ϵ'/ϵ: Lattice Computations of $K \to \pi\pi$ Decay Amplitudes

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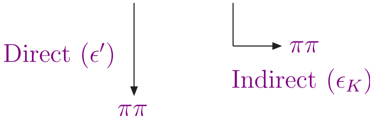
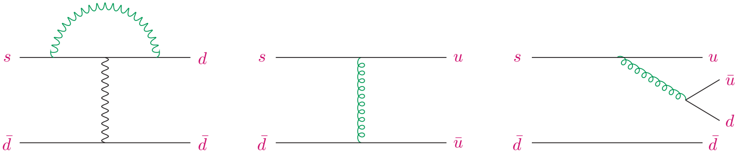
- $K \to \pi\pi$ decays are a very important class of processes for standard model phenomenology with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.
- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.
- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule (Re A_0/Re $A_2 \simeq 22.5$) and an understanding of the experimental value of ε'/ε, the parameter which was the first experimental evidence of direct CP-violation.
- The evaluation of $K \to \pi\pi$ matrix elements requires an extension of the standard computations of $\langle 0 \,|\, O(0) \,|\, h \rangle$ and $\langle h_2 \,|\, O(0) \,|\, h_1 \rangle$ matrix elements with a single hadron in the initial and/or final state.

- Directly CP-violating decays are those in which a CP-even (-odd) state decays into a CP-odd (-even) one: $K_L \;\propto\; K_2 \;+\; \bar\epsilon K_1\,.$



- Consider the following contributions to $K\to\pi\pi$ decays:



$I=0$, Complex (a) | $I=0$, Real (b) | $I=0$ or 2, Real (c)

Direct CP-violation in kaon decays manifests itself as a non-zero relative phase between the $I=0$ and $I=2$ amplitudes.

- We also have *strong phases*, δ_0 and δ_2 which are independent of the form of the weak Hamiltonian.

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\, V_{ud}V_{us}^* \sum_{i=1}^{10} \left[z_i(\mu) + \tau\, y_i(\mu)\right] Q_i\,, \quad \text{where } \tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \quad \text{and}$$

Current – Current Operators

$$Q_1 = (\bar{s}d)_L(\bar{u}u)_L \qquad Q_2 = (\bar{s}^i d^j)_L(\bar{u}^j u^i)_L$$

QCD Penguin Operators

$$Q_3 = (\bar{s}d)_L \textstyle\sum_{q=u,d,s}(\bar{q}q)_L \qquad Q_4 = (\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s}(\bar{q}^j q^i)_L$$

$$Q_5 = (\bar{s}d)_L \textstyle\sum_{q=u,d,s}(\bar{q}q)_R \qquad Q_6 = (\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s}(\bar{q}^j q^i)_R$$

Electroweak Penguin Operators

$$Q_7 = \tfrac{3}{2}(\bar{s}d)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}q)_L \qquad Q_8 = \tfrac{3}{2}(\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}^j q^i)_L$$

$$Q_9 = \tfrac{3}{2}(\bar{s}d)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}q)_R \qquad Q_{10} = \tfrac{3}{2}(\bar{s}^i d^j)_L \textstyle\sum_{q=u,d,s} e_q(\bar{q}^j q^i)_R$$

This 10 operator basis is very natural but over-complete:

$$\begin{aligned} Q_{10} - Q_9 &= Q_4 - Q_3 \\ Q_4 - Q_3 &= Q_2 - Q_1 \\ 2Q_9 &= 3Q_1 - Q_3\,. \end{aligned}$$

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- In 2015 RBC-UKQCD published our first result for ϵ'/ϵ computed at physical quark masses and kinematics, albeit still with large relative errors:

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

to be compared with

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

RBC-UKQCD, arXiv:1505.07863

- This is by far the most complicated project that I have ever been involved with.
- This single result hides much important (and much more precise) information which we have determined along the way.
- In this talk I will review the main obstacles to computing $K \to \pi\pi$ decay amplitudes, the techniques used to overcome them, our main results and the prospects for the reduction of the uncertainties.

1. A_0 and A_2 amplitudes with unphysical quark masses and with the pions at rest.

   "K to $\pi\pi$ decay amplitudes from lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D. Mawhinney, C.T.S, A.Soni, C.Sturm, H.Yin and R. Zhou, Phys. Rev. D **84** (2011) 114503 [arXiv:1106.2714 [hep-lat]].

   "Kaon to two pions decay from lattice QCD, $\Delta I = 1/2$ rule and CP violation"

   Q.Liu, Ph.D. thesis, Columbia University (2010)

2. A_2 at physical kinematics and a single coarse lattice spacing.

   "The $K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.S., A.Soni, and C.Sturm

   Phys. Rev. Lett. **108** (2012) 141601 [arXiv:1111.1699 [hep-lat]],

   "Lattice determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude A_2"

   Phys. Rev. D **86** (2012) 074513 [arXiv:1206.5142 [hep-lat]]

   "Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD,"

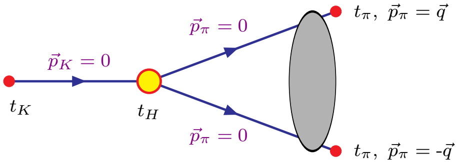
   P.A. Boyle, N.H. Christ, N. Garron, E.J. Goode, T. Janowski, C. Lehner, Q. Liu, A.T. Lytle, C.T. Sachrajda, A. Soni, and D.Zhang, Phys. Rev. Lett. **110** (2013) 15, 152001 [arXiv:1212.1474 [hep-lat]].

3. A_2 at physical kinematics on two finer lattices $\Rightarrow$ continuum limit taken.
   "$K \to \pi\pi \; \Delta I = 3/2$ decay amplitude in the continuum limit,"
   T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Janowski, C.Jung, C.Kelly, C.Lehner, A.Lytle, R.D.Mawhinney, C.T.S., A.Soni, H.Yin, and D.Zhang
   Phys. Rev. D **91** (2015) 7, 074502 [arXiv:1502.00263 [hep-lat]].

4. A_0 at physical kinematics and a single coarse lattice spacing.
   "Standard-model prediction for direct CP violation in $K \to \pi\pi$ decay,"
   Z.Bai, T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, C.T.S, A. Soni, and D. Zhang,
   Phys. Rev. Lett. **115** (2015) 21, 212001 [arXiv:1505.07863 [hep-lat]].

See also: "Calculation of ϵ'/ϵ on the lattice" C.Kelly et al. PoS FPCP2016 (2017) 017

The story continues!



- $K \to \pi\pi$ correlation function is dominated by lightest state, i.e. the state with two-pions at rest. Maiani and Testa, PL B245 (1990) 585

$$C(t_\pi) = A + B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots$$

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003

For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.

- For A_2, there is no vacuum subtraction and we can use the Wigner-Eckart theorem to write

$$\underbrace{\langle(\pi\pi)^{I=2}_{I_3=1}\,|}_{\frac{1}{\sqrt{2}}(\langle\pi^+\pi^0|+\langle\pi^0\pi^+|)}\quad Q^{\Delta I=3/2}_{\Delta I_3=1/2,i}\;|\;K^+\rangle=\frac{3}{2}\underbrace{\langle(\pi\pi)^{I=2}_{I_3=2}\,|}_{\langle\pi^+\pi^+|}Q^{\Delta I=3/2}_{\Delta I_3=3/2,i}\;|\;K^+\rangle\,,$$

  and impose anti-periodic conditions on the d-quark in one or more directions.

- If we impose the anti-periodic boundary conditions in all 3 directions then the ground state is

$$\Big|\pi\left(\frac{\pi}{L},\frac{\pi}{L},\frac{\pi}{L}\right)\,\pi\left(\frac{-\pi}{L},\frac{-\pi}{L},\frac{-\pi}{L}\right)\Big\rangle\,.$$

- With an appropriate choice of L and the number of directions, we can arrange that $E_{\pi\pi}=m_K$.

- Isospin breaking by the boundary conditions is harmless (exponentially small in the volume) here. CTS & G.Villadoro, hep-lat/0411033

- These are based on the Poisson summation formula:

$$\frac{1}{L}\sum_{n=-\infty}^{\infty} f(p_n^2) = \int_{-\infty}^{\infty}\frac{dp}{2\pi}\, f(p^2) + \sum_{n\neq 0}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\, f(p^2)e^{inpL}\,,$$

- For single-hadron states the finite-volume corrections decrease exponentially with the volume $\propto e^{-m_\pi L}$. For multi-hadron states, the finite-volume corrections generally fall as powers of the volume.
- For two-hadron states, there is a huge literature following the seminal work by Lüscher and the effects are generally understood.
  - The spectrum of two-pion states in a finite volume is given by the scattering phase-shifts. M. Luscher, Commun. Math. Phys. 105 (1986) 153, Nucl. Phys. B354 (1991) 531.
  - The $K\to\pi\pi$ amplitudes are obtained from the finite-volume matrix elements by the Lellouch-Lüscher factor which contains the derivative of the phase-shift. L.Lellouch & M.Lüscher, hep-lat/:0003023, C.h.Kim, CTS & S.R.Sharpe, hep-lat/0507006 $\cdots$
  - Recently we have also determined the finite-volume corrections for $\Delta m_K = m_{K_L} - m_{K_S}$. N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170
- For three-hadron states, there has been a major effort by Hansen and Sharpe leading to much theoretical clarification.

M.Hansen & S.Sharpe, arXiv:1408.4933, 1409.7012, 1504.04248

Error budgets in our calculation of A_2

RBC-UKQCD, T.Blum et al., arXiv:1502:00263

Source	ReA_2	ImA_2
NPR (nonperturbative)	0.1%	0.1%
NPR (perturbative)	2.9%	7.0%
Finite volume corrections	2.4%	2.6%
Unphysical kinematics	4.5%	1.1%
Wilson coefficients	6.8%	10%
Derivative of the phase shift	1.1%	1.1%
Total	9%	12%

- *Wilson Coefficients* and *NPR(perturbative*) errors are not from our lattice calculation.
- Step-scaling can be used to increase the scale at which the matching is performed.

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- Our first results for A_2 at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%. arXiv:1111.1699, arXiv:1206.5142
- Our recent results were obtained on two new ensembles, 48^3 with $a \simeq 0.11$ fm and 64^3 with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

$$\begin{aligned}\mathrm{Re}(A_2) &= 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8}\ \mathrm{GeV}.\\ \mathrm{Im}(A_2) &= -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13}\ \mathrm{GeV}.\end{aligned}$$

arXiv:1502.00263

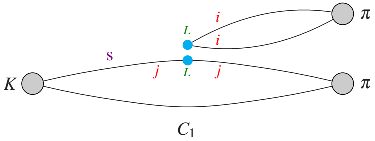
- The experimentally measured value is $\mathrm{Re}(A_2) = 1.479(4) \times 10^{-8}$ GeV.
- Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of A_2 at physical kinematics can now be considered as standard.

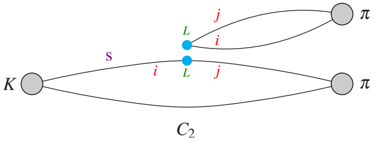
RBC-UKQCD Collaboration, arXiv:1212.1474

- Re A_2 is dominated by a simple operator:

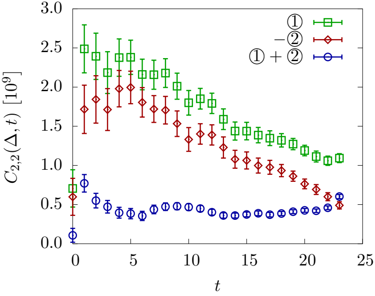
$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\right\} + (\bar{s}^i u^i)_L\,(\bar{u}^j d^j)_L$$

and two diagrams:

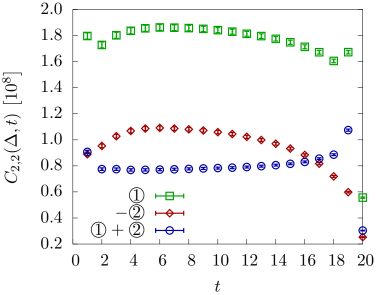




- Re A_2 is proportional to $C_1 + C_2$.
- The contribution to Re A_0 from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3}C_1$.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- **We believe that the strong suppression of Re A_2 and the (less-strong) enhancement of Re A_0 is a major factor in the $\Delta I = 1/2$ rule.**



Physical Kinematics



$m_\pi \simeq 330$ MeV at threshold.

- Notation ⓘ $\equiv C_i,\ i = 1, 2$.
- Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we needed to compute Re A_0 at physical kinematics and reproduce the experimental value of 22.5.
- Much early phenomenology was based on the vacuum insertion approach. although the qualitative picture we find had been suggested by Bardeen, Buras and Gerard in 1987.

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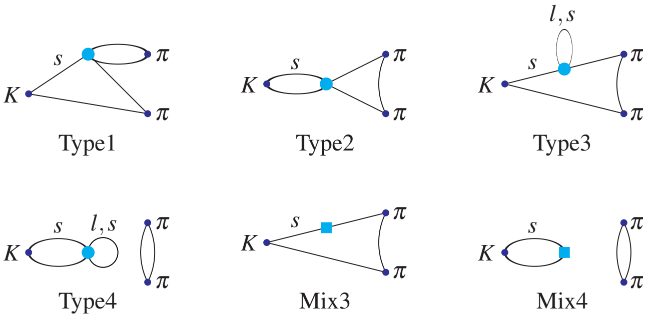
- The calculation is much more difficult for the $K \to (\pi\pi)_{I=0}$ amplitude A_0:
  - The presence of disconnected diagrams, vacuum subtraction, ultra-violet power divergences, $\cdots$



  - $|\pi^+(\pi/L)\pi^-(\text{-}\pi/L)\rangle$ has a different energy from $|\pi^0(\vec{0})\pi^0(\vec{0})\rangle$.
  - We have developed the implementation of G-parity boundary conditions in which $(u,d) \to (\bar{d}, -\bar{u})$ at the boundary . Key theoretical development.

U. Wiese, Nucl.Phys. B375 (1992) 45 , RBC-UKQCD, C.h.Kim hep-lat/0311003

- Computations were performed on a $32^3 \times 64$ lattice with the Iwasaki and DSDR gauge action and $N_f = 2+1$ flavours of Möbius Domain Wall Fermions:

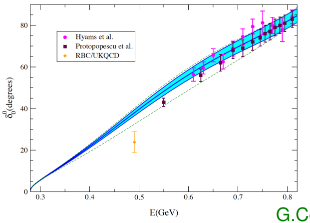
$$a^{-1} = 1.379(7)\,\text{GeV}, m_\pi = 143.2(2.0)\,\text{MeV}, (E_\pi = 274.8(1.4)\,\text{MeV})$$

- The $\pi\pi$ energies are

$$E_{\pi\pi}^{I=0} = (498 \pm 11)\,\text{MeV}, \quad E_{\pi\pi}^{I=2} = (565.7 \pm 1.0)\,\text{MeV}$$

  to be compared with $m_K = (490.6 \pm 2.4)$ MeV.

- Lüscher's quantisation condition $\Rightarrow E_{\pi\pi}^{I=0}$ corresponds to $\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ$, which is somewhat smaller than phenomenological expectations.



G.Colangelo

- The phenomenological estimate of $\delta_0^0(m_K^2) = 38.0(1.3)^\circ$ corresponds to $E_{\pi\pi} = 470$ MeV.
- While this discrepancy does not significantly affect the $K \to \pi\pi$ matrix elements, it is important to resolve it. The lattice Lüscher formalism is relatively straightforward.

$$H_W = \frac{G_F}{\sqrt{2}} V^*_{us} V_{ud} \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu). \qquad \left(\tau = -\frac{V^*_{ts} V_{td}}{V^*_{us} V_{ud}} \right)$$

Wilson coefficients from Buchalla, Buras, Lautenbacher, hep-ph/9512380

i	Re(A_0)(GeV)	Im(A_0)(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.60(0.90)(0.28) \times 10^{-7}$	0
3	$-1.28(1.69)(1.20) \times 10^{-10}$	$1.53(2.03)(1.44) \times 10^{-12}$
4	$-2.01(0.69)(0.36) \times 10^{-9}$	$1.80(0.61)(0.32) \times 10^{-11}$
5	$-8.93(2.23)(1.84) \times 10^{-10}$	$1.54(0.38)(0.32) \times 10^{-12}$
6	$3.51(0.89)(0.23) \times 10^{-9}$	$-3.56(0.90)(0.24) \times 10^{-11}$
7	$2.38(0.40)(0.00) \times 10^{-11}$	$8.49(1.44)(0.00) \times 10^{-14}$
8	$-1.28(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.38(1.97)(0.48) \times 10^{-12}$	$-2.41(0.64)(0.16) \times 10^{-12}$
10	$7.29(2.62)(0.68) \times 10^{-12}$	$-4.72(1.69)(0.44) \times 10^{-13}$
Total (stat only)	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.90(1.19)(0.32) \times 10^{-11}$
Final (incl. syst)	$4.66(1.00)(1.21) \times 10^{-7}$	$-1.90(1.23)(1.04) \times 10^{-11}$

- Representative Errors

Description	Error	Description	Error
Finite lattice spacing	8%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			26%

- Two groups have used our matrix elements $\langle(\pi\pi)_{I=0}|Q_6|K\rangle$ and $\langle(\pi\pi)_{I=2}|Q_8|K\rangle$ and smaller ranges for the smaller contributions to obtain ϵ'/ϵ:

RBC-UKQCD	$(1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$	arXiv:1505.07863
A.Buras, M.Gorbahn, S.Jäger & M.Jamin	$(1.9 \pm 4.5) \times 10^{-4}$	arXiv:1507.06345
T.Kitahara, U.Nierste & P.Tremper	$(1.06 \pm 5.07) \times 10^{-4}$	arXiv:1607.06727

and recall the experimental value is $(16.6 \pm 2.3) \times 10^{-4}$.

- Our computed value of Re $A_0 = 4.66(1.00)(1.21) \times 10^{-7}$ to be compared to the experimental value of Re $A_0 = 3.3201(18) \times 10^{-7}$.

- It is necessary to improve the statistics to establish that the results are robust.
  - 2015 PRL - Measurements were performed on 216 configurations. We currently (25th May 2017) have 836 additional independent configurations on 304 of which measurements have been made.
    - June 7th 2017 - 889 independent configurations on which measurements have been made on 352.
  - Each additional independent G-parity configuration took 31.2 hours to generate on 512 nodes BG/Q and a set of measurements on one configuration takes 18.8 hours.
  - The gauge configuration generation has been reduced to 7.6 hours per independent configuration by the use of an *exact one flavour algorithm*.

    Y-C Chen & T-W Chiu, arXiv1403.1683; D.J.Murphy, arXiv:1611.00298
  - We envisage presenting updated results from > 1000 configurations by late 2017/early 2018, including some of the systematic improvements mentioned below.

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- Preliminary comparison of matrix elements of Q_2 and Q_6 obtained from 216+269 = 485 configurations and the original set of 216 configurations.

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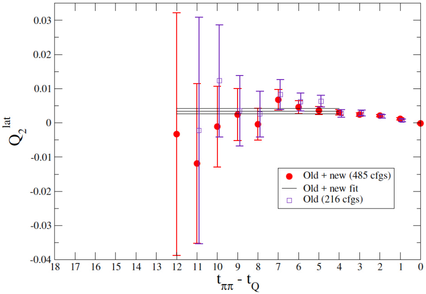
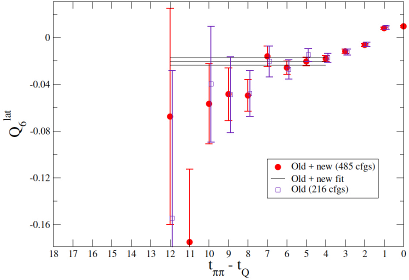
- Preliminary





- A major component of our systematic error is due to the truncation of the perturbation series in both the Wilson Coefficients and in the matching factors relating the non-perturbative renormalization (NPR) factors to those in $\overline{\text{MS}}$.

$$\begin{aligned} A_I &= F\,\frac{G_F}{\sqrt{2}}\,V_{ud}V_{us}^*\sum_i C_i^{\overline{\text{MS}}}(\mu)\langle(\pi\pi)_I\,|\,Q_i^{\overline{\text{MS}}}(\mu)\,|K\rangle \\ &= F\,\frac{G_F}{\sqrt{2}}\,V_{ud}V_{us}^*\sum_i C_i^{\overline{\text{MS}}}(\mu)Z_{ij}^{\text{RI-SMOM}\to\overline{\text{MS}}}\langle(\pi\pi)_I\,|\,Q_j^{\text{RI-SMOM}}(\mu)\,|K\rangle \end{aligned}$$

  where F is the Lellouch-Lüscher factor correcting for the leading finite-volume effects and the C_i are the Wilson coefficient functions.

  - The C_i are computed at NLO in the $\overline{\text{MS}}$ scheme but the $Q_i^{\overline{\text{MS}}}$ are not defined non-perturbatively.
  - Thus the calculation of the $C_i^{\overline{\text{MS}}}$ is not the end of perturbation theory.
  - We therefore calculate the $\langle(\pi\pi)_I\,|\,Q_j^{\text{RI-SMOM}}(\mu)\,|K\rangle$ non-pertubatively and match to $\overline{\text{MS}}$ at one-loop order.

$$A_I = F\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*\sum_i C_i^{\overline{\rm MS}}(\mu)Z_{ij}^{\text{RI-SMOM}\to\overline{\rm MS}}\langle(\pi\pi)_I\,|\,Q_j^{\text{RI-SMOM}}(\mu)\,|K\rangle$$

- The obvious way to reduce the perturbative error is for the experts to calculate the $C_i^{\text{RI-SMOM}}$ more precisely. However,
- We can use step scaling to increase the scale μ at which the $Q_j^{\text{RI-SMOM}}(\mu)$ are defined.
  - As a first application of step-scaling we step from the 32^3 lattice at which we used $\mu = 1.53$ GeV to a finer lattice, increasing the scale to $\mu = 2.29$ GeV, corresponding of a decrease of about a factor of 2 in α_s^2.
- As an estimate of the truncation error in $Z^{\text{RI-SMOM}\to\overline{\rm MS}}$ we compare the results from two RI-SMOM schemes.
- Ultimately we might aim also to calculate the Wilson Coefficients non-perturbatively, by performing the OPE at fictitious lower values M_W and m_t, *OPE-without-OPE*, eliminating the need for perturbation theory. Exploratory studies are under way.
- The operator $G_1 \propto \bar{s}\gamma_\mu(1-\gamma_5)(D_\nu G_{\nu\mu})d$ mixes with the Q_i and hence affects the renormalisation. On-shell G_1 is a linear combination of the Q_i. Exploratory studies show that this effect is small. G.McGlynn, arXiv:1605.08807, C.Kelly, PoS(Lattice 2016) 308

- We have introduced a number of two-pion interpolating operators into the two-pion correlation functions in order to control better the contamination from excited states.
  - These include $\bar{q}q$ operators as well as a variety of two-pion operators with a variety of wavefunctions.
- Developing and testing new methods
  - Use position-space NPR to cross charm threshold non-perturbatively.
  - Use modern multi-source methods to access excited finite-volume π-π states using periodic boundary conditions.

- Isospin Breaking (including Electromagnetism)
  - For most physical quantities IB effects $\sim O(1\%) \ll$ precision of present calculations of $K \to \pi\pi$ amplitudes.
  - However, the $\Delta I = 1/2$ rule, suggests that there may be an enhanced effect from A_0 feeding into A_2.
  - There is a considerable effort in including IB effects in the spectrum and simple matrix elements.
  - For $K \to \pi\pi$ decays, there is a ChPT-based phenomenological study e.g.:

$$\frac{\epsilon'}{\epsilon} = \frac{\omega_+}{\sqrt{2}|\epsilon|\mathrm{Re}\,A_0}\left\{\frac{1}{\omega_+}\mathrm{Im}\,A_2 - (1-\Omega_{\mathrm{eff}})\mathrm{Im}\,A_0\right\}$$

    with $\Omega_{\mathrm{eff}} = (14.8 \pm 8.0)\,10^{-2}$. V.Cirigliano, G.Ecker, H.Neufeld & A. Pich, hep-ph/0310351
    "Within the uncertainties [...] the IB corrections to ϵ' is below 15%." This would be a significant effect, but is still well within the error on ϵ'/ϵ.
  - Ultimately we will also calculate the IB effects non-perturbatively.

Conclusions

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- As a results of our work, the computation of A_2 is now “standard".
- It appears that the explanation of the $\Delta I = 1/2$ rule has a number of components, of which the significant cancelation between the two dominant contributions to ReA_2 is a major one.
- We have completed the first calculation of ϵ'/ϵ with controlled errors $\Rightarrow$ motivation for further refinement (systematic improvement by collecting more statistics, working on larger volumes, $\geq$2 lattice spacings etc.)
  - I stress that our particular direct contribution is the determination of the matrix elements $\langle \pi\pi | Q_i^{\text{RI-SMOM}} | K \rangle$. These contain all the NP QCD effects and are then processed to give ϵ'/ϵ.
- ϵ'/ϵ is now a quantity which is amenable to lattice computations.
- Other non-standard calculations of the RBC-UKQCD collaborations include the evaluation of Δm_K, the long-distance contribution to ϵ_K and the study of long-distance contributions to rare kaon decays.

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