$\bar{B} \to D^{(\star)}\ell\bar{\nu}$ decays at non-zero recoil on the lattice

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On behalf of the Fermilab/MILC collaborations, with:

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Introduction: The $|V_{cb}|$ CKM matrix element

- Long-standing tension between inclusive and exclusive $|V_{cb}|$ determinations

$$|V_{cb}|_{inc,\mathrm{kinetic}} = (42.2 \pm 0.8) \times 10^{-3}, \qquad |V_{cb}|_{B\to D^*} = (39.0 \pm 0.8) \times 10^{-3}$$

$$|V_{cb}|_{inc,1\mathrm{S}} = (42.0 \pm 0.5) \times 10^{-3}, \qquad |V_{cb}|_{B\to D} = (40.5 \pm 1.0) \times 10^{-3}$$

$$|V_{cb}|_{\mathrm{UT}} = (42.0 \pm 0.7) \times 10^{-3}$$

Bigi and Gambino, arXiv:1606.08030
Amhis et al., arXiv:1612.07233
Bauer et al., hep-ph/0408002

The z fit approach

Fit experimental and lattice data in terms of z expansion. Determine and compare slopes (and curvature) in z. If consistent, do combined fit to determine $|V_{cb}|$.

- Model-independent
- Can quantify improvement between lattice and experimental measurements
- Systematically improvable - as data gets more precise, can add more terms in z
- Minimizes error in $|V_{cb}|$ by using all lattice and experimental data in a single fit
- Caprini, Lellouch, Neubert (CLN) uses dispersion relations and HQET at $1/m_Q$. Could be underestimated systematics associated with HQET expansion.

Fits to $B \to D^*\ell\nu$ data

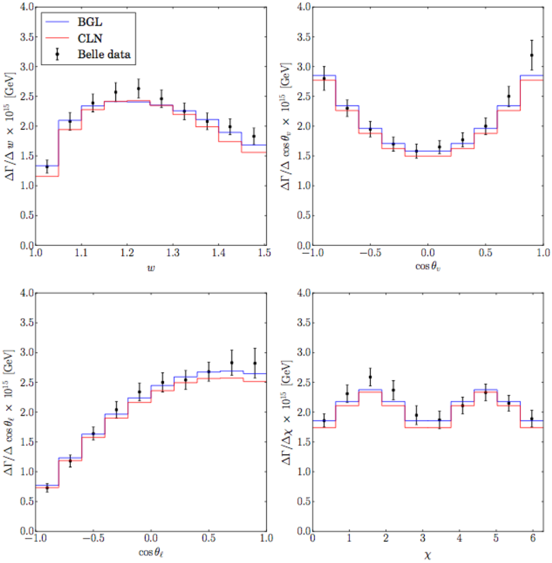

FIG. 2: The Belle data (black points) compared to the results of our fit using the BGL parameterization (blue), and the results of the Belle analysis using the CLN parameterization (red).

Results of fit

Both Grinstein and Kobach (1703.0817)
and Bigi, Gambino, and Schacht (1703.06124)
independently find

$$|V_{cb}|_{B\to D^*,\mathrm{BGL}} = (41.9^{+2.0}_{-1.9}) \times 10^{-3}$$

compared to

$$|V_{cb}|_{B\to D^*,\mathrm{CLN}} = (37.4 \pm 1.3) \times 10^{-3},$$

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Lattice QCD Calculations

Calculate expectation values on an ensemble of gauge fields $[\mathcal{U}]$ with an exponential weight

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D\mathcal{U} D\psi_{\text{sea}} D\overline{\psi}_{\text{sea}} e^{-S_{\text{QCD}}[\mathcal{U},\psi_{\text{sea}},\overline{\psi}_{\text{sea}}]} \mathcal{O}[\mathcal{U},\psi_{\text{val}},\overline{\psi}_{\text{val}}], \quad (1)$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D\mathcal{U} \prod_{f=1}^{n_f} \det(\not{D} + m_f) e^{-S_{\text{QCD}}[\mathcal{U}]} \mathcal{O}[\mathcal{U},\psi_{\text{val}},\overline{\psi}_{\text{val}}], \quad (2)$$

The action is discretized, so that derivatives become finite differences. Integral is still too large to do directly ($N_s^3 \times N_t \times 4 \times N_f \times N_c$), so we use Monte Carlo importance sampling.

Types of Errors

Because QCD with physical quark masses is a nonlinear multiscale problem ($\Lambda_{QCD} \approx 100 - 200$ MeV, $m_{u,d} \approx 2 - 6$ MeV, $m_b \approx 4.3$ GeV), it is very expensive to simulate at the physical quark masses.

1.) Statistics and fitting

2.) Tuning lattice spacing, a, and quark masses

3.) Matching lattice gauge theory to continuum QCD

4.) Extrapolation to continuum

5.) Chiral extrapolation to physical up, down quark masses

6.) Quenching. Uncontrolled!

7.) Neglecting QED and/or isospin effects

Some types of lattice fermions

Lots of ways to solve the lattice fermion doubling problem:

- Wilson Fermions: Introduces an additional "irrelevant" term to the action. Improved variants, i.e. "clover" used in practice. Fairly cheap.
- Staggered Fermions: Identifies some of the extra fermions with the different spin components of a single fermion. There are still 4 extra species of fermions, and these are eliminated by taking the 4th root of the determinant. Some open theoretical issues with this, though theoretical progress has been made on this front. Very cheap.
- Domain Wall Fermions: Solves chiral symmetry problem by using Wilson type quarks in five dimensions. More costly because of the extra dimension. There is a small chiral symmetry breaking due to the finiteness of the fifth dimension. Expensive.
- Overlap Fermions: Exact lattice chiral symmetry. Very expensive.

Effective field theories

Effective field theories can be used to quantify systematic errors due to extrapolations in light quark masses (chiral perturbation theory), or the treatment of heavy-quarks like charm and bottom (Heavy-Quark Effective Theory).

Symanzik effective theory can be used to quantify systematic errors due to finite-lattice spacing.

Chiral perturbation theory

Chiral perturbation theory (ChPT) is an expansion about small quark masses and momenta.

At each order new terms must be introduced to cancel the renormalization scale dependence. These terms are not determined within ChPT.

When combined with lattice calculations, these constants can be determined.

It is possible to account for lattice artifacts in the ChPT by introducing the appropriate symmetry breaking terms in the chiral lagrangian (finite lattice spacing) or restricting the Feynman integrals to finite volume.

Heavy quarks on the lattice

The lattice cut-off is smaller than the heavy quark masses for realistic lattices. A solution: heavy quark effective theory(HQET)

Fermilab Method:

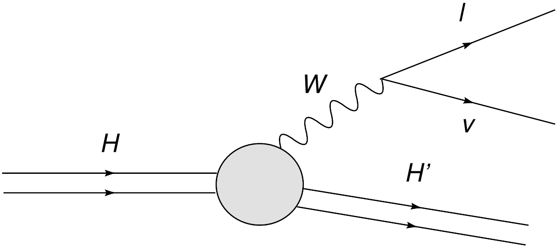
Continuum QCD $\rightarrow$ Lattice gauge theory
(using HQET)

- Requires tuning parameters of the lattice action. Can be systematically improved by adding higher dimensional operators to the action.
- The currents and 4-quark operators must also be matched to continuum QCD. Typically this is done using lattice perturbation theory. FNAL/MILC use a "mostly nonperturbative" method where diagonal flavor currents are calculated nonperturbatively, and the ratio of the off-diagonal matching factor and the diagonal matching factors is close to 1 and computed using lattice perturbation theory.

Lattice calculation

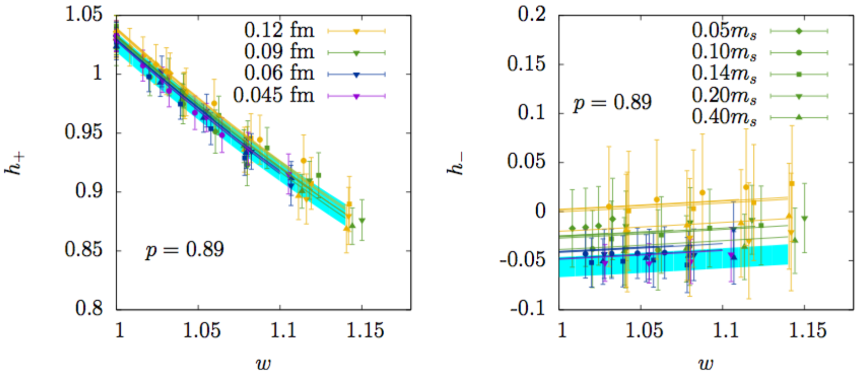
- Done on MILC lattices with improved staggered (asqtad) sea quarks
- Heavy quarks are treated using the Fermilab action (and heavy quark effective theory)
- Light valence quarks are also asqtad staggered
- Many MILC lattice ensembles exist with a wide range of lattice spacings and light quark masses.

Heavy-light semileptonic decays

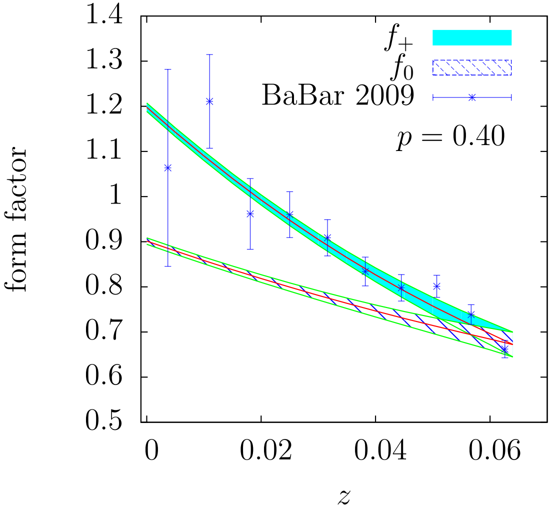


Vertex proportional to $|V_{qq'}|$. In order to extract it, a nonperturbative determination of the form factors is needed.

Chiral/Continuum Extrapolation for $B \to D\ell\nu$



Combined z-fit to $B \to D\ell\nu$ form factors



z-fit to lattice (FNAL/MILC, 1503.07237) plus experiment

$B \to D\ell\nu$

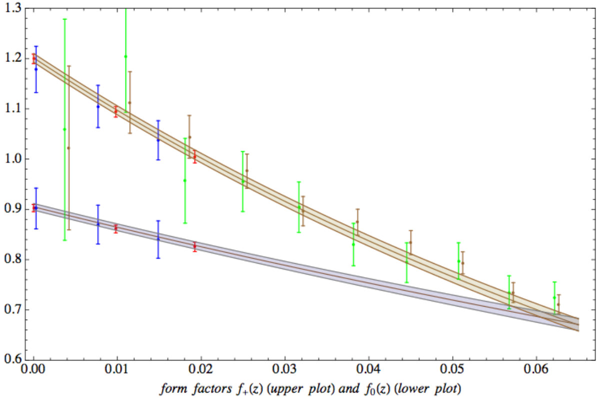


Figure 1: Form factors in the $N = 4$ fit with data points. FNAL/MILC synthetic data are shown in red, HPQCD in blue, Belle data in brown, BaBar in green.

Bigi and Gambino, arXiv:1606.08030 $\qquad |V_{cb}| = (40.5 \pm 1.0) \times 10^{-3}$

Introduction: The weak decay $\bar{B} \to D^{\star}\ell\bar{\nu}$

- Form factors

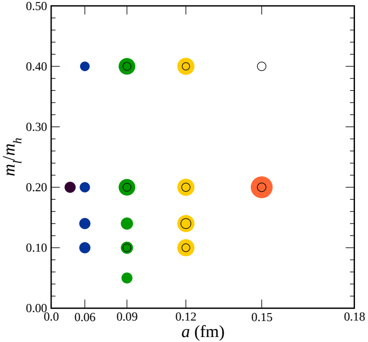
$$\frac{\langle D^{\star}(p_{D^{\star}},\epsilon^{\nu})|\,\mathcal{V}^{\mu}\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^{\star}}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\ \ \rho\sigma}v_B^{\rho}v_{D^{\star}}^{\sigma}h_V(w)$$

$$\frac{\langle D^{\star}(p_{D^{\star}},\epsilon^{\nu})|\,\mathcal{A}^{\mu}\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^{\star}}}} = \frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\,(1+w)\,h_{A_1}(w) - v_B^{\nu}\left(v_B^{\mu}h_{A_2}(w) + v_{D^{\star}}^{\mu}h_{A_3}(w)\right)\right]$$

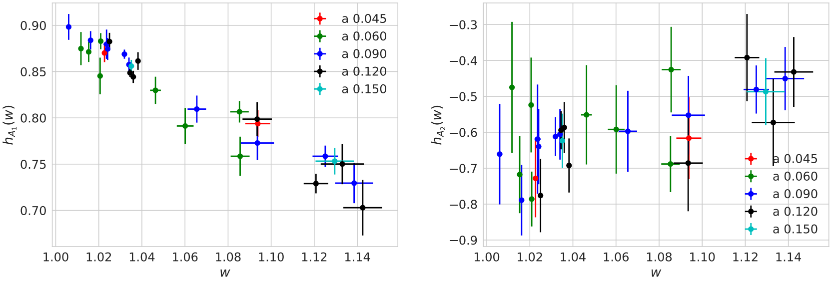
- Playing with the polarization/momentum of the $D^{\star}$ we can calculate the different form factors
- From the differential decay rate and the form factors (encoded in $\mathcal{F}(w)$) we can extract V_{cb}

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{4\pi^3} r^3(1-r^2)(w^2-1)^{\frac{1}{2}}\,|\eta_{EW}|^2\,|V_{cb}|^2\,\chi(w)\,|\mathcal{F}(w)|^2$$

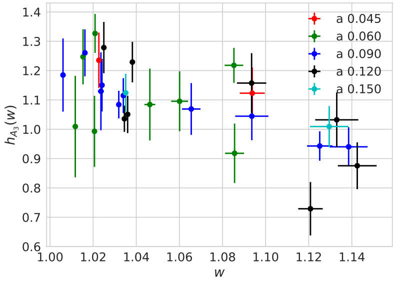
- Analysis of weak semileptonic decay $\bar{B} \to D^{\star} \ell \bar{\nu}$
- Using 15 $N_f = 2 + 1$ MILC ensembles of improved staggered (asqtad) quarks
- The heavy quarks are treated using the Fermilab action

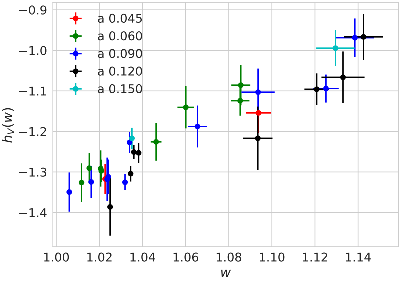


$B \to D^*\ell\nu$ form factors



$B \to D^*\ell\nu$ form factors





Conclusions

- Most likely the exclusive/inclusive conflict for $|V_{cb}|$ is resolved.
- New lattice calculations and data from Belle II should improve the precision of $|V_{cb}|$ from exclusive $B \to D^{(*)}\ell\nu$ decays
- Lattice calculations at non-zero recoil will be available soon for $B \to D^*\ell\nu$. This should decrease the error on $|V_{cb}|$. Will also provide useful cross check of $R(D^*)$.