B Physics Beyond the Standard Model at One Loop

M. Fael, J. Aebischer, A. Crivellin, C. Greub & J. Virto
Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern.

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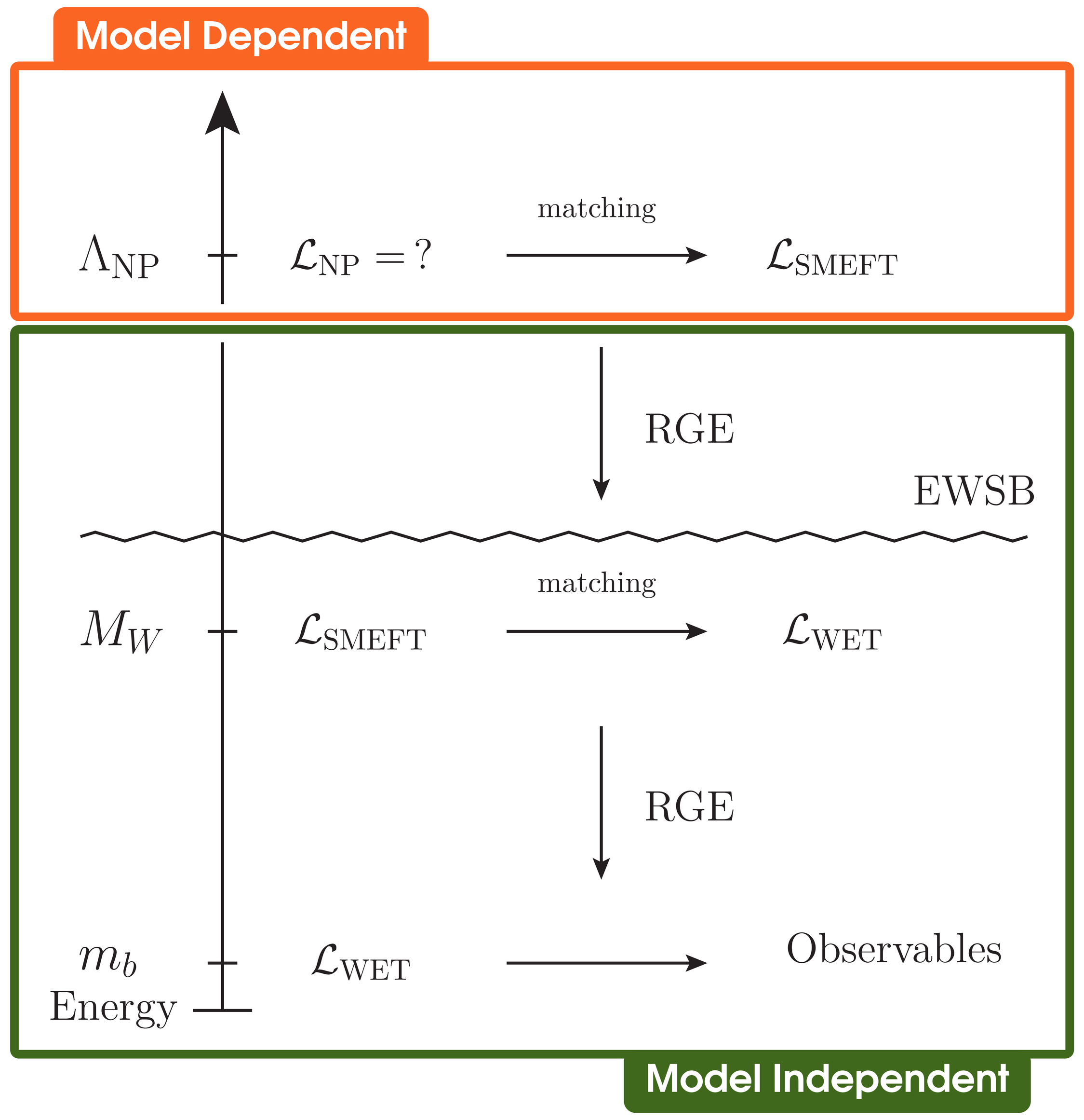
UNIVERSITÄT BERN
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

Introduction

- New Physics (NP) realized above the ElectroWeak (EW) scale can be encoded in a model independent way in the Wilson coefficients of higher dimensional operators which are invariant under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group of the SM.
- B physics is the physics related to the decay and mixing or B mesons. These processes require a change of flavour number, and therefore must be mediated by Weak interactions or by NP.
- B physics within and beyond the SM is well described by an effective Lagrangian which include QCD and QED coupled to all leptons and the five lightest quarks, plus a full set of *local* operators.
- Features of the Effective Field Theory framework:
  - Independent on the specific UV completion.
  - It provides for the resummation of large logarithms:

$$\text{SMEFT: } \log^n\left(\frac{M_W}{\Lambda_{\rm NP}}\right) \qquad \text{WET: } \log^n\left(\frac{m_b}{M_W}\right)$$

- The resummation can be performed with the Renormalization Group method within the Effective Theory.
- The renormalization scale dependence of the Wilson coefficients if given by the *anomalous dimensions*.



The SM Effective Field Theory (SMEFT)

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}^{(4)}_{\rm SM} + \frac{1}{\Lambda_{\rm NP}} C^{(5)}_{\nu\nu} Q^{(5)}_{\nu\nu} + \frac{1}{\Lambda^2_{\rm NP}} \sum_k C^{(6)}_k Q^{(6)}_k$$

- The most general Lagrangian invariant under the SM gauge group.
- $\mathcal{L}^{(4)}_{\rm SM}$: the usual renormalizable SM Lagrangian.
- NP encoded in higher-dimensional operators suppressed by powers of $\Lambda_{\rm NP}$.
- $Q^{(5)}_{\nu\nu}$: Weinberg operator giving rise to neutrino masses [1].
- $Q^{(6)}_k$ and $C^{(6)}_k$: the 2499 (+ baryon violating) dimension-six operators and their corresponding Wilson coefficients in the Warsaw basis [2].
- Well-known RGE evolution at one-loop [3]

The Weak Effective Theory (WET)

$$\mathcal{L}_{\rm WET} = \mathcal{L}^{(u,d,c,s,b,e,\mu,\tau)}_{\rm QCD+QED} + \mathcal{L}_{\rm EW} + \mathcal{L}_{\rm BSM}$$

- Invariant under $SU(3)_C \times U(1)_{\rm em}$.
- $\mathcal{L}_{\rm EW}$ and $\mathcal{L}_{\rm BSM}$: the effective Lagrangian after integrating out the SM and the BSM heavy degrees of freedom.
- $\mathcal{L}_{\rm EW}$ is determined by the well-known matching conditions in the SM. The *anomalous dimensions* of the SM operator basis are known to high perturbative orders [4].
- $\mathcal{L}_{\rm BSM}$ contains all BSM effects but no pure-SM ones. The operator basis is larger than the SM one.

The BSM operators can be grouped into classes according to their flavour quantum numbers:

Class	Flavour structure	Number of Ops.	Other flavours	ADM	Example process
Class I	$\bar{s}b\,\bar{s}b$	5+3	$\bar{d}b\,\bar{d}b$	$\hat\gamma_{\rm I}$	$B_q - \overline{B}_q$ mixing
Class II	$\bar{u}b\,\bar{\ell}\nu_{\ell'}$	2+3	$\bar{c}b\,\bar{\ell}\nu_{\ell'}$	$\hat\gamma_{\rm II}$	$\overline{B}_d \to \pi^+\mu^-\nu$
Class III	$\bar{s}b\,\bar{u}c$	10+10	$\bar{s}b\,\bar{c}u$ $\bar{d}b\,\bar{u}c$ $\bar{d}b\,\bar{c}u$	$\hat\gamma_{\rm III}$	$B^- \to \overline{D}^0 K^-$
Class III	$\bar{s}b\,\bar{u}c$	10+10	$\bar{s}b\,\bar{c}u$ $\bar{d}b\,\bar{u}c$ $\bar{d}b\,\bar{c}u$	$\hat\gamma_{\rm III}$	$B^- \to \overline{D}^0 K^-$
Class IV	$\bar{s}b\,\bar{s}d$	5+5	$\bar{d}b\,\bar{d}s$	$\hat\gamma_{\rm IV}$	$B^- \to \overline{K}^0 K^-$
Class V	$\bar{s}b\,\bar{q}q$ $\bar{s}b\,F$, $\bar{s}b\,G$ $\bar{s}b\,\bar{\ell}\ell$	57+57	$\bar{d}b\,\bar{q}q$ $\bar{d}b\,F$, $\bar{d}b\,G$ $\bar{d}b\,\bar{\ell}\ell$	$\hat\gamma_{\rm V}$	$\overline{B}_d \to D^+D^-_s$ $\overline{B}_d \to X_s\gamma$ $B^- \to K^-\mu^+\mu^-$
Class Vb	$\bar{s}b\,\bar{\ell}\ell'$, $\ell \neq \ell'$	$(5+5)\times 6$	$\bar{d}b\,\bar{\ell}\ell'$	$\hat\gamma_{\rm Vb}$	$\overline{B}_s \to \mu^-\tau^+$
Class Vν	$\bar{s}b\,\bar{\nu}_\ell\nu_{\ell'}$	$(1+1)\times 9$	$\bar{d}b\,\bar{\nu}_\ell\nu_{\ell'}$	zero	$B^- \to K^-\nu\nu$
Class VI	$\bar{\ell}^c b\,\bar{c}^c u$	$(5+5)\times 3$	None	$\hat\gamma_{\rm VIa}$	$\overline{B}_d \to \Lambda^-_c e^+$
	$\bar{\ell}^c b\,\bar{u}^c u$	$(2+2)\times 3$	$\bar{\ell}^c b\,\bar{c}^c c$	$\hat\gamma_{\rm VIb}$	$B_d \to p\,e^-$
	$\bar{\nu} b\,\bar{u}^c s$	5×3	$\bar{\nu} b\,\bar{u}^c d$ $\bar{\nu} b\,\bar{c}^c s$ $\bar{\nu} b\,\bar{c}^c d$	$\hat\gamma_{\rm VIc}$	$B^+ \to p\,\nu$
	$\bar{\nu} b\,\bar{u}^c b$	2×3	$\bar{\nu} b\,\bar{c}^c b$	$\hat\gamma_{\rm VId}$	$\Lambda^0_b \to B_d\,\nu$
	$\bar{\ell} b\,\bar{s}^c d$	$(5+5)\times 3$	None	$\hat\gamma_{\rm VIe}$	$\Lambda^0_b \to K^+e^-$
	$\bar{\ell}^c b\,\bar{s}^c s$	$(2+2)\times 3$	$\bar{\ell}^c b\,\bar{d}^c d$	$\hat\gamma_{\rm VIf}$	$\Lambda^0_b \to \pi^+e^-$
	$\bar{\ell}^c b\,\bar{s}^c b$	$(2+2)\times 3$	$\bar{\ell}^c b\,\bar{d}^c b$	$\hat\gamma_{\rm VIg}$	$\Xi^0_b \to B^+e^-$

Table 1: Summary list of non-redundant operators. The number of operators in each class is indicated by $(n+n')\times n_\ell$, where n is the number of different operators modulo lepton flavours, n' is the number of operators with opposite chirality, and n_ℓ accounts for the different leptonic flavours. The last column lists an example of a process to which the corresponding class of operators contributes.

Example: Class V $|\Delta B| = |\Delta S| = 1$

Magnetic Operators:

$$\mathcal{O}^s_{7\gamma} = \frac{e}{g_s^2} m_b\,(\bar{s}\,P_R\,\sigma_{\mu\nu}\,b)\,F^{\mu\nu}, \qquad \mathcal{O}^s_{8g} = \frac{1}{g_s} m_b\,(\bar{s}\,P_R\,\sigma_{\mu\nu}\,T^A\,b)\,G^{\mu\nu}_A\,.$$

Semi-leptonic:

$$\mathcal{O}^{sb\ell\ell'}_1 = (\bar{s}\,P_R\,\gamma_\mu\,b)\,(\bar{\ell}\gamma^\mu\ell'), \qquad \mathcal{O}^{sb\ell\ell'}_3 = (\bar{s}\,P_R\,\gamma_{\mu\nu\rho}\,b)\,(\bar{\ell}\gamma^{\mu\nu\rho}\ell'),$$
$$\mathcal{O}^{sb\ell\ell'}_5 = (\bar{s}\,P_R\,b)(\bar{\ell}\,\ell'), \qquad \mathcal{O}^{sb\ell\ell'}_7 = (\bar{s}\,P_R\,\sigma^{\mu\nu}\,b)(\bar{\ell}\,\sigma_{\mu\nu}\,\ell'),$$
$$\mathcal{O}^{sb\ell\ell'}_9 = (\bar{s}\,P_R\,\gamma_{\mu\nu\rho\sigma}\,b)\,(\bar{\ell}\gamma^{\mu\nu\rho\sigma}\ell'),$$
$$\mathcal{O}^{sb\ell\ell'}_L = (\bar{s}\,P_R\,\gamma_\mu\,b)\,(\bar{\nu}_\ell\gamma^\mu\nu_{\ell'}), \qquad \mathcal{O}^{sb\ell\ell'}_R = (\bar{s}\,P_L\,\gamma_\mu\,b)\,(\bar{\nu}_\ell\gamma^\mu\nu_{\ell'})\,.$$

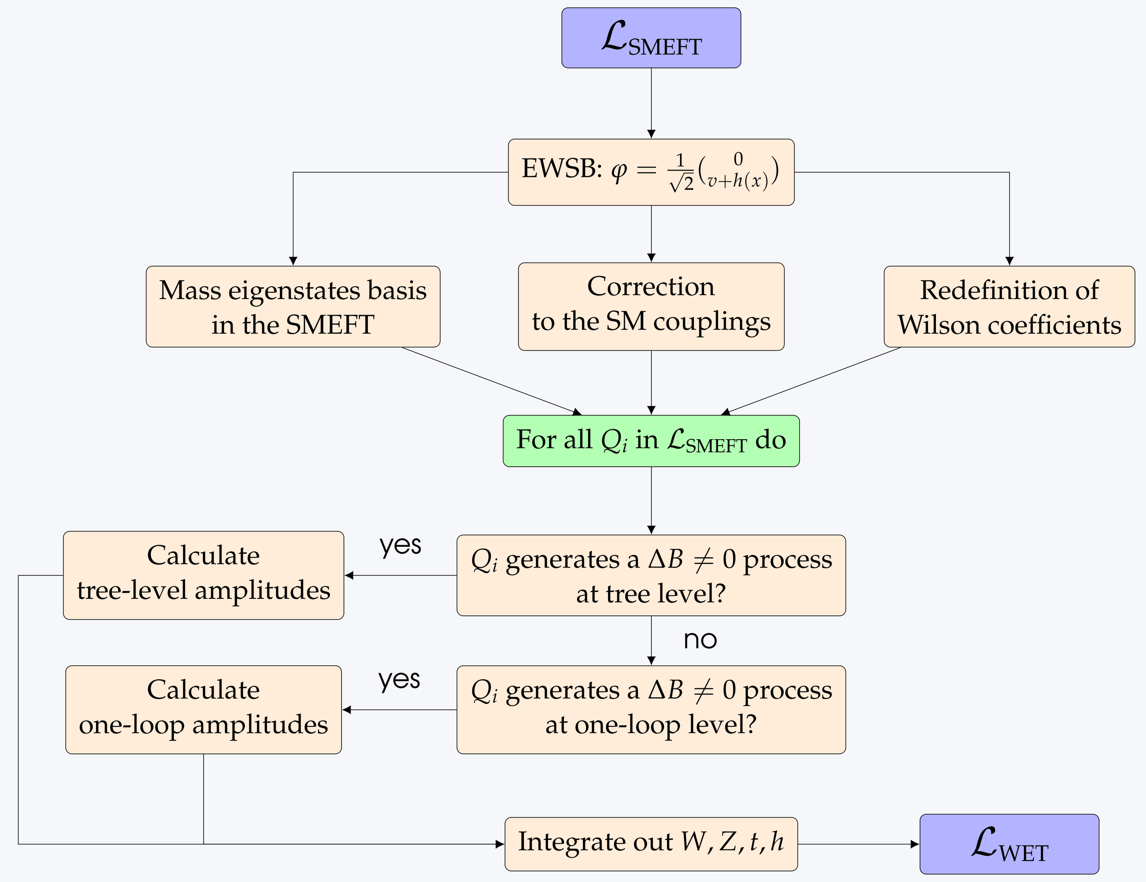
Four-quarks: (remove operators with even indices for $q = s, b$)

$$\mathcal{O}^{sbqq}_1 = (\bar{s}\,P_R\,\gamma_\mu b)\,(\bar{q}\gamma^\mu q), \qquad \mathcal{O}^{sbqq}_2 = (\bar{s}\,P_R\,\gamma_\mu\,T^A b)\,(\bar{q}\gamma^\mu\,T^A q),$$
$$\mathcal{O}^{sbqq}_3 = (\bar{s}\,P_R\,\gamma_{\mu\nu\rho}\,b)\,(\bar{q}\gamma^{\mu\nu\rho} q), \qquad \mathcal{O}^{sbqq}_4 = (\bar{s}\,P_R\,\gamma_{\mu\nu\rho} T^A b)\,(\bar{q}\gamma^{\mu\nu\rho}\,T^A q),$$
$$\mathcal{O}^{sbqq}_5 = (\bar{s}\,P_R\,b)(\bar{q}\,q), \qquad \mathcal{O}^{sbqq}_6 = (\bar{s}\,P_R\,T^A b)(\bar{q}\,T^A q),$$
$$\mathcal{O}^{sbqq}_7 = (\bar{s}\,P_R\,\sigma^{\mu\nu}\,b)(\bar{q}\,\sigma_{\mu\nu}\,q), \qquad \mathcal{O}^{sbqq}_8 = (\bar{s}\,P_R\,\sigma^{\mu\nu}\,T^A b)(\bar{q}\sigma_{\mu\nu}\,T^A q),$$
$$\mathcal{O}^{sbqq}_9 = (\bar{s}\,P_R\,\gamma_{\mu\nu\rho\sigma}\,b)\,(\bar{q}\gamma^{\mu\nu\rho\sigma} q), \qquad \mathcal{O}^{sbqq}_{10} = (\bar{s}\,P_R\,\gamma_{\mu\nu\rho\sigma}\,T^A b)\,(\bar{q}\gamma^{\mu\nu\rho\sigma}\,T^A q)\,.$$

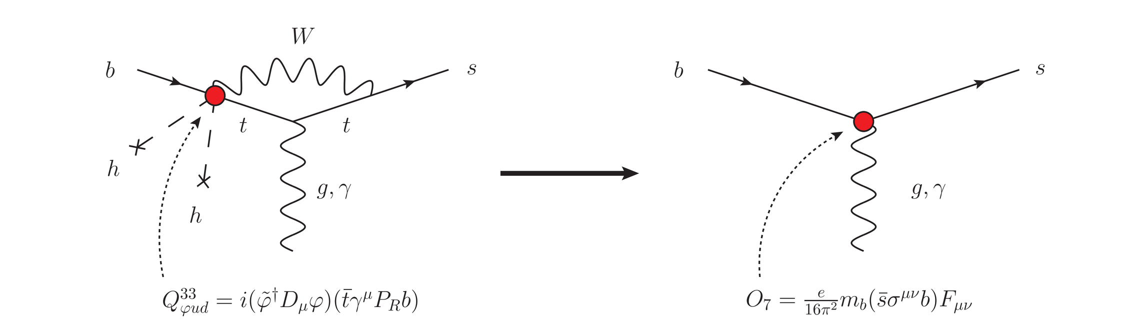
plus the analogous set with opposite chirality:

$$\mathcal{O}_{i'} = \mathcal{O}_i|_{P_{L,R}\to P_{R,L}}$$

$\mathcal{L}_{\rm SMEFT} \longrightarrow \mathcal{L}_{\rm WET}$: The Matching



Example: mathing $Q_{\varphi ud}$



Matching results: $C_7 = \tilde{C}^{33}_{\varphi ud}\,\frac{m_t}{m_b}\frac{v^2}{\Lambda^2} V^\star_{ts} E^7_{\varphi ud}(x_t)$, $\quad C_8 = \tilde{C}^{33}_{\varphi ud}\,\frac{m_t}{m_b}\frac{v^2}{\Lambda^2} V^\star_{ts} E^8_{\varphi ud}(x_t)$,

where $x_t = m_t^2/M_W^2$ and

$$E^7_{\varphi ud}(x_t) = \frac{-5x_t^2 + 31x_t - 20}{24(x_t-1)^2} + \frac{x_t(2-3x_t)}{4(x_t-1)^3}\ln(x_t)\,,$$
$$E^8_{\varphi ud}(x_t) = -\frac{x_t^2 + x_t + 4}{8(x_t-1)^2} + \frac{3x_t}{4(x_t-1)^3}\ln(x_t)\,.$$

$(\bar{L}R)(\bar{R}L)$ or $(\bar{L}R)(\bar{L}R)$

$Q_{\ell edq}$	$(\bar{\ell}^a_i e_j)(\bar{d}_k q^a_l)$
$Q^{(1)}_{quqd}$	$(\bar{q}^a_i u_j)\varepsilon_{ab}(\bar{q}^b_k d_l)$
$Q^{(8)}_{quqd}$	$(\bar{q}^a_i T^A u_j)\varepsilon_{ab}(\bar{q}^b_k T^A d_l)$
$Q^{(1)}_{\ell equ}$	$(\bar{\ell}^a_i e_j)\varepsilon_{ab}(\bar{q}^b_k u_l)$
$Q^{(3)}_{\ell equ}$	$(\bar{\ell}^a_i \sigma^{\mu\nu} e_j)\varepsilon_{ab}(\bar{q}^b_k \sigma_{\mu\nu} u_l)$

$(\bar{L}L)(\bar{L}L)$

$Q^{(1)}_{qq}$	$(\bar{q}_i\gamma_\mu q_j)(\bar{q}_k\gamma^\mu q_l)$
$Q^{(1)}_{\ell q}$	$(\bar{\ell}_i\gamma_\mu \ell_j)(\bar{q}_k\gamma^\mu q_l)$
$Q^{(3)}_{qq}$	$(\bar{q}_i\gamma_\mu\tau^I q_j)(\bar{q}_k\gamma^\mu\tau^I q_l)$
$Q^{(3)}_{\ell q}$	$(\bar{\ell}_i\gamma_\mu\tau^I \ell_j)(\bar{q}_k\gamma^\mu\tau^I q_l)$

$\psi^2 X\varphi$

Q_{dW}	$(\bar{q}_i\sigma^{\mu\nu}d_j)\tau^{\rm I}\varphi W^{\rm I}_{\mu\nu}$
Q_{dB}	$(\bar{q}_i\sigma^{\mu\nu}d_j)\varphi B_{\mu\nu}$
Q_{dG}	$(\bar{q}_i\sigma^{\mu\nu}T^{\rm A}d_j)\varphi G^{\rm A}_{\mu\nu}$

$(\bar{L}L)(\bar{R}R)$

$Q_{\ell d}$	$(\bar{\ell}_i\gamma_\mu \ell_j)(\bar{d}_k\gamma^\mu d_l)$
Q_{qe}	$(\bar{q}_i\gamma_\mu q_j)(\bar{e}_k\gamma^\mu e_l)$
$Q^{(1)}_{qu}$	$(\bar{q}_i\gamma_\mu q_j)(\bar{u}_k\gamma^\mu u_l)$
$Q^{(1)}_{qd}$	$(\bar{q}_i\gamma_\mu q_j)(\bar{d}_k\gamma^\mu d_l)$
$Q^{(8)}_{qu}$	$(\bar{q}_i\gamma_\mu T^{\rm A} q_j)(\bar{u}_k\gamma^\mu T^{\rm A} u_l)$
$Q^{(8)}_{qd}$	$(\bar{q}_i\gamma_\mu T^{\rm A} q_j)(\bar{d}_k\gamma^\mu T^{\rm A} d_l)$

$(\bar{R}R)(\bar{R}R)$

Q_{dd}	$(\bar{d}_i\gamma_\mu d_j)(\bar{d}_k\gamma^\mu d_l)$
Q_{ed}	$(\bar{e}_i\gamma_\mu e_j)(\bar{d}_k\gamma^\mu d_l)$
$Q^{(1)}_{ud}$	$(\bar{u}_i\gamma_\mu u_j)(\bar{d}_k\gamma^\mu d_l)$
$Q^{(8)}_{ud}$	$(\bar{u}_i\gamma_\mu T^{\rm A} u_j)(\bar{d}_k\gamma^\mu T^{\rm A} d_l)$

$\psi^2\varphi^2 D$

$Q^{(1)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_i\gamma^\mu q_j)$
$Q^{(3)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}^{\rm I}_\mu\varphi)(\bar{q}_i\tau^{\rm I}\gamma^\mu q_j)$
$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_i\gamma^\mu d_j)$
$Q_{\varphi ud}$	$i(\tilde\varphi^\dagger D_\mu\varphi)(\bar{u}_i\gamma^\mu d_j)$

$\psi^2\varphi^3$

$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_i d_j\varphi)$

Table 2: Complete list of the dimension-six operators that contribute to $b \to s$ and $b \to c$ transitions at tree level.

$\psi^2 X\varphi$

Q_{uW}	$(\bar{q}_i\sigma^{\mu\nu}u_j)\,\tau^{\rm I}\,\tilde\varphi\,W^{\rm I}_{\mu\nu}$
Q_{uB}	$(\bar{q}_i\sigma^{\mu\nu}u_j)\,\tilde\varphi\,B_{\mu\nu}$
Q_{uG}	$(\bar{q}_i\sigma^{\mu\nu}T^A u_j)\tilde\varphi G^A_{\mu\nu}$

$(\bar{L}L)(\bar{R}R)$

$Q_{\ell u}$	$(\bar{\ell}_i\gamma_\mu \ell_j)(\bar{u}_k\gamma^\mu u_l)$
$Q^{(1)}_{qu}$	$(\bar{q}_i\gamma_\mu q_j)(\bar{u}_k\gamma^\mu u_l)$
$Q^{(8)}_{qu}$	$(\bar{q}_i\gamma_\mu T^A q_j)(\bar{u}_k\gamma^\mu T^A u_l)$

$(\bar{R}R)(\bar{R}R)$

Q_{eu}	$(\bar{e}_i\gamma_\mu e_j)(\bar{u}_k\gamma^\mu u_l)$
Q_{uu}	$(\bar{u}_i\gamma_\mu u_j)(\bar{u}_k\gamma^\mu u_l)$
$Q^{(1)}_{ud}$	$(\bar{u}_i\gamma_\mu u_j)(\bar{d}_k\gamma^\mu d_l)$
$Q^{(8)}_{ud}$	$(\bar{u}_i\gamma_\mu T^A u_j)(\bar{d}_k\gamma^\mu T^A d_l)$

$\psi^2\varphi^2 D$

$Q_{\varphi ud}$	$(\tilde\varphi^\dagger iD_\mu\varphi)(\bar{u}_i\gamma^\mu d_j)$
$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_i\gamma^\mu u_j)$

$(\bar{L}R)(\bar{L}R)$

$Q^{(1)}_{quqd}$	$(\bar{q}^a_i u_j)\varepsilon_{ab}(\bar{q}^b_k d_l)$
$Q^{(8)}_{quqd}$	$(\bar{q}^a_i T^{\rm A} u_j)\varepsilon_{ab}(\bar{q}^b_k T^{\rm A} d_l)$

Table 3: Dimension-six operators that contribute to $b \to s$ transitions at the one-loop level.

Complete RGE below the EW scale

The Wilson coefficients and the operators are bare quantities and need to be renormalized:

$$C_i^{(0)} = Z^c_{ij}\,C_j \text{ and } \mathcal{O}_i^{(0)} = Z^{\mathcal{O}}_{ki}\,\mathcal{O}_k$$

- $\hat{Z}^{\mathcal{O}}$: field renormalization, and possibly the renormalization of masses and couplings in the operator definition.
- $\hat{Z}^c$: renormalization of the Wilson coefficients, responsible for the operator mixing.
- Bare quantities must be independent on the renormalization scale μ_R: $\frac{dC_i^{(0)}}{d\log\mu_R} = 0$.

RGE Equation

$$\frac{d\vec{C}}{d\log\mu_R} = -\hat{Z}_c^{-1}\frac{d\hat{Z}_c}{d\log\mu_R}\vec{C} = \hat\gamma^T\vec{C}$$

The Anomalous Dimension Matrix $\hat\gamma$

$$\hat\gamma = \frac{\alpha_s}{4\pi}\hat\gamma_s + \frac{\alpha_{\rm em}}{4\pi}\hat\gamma_e + O(\alpha_s^2, \alpha^2_{\rm em}, \alpha_s\alpha_{\rm em})$$

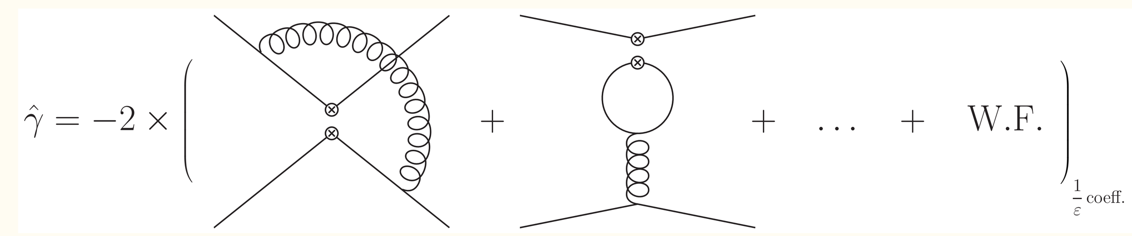
The RGE solution

$$\vec{C}(\mu) = \left[\hat{U}_s(\mu,\mu_0) + \Delta\hat{U}_{\rm em}(\mu,\mu_0)\right]\vec{C}(\mu_0)$$

- $\hat{\mathbf{U}}_{\mathbf{s}}(\boldsymbol{\mu}, \boldsymbol{\mu_0})$: responsible for the evolution in pure QCD.
- $\boldsymbol{\Delta}\hat{\mathbf{U}}_{\mathbf{em}}(\boldsymbol{\mu}, \boldsymbol{\mu_0})$: responisible for the extra evolution in presence of QED interactions.

The Matrix

- We collected and calculated the complete one-loop ADM in QCD and QED for the full operator basis in Tab. 1.



- The full ADM has the following block-diagonal form:

$$\hat\gamma = \text{Diagonal}\left\{\hat\gamma_{\rm I}, \hat\gamma_{\rm II}, \hat\gamma_{\rm III}, \hat\gamma_{\rm IV}, \hat\gamma_{\rm V}, \hat\gamma_{\rm Vb}, \hat{0}_{2\times2}, \hat\gamma_{\rm VIa}, \hat\gamma_{\rm VIb}, \hat\gamma_{\rm VIc}, \hat\gamma_{\rm VId}, \hat\gamma_{\rm VIe}, \hat\gamma_{\rm VIf}, \hat\gamma_{\rm VIg}\right\}.$$

- The different blocks $\hat\gamma_J$ have dimensions specified in Tab. 1; their explicit expression is given in Sec. 4 of [6].
- The evolution matrices $\hat{U}_s(\mu,\mu_0)$ and $\Delta\hat{U}_{\rm em}(\mu,\mu_0)$ are also presented in [6]. They are also given for convenience in electronic format as a Mathematica package.

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