



# Rare Kaon Decays and Minimal SUSY with Large $\tan \beta$

T. Blažek, Z. Kučerová, P. Maták, Z. Šinská

*Comenius University Bratislava*

*5 June 2017, Prague*



# Outline

Motivation for large  $\tan \beta$

Beyond SM Signals in

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

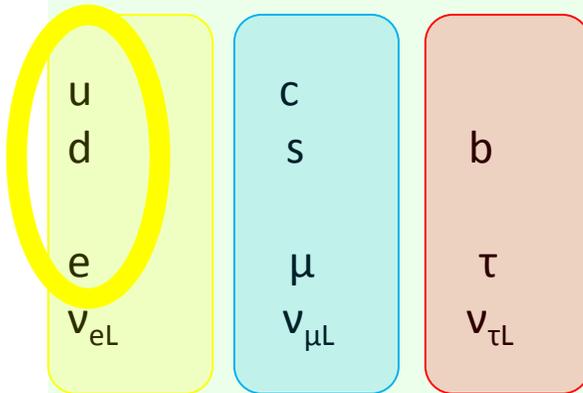
$$R_K$$

Conclusions

# Standard Model as Low-Energy Theory

$1 < E \ll 100 \text{ GeV}$

<b>symmetry</b>	$SU(3)_c$	$\times$	$U(1)_{em}$	<b>non-chiral</b> theory (the left and right transform the same way)
<b>forces</b>	<i>strong</i>		<i>electromagnetism</i>	
gluons	octet		singlet	
photon	singlet		singlet	
<b>particle states</b>	representations		electric charge $Q_{el}$	

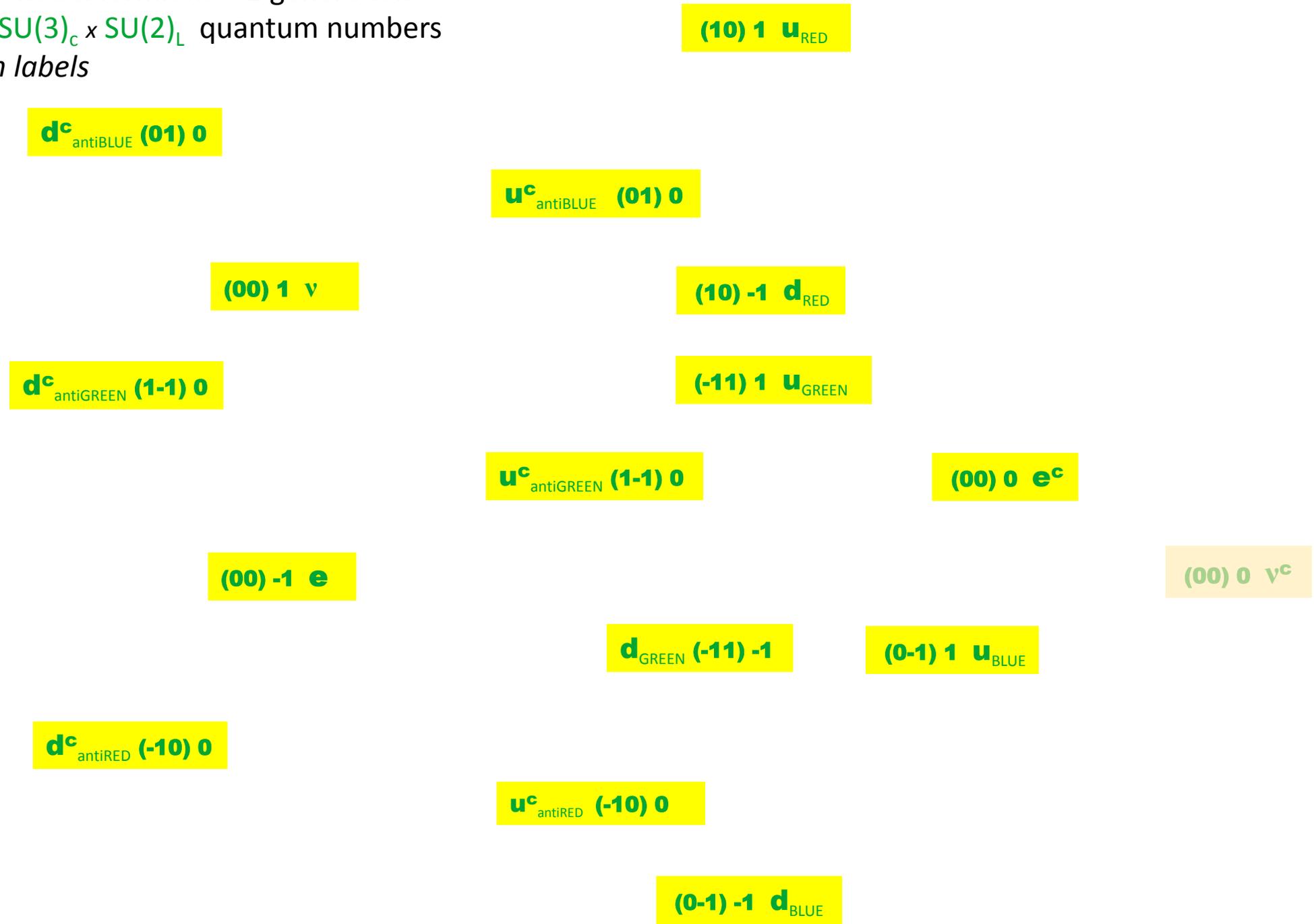


triplet	+ 2/3
triplet	- 1/3
singlet	- 1
singlet	0

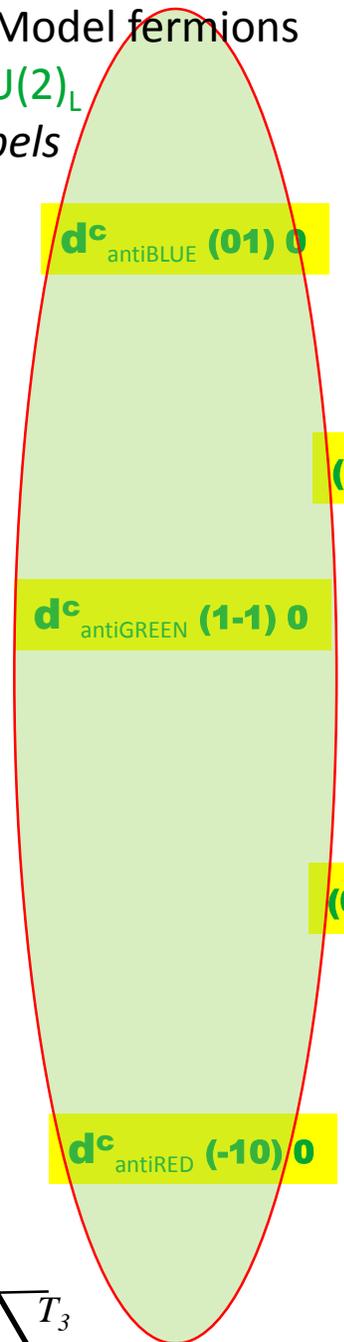
Non-Renormalizable Effective Operators effectively make up for disappearance of W, Z, H, t

At 1 GeV  $SU(3)_c$  becomes strongly coupled  $\rightarrow$  quark confinement  $\rightarrow$  baryons and mesons  
 Only protons, neutrons & electrons stay stable and form bound states  $\rightarrow$  atoms in our world

Standard Model fermions – 1 generation  
*showing*  $SU(3)_C \times SU(2)_L$  quantum numbers  
*as Dynkin labels*



Standard Model fermions  
 $SU(3)_c \times SU(2)_L$   
 Dynkin labels



$(00) 1 \nu$

$(00) -1 e$



$(10) 1 u_{\text{RED}}$

$(10) -1 d_{\text{RED}}$

$(-11) 1 u_{\text{GREEN}}$

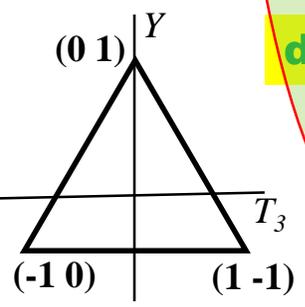
$(00) 0 e^c$

$d_{\text{GREEN}} (-11) -1$

$(0-1) 1 u_{\text{BLUE}}$

$(0-1) -1 d_{\text{BLUE}}$

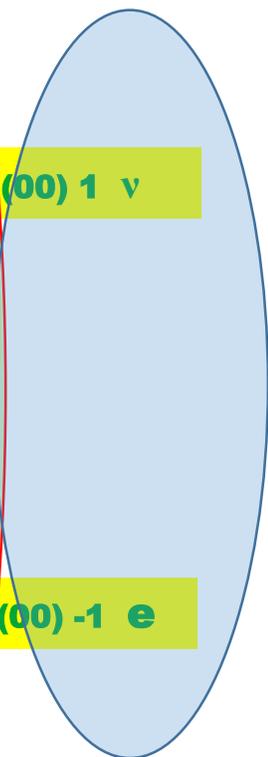
Reminder:  
 $SU(3)$   
 antitriplet



# Standard Model fermions

$SU(3)_c \times SU(2)_L$

*Dynkin labels*

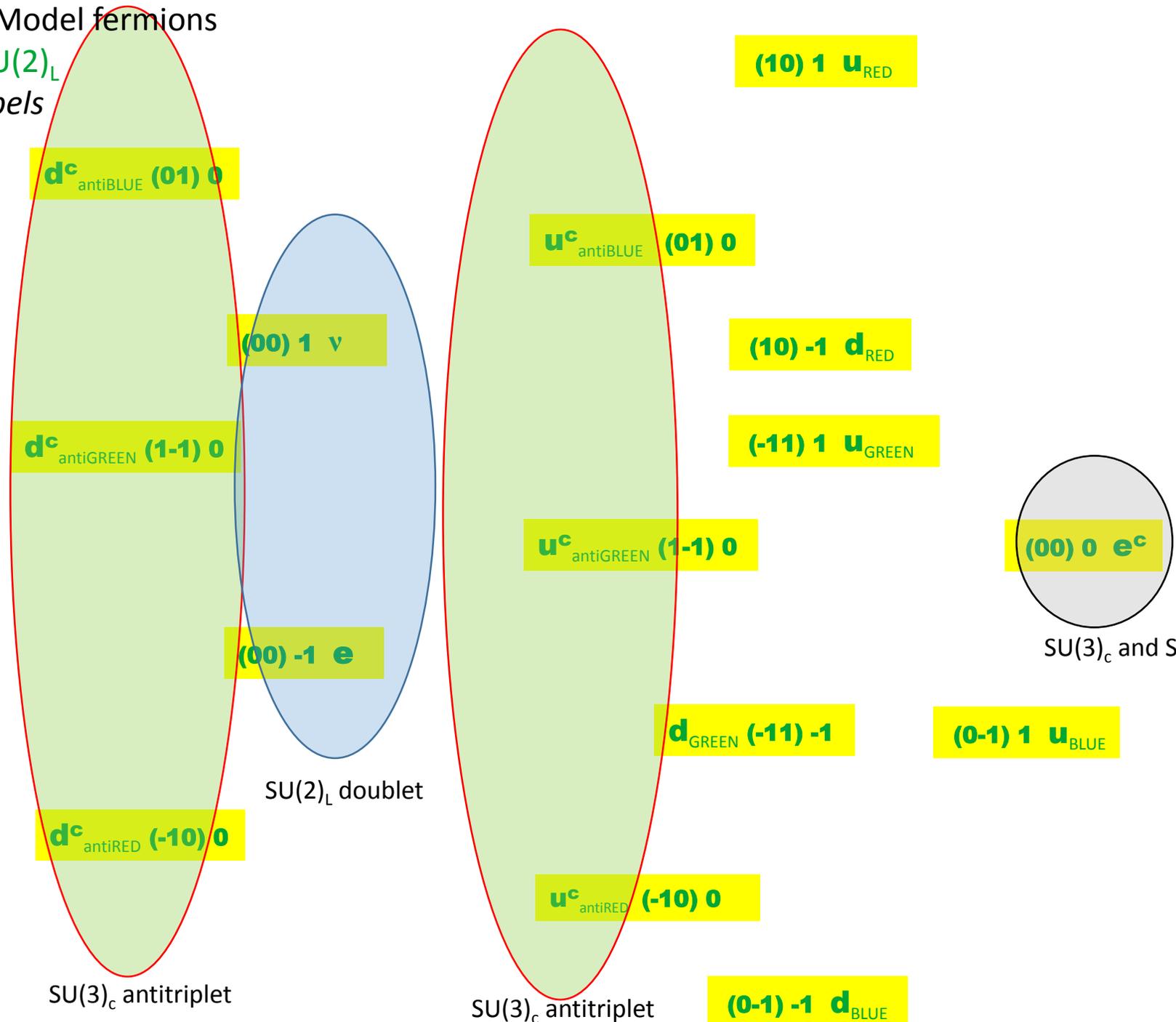


$SU(3)_c$  antitriplet

$SU(3)_c$  antitriplet

$SU(2)_L$  doublet

Standard Model fermions  
 $SU(3)_c \times SU(2)_L$   
 Dynkin labels



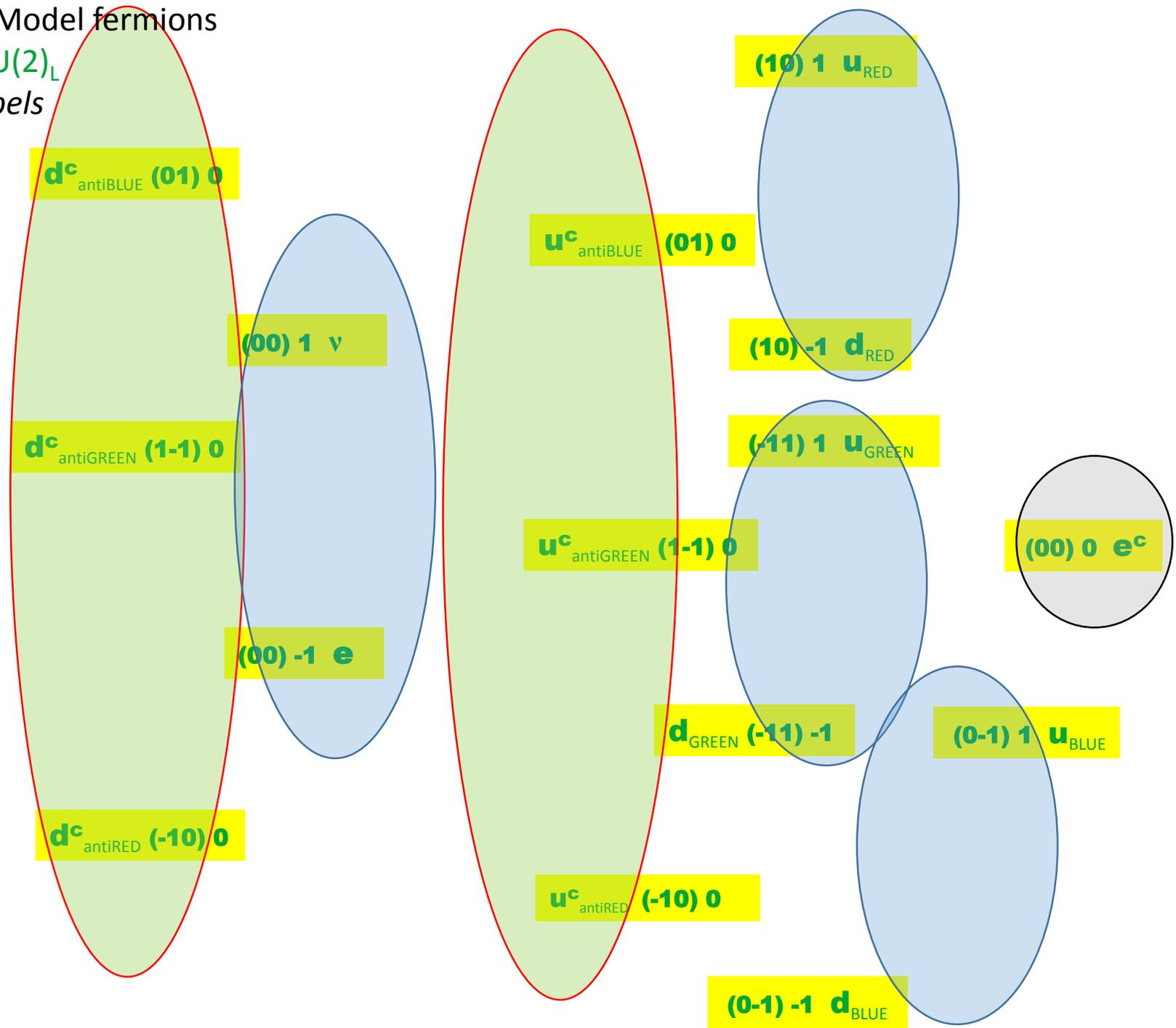
$SU(3)_c$  antitriplet

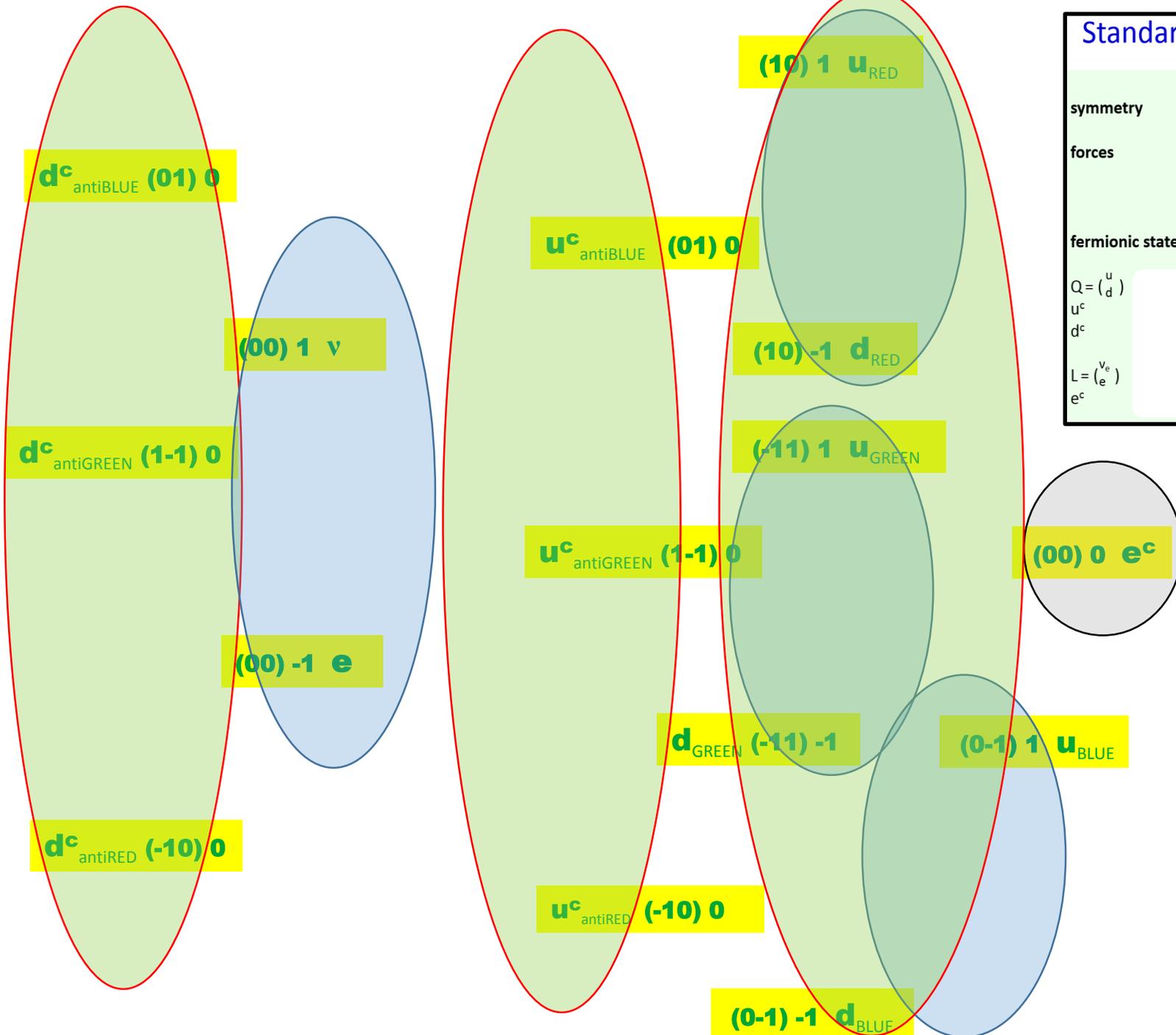
$SU(2)_L$  doublet

$SU(3)_c$  antitriplet

$SU(3)_c$  and  $SU(2)_L$  singlet

Standard Model fermions  
 $SU(3)_c \times SU(2)_L$   
 Dynkin labels





# Standard Model

$E \gg 100 \text{ GeV}$

<b>symmetry</b>	SU(3) <sub>c</sub> x SU(2) <sub>L</sub> x U(1) <sub>Y</sub>			chiral theory (the left and right transform different)
<b>forces</b>	strong gluons	electroweak W-bosons B-boson		
<b>fermionic states of one generation</b>	representations	hypercharge Y (Y=Q <sub>el</sub> -T <sub>3</sub> )		
Q = $\begin{pmatrix} u \\ d \end{pmatrix}$	triplet	doublet	+1/6	
u <sup>c</sup>	antitriplet	singlet	-2/3	
d <sup>c</sup>	antitriplet	singlet	+1/3	
L = $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	singlet	doublet	-1/2	
e <sup>c</sup>	singlet	singlet	+1	

$(10) 1 u_{\text{RED}}$

$d_{\text{antiBLUE}}^c (01) 0$

$u_{\text{antiBLUE}}^c (01) 0$

$(00) 1 v$

$(10) -1 d_{\text{RED}}$

$d_{\text{antiGREEN}}^c (1-1) 0$

$(-11) 1 u_{\text{GREEN}}$

$u_{\text{antiGREEN}}^c (1-1) 0$

$(00) 0 e^c$

$(00) -1 e$

$(00) 0 v^c$

$d_{\text{GREEN}} (-11) -1$

$(0-1) 1 u_{\text{BLUE}}$

$d_{\text{antiRED}}^c (-10) 0$

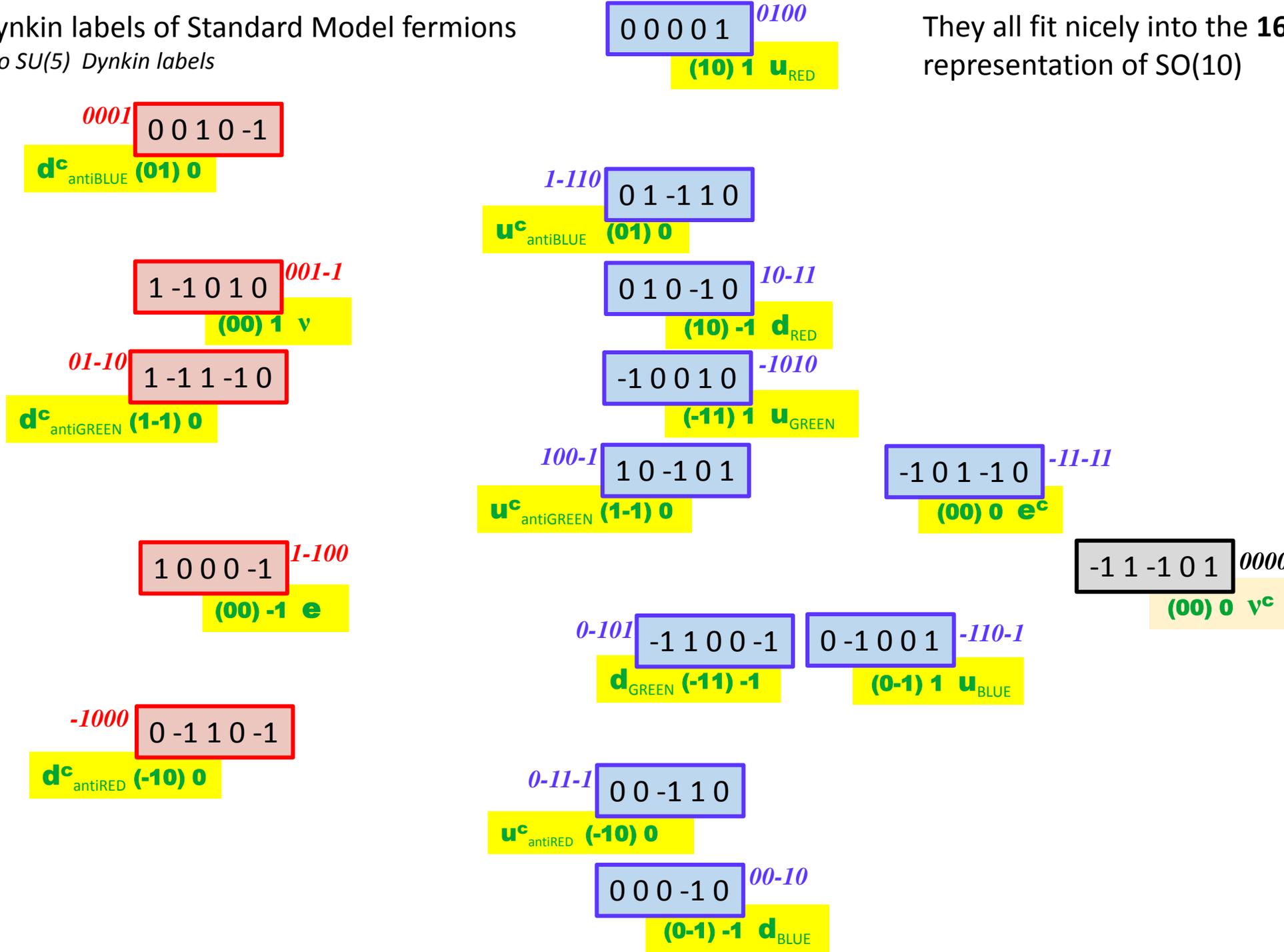
$u_{\text{antiRED}}^c (-10) 0$

$(0-1) -1 d_{\text{BLUE}}$

# SO(10) Dynkin labels of Standard Model fermions

showing also SU(5) Dynkin labels

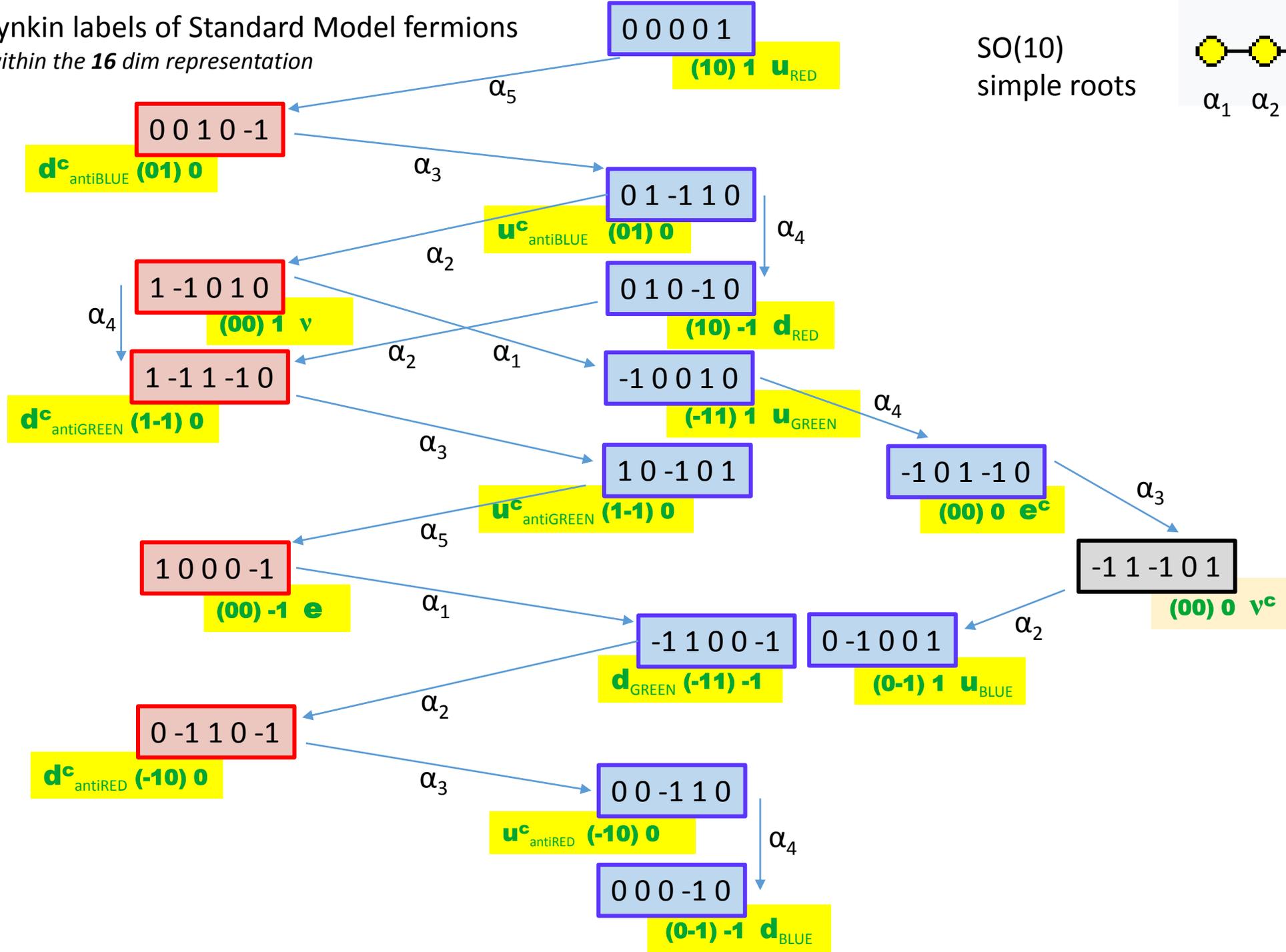
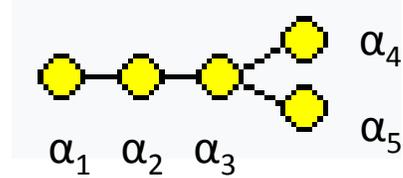
They all fit nicely into the **16** dim representation of SO(10)



# SO(10) Dynkin labels of Standard Model fermions

lowerings within the **16** dim representation

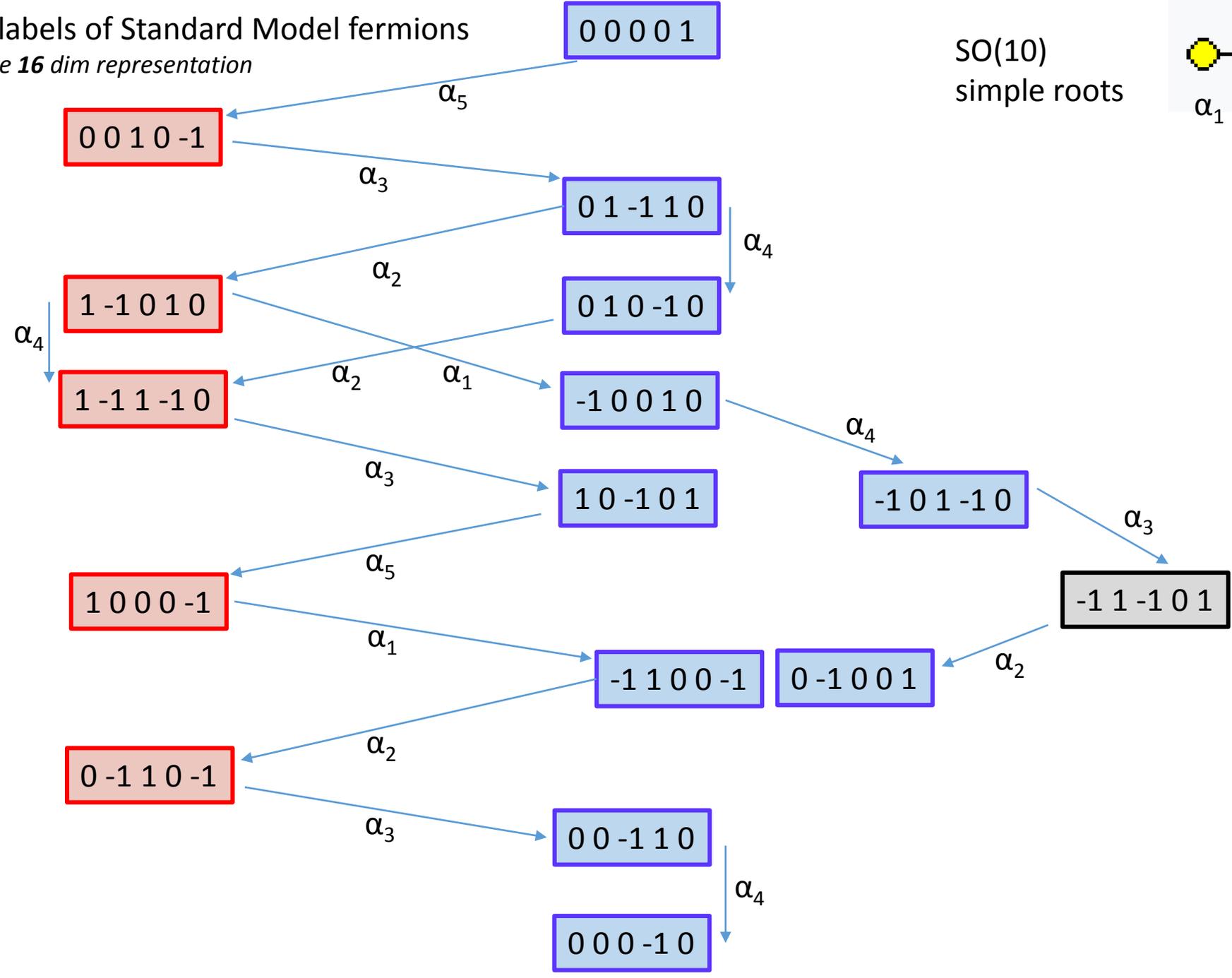
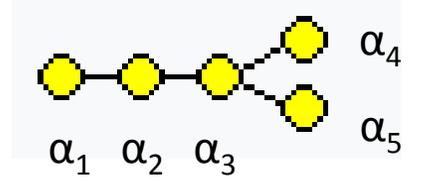
SO(10)  
simple roots



# SO(10) Dynkin labels of Standard Model fermions

lowerings within the **16** dim representation

SO(10)  
simple roots



# Indirect Searches

Low Energy processes

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

with P.Maták, Intl. J. Phys. 2014, PhD Thesis 2015

$$R_K = \Gamma(K^+ \rightarrow e^+ \nu_e) / \Gamma(K^+ \rightarrow \mu^+ \nu_\mu)$$

with P.Maták, Z.Kučerová  
and Z.Šinská, M.S. Thesis 2016

$$BR(\tau \rightarrow e \gamma)$$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$  in SM

$$\langle \pi^+ \nu \bar{\nu} | \bar{s} \gamma_\mu P_{L,R} d | K^+ \rangle \doteq \sqrt{2} \langle \pi^0 e^+ \nu_e | \bar{s} \gamma_\mu P_L u | K^+ \rangle$$

where, as far as the electron is treated massless, the effect of the leptonic current is the same for both decays. After the next-to-leading order (NLO) isospin breaking corrections are included, the hadronic matrix elements enter the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  branching ratio through the parameter  $\kappa_+$

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+ \left[ \left( \frac{\text{Im } \lambda_t}{\lambda^5} X \right)^2 + \left( \frac{\text{Re } \lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{\text{Re } \lambda_t}{\lambda^5} X \right)^2 \right]$$

NNLO result<sup>13</sup>

while long distance effects have been included in  $\delta P_{c,u} = 0.04 \pm 0.02$ .<sup>3</sup>

$$P_c = (0.372 \pm 0.015) \times \left( \frac{0.2255}{\lambda} \right)$$

In our notation  $X = X_L + X_R$  stands for the top quark and short distance contributions. In the SM  $X_R^{\text{SM}} = 0$  and  $X_L^{\text{SM}} \simeq X_0(x_t)$ , where  $x_t = m_t^2/M_W^2$ . The loop function  $X_0(x_t)$  represents the sum of the SM top quark diagrams and is equal to<sup>14</sup>

$$X_0(x_t) = \frac{x_t [x_t^2 + x_t - 2 + 3(x_t - 2) \ln x_t]}{8(1 - x_t)^2}.$$

Inclusion of NLO QCD corrections<sup>5</sup> and two-loop electroweak<sup>15</sup> corrections lead to  $X(x_t) = 1.469 \pm 0.02$  and resulting branching fraction<sup>15</sup>

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (7.81_{-0.71}^{+0.80} \pm 0.29) \times 10^{-11}$$

## $K^+ \rightarrow \pi^+ \nu \nu$ in MSSM

In the super-CKM<sup>b</sup> basis, we assume the squark mass matrix

$$M_{\tilde{q}}^2 = \begin{pmatrix} M_{\tilde{q},LL}^2 & M_{\tilde{q},LR}^2 \\ M_{\tilde{q},LR}^{2\dagger} & M_{\tilde{q},RR}^2 \end{pmatrix}$$

with<sup>c</sup>

$$M_{\tilde{q},LL}^2 = V_{qL} \mathbf{m}_{\tilde{Q}}^2 V_{qL}^\dagger + \mathbf{m}_q^2 + (T_q^3 - Q_q s_W^2) M_Z^2 \cos 2\beta \mathbf{1},$$

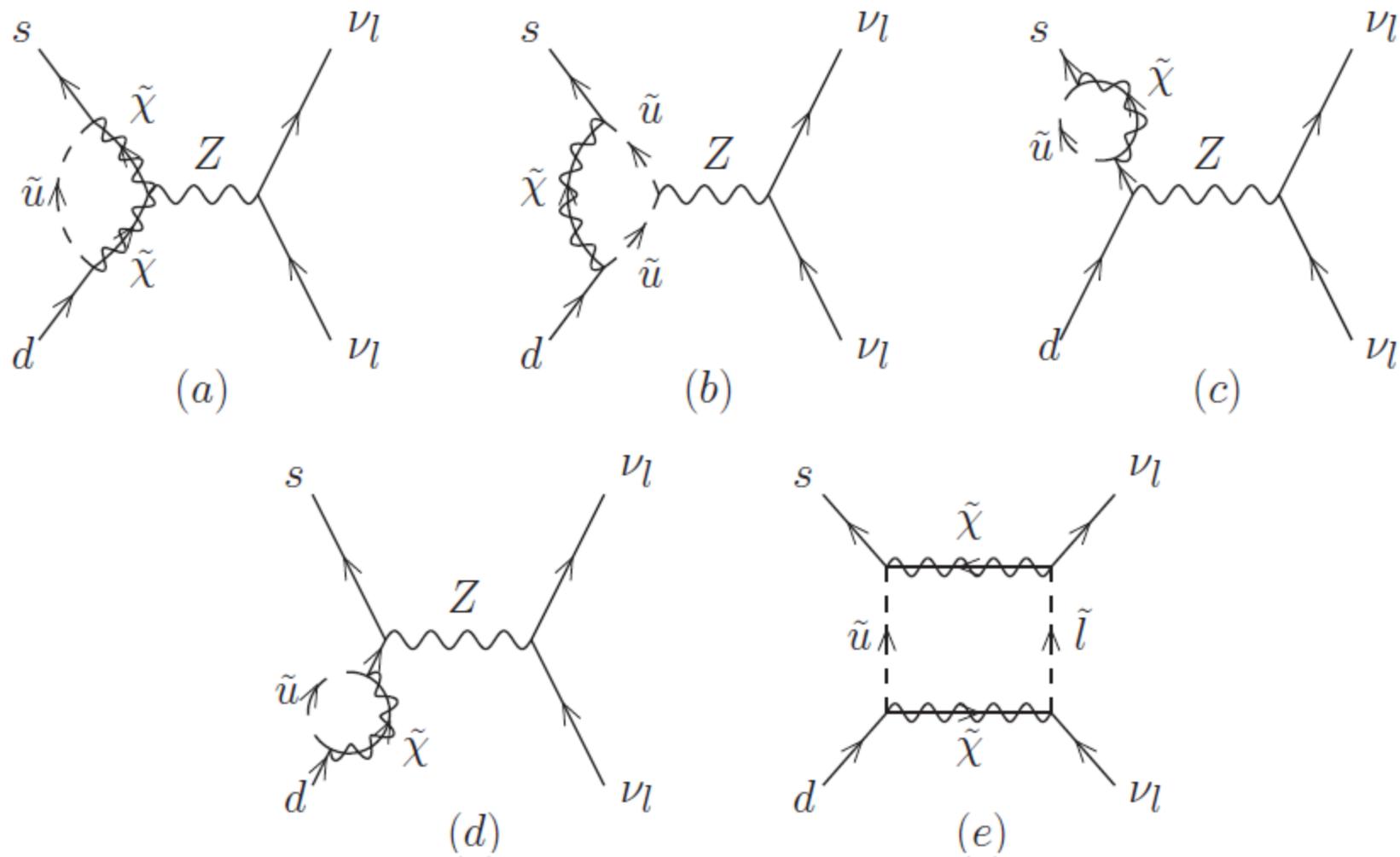
$$M_{\tilde{q},RR}^2 = V_{qR} \mathbf{m}_{\tilde{q}}^2 V_{qR}^\dagger + \mathbf{m}_q^2 + Q_q s_W^2 M_Z^2 \cos 2\beta \mathbf{1},$$

$$M_{\tilde{q},LR}^2 = (\mathbf{A}_{\tilde{q}} - \mu^* \cot \beta) \mathbf{m}_q.$$

**MSSM squark mass matrix: new sources of flavor violation *parametrized by***

$$\delta_{\tilde{q}XY}^{ij} = \frac{(M_{\tilde{q},XY}^2)^{ij}}{\sqrt{(M_{\tilde{q},XX}^2)^{ii} (M_{\tilde{q},YY}^2)^{jj}}}, \quad i \neq j$$

# $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in MSSM



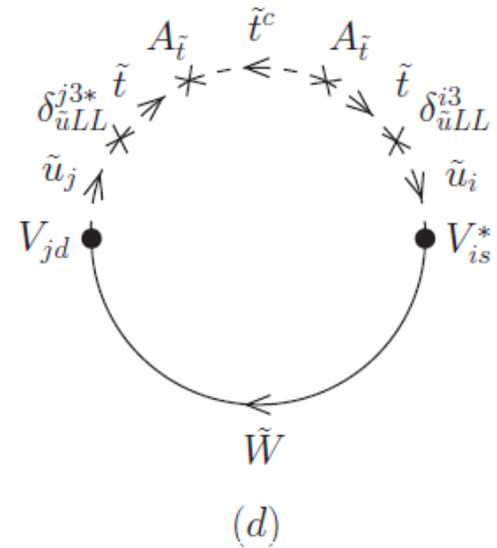
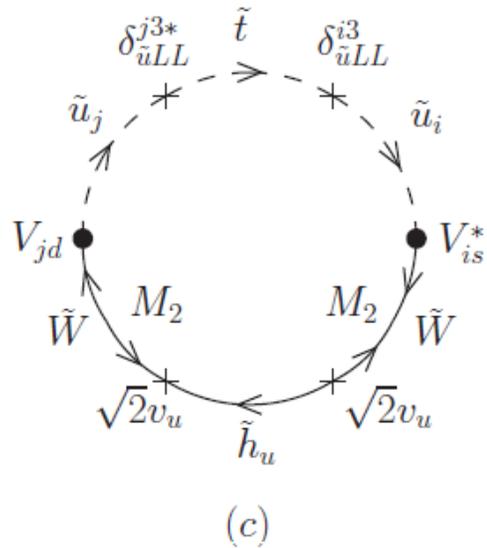
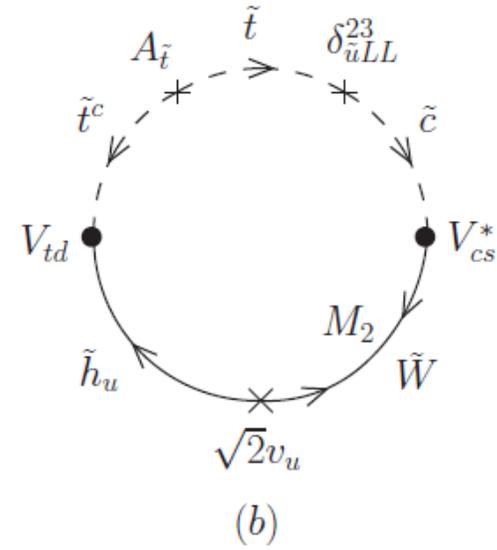
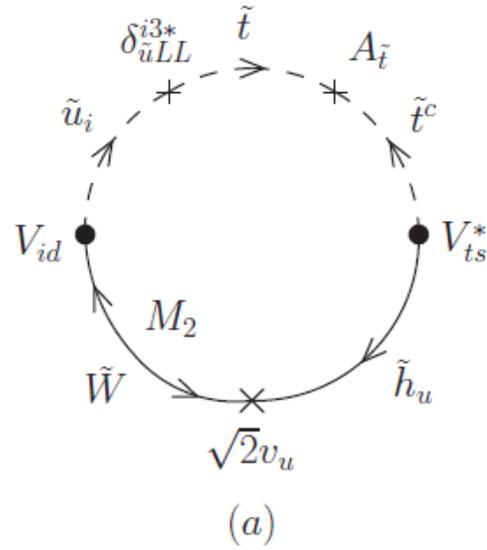
MSSM chargino diagrams for the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ .

# $K^+ \rightarrow \pi^+ \nu \nu_{\text{bar}}$ in MSSM

Limits on left–left mass insertions for squark and gluino masses given in TeV.

$\delta_{\tilde{q}LL}^{ij}$	Constraining observables	Upper bound	$\tilde{m}$	$M_3$
$ \delta_{\tilde{u}LL}^{12} $	$D^0 - \bar{D}^0$	0.10 (Refs. 28 and 29)	$< 1.0$	$< 1.0$
		0.14 (Ref. 28)	0.5	1.0
		0.06 (Refs. 23, 28 and 30)	$< 0.6$	$< 0.6$
$ \delta_{\tilde{d}LL}^{12} $	$K^0 - \bar{K}^0$	0.14 (Ref. 18)	$< 1.0$	$< 2.0$
$ \text{Re}(\delta_{\tilde{d}LL}^{12}) $		0.03 (Refs. 23 and 30)	$< 0.6$	$< 0.6$
$ \text{Im}(\delta_{\tilde{d}LL}^{12}) $		0.003 (Ref. 23)	0.5	0.5
$ \text{Re}(\delta_{\tilde{d}LL}^{13}) $	$\Delta M_d, S_{\psi K_S}$	0.1 (Refs. 23 and 30)	$< 0.6$	$< 0.6$
$ \text{Im}(\delta_{\tilde{d}LL}^{13}) $		0.03 (Ref. 30)	$< 0.6$	$< 0.6$
			$\tilde{m}$	$M_2$
$ \delta_{\tilde{d}LL}^{13} $	$B \rightarrow X_s \gamma, X_s \bar{l}l$	0.24 (Ref. 31)	0.5	0.6
$ \delta_{\tilde{d}LL}^{23} $		0.11 (Ref. 31)	0.5	0.6
$ \text{Re}(\delta_{\tilde{d}LL}^{23}) $		0.1 (Ref. 30)	$< 0.6$	$< 0.16$
$ \text{Im}(\delta_{\tilde{d}LL}^{23}) $		0.2 (Ref. 30)	$< 0.6$	$< 0.16$

# $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in MSSM



Dominant  $\delta_{\tilde{u}LL}^{i3}$  loops contributing to  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay in the large  $\tan \beta$  regime.  $\tilde{W}$  and  $\tilde{h}_u$  are represented by two-component Weyl spinors.

# $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in MSSM

## Numerical Results

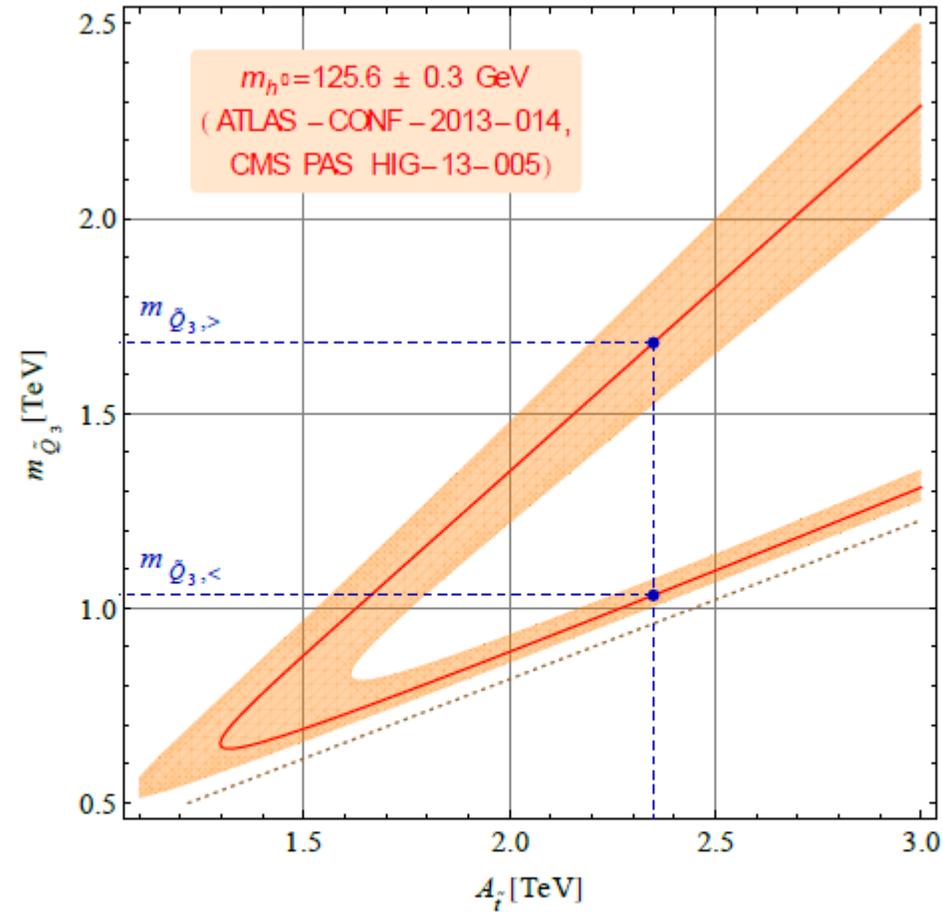


Figure 3: Allowed values of scalar mass and left-right mixing corresponding to  $\tan \beta = 50$  and average of Higgs mass measured by CMS [2] and ATLAS

# $K^+ \rightarrow \pi^+ \nu \nu_{\text{bar}}$ in MSSM

## *Numerical Results*

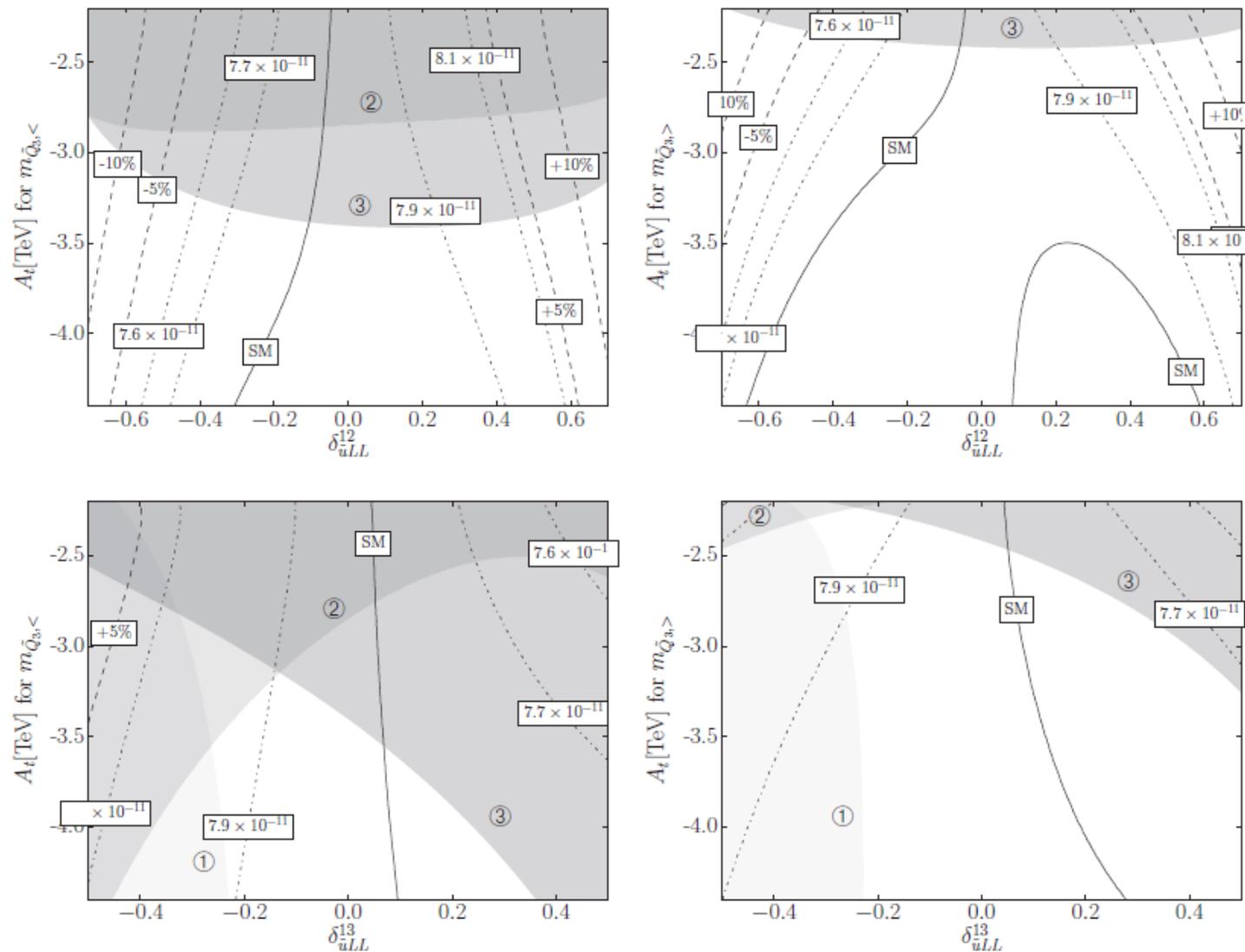
Table 2: Used values of the MSSM parameters. All masses are in TeV.

$M_2$	$\mu$	$M_{A^0}$	$\tan \beta$	$m_{\tilde{Q}_1}$	$m_{\tilde{u}_1, \tilde{d}_1}$	$m_{\tilde{u}_3, \tilde{d}_3}$
1.0	0.11	1.5	50	$1.2 \times m_{\tilde{Q}_3}$	$1.2 \times m_{\tilde{Q}_3}$	$m_{\tilde{Q}_3}$

# $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in MSSM

T. Blažek & P. Maták

## Numerical Results



The effect of  $A_{\tilde{t}}$  and  $\delta_{\tilde{u}LL}$  in the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay branching ratio.

$$R_K = \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)}$$

$$R_K^{SM} = (2.477 \pm 0.001) \times 10^{-5}$$

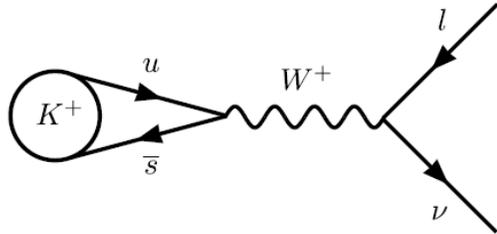
$$R_K^{exp} = (2.488 \pm 0.010) \times 10^{-5}$$

$$R_K = R_K^{SM} (1 + \Delta r), \quad \Delta r \equiv \frac{R_K}{R_K^{SM}} - 1$$

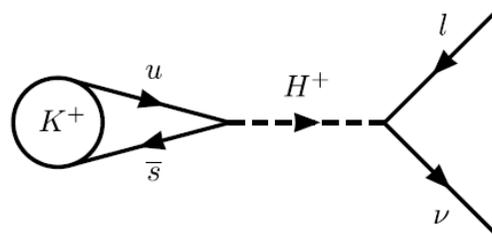
$$R_K = \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)}$$

AT TREE LEVEL

SM



MSSM



$$i\mathcal{M}_{fi} = \langle 0 | -i \frac{g_2}{\sqrt{2}} V_{us}^* \bar{v}_s \gamma^\mu P_L u_u | K^+ \rangle \left( \frac{i}{M_W^2} \right) \left[ -i \frac{g_2}{\sqrt{2}} \bar{u}_\nu \gamma_\mu P_L v_l \right] +$$

$$+ \langle 0 | i \sin \beta y_s V_{us}^* \bar{v}_s P_L u_u | K^+ \rangle \left( \frac{-i}{M_H^2} \right) [i \sin \beta y_e \bar{u}_\nu P_L v_l]$$

SM

MSSM

MSSM contrib.  
is Flavor  
Independent

...

AT TREE LEVEL

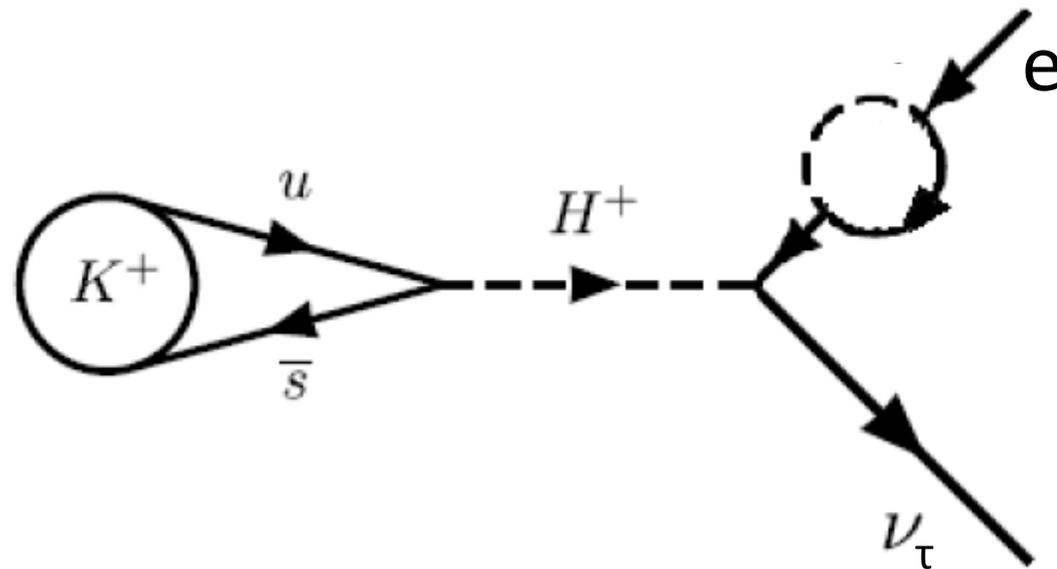
$$\Gamma(K^+ \rightarrow l^+ \nu) = \frac{1}{4\pi} [G_F F_0 V_{us}^* m_l]^2 \left[ 1 - \frac{m_K^2}{M_H^2} (\tan \beta)^2 \right]^2 m_K \left( 1 - \frac{m_l^2}{m_K^2} \right)^2$$

$$R_K = \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)}$$

AT LOOP LEVEL

$$R_K = \frac{\Gamma(K \rightarrow e \nu_e) + \Gamma(K \rightarrow e \nu_\mu) + \Gamma(K \rightarrow e \nu_\tau)}{\Gamma(K \rightarrow \mu \nu_\mu) + \Gamma(K \rightarrow \mu \nu_e) + \Gamma(K \rightarrow \mu \nu_\tau)}$$

**DOMINANT  
MSSM LOOP  
LEVEL CONTRIBUTION**



$$R_K = \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)}$$

AT LOOP LEVEL

$$\Gamma(K^+ \rightarrow l^+ \nu) = \Gamma^{SM, tree}(K^+ \rightarrow l^+ \nu) \times$$

$$\times \left[ \overset{\text{ŠM}}{\uparrow} \overset{\text{MSSM-tree}}{\uparrow} \underbrace{1 - \frac{m_K^2}{M_H^2} (\tan \beta)^2}_a + \frac{m_K^2}{M_H^2} (\tan \beta)^2 \underbrace{\frac{i}{m_l} m_A F(C_\tau^R C_\tau^{L*} + N_\tau^R N_\tau^{L*})}_{b(l)} \right]^2$$

$$R_K = \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)} = R_K^{SM} \frac{|1 - a + ab(e)|^2}{|1 - a + ab(\mu)|^2}$$

$$\Delta r = \frac{R_K}{R_K^{SM}} - 1 = \frac{|1 - a + ab(e)|^2}{|1 - a + ab(\mu)|^2} - 1$$

$$R_K = \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)}$$

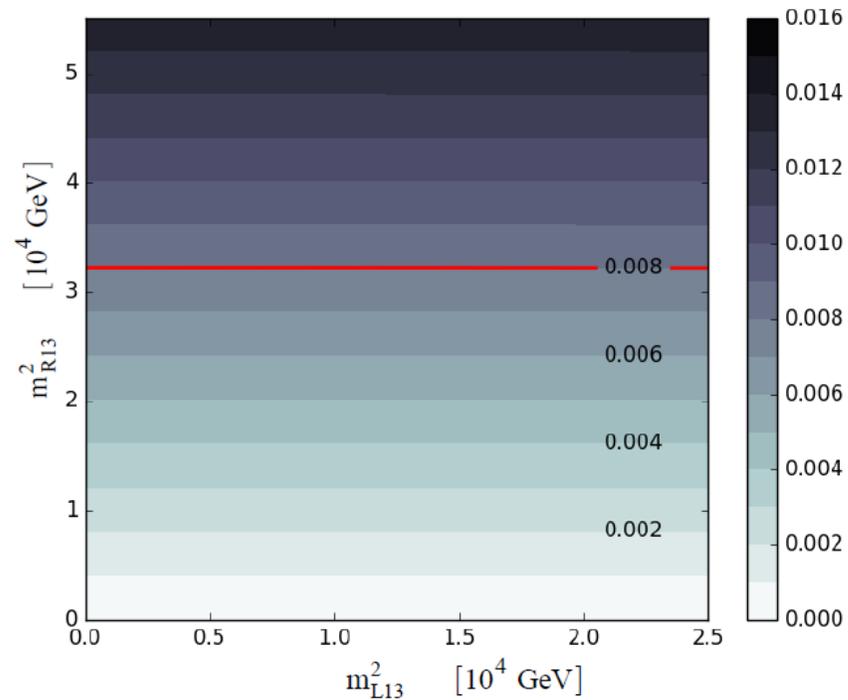
$$\mu = 300 \text{ GeV}$$

$$m_{L33}^2 = 225000 \text{ GeV}, m_{R33}^2 = 210000 \text{ GeV}$$

$$\text{fixované: } m_{L23}^2 = 0 \text{ GeV}, m_{R23}^2 = 0 \text{ GeV}$$

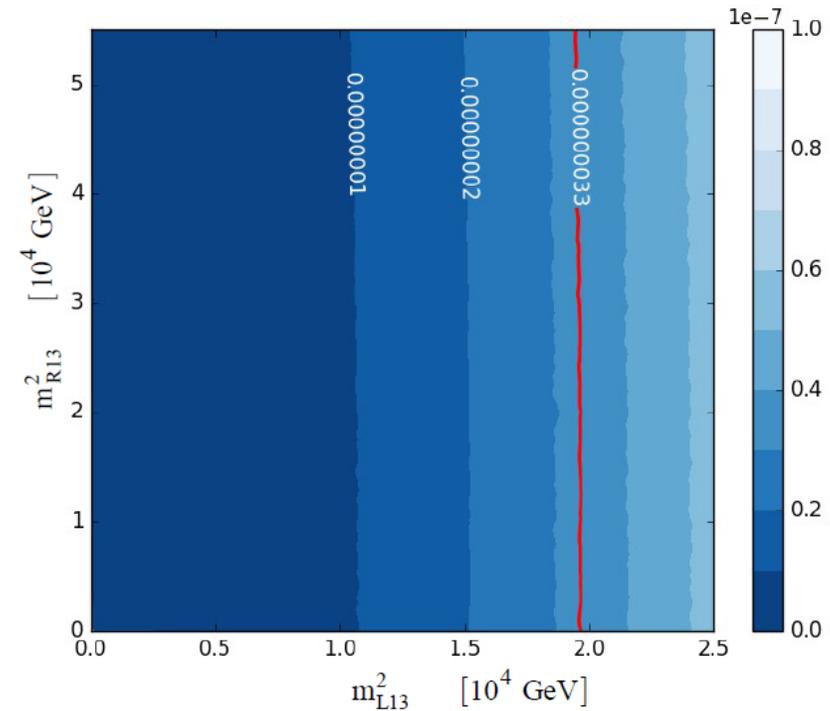
(4.4.1)

$\Delta r$



(4.4.2)

$BR_e$



$$R_K = \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)}$$

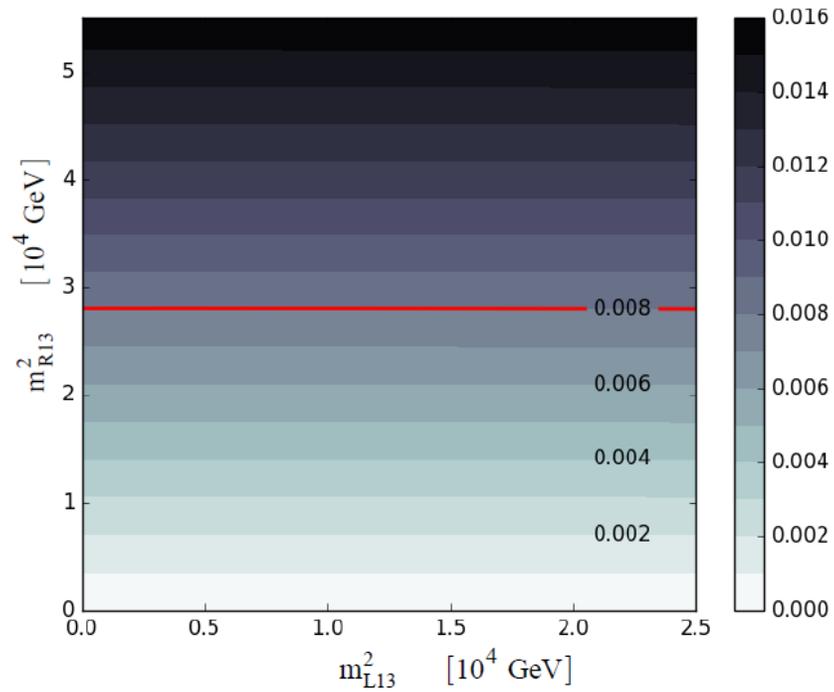
$\mu = 300 \text{ GeV}$

$m_{L33}^2 = 180000 \text{ GeV}^2, m_{R33}^2 = 160000 \text{ GeV}^2$

fixované:  $m_{L23}^2 = 0 \text{ GeV}^2, m_{R23}^2 = 0 \text{ GeV}^2$

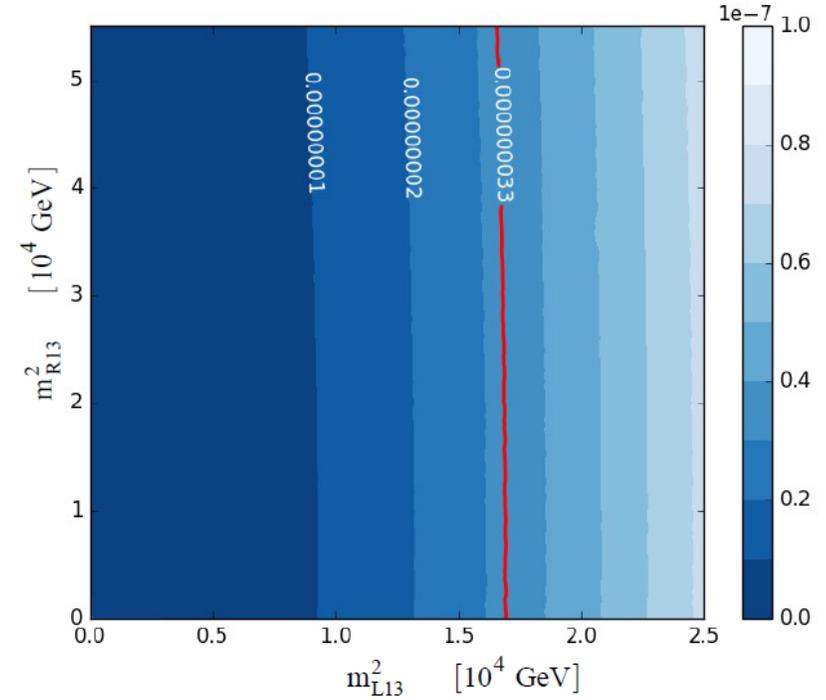
(4.6.1)

$\Delta r$



(4.6.2)

$BR_e$



## CONCLUSIONS

Rare  $K^+$  decays could provide signals of BSM Physics provided  
MSSM is the SM extension

Showed results for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and for  $\Delta r$  in  $R_K$  ratio for  
MSSM with large tan beta

10% or better precision may be needed for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  exp. value

$\Delta r$  in  $R_K$  ratio could easily be as large as the current one sigma error  
=> remains a good observable to look for BSM Physics



