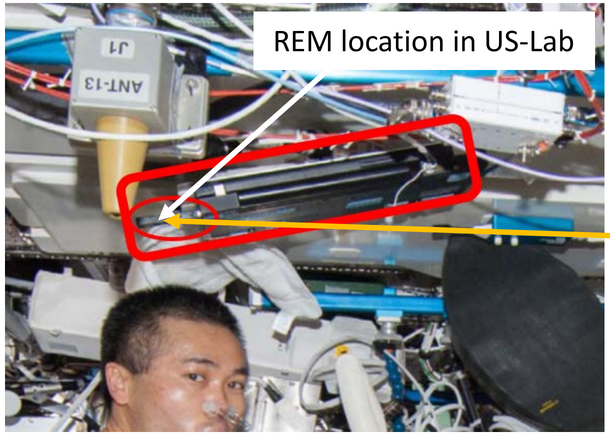
21600 data pts. (4 s. intervals)
720 data pts. (2 m. interval)
720 transport runs

Validation Underestimation in Dose Rate at US-Lab REM - III

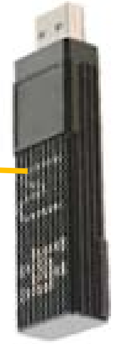


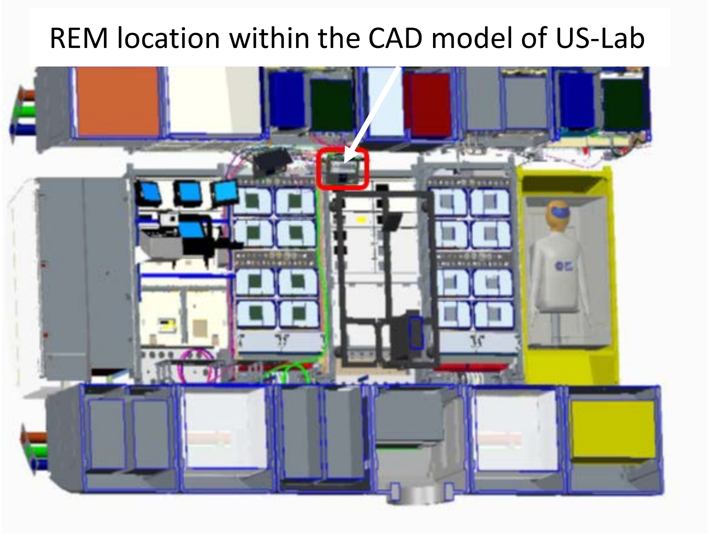
ISS US-Lab REM Target Point Details

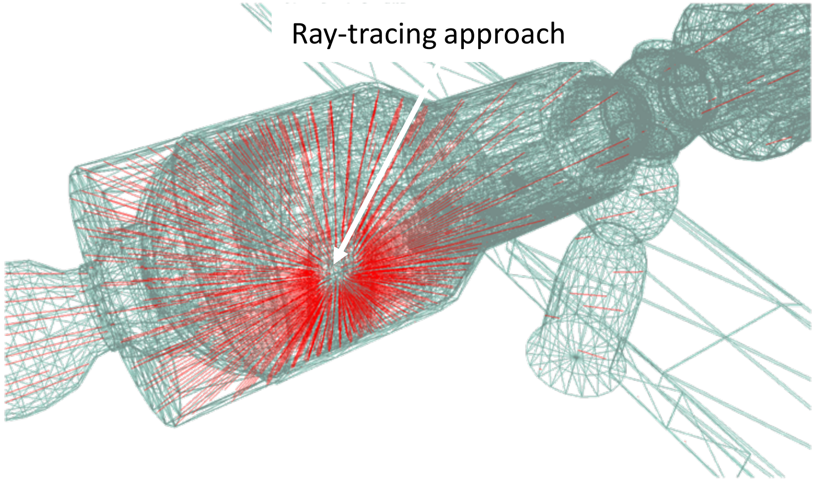


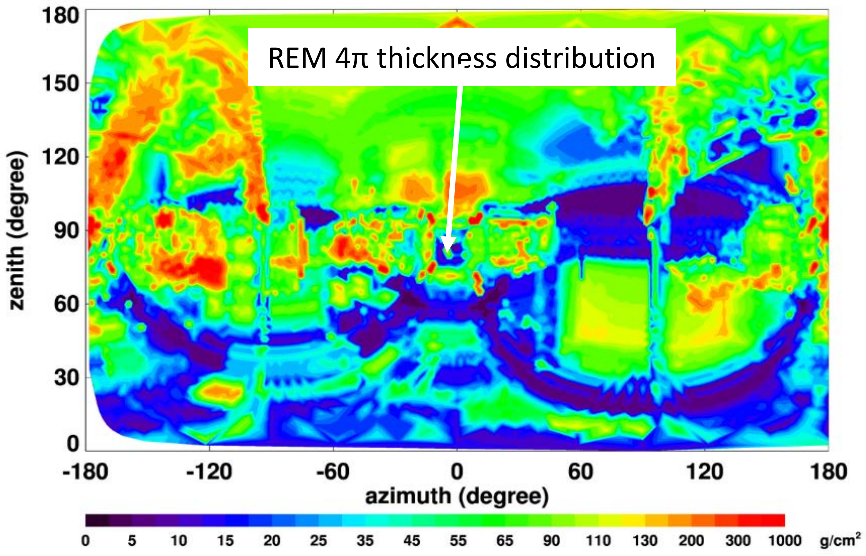


REM detector with USB interface



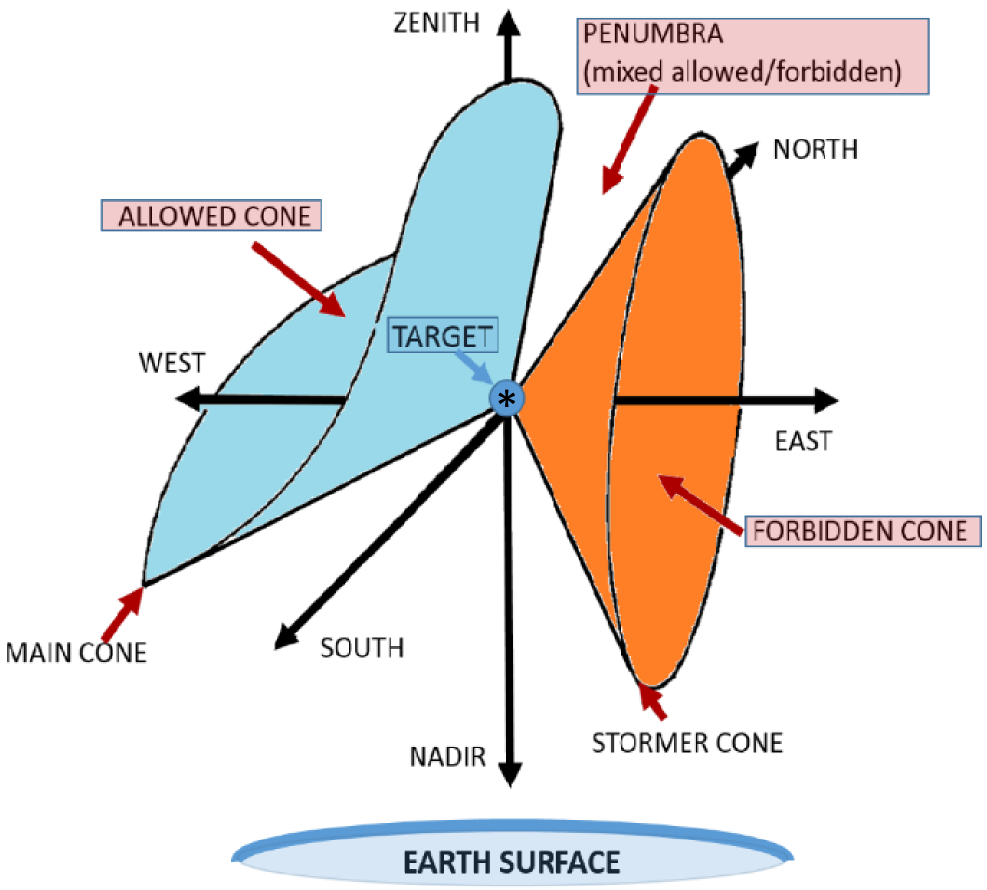




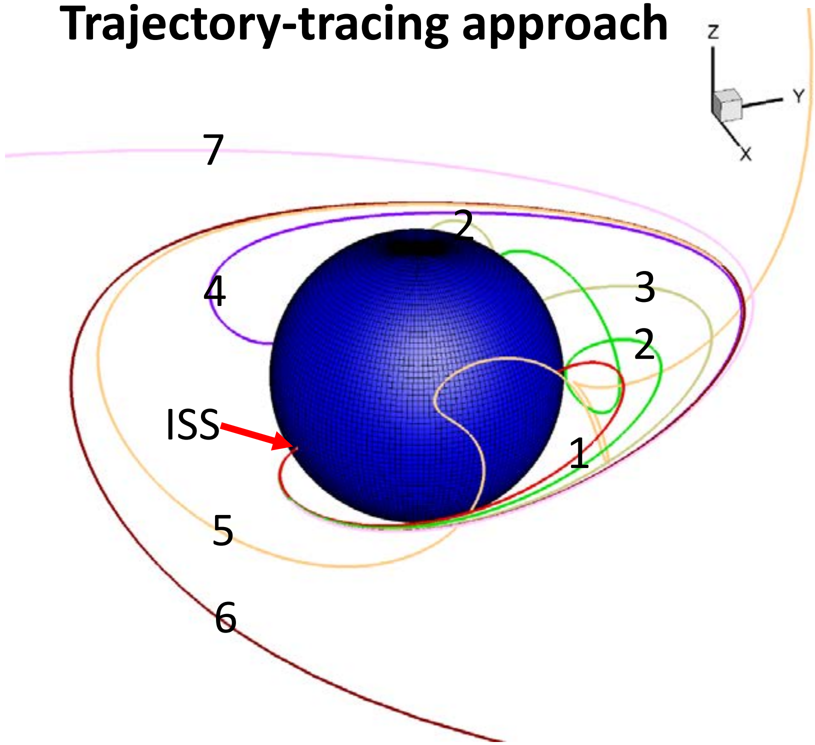


Cutoff Computation Using Stormer and Trajectory-Tracing Approaches

Stormer approach



Trajectory-tracing approach



For a specific ISS trajectory point at 400 km., build a **rigidity grid** in the range 1-50 GV for proton

Altitude 400 km.

Traj1: 11.0000 GV (0, 0, -10.5, 288.4), looking at Zenith
Traj2: 11.1250 GV (0, 0, -10.5, 288.4), looking at Zenith
Traj3: 11.2500 GV (0, 0, -10.5, 288.4), looking at Zenith
Traj4: 11.3070 GV (0, 0, -10.5, 288.4), looking at Zenith
Traj5: 11.3102 GV (0, 0, -10.5, 288.4), looking at Zenith
Traj6: 11.3107 GV (0, 0, -10.5, 288.4), looking at Zenith
Traj7: 11.3250 GV (0, 0, -10.5, 288.4), looking at Zenith

*Smart, D.F., et al., "Geomagnetic cutoffs: a review for space dosimetry applications", *Adv. in Space Res.*, 1994, v. 14, pp. 787-796.

Background on Stormer Approach

$$R(r,\varepsilon,\beta,\psi)=\frac{M}{r^2}\frac{\cos^4\psi}{\left[1+\sqrt{1+\sin\varepsilon\sin\beta\cos^3\psi}\right]^2}$$

ε: Zenith angle

β: Azimuth angle measured clockwise from magnetic north

ψ: Magnetic latitude

r: Distance from effective central dipole

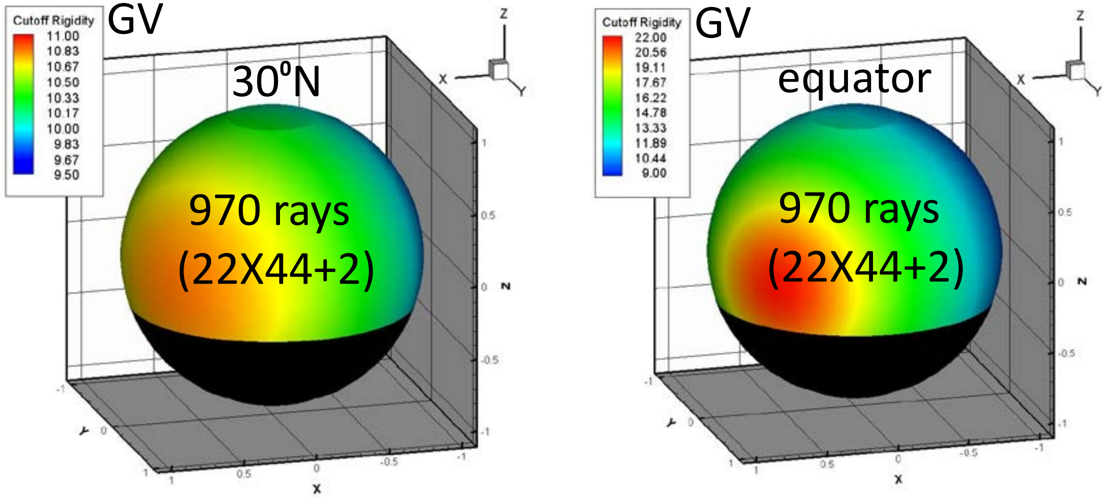
$$R_{east}(r_{Earth})=59.9\,(\mathrm{GV})\ \ (\varepsilon=270^\circ,\beta=90^\circ,\psi=0^\circ)$$

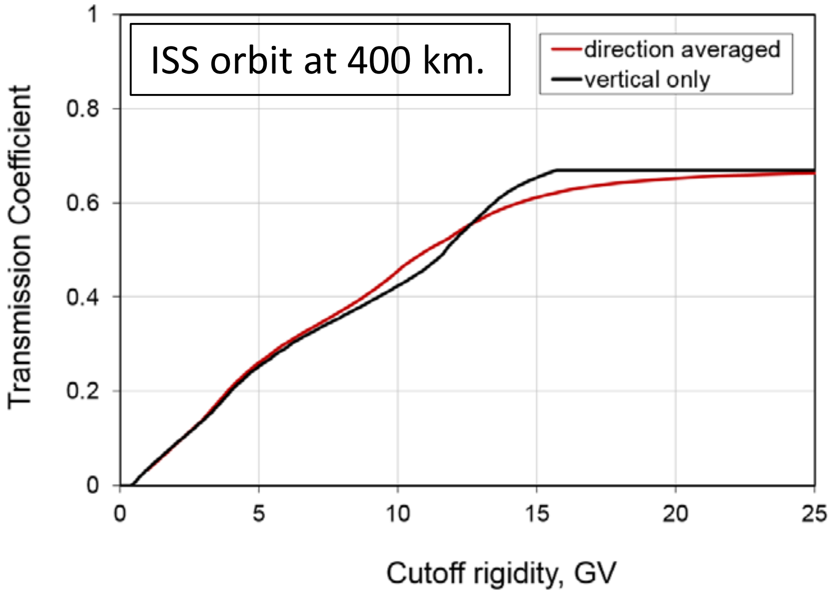
$$R_{west}(r_{Earth})=10.3\,(\mathrm{GV})\ \ (\varepsilon=270^\circ,\beta=270^\circ,\psi=0^\circ)$$

$$R_v(r,\psi)=\frac{M}{4r^2}\cos^4\psi\ \ (\varepsilon=180^\circ,\beta=0^\circ)$$

$$R_v(r_{Earth},\psi)=14.9\cos^4\psi\,(\varepsilon=180^\circ,\beta=0^\circ)$$

Directional cutoff rigidity in GV for ISS orbit at 400 km.





Background on Trajectory-Tracing Approach

$$\frac{d^2\mathbf{R}_{r,\theta,\phi}}{dt^2} = (e/mc)\frac{d\mathbf{R}_{r,\theta,\phi}}{dt} \times \mathbf{B}_{r,\theta,\phi}$$

Vector form

In (r,θ,ϕ) system, particle velocity terms are

$$\frac{dr}{dt} = v_r, \quad \frac{d\theta}{dt} = \frac{v_\theta}{r}, \quad \frac{d\phi}{dt} = \frac{v_\phi}{r\sin\theta}$$

Resulting in three **coupled** simultaneous ODEs with six unknowns $(r,\theta,\phi,\frac{dr}{dt},\frac{d\theta}{dt},\frac{d\phi}{dt})$

$$\frac{dv_r}{dt} = \frac{e}{mc}\left(v_\theta B_\phi - v_\phi B_\theta\right) + \frac{v_\theta^2}{r} + \frac{v_\phi^2}{r}$$

$$\frac{dv_\theta}{dt} = \frac{e}{mc}\left(v_\phi B_r - v_r B_\phi\right) - \frac{v_r v_\theta}{r} + \frac{v_\phi^2}{r\tan\theta}$$

$$\frac{dv_\phi}{dt} = \frac{e}{mc}\left(v_r B_\theta - v_\theta B_r\right) - \frac{v_r v_\phi}{r} - \frac{v_\theta v_\phi}{r\tan\theta}$$

Scalar form

$$B_r = -\frac{\partial U(r,\theta,\phi)}{\partial r}, \quad B_\theta = -\frac{1}{r}\frac{\partial U(r,\theta,\phi)}{\partial \theta}, \quad B_\phi = -\frac{1}{r\sin\theta}\frac{\partial U(r,\theta,\phi)}{\partial \phi}$$

$$U(r,\theta,\phi) = a\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(g_n^m \cos m\phi + h_n^m \sin m\phi)P_n^m(\cos\theta)\left(\frac{a}{r}\right)^{n+1}$$

g_n^m, h_n^m are **Gauss** coefficient of magnetic field, and $P_n^m(\cos\theta)$ is Schmidt-normalized associated **Legendre** polynomial

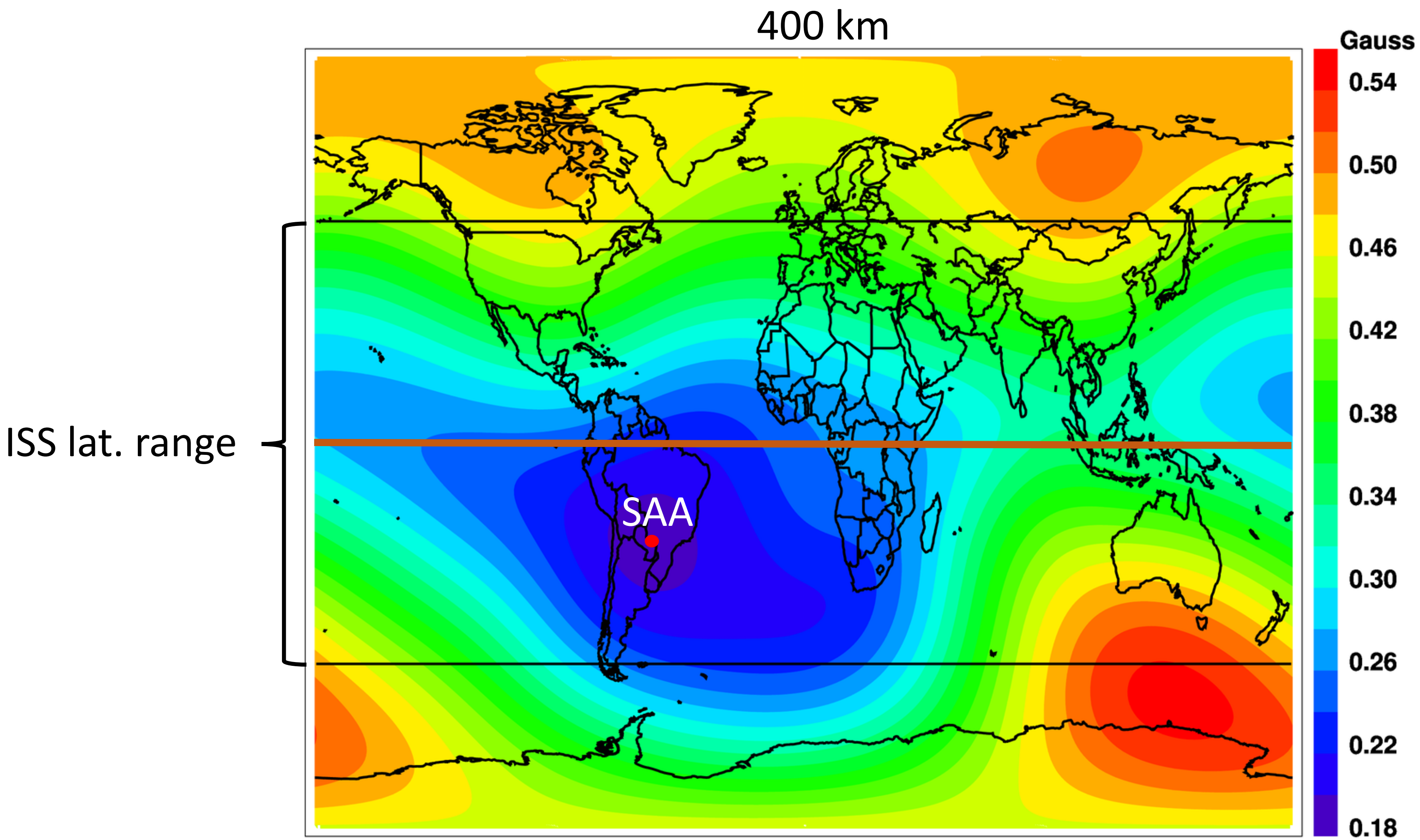
$$g_n^m(t) = g_n^m(T_0) + (t - T_0)\frac{\partial g_n^m}{\partial t}$$

$$h_n^m(t) = h_n^m(T_0) + (t - T_0)\frac{\partial h_n^m}{\partial t}$$

Recipe for the computation of trajectory-tracing cutoffs:

1. Compute (B_r, B_ϑ, B_ϕ) components from potential $U(r,\theta,\phi)$
2. Put the (B_r, B_ϑ, B_ϕ) into ODE and solve for velocity (v_r, v_ϑ, v_ϕ) and position (R_r, R_ϑ, R_ϕ)
3. Velocity is related to momentum, and momentum is related to rigidity
4. You now have rigidity and position of an ion

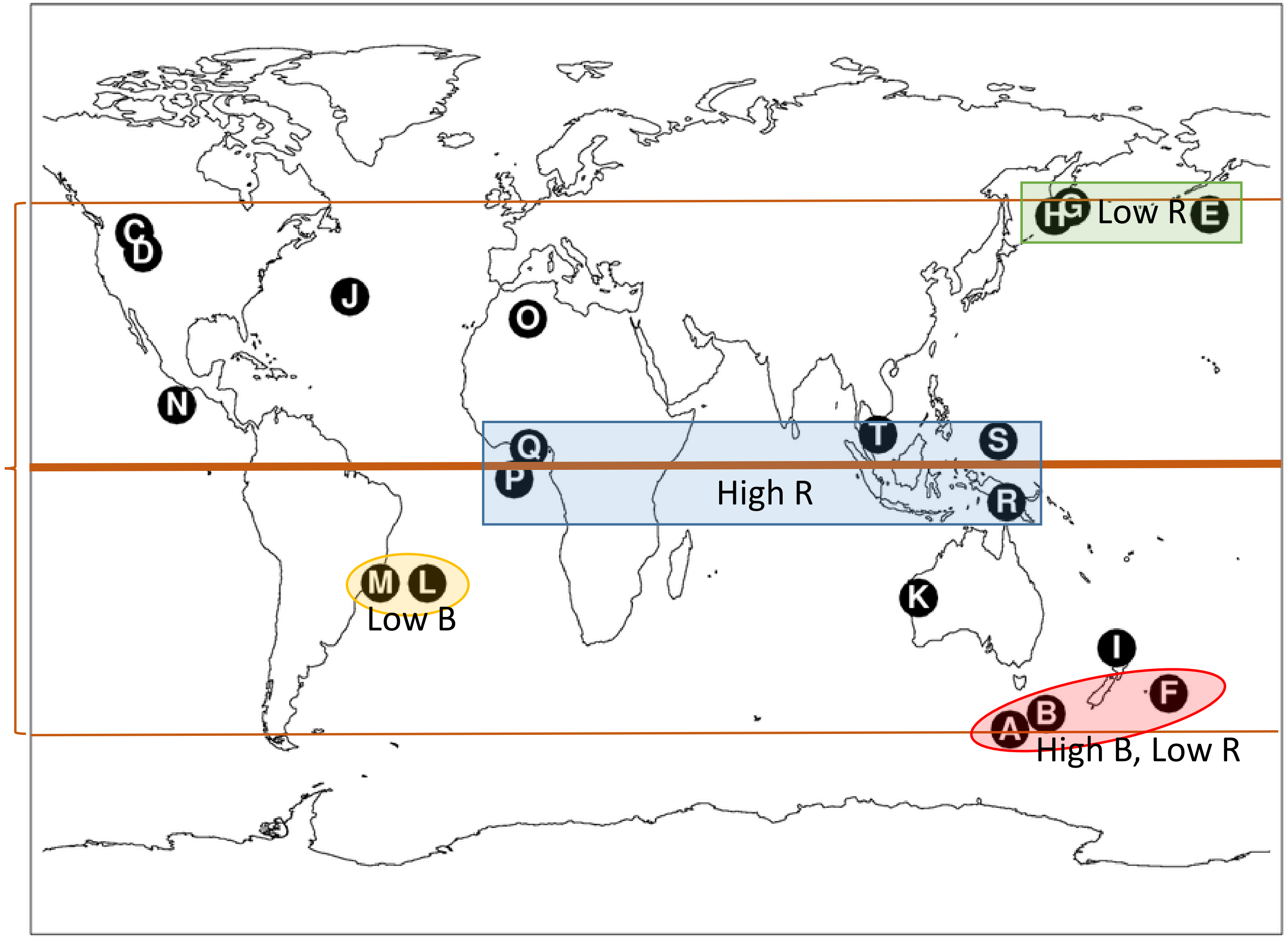
Global B Field for IGRF 2015



Operational ISS Trajectory at 20 Geographical Locations

File Edit Format View Help

	lat	lon	alt	year	GV
0	-51.3670	143.7630	434.3850	2013.890	0.4896
1	-48.2800	154.3890	433.0340	2013.890	1.0007
2	46.7320	246.8310	420.4920	2013.890	1.4774
3	42.7030	249.3540	419.4200	2013.890	2.0077
4	50.2740	202.3090	421.1150	2013.890	2.5091
5	-44.2850	190.3460	431.3360	2013.890	2.9983
6	51.8010	161.6700	421.4900	2013.890	3.5282
7	49.9730	156.7690	420.9840	2013.890	4.0076
8	-35.2780	175.0590	427.4680	2013.890	4.5044
9	34.0400	310.0560	417.7150	2013.890	5.4637
10	-25.4140	116.9370	423.4610	2013.890	6.4767
11	-22.5350	332.7520	422.4420	2013.890	7.5385
12	-22.4880	319.0040	422.9580	2013.890	8.5133
13	12.6630	259.2990	415.6560	2013.890	9.4866
14	29.6720	2.2940	417.0130	2013.890	10.4933
15	-1.9810	358.3040	417.4960	2013.890	11.5185
16	4.1540	2.6310	416.6810	2013.890	12.5099
17	-6.4990	142.7840	418.2160	2013.890	13.5009
18	5.6000	140.3700	415.9120	2013.890	14.4941
19	6.8240	105.0410	416.3210	2013.890	15.4970



For all 20 points, compute dose rate at US-Lab REM detector location according to the following recipe:

1. Compute cutoffs using both Stormer and full trajectory-tracing
2. Use computed cutoffs to get transmission coefficients
3. Use computed transmission coefficients to attenuate GCR ions
4. Use attenuated GCR ions to perform particle transport
5. Use US-Lab REM location ray-traced thickness file to get flux
6. Use flux to get dose_eq rate at REM location

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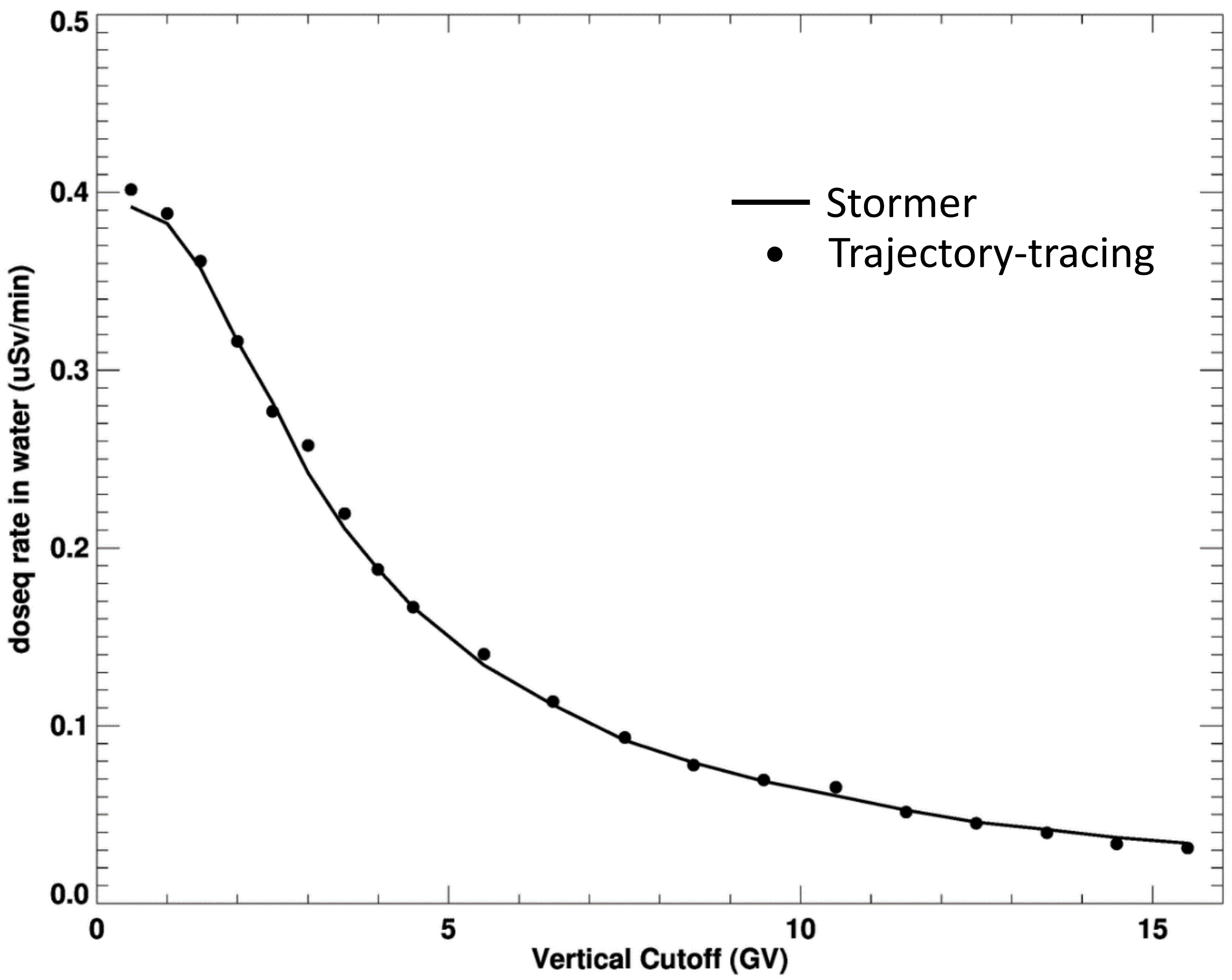
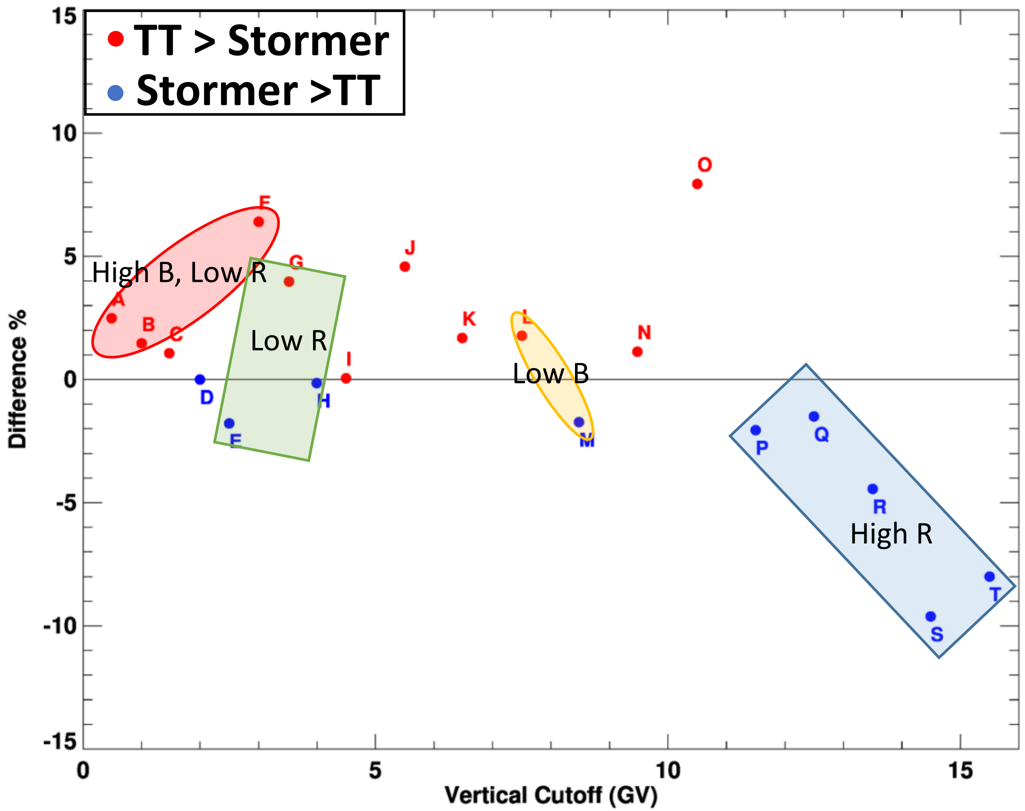
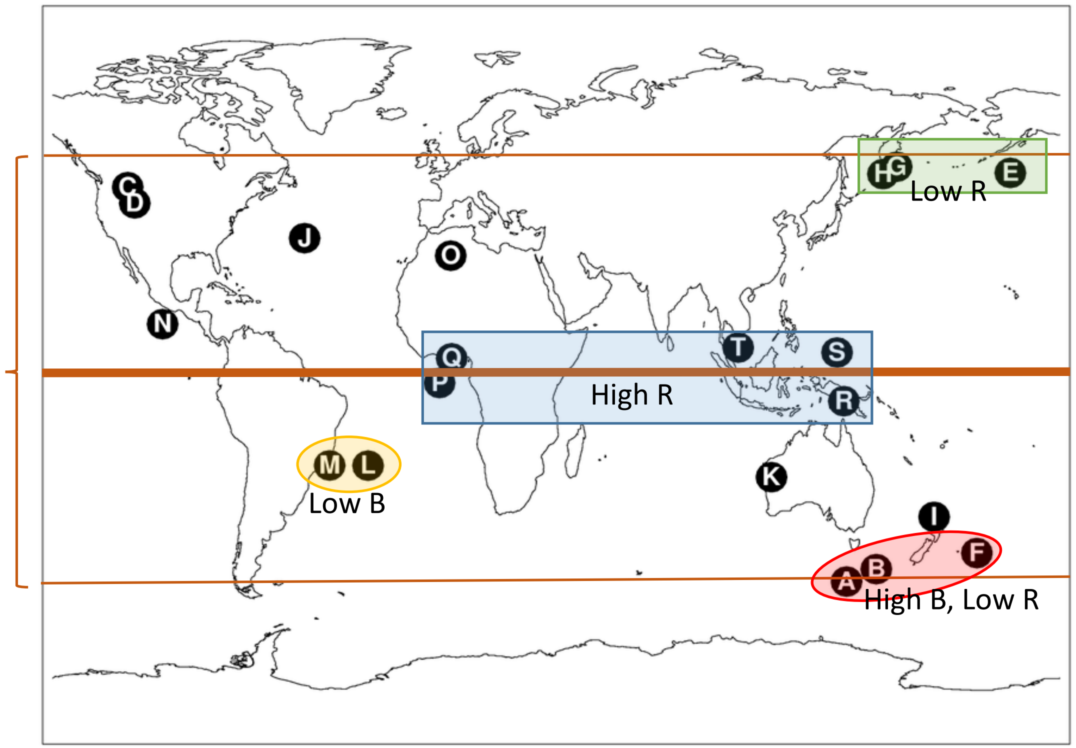
7. Figure out how long it takes to do steps 1-6 using Stormer and full trajectory-tracing methods

Dose_eq Rate Comparison at ISS US-Lab REM Location



Dose_eq Rate % Relative Difference Comparison at ISS US-Lab REM Location





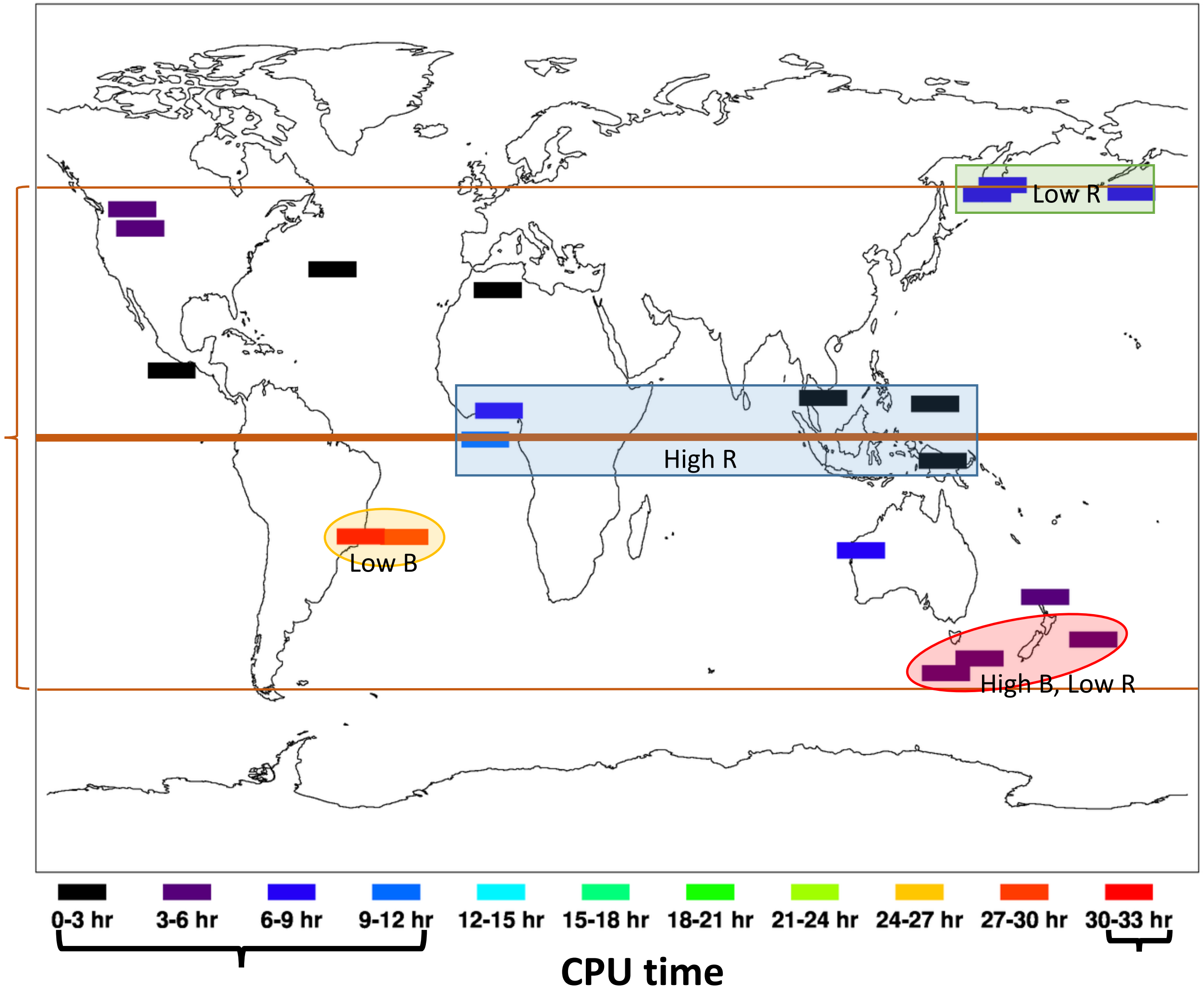
Conclusions:

B field { 1. At low B (i.e. SAA), there is little difference between Stormer and trajectory-tracing in dose_eq
2. At high B (i.e. south of Australia) Stormer is smaller than trajectory-tracing in dose_eq

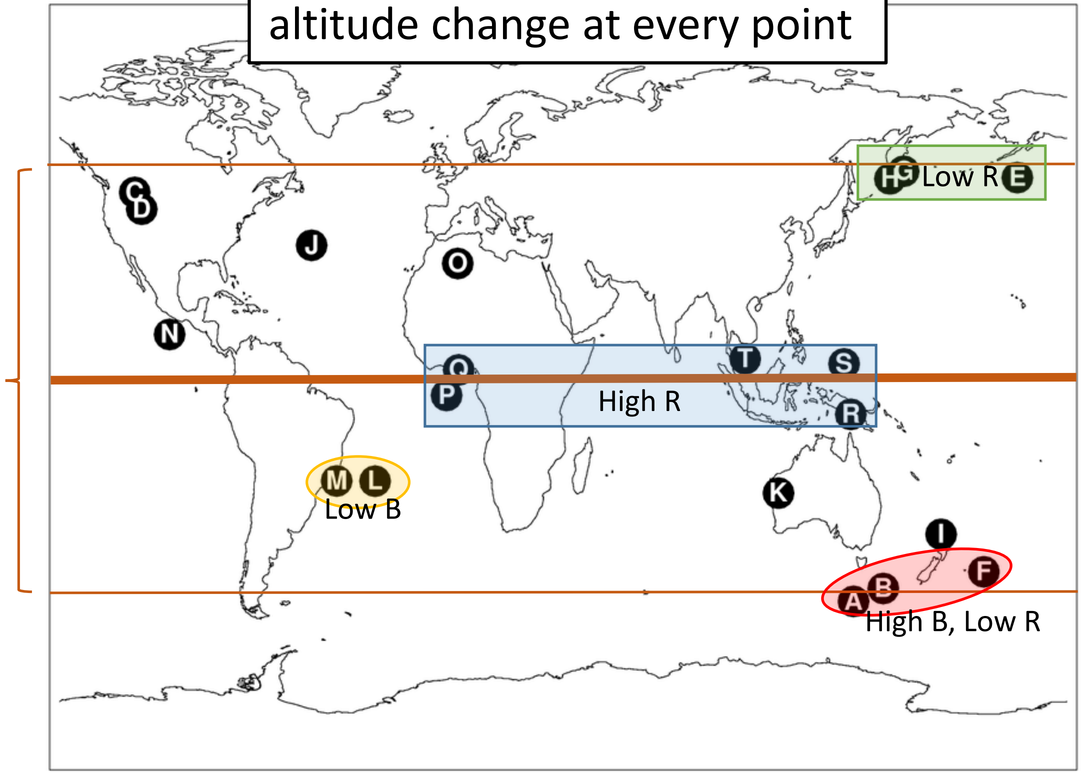
R field { 3. At low R (i.e. east of Japan) there is small difference between Stormer and trajectory-tracing in dose_eq
4. At high R (i.e. equator) Stormer is larger than trajectory-tracing in dose_eq

5. At mid-latitude (i.e. points C, D, J, N, O, I, K) Stormer is smaller than trajectory-tracing in dose_eq

Dose Rate Calculation CPU Time at 20 Locations for ISS US-Lab REM

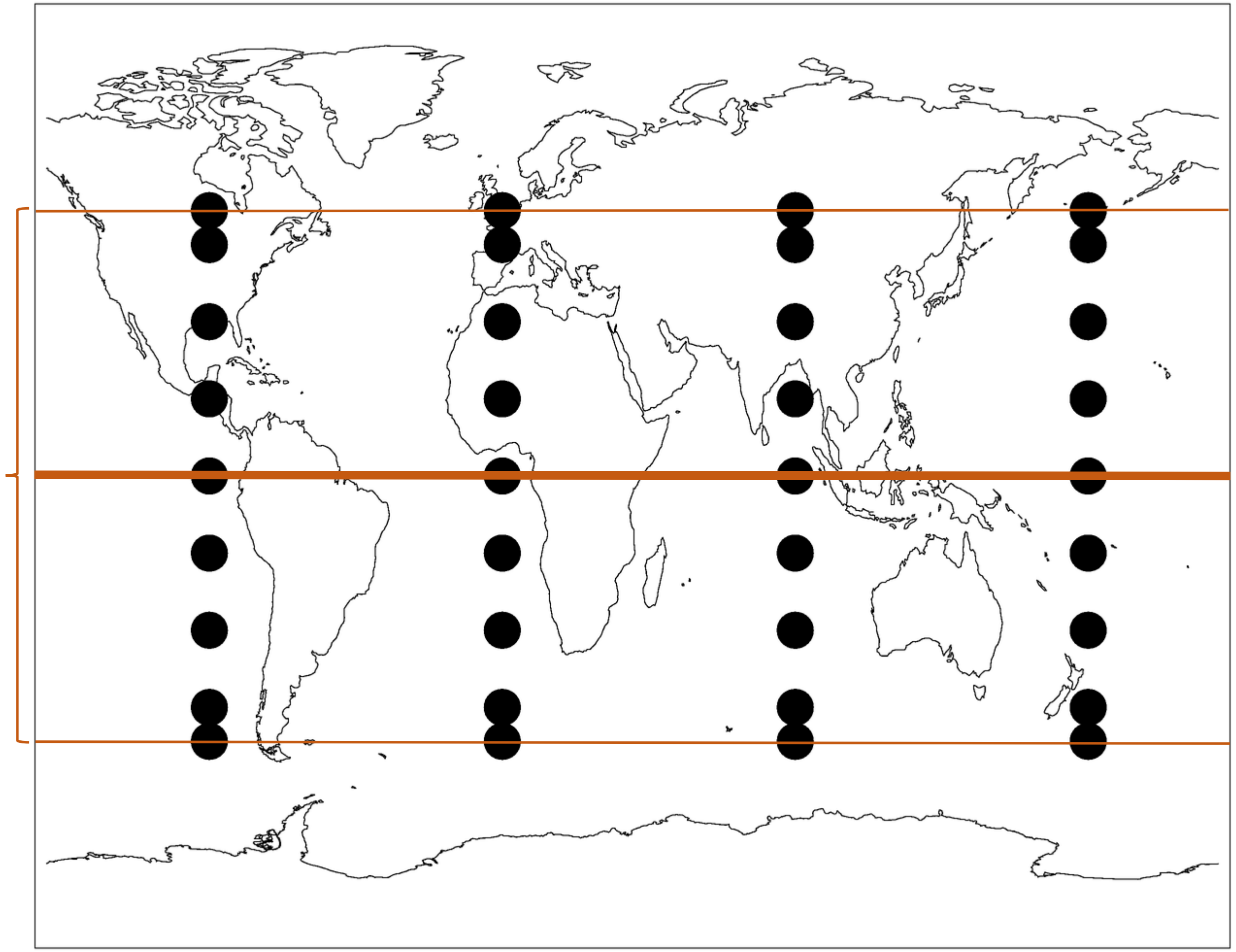


Note: Longitude, latitude and altitude change at every point



Hardware: Linux-workstation using Intel fortran with single-threading

Simplified ISS Trajectory Points at 36 Geographical Locations



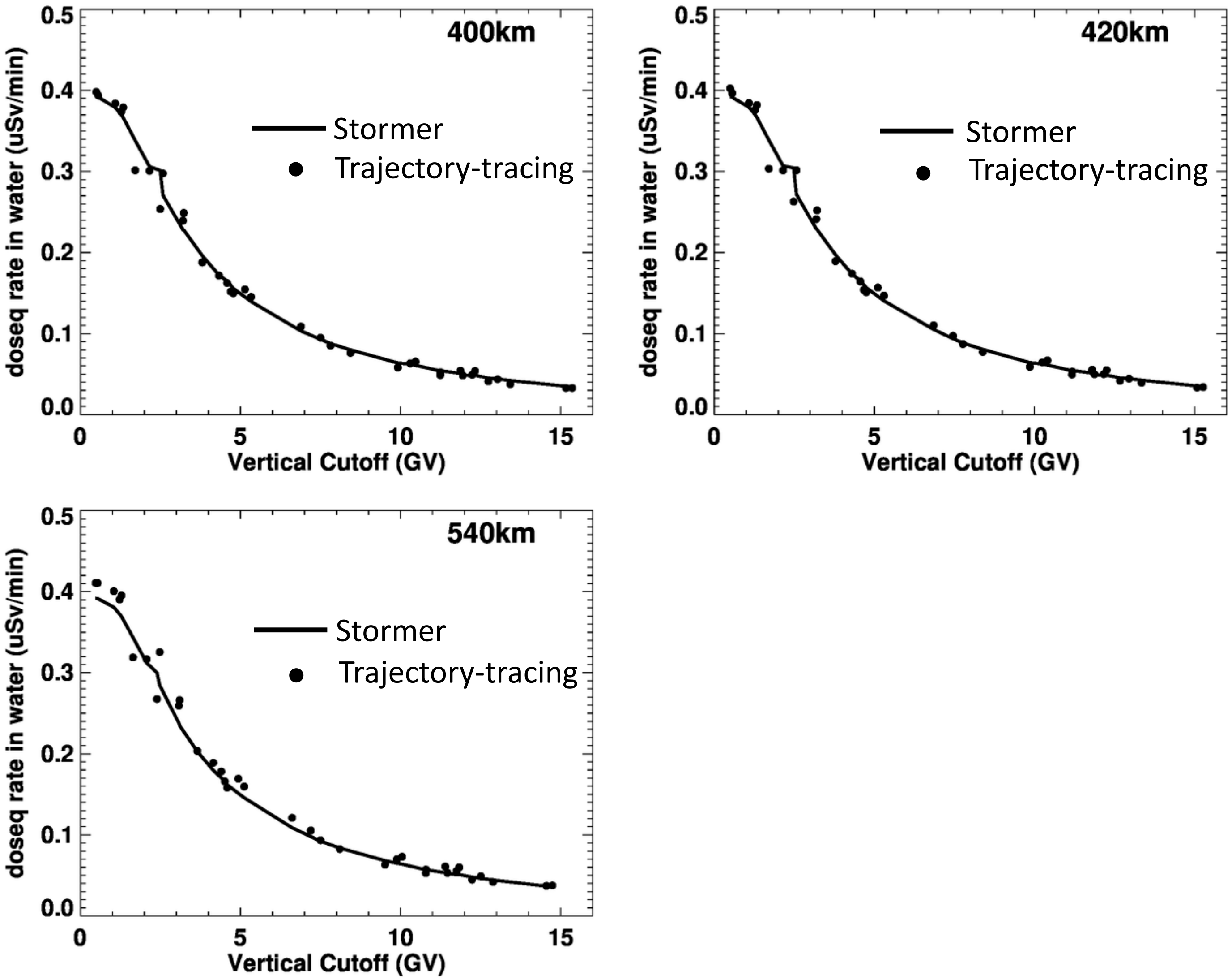
- For all 36 points, use altitudes of **400, 420** and **540 km.** to compute dose rate at US-Lab REM

- **For now** compute dose rate in a **static field** only (**IGRF**)

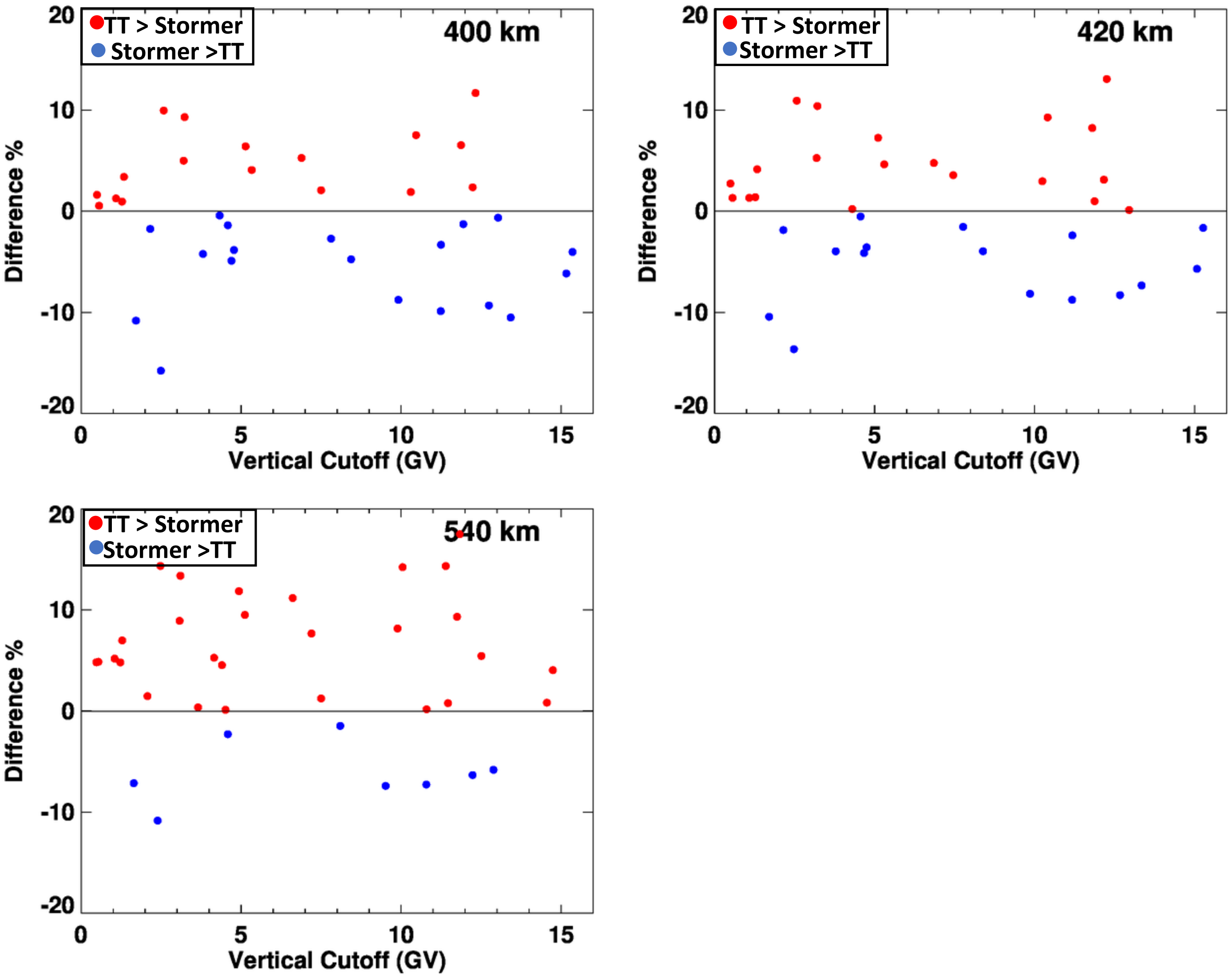
- **In future** compute dose rate in a **dynamic field**
  (**IGRF** + **dynamic model** [FM[&], OPq[#], Ts[*],…])
  The dynamic models account for the presence of **magnetospheric current systems** such as **Chapman-Ferraro current**, **magnetotail current sheet**, **ring** and **partial ring current**, and **Birkeland** currents

[&]FM: Fairfield-Mead (1975)
[#]OPq: Olson-Pfitzer (1977)
[*]Ts: N.A. Tsyganenko (1987-2003)

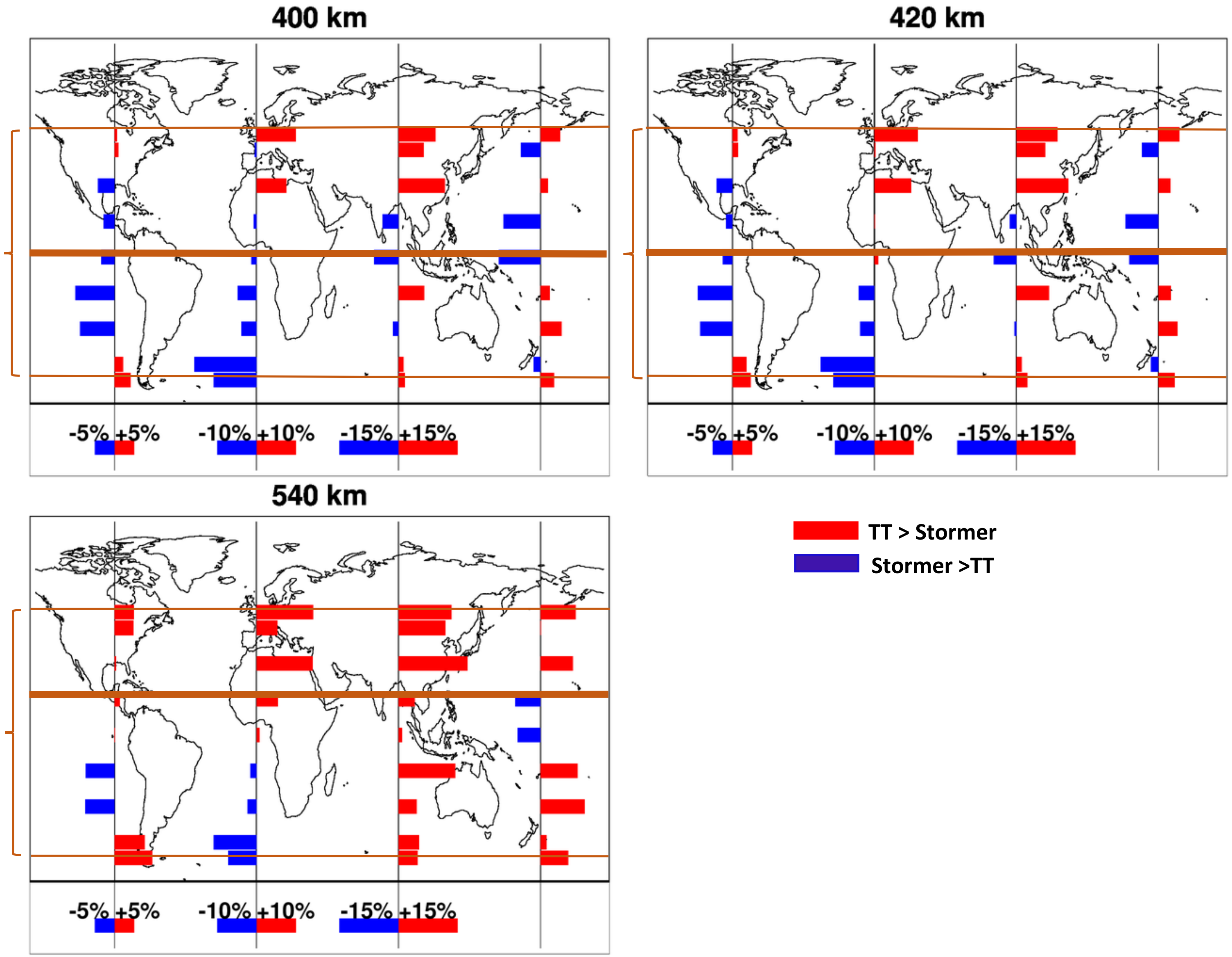
Dose_eq Rate Comparison at ISS US-Lab REM Location



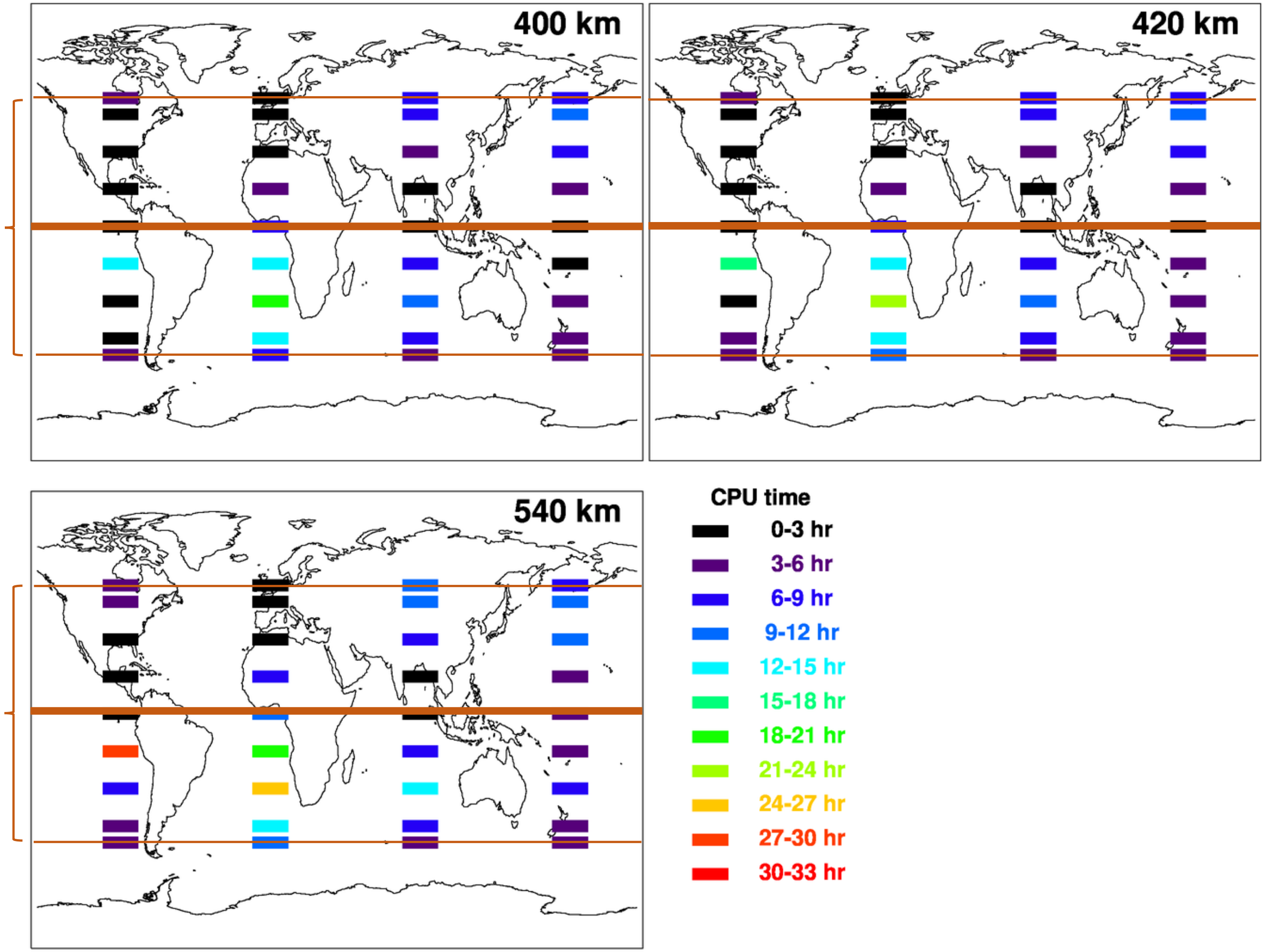
Dose_eq Rate % Relative Difference Comparison at ISS US-Lab REM Location



Dose_eq Rate % Relative Difference Comparison at ISS US-Lab REM Location



Dose_eq Calculation CPU Time at ISS US-Lab REM Location



More Efficient (i.e. Hybrid) Cutoff Computation Using Trajectory-Tracing

Stormer approach

$$R(r,\varepsilon,\beta,\psi) = \frac{M}{r^2}\frac{\cos^4\psi}{\left[1+\sqrt{1+\sin\varepsilon\sin\beta\cos^3\psi}\right]^2}$$

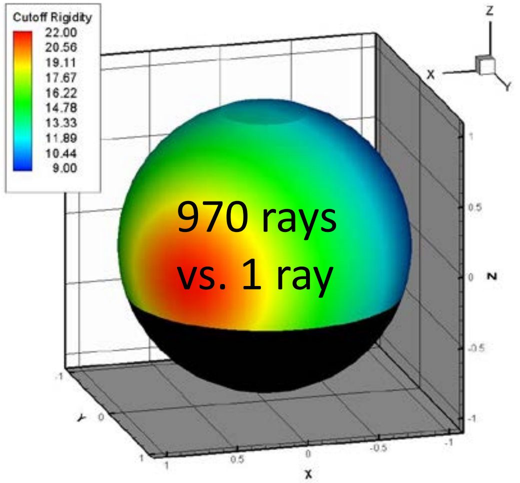
$$R_v(r,\psi) = \frac{M}{4r^2}\cos^4\psi \qquad (\varepsilon = 180^\circ, \beta = 0^\circ)$$

Trajectory-tracing approach

$R(r,\varepsilon,\beta,\psi) = \text{Numerical trajectory}$ **(full (970 rays))**

$R_v(r,\psi) = \text{Numerical trajectory}$ **(vertical (1 ray))**

$$R(r,\varepsilon,\beta,\psi) = \frac{4R_v(r,\psi)}{\left[1+\sqrt{1+\sin\varepsilon\sin\beta\cos^3\psi}\right]^2}$$



Plan to Improve Trajectory-Tracing Numerical Procedure

I →

1. Optimization of rigidity grid (upper/lower boundaries and adaptive step size, etc.)
2. Re-evaluation of constraints on numerical solution (conservation of energy, maximum allowed increment in Lorentz force per solution step, etc.)
3. Trade studies of accuracy versus computation efficiency in the use of pre-computed B-field components
4. Alternative numerical ODE solvers

II →

5. Once steps 1 - 4 are optimized, then configure the code(s) for multi-node, multi-core, hyper-threaded hardware (i.e. run on a cluster)

III →

6. Coupling of IGRF (i.e. static field) and geomagnetic current system
7. To speed up the code, provide the **option** of choosing **vertical cutoff** (hybrid) or **directional cutoff**

Summary

- Reviewed the current status of dosimetric **underestimation** in the GCR validation work
- Provided a brief description of **Stormer** and **trajectory-tracing** cutoff calculation methodologies
- Presented a dose_eq comparison at the US-Lab REM location using **operational** and **simplified** ISS trajectories
    - Showed that for ISS altitudes, over or under prediction of Stormer compared to trajectory-tracing is location dependent and **differences tend to be <15%**
    - Showed that trajectory-tracing calculations require considerable computational resources
    - Showed the advantage of trajectory-tracing over Stormer at **higher altitudes**
- Presented a hybrid approach for rapid trajectory-tracing analysis
- Presented steps which should be investigated to speed up the trajectory-tracing calculations