Importance of Magnetic Field on Hydrodynamics evolution in Heavy Ion Collisions

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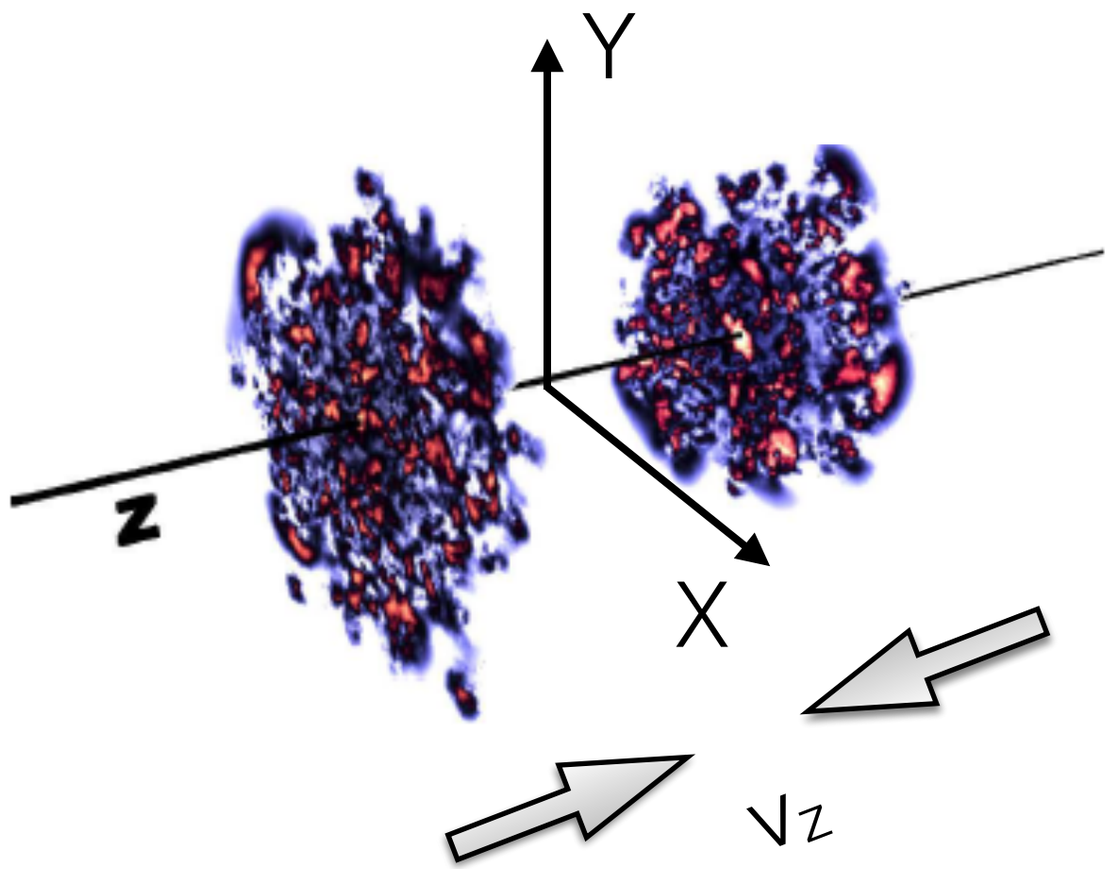
based on arXiv: 1706.05326

Outline

- Motivation
- 2+1 dimensional reduced magnetohydrodynamics
- results
- Conclusion

Motivation

In high energy heavy ion collisions +ve charged nucleus collide at a speed~c -> produce intense magnetic fields



Mid central $\sqrt{s_{\mathrm{NN}}}$=200GeV Au+Au collisions -> B~10^{18}Gauss

Few theoretical predictions

- With strong background magnetic field one can observe charge separation in a chiral imbalanced medium. This is also known as Chiral Magnetic Effect (CME)

kharzeev et al Nucl Phys. A803

- Di-lepton and photon production are altered under strong electromagnetic field

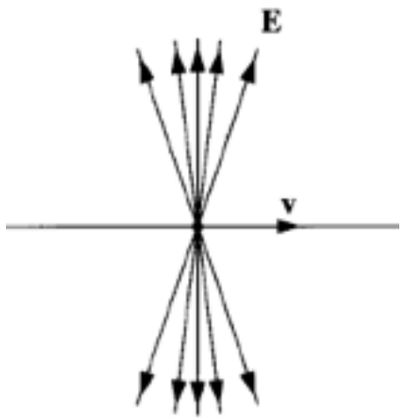
K Tuchin PRC 88.024910
PRC 83,017901

- It is also interesting (and important) to investigate the effect of electromagnetic field on the hydrodynamics evolution and on the flow harmonics

L G Pang et al PRC 93,044919
G Inghirami et al EPJC 12,659

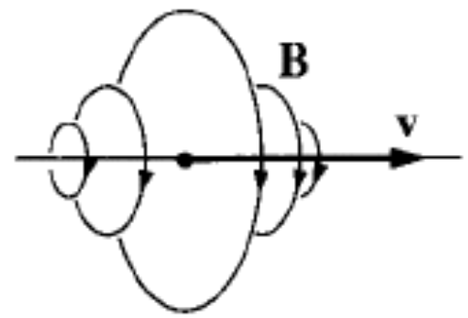
Field of a moving charge

Electric field



$$\vec{E}(\vec{r},t) = \frac{e}{4\pi} \frac{\vec{R}_i(1-v_i^2)}{R_i^3\left(1-\left[\vec{R}_i \times \vec{v_i}\right]^2/R_i^2\right)^{3/2}}$$

Magnetic field



$$\vec{B}\left(\vec{r},t\right) = \vec{v_i} \times \vec{E}_i$$

$$\hbar = c = \epsilon_0 = 1$$

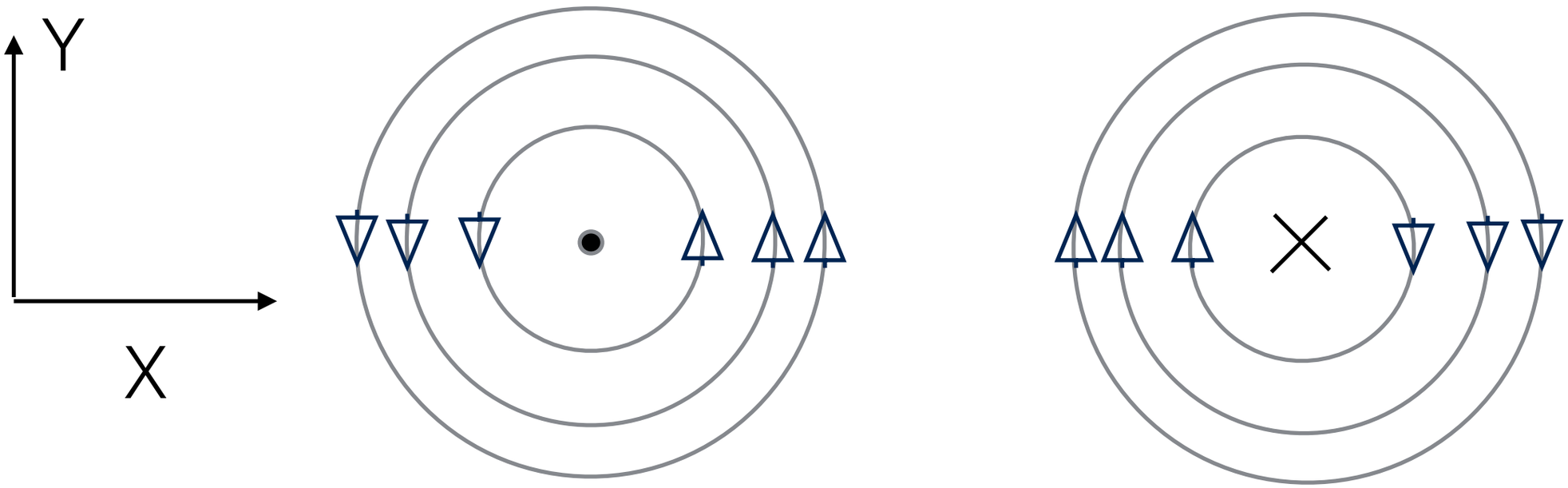
$$[eB] = \mathrm{GeV}^2$$

$$1\mathrm{GeV}^2 = 5.128 \times 10^{19}\mathrm{Gauss}$$

At relativistic speed

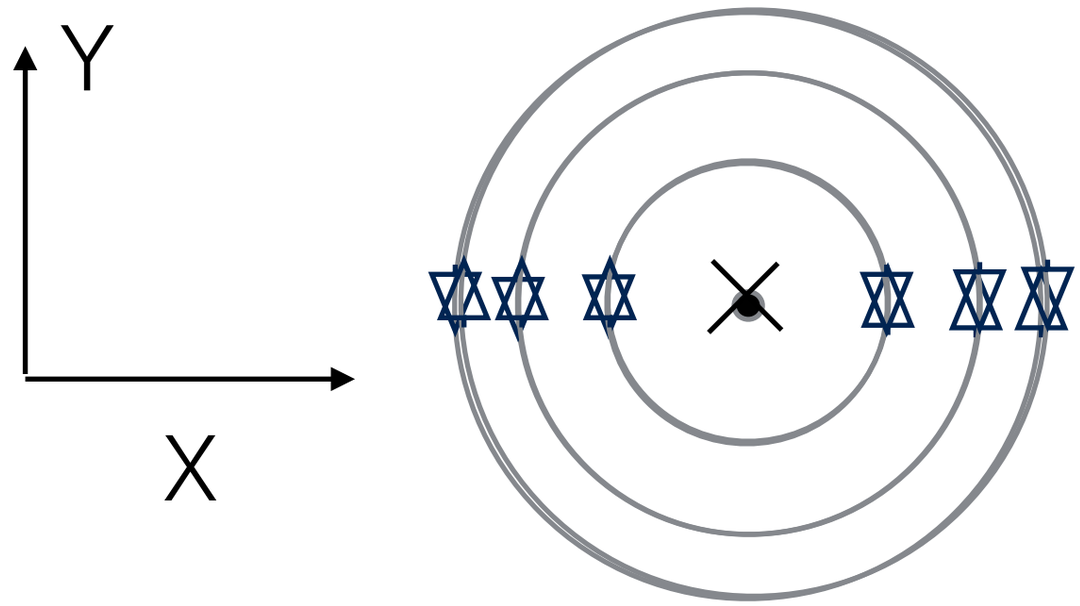
- Transverse components dominates
- Field rapidly fall off as we move away from charge along z

Magnetic field in transverse plane



Moving nucleus ->electric current along z ->magnetic field

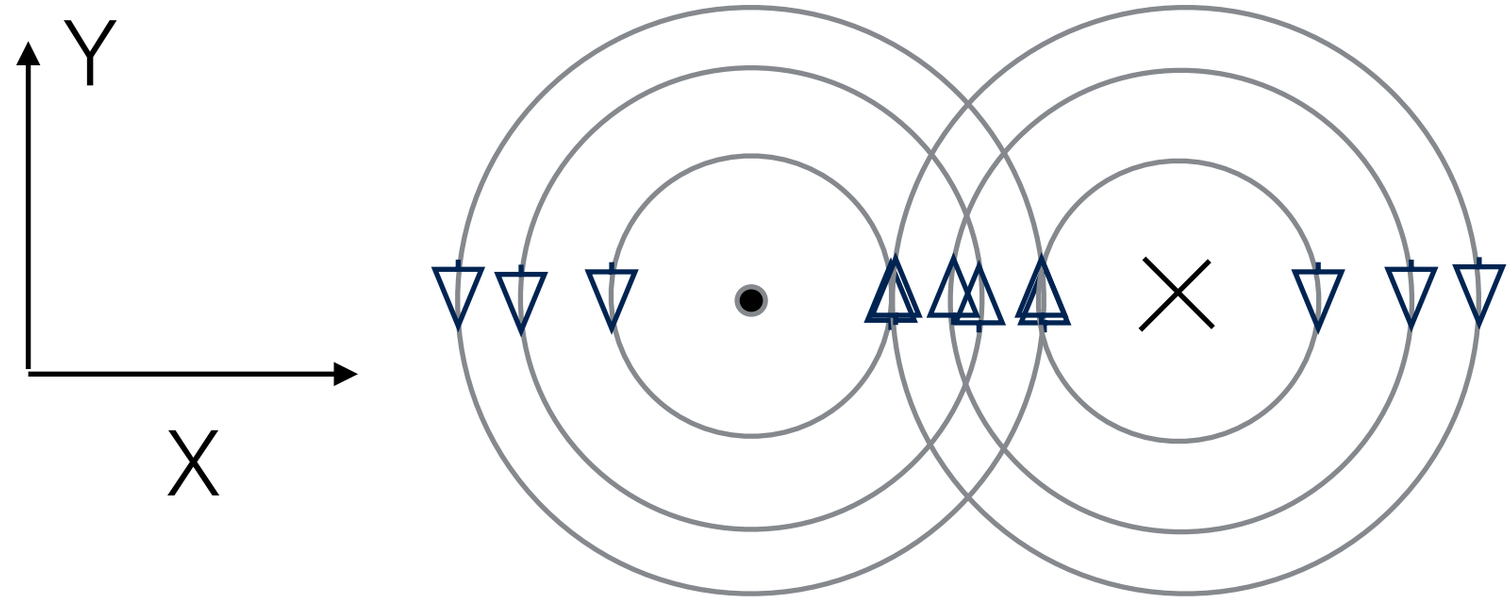
Magnetic field in transverse plane



Moving nucleus ->electric current along z ->magnetic field

Impact parameter ~ 0 -> vanishing mag field

Magnetic field in transverse plane



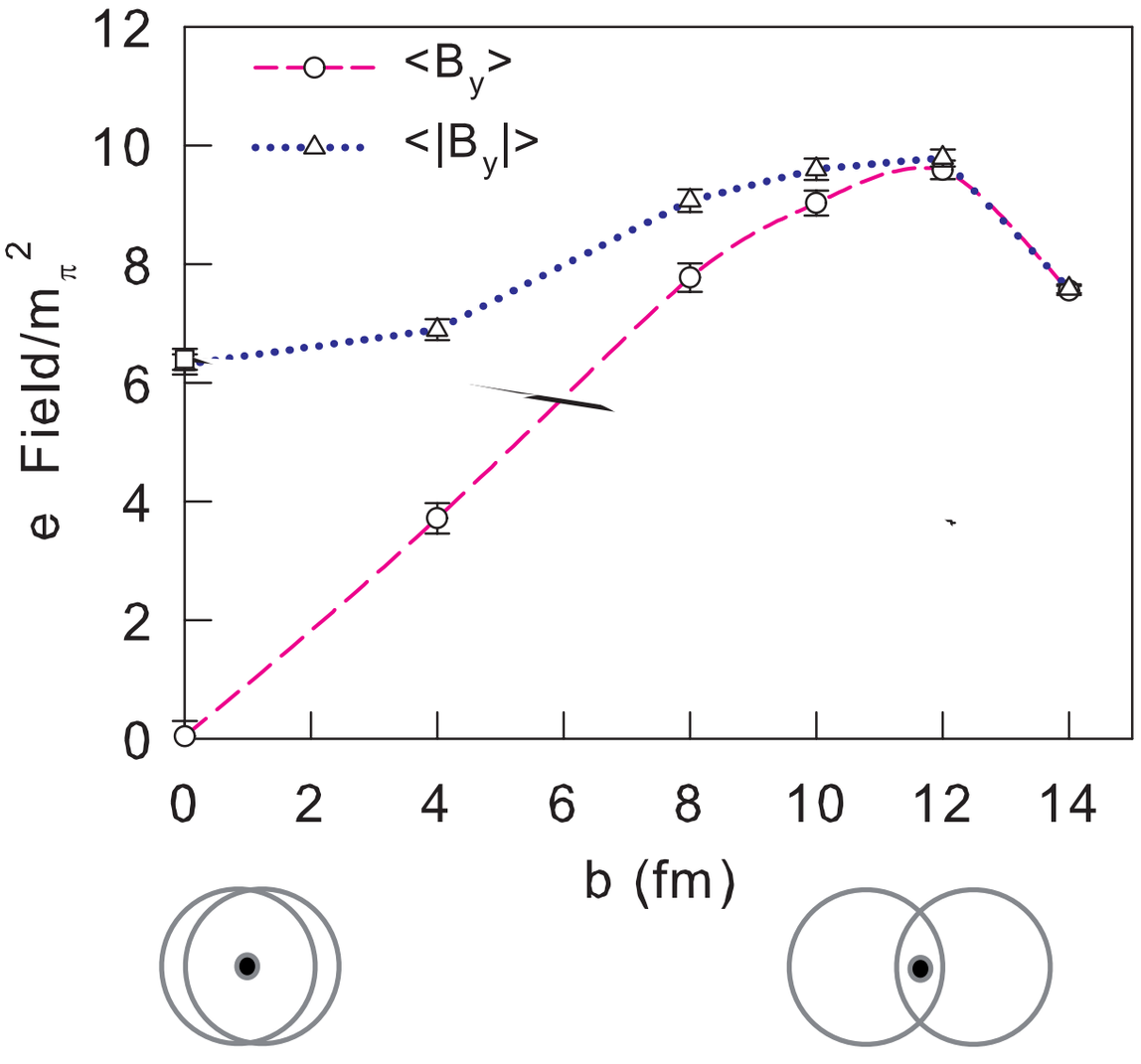
Moving nucleus ->electric current along z ->magnetic field

Impact parameter ~ 0 -> vanishing mag field

Non-zero impact parameter -> mag field along Y

Impact parameter dependence of magnetic field

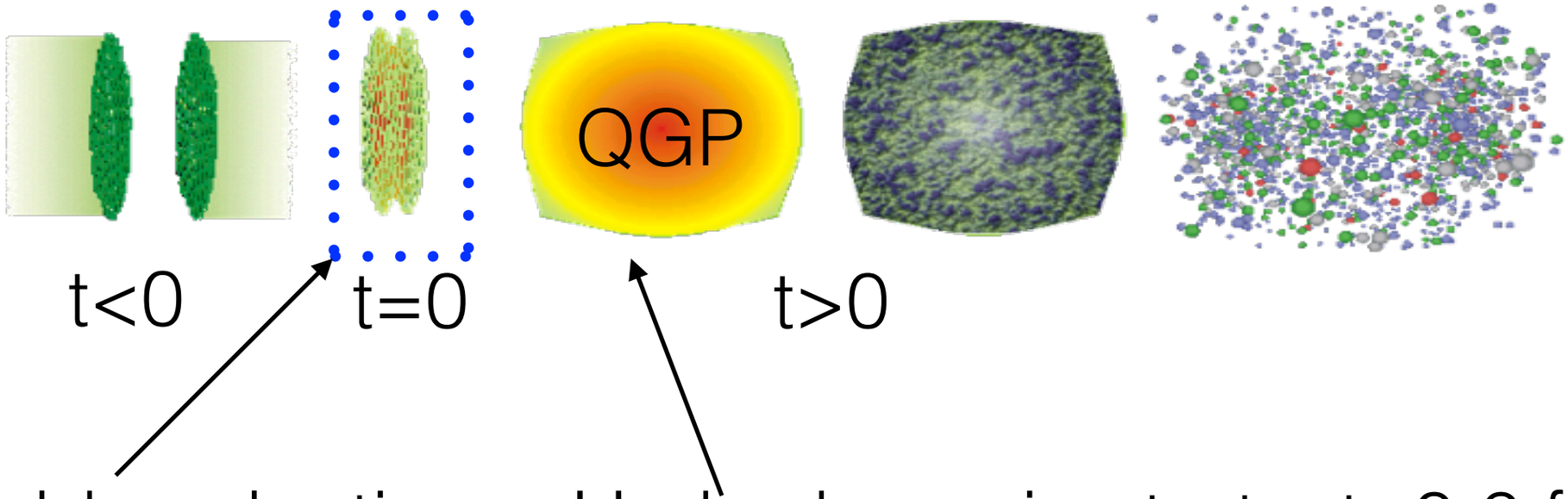
Au+Au @ 200 GeV

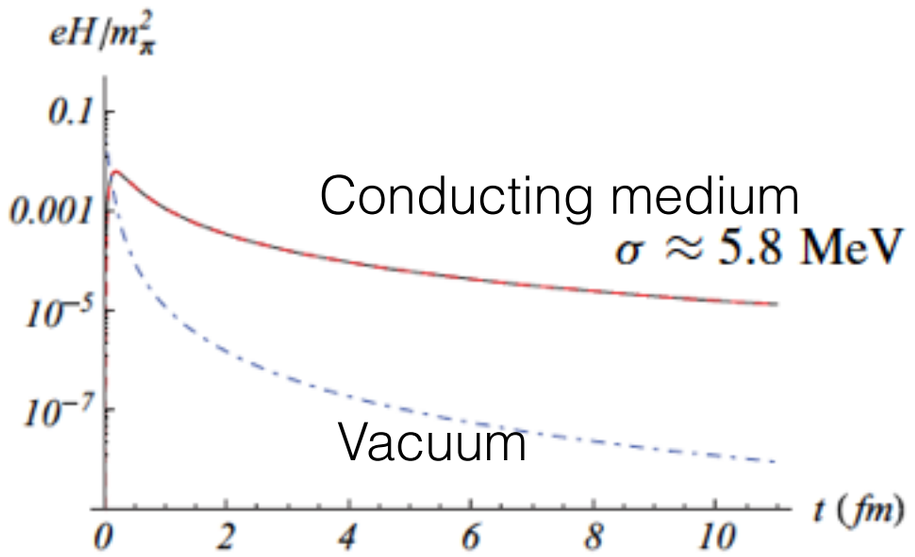


- $\langle B_y \rangle$ —> increases with b ~ 10 at b =10-12 fm
- central collisions $\langle B \rangle \neq \langle |B| \rangle$
- energy density~2.5 GeV/fm^3 !!

$$\varepsilon_{\mathrm{magfield}} = \frac{1}{2}(eB)^2$$

Temporal evolution of fields



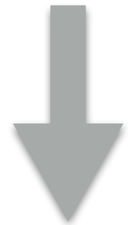


Tuchin, PRC 88,024911 (2013)

We consider
zero and
non-zero conductivity

Relativistic magnetohydrodynamics

fluid consists of charged particles moving under external electric/magnetic field experience force which changes fluid velocity



fluid constituents are electrically charged hence its motion generates electromagnetic field which changes the external field

In Magnetohydrodynamics we solve energy-momentum conservation and Maxwell equations self consistently

Formulation of Relativistic magnetohydrodynamics

conservation equations

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu F^{\mu\nu} = j^\mu$$

$$\nabla_\mu N^\mu = 0 \qquad \nabla_\mu \mathcal{F}^{\mu\nu} = 0$$

$$N^\mu = nu^\mu \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \mathcal{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$j^\mu = \rho u^\mu + \sigma^{\mu\beta}E_\beta$$

In case of vanishing net charge and isotropic system $\sigma^{\mu\beta} = \sigma g^{\mu\beta}$, $\rho \to 0$

$$j^\mu = \sigma E^\mu$$

Formulation of Relativistic ideal MHD

$$E^{\mu} = F^{\mu\nu} u_{\nu} = \gamma \left[\left(\vec{v} \cdot \vec{E} \right), \vec{E} + \left(\vec{v} \times \vec{B} \right) \right]$$

In the fluid rest frame

$$\vec{j} = \sigma \left[\vec{E} + \left(\vec{v} \times \vec{B} \right) \right]$$

when $\sigma \longrightarrow \infty$ but for finite $\vec{j}$

$$\vec{E} = -\vec{v} \times \vec{B}$$

Ideal MHD

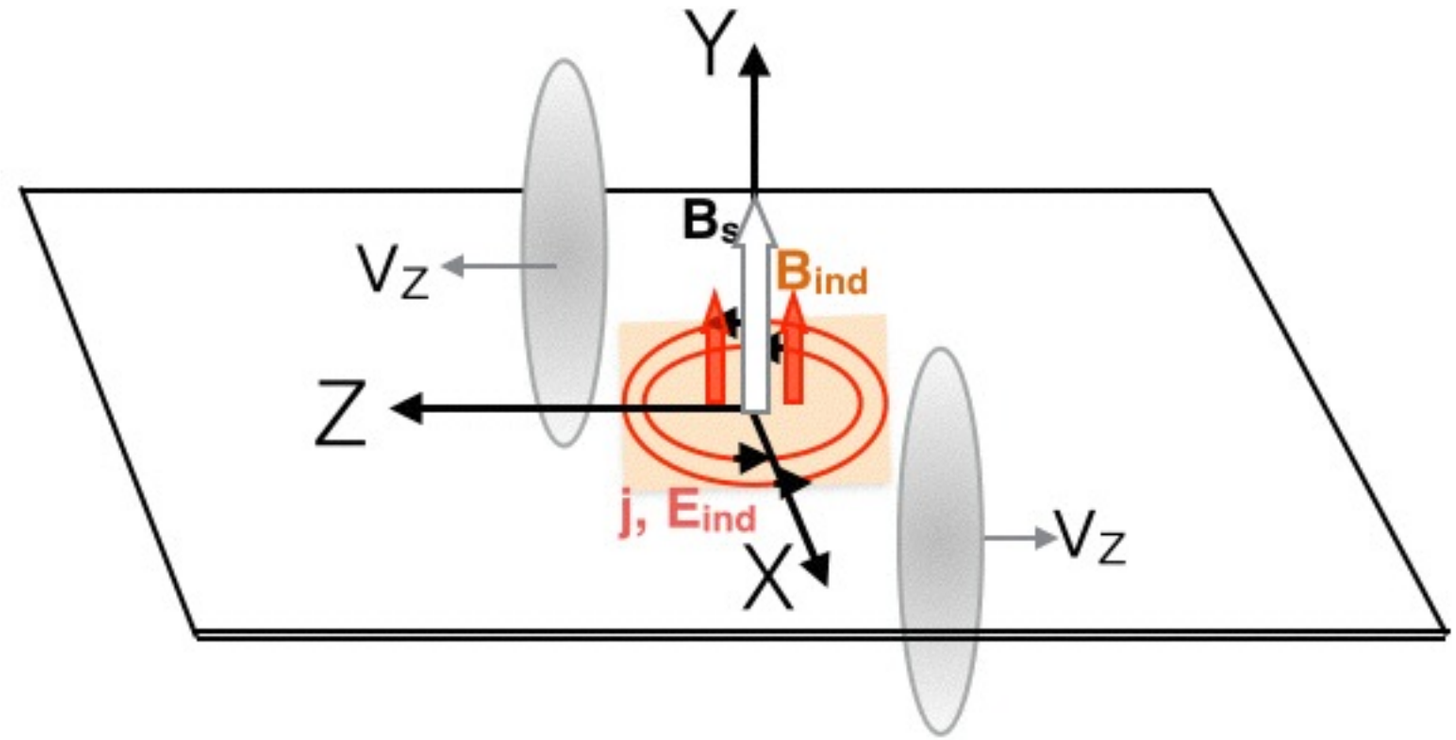
$$T^{\mu\nu}_{matter} = (\varepsilon + p)\, u^{\mu} u^{\nu} - p g^{\mu\nu}$$

$$T^{\mu\nu}_{field} = -F^{\mu\alpha} F^{k}_{\alpha} + \frac{1}{4} g^{\mu\nu} F_{\gamma\delta} F^{\gamma\delta} \quad \Longrightarrow \quad B^2 u^{\mu} u^{\nu} - \frac{1}{2} B^2 g^{\mu\nu} - B^2 b^{\mu} b^{\nu}$$

Energy momentum tensor for ideal MHD

$$T^{\mu\nu} = \left(\varepsilon + p + B^2 \right) u^{\mu} u^{\nu} - \left(p + \frac{1}{2} B^2 \right) g^{\mu\nu} - B^2 b^{\mu} b^{\nu}$$

Initial magnetic field



We numerically solve energy-momentum tensor on a space-time lattice.
The space-time evolution of magnetic field (in lab frame) is taken as parametrised

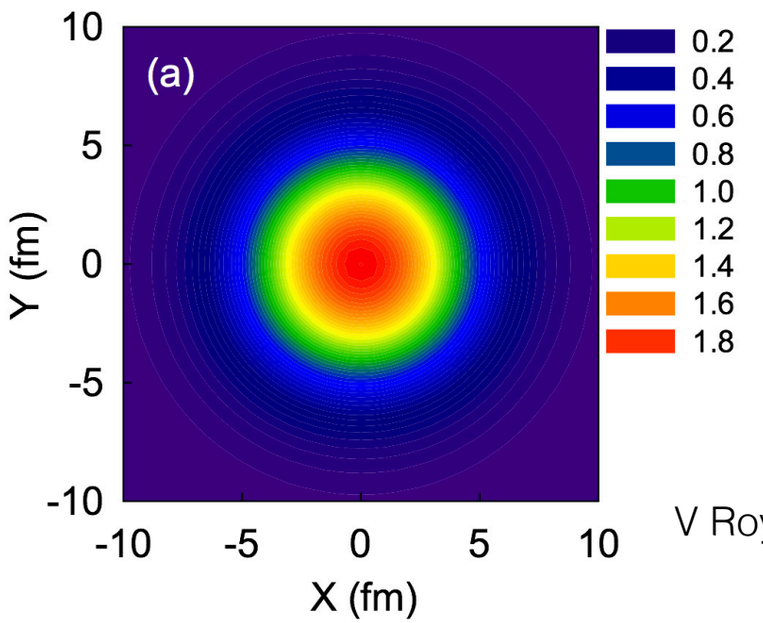
Fluid frame ⟶ $B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$ ⟵ Lab frame B

$$B^\mu B_\mu = -B^2$$

Lab frame: $\frac{eB_y(\tau,x,y)}{m_\pi^2} = f(\tau)\,g(x,y)$

Magnetic field in the transverse plane

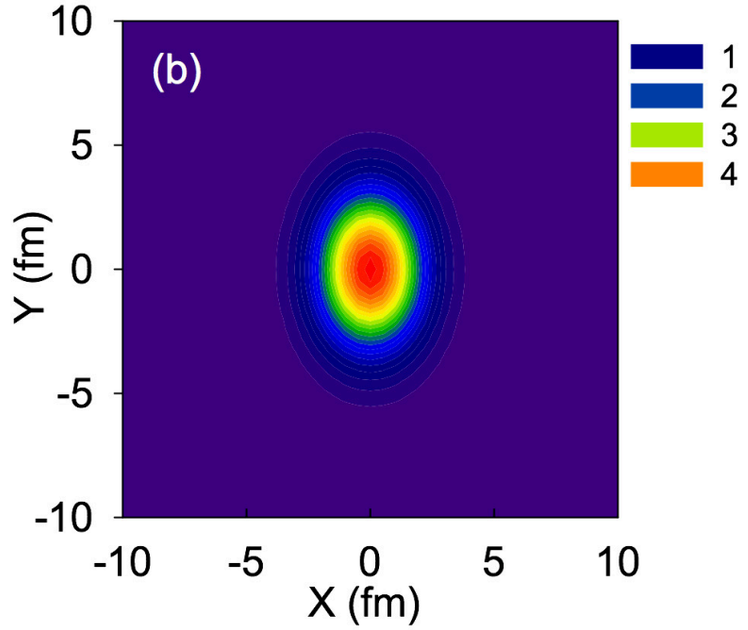
b=0 fm



V Roy et al PRC, 92,064902 (2015)

$\sigma_x = \sigma_y = 3.5\text{fm}$

b=10 fm

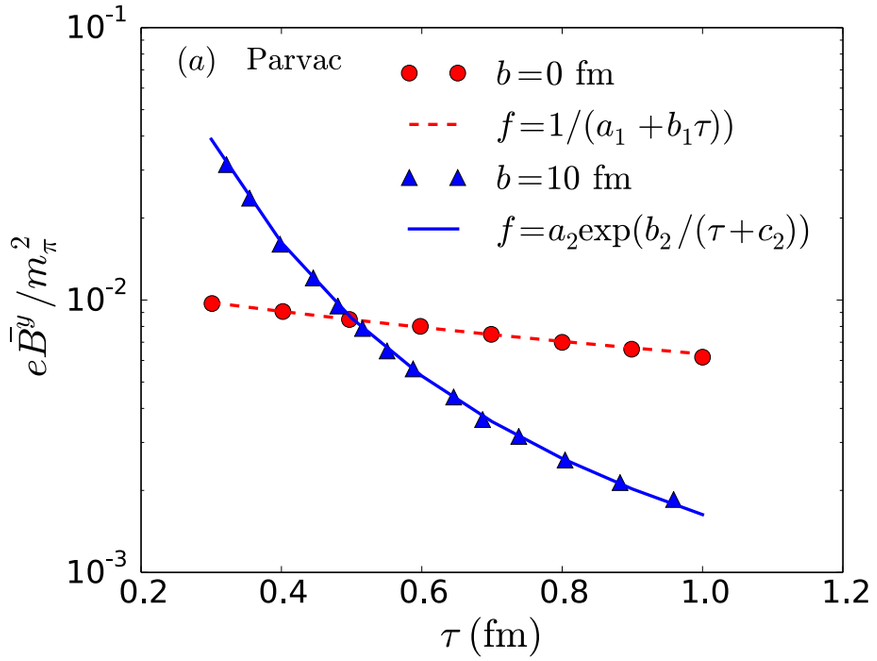


$\sigma_x = 1.5\text{fm} \;\; ; \;\; \sigma_y = 2.2\text{fm}$

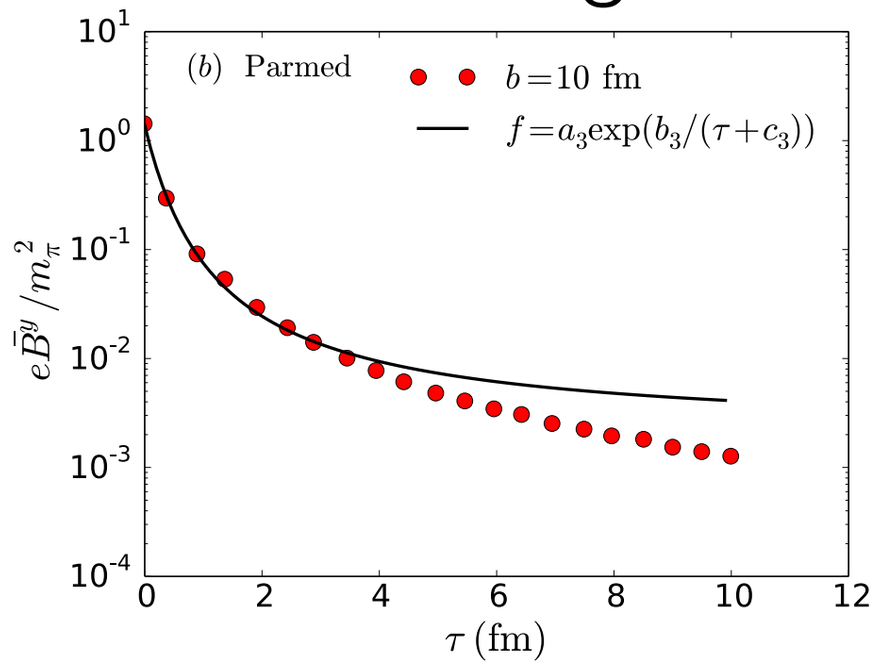
$$g\left(x,y\right) = exp\left(-\frac{\left(x-x_0\right)^2}{4\sigma_x^2} - \frac{\left(y-y_0\right)^2}{4\sigma_y^2}\right)$$

Time evolution of magnetic field

Vacuum

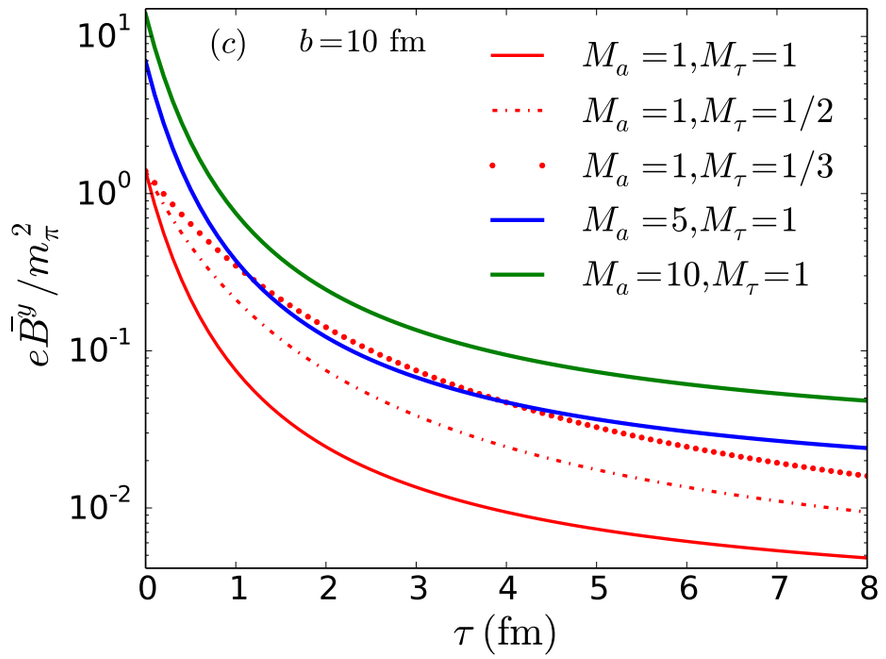


Conducting QGP



$$f(\tau) = M_a a_3 e^{(b_3/(M_\tau \tau + c_3))}$$

$$M_a = 1 \Longrightarrow \frac{eB}{m_\pi^2} = 1 \text{ at } \tau = 0$$



Other inputs to RMHD model

Initial energy density : $\varepsilon(x,y,b) = \varepsilon_0 \left[x_h N_{part}(x,y,b) + (1-x_h) N_{coll}(x,y,b)\right]$

Equation of State : Lattice+HRG (s95p-PCE165-v0)

Thermalisation time : 0.6 fm

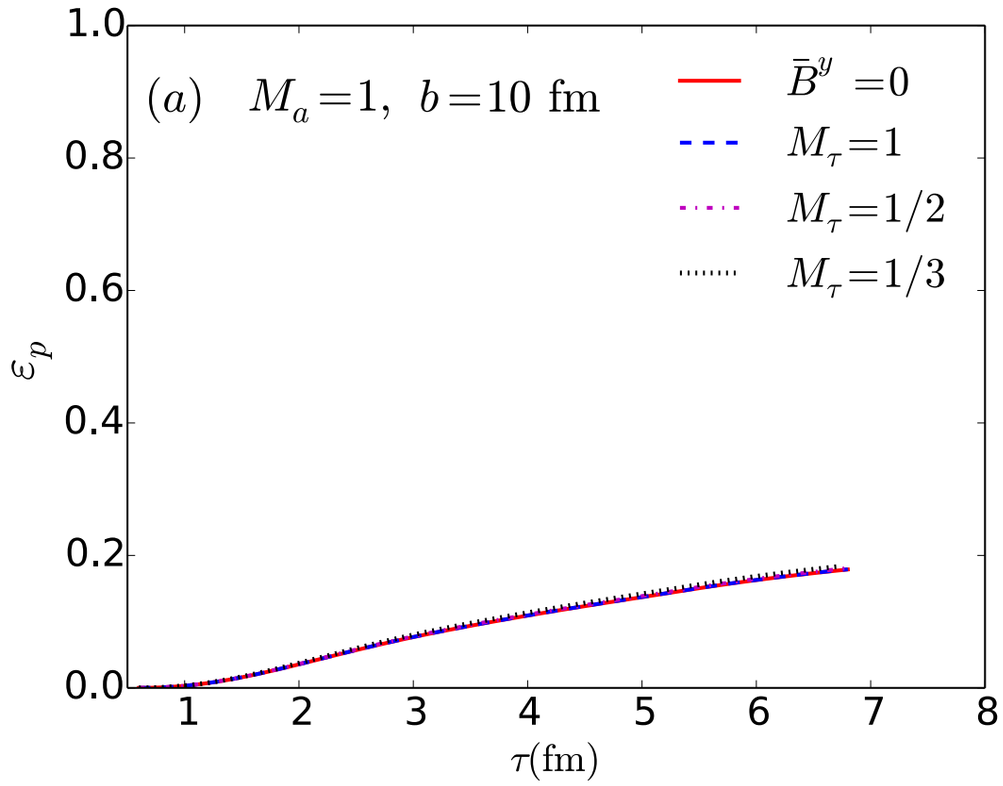
Initial fluid velocity : $v_x(x,y) = v_y(x,y) = 0$

Freeze out temperature: 130 MeV

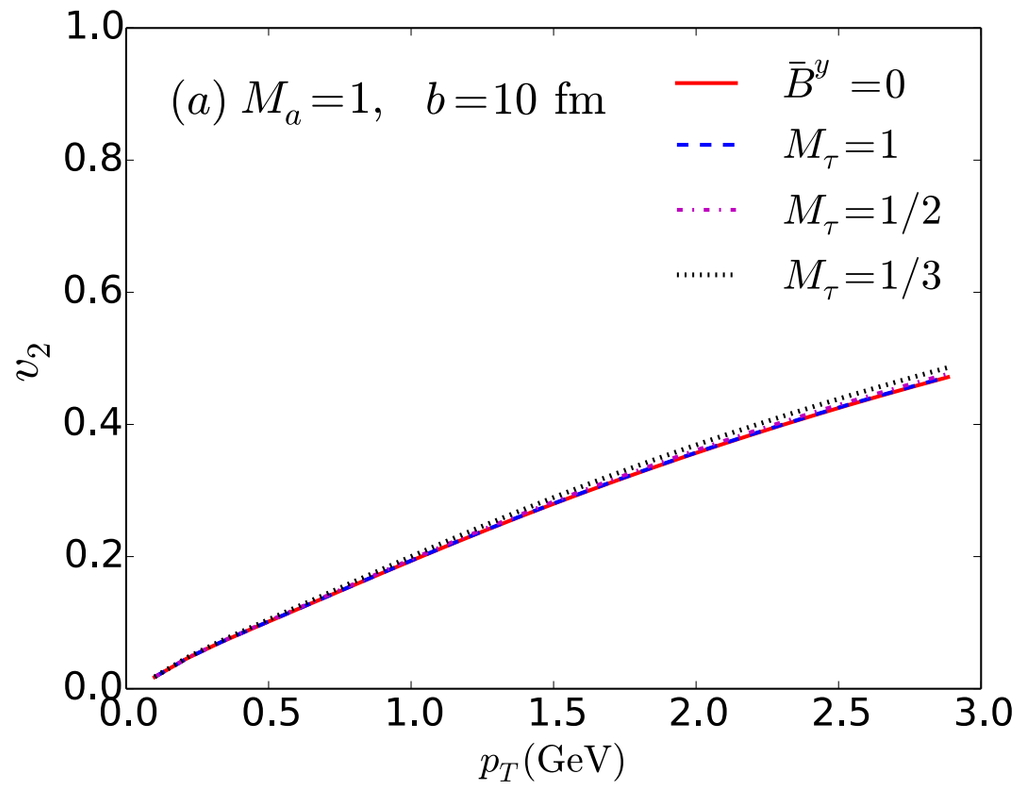
Multidimensional Flux-Corrected algorithm SHASTA is used for the numerical solution of conservation equations

Momentum anisotropy and v_2

$$eB = m_\pi^2$$



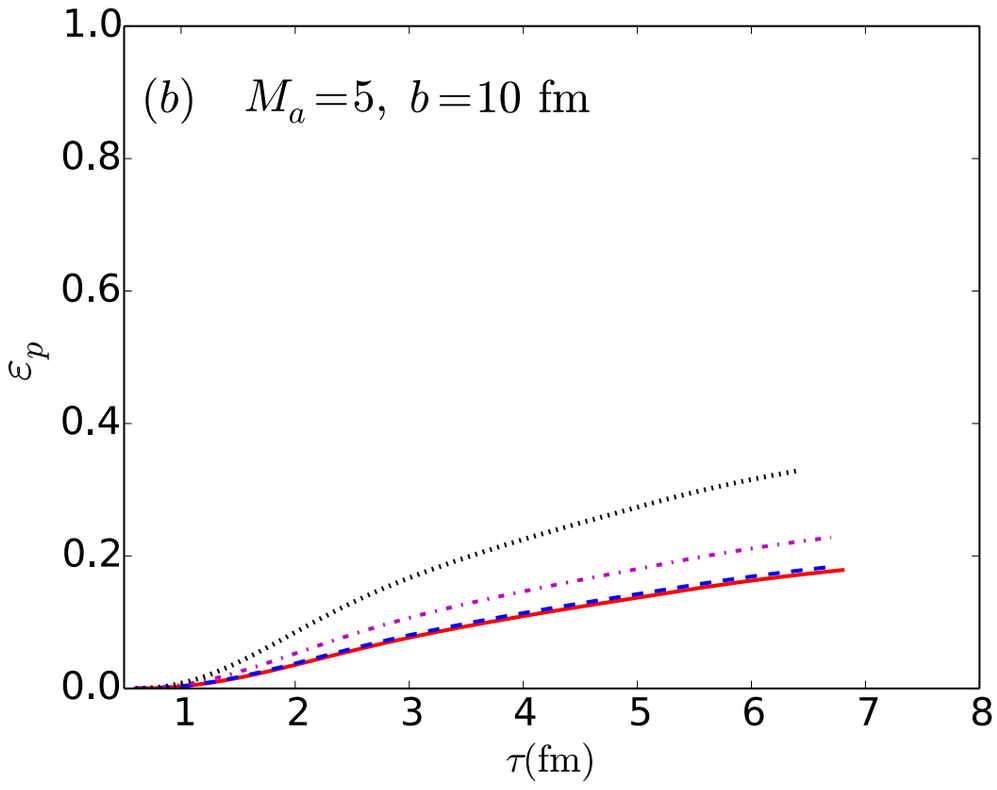
$$\varepsilon_p = \frac{< T^{xx} - T^{yy} >}{< T^{xx} + T^{yy} >}$$



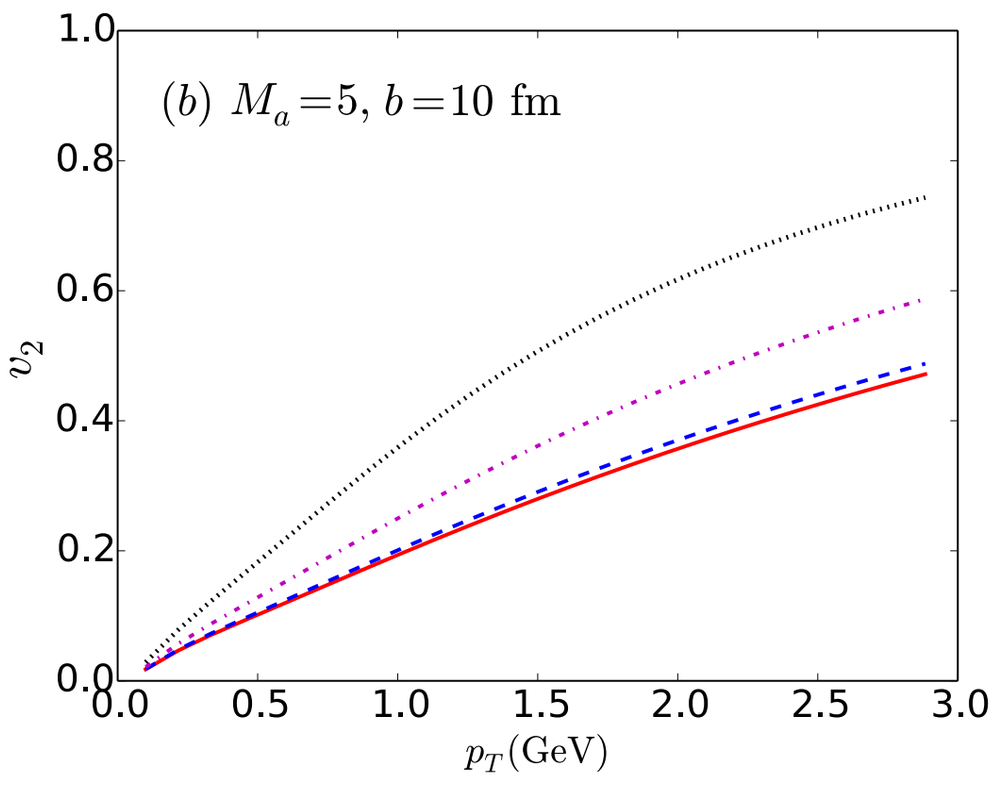
$$v_2 =< cos(2\phi) >$$

Momentum anisotropy and v_2

$$eB = 5m_\pi^2$$



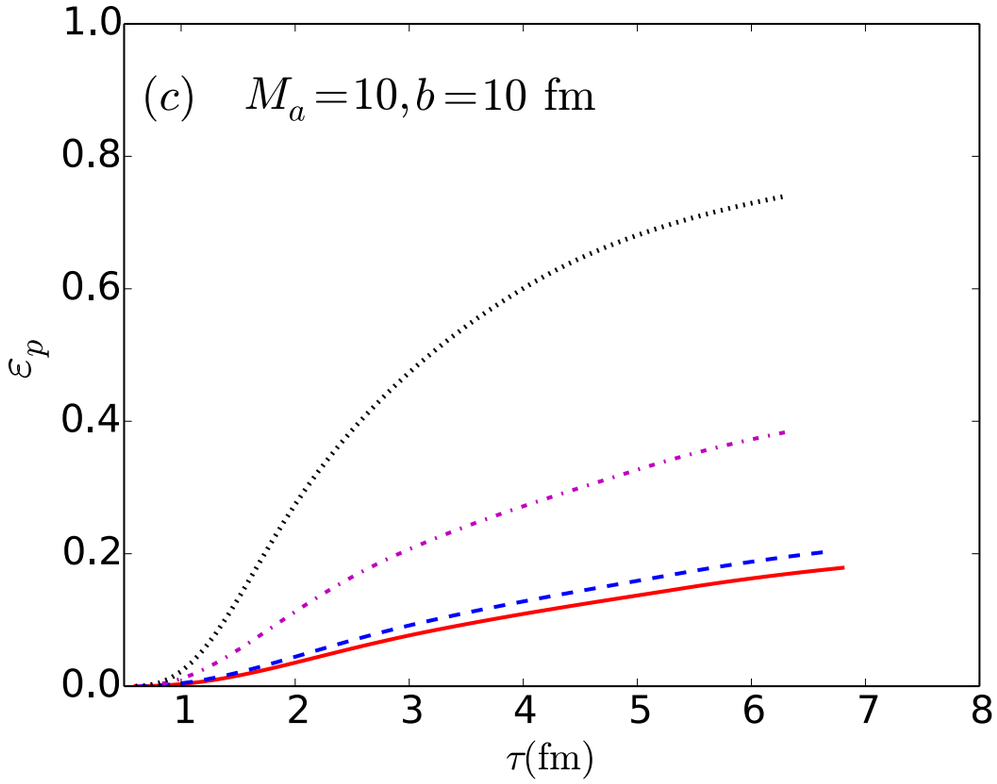
Momentum anisotropy



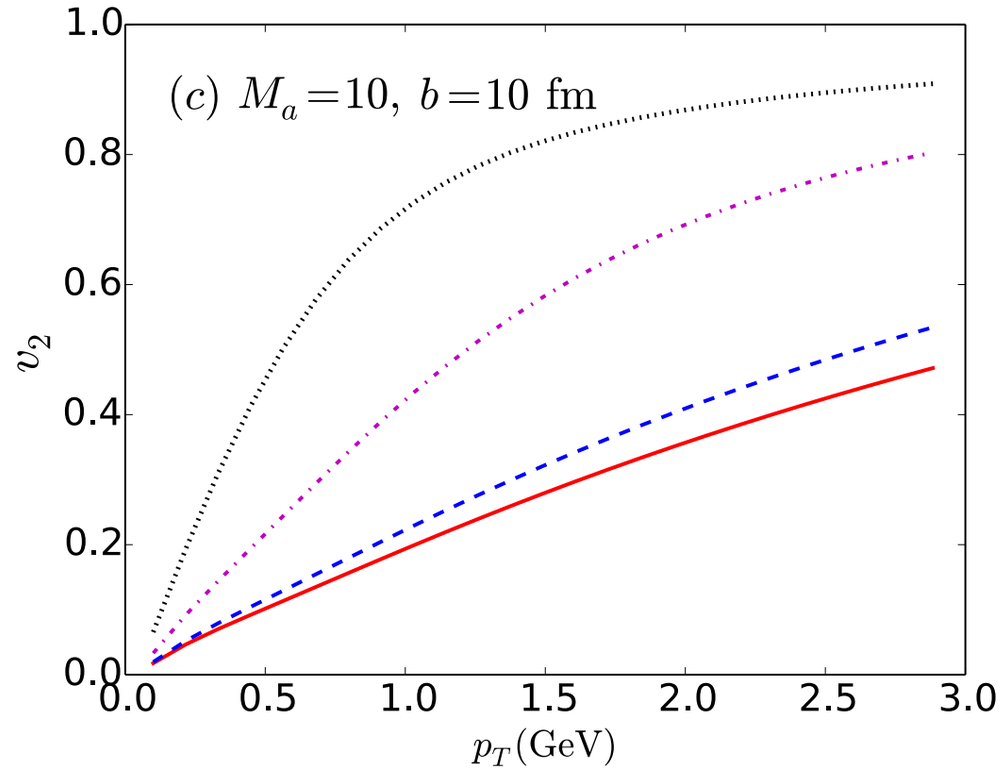
Elliptic flow

Momentum anisotropy and v_2

$$eB = 10m_\pi^2$$



Momentum anisotropy



Elliptic flow

$$T^{\tau x} \Longrightarrow \partial_y \left[\tilde{B}^y \tilde{B}^x\right] - \partial_x \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{x\,2}\right]$$

$$T^{\tau y} \Longrightarrow \partial_x \left[\tilde{B}^y \tilde{B}^x\right] - \partial_y \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{y\,2}\right]$$

Summary & outlook

- For realistic values of magnetic field we observe enhanced elliptic flow within reduced MHD formalism.

- In order to extract transport coefficient of QGP more precisely it is important to consider the effect of magnetic field on flow harmonics.

- A self consistent Relativistic non-ideal magnetohydrodynamics code is needed in order to more accurately study the effect of magnetic field.

Extra slides

Finding velocity in each time step

$$
\begin{aligned}
E :=& T^{\tau\tau} = w\gamma^2 - p_B - B^2(b^\tau)^2 \ , \\
M^x :=& T^{\tau x} = w\gamma^2 v^x - B^2 b^\tau b^x \ , \\
M^y :=& T^{\tau y} = w\gamma^2 v^y - B^2 b^\tau b^y \ .
\end{aligned}
$$

$$
\begin{aligned}
E^{'} :=& \ E + B^2(b^\tau)^2 = w\gamma^2 - p_B \ , \\
M^{x^{'}} :=& \ M^x + B^2 b^\tau b^x = w\gamma^2 v^x \ , \\
M^{y^{'}} :=& \ M_y + B^2 b^\tau b^y = w\gamma^2 v^y \ ,
\end{aligned}
$$

$$
\begin{aligned}
M^{'} &= \left(E^{'} + p_B\right) v \ , \\
\varepsilon &= E^{'} - M^{'} v - \frac{B^2}{2} \ ,
\end{aligned}
$$

$$
v = \left. \frac{M^{'}}{E^{'} + p(\varepsilon)} \right|_{\varepsilon = E^{'} - M^{'} v - B^2/2} \ .
$$

Energy momentum conservation

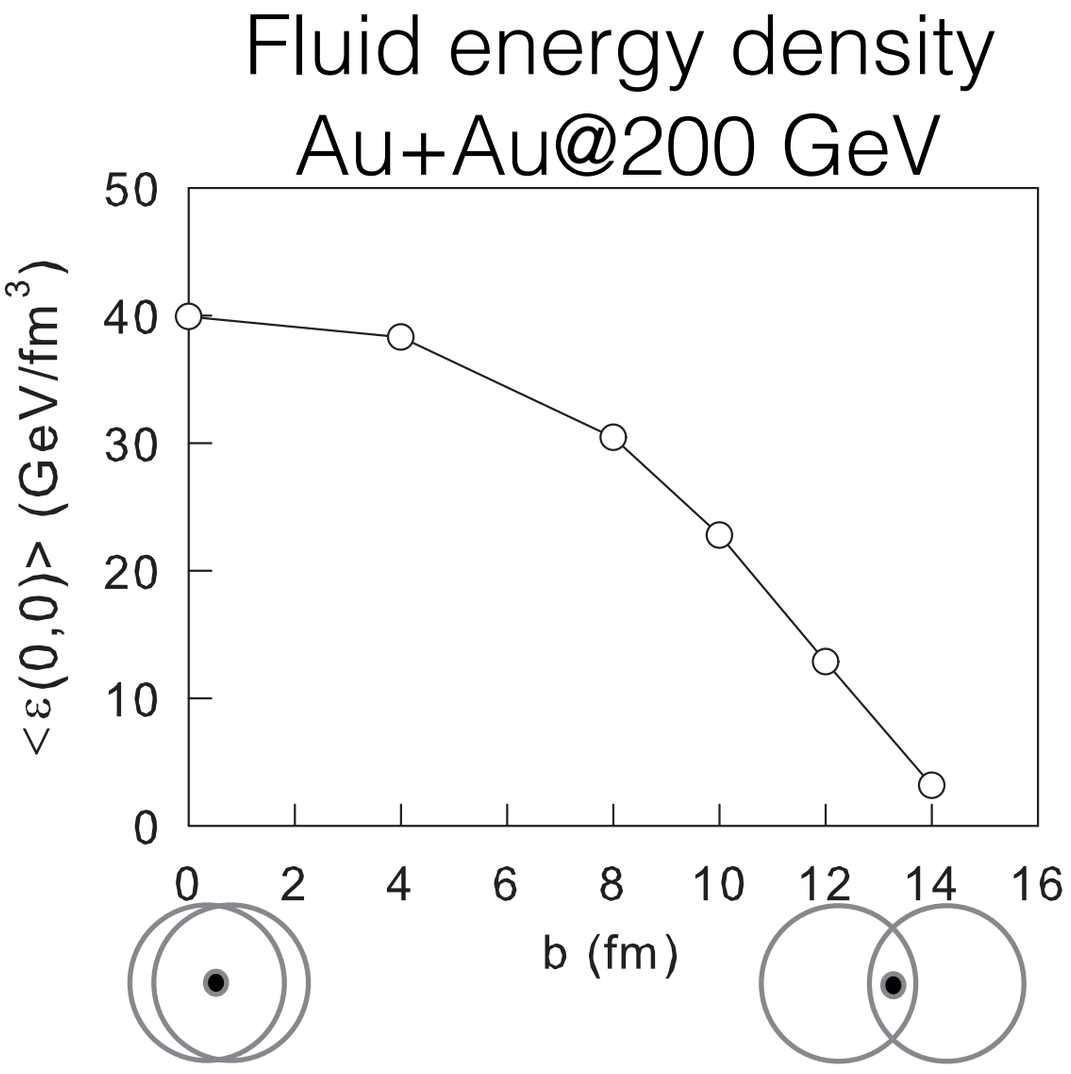
$$\partial_\tau \left(\tilde{T}^{\tau\tau}\right) + \partial_x \left(\tilde{T}^{\tau\tau}\tilde{v_x}\right) + \partial_y \left(\tilde{T}^{\tau\tau}\tilde{v_y}\right) = -\left(p + \frac{B^2}{2}\right)$$

$$\begin{aligned}
\partial_\tau \left(\tilde{T}^{\tau x}\right) + \partial_x \left[\left(\tilde{T}^{\tau x} + \tilde{B}^\tau \tilde{B}^x\right) v_x\right] + \partial_y \left[\left(\tilde{T}^{\tau x} + \tilde{B}^\tau \tilde{B}^x\right) v_y\right] &= \partial_y \left[\tilde{B}^y \tilde{B}^x\right] - \partial_x \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{x\,2}\right] \\
\partial_\tau \left(\tilde{T}^{\tau y}\right) + \partial_x \left[\left(\tilde{T}^{\tau y} + \tilde{B}^\tau \tilde{B}^y\right) v_x\right] + \partial_y \left[\left(\tilde{T}^{\tau y} + \tilde{B}^\tau \tilde{B}^y\right) v_y\right] &= \partial_x \left[\tilde{B}^y \tilde{B}^x\right] - \partial_y \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{y\,2}\right]
\end{aligned}$$

- $T^{\tau\tau} = w\gamma^2 - p_B - B^\tau B^\tau$
- $T^{x\tau} = w\gamma^2 v^x - B^\tau B^x$
- $T^{y\tau} = w\gamma^2 v^y - B^\tau B^y$

$w = \left(\varepsilon + p + \frac{B^2}{2}\right)$ and $p_B = \left(p + \frac{B^2}{2}\right)$.

Energy density of fluid



$$\varepsilon_{\text{fluid}} = k \sum_{nucleon} e^{-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_g^2}}$$

for initial time ~0.5 fm

$$k = 6$$
$$\sigma_g = 0.5fm$$

$$\sigma = \frac{\varepsilon_{\text{magfield}}}{\varepsilon_{\text{fluid}}}$$

$$\langle\sigma\rangle = \langle\frac{\varepsilon_{magfield}}{\varepsilon_{fluid}}\rangle \neq \langle\sigma\rangle = \frac{\langle\varepsilon_{magfield}\rangle}{\langle\varepsilon_{fluid}\rangle}$$

Ratio of field to fluid energy density

$$\frac{\text{energy of field}}{\text{energy of fluid}}$$

$\sigma >\sim 1$ —> **B** Important

$\sigma \ll 1$ —> **B** may be neglected

Consensus : since mag field decay quickly

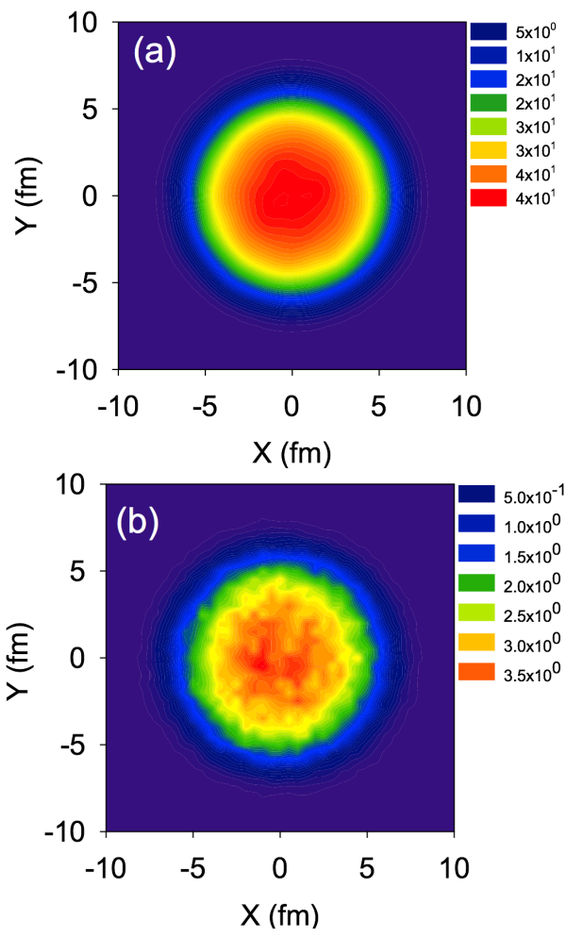
$$\langle\sigma\rangle = \frac{\langle\varepsilon_{magfield}\rangle}{\langle\varepsilon_{fluid}\rangle} \quad <<1$$

Reality: not always true because of the fluctuating IC

$$\langle\sigma\rangle = \langle\frac{\varepsilon_{magfield}}{\varepsilon_{fluid}}\rangle \quad \sim 1$$

Event average energy

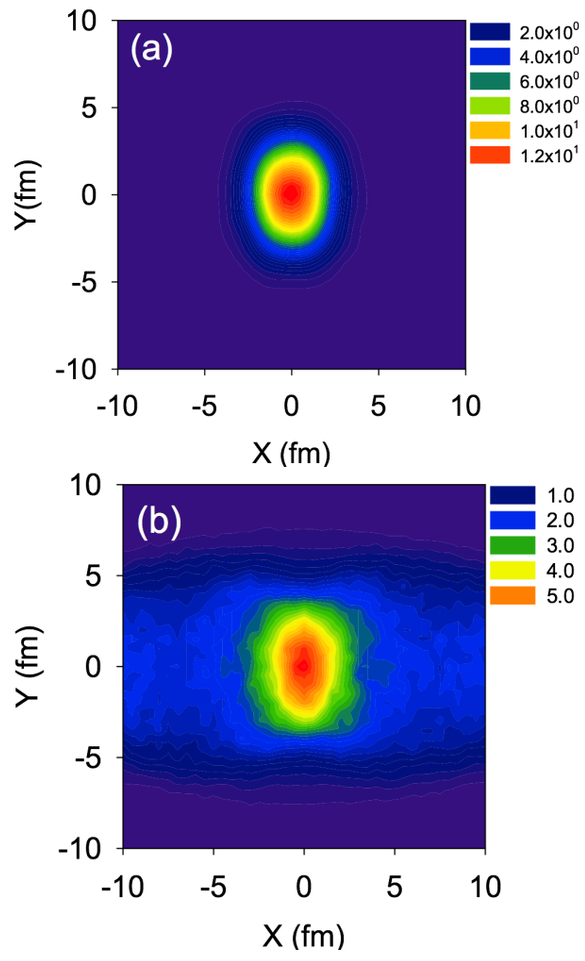
b=0 fm



Fluid energy

$$\frac{(eB_y)^2}{2}$$

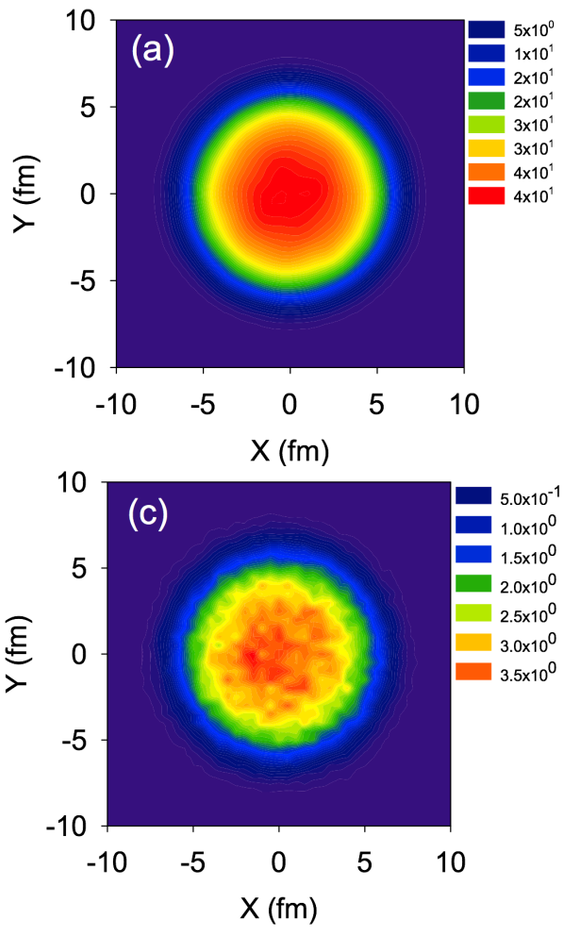
b=12 fm



V Roy et al PRC, 92,064902 (2015)

Event average energy

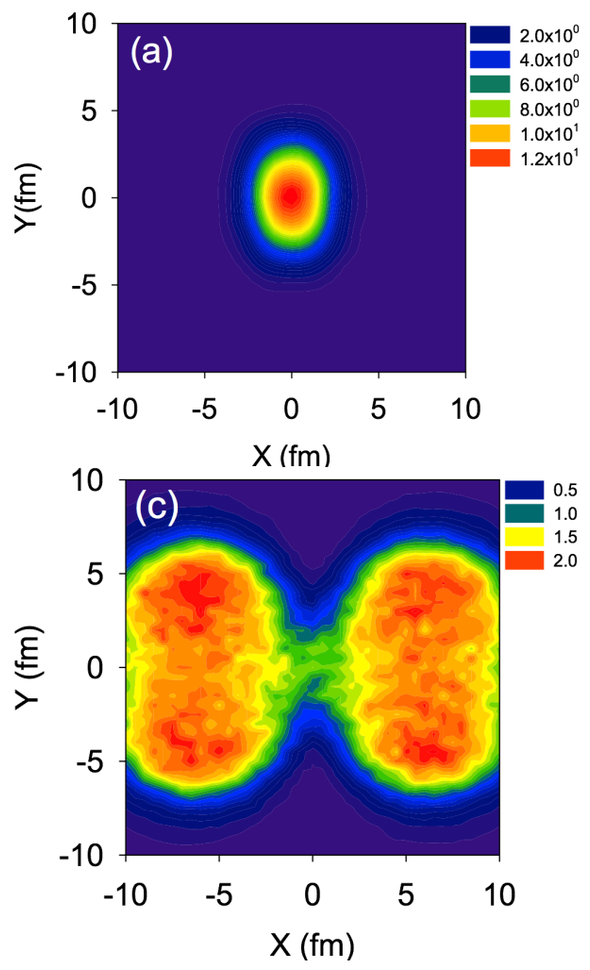
b=0 fm



Fluid energy

$$\frac{(eB_x)^2}{2}$$

b=12 fm



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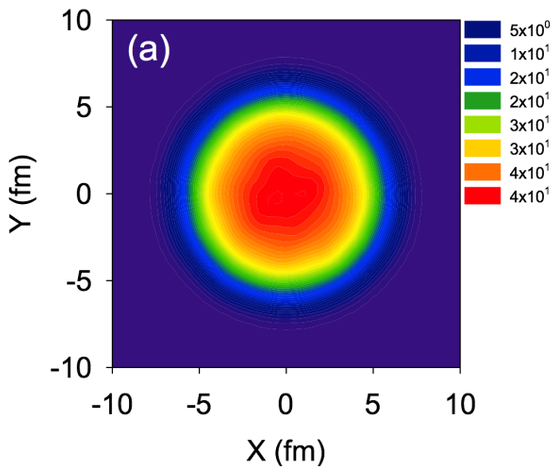
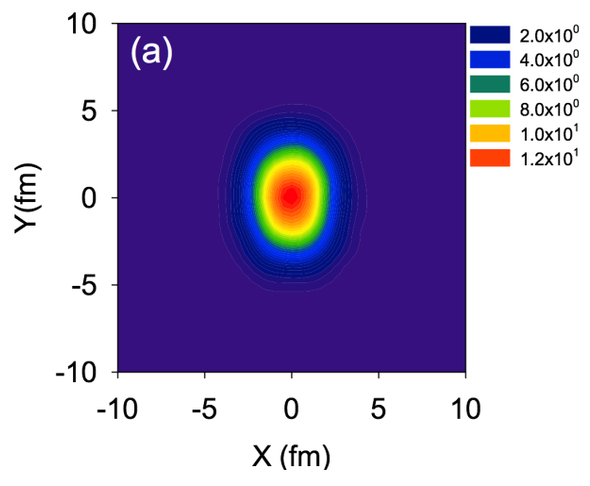
Event average sigma

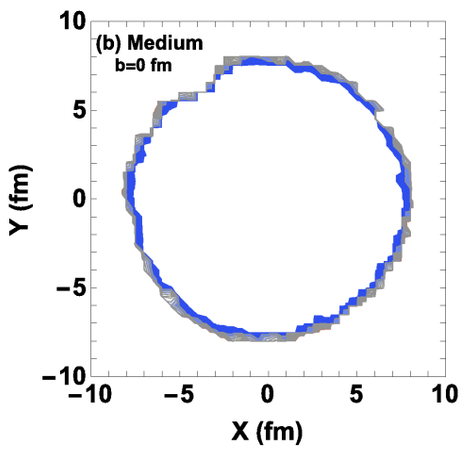
b=0 fm



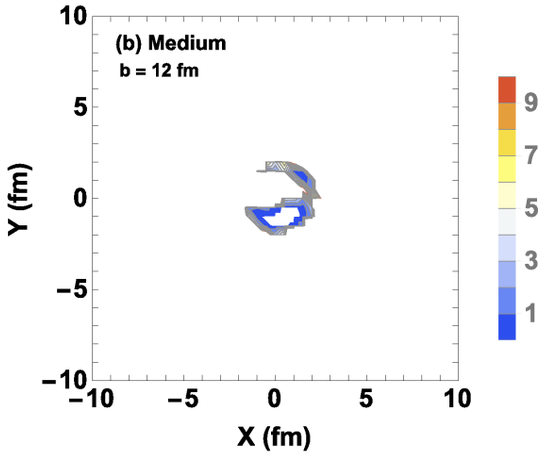
Fluid energy

b=12 fm





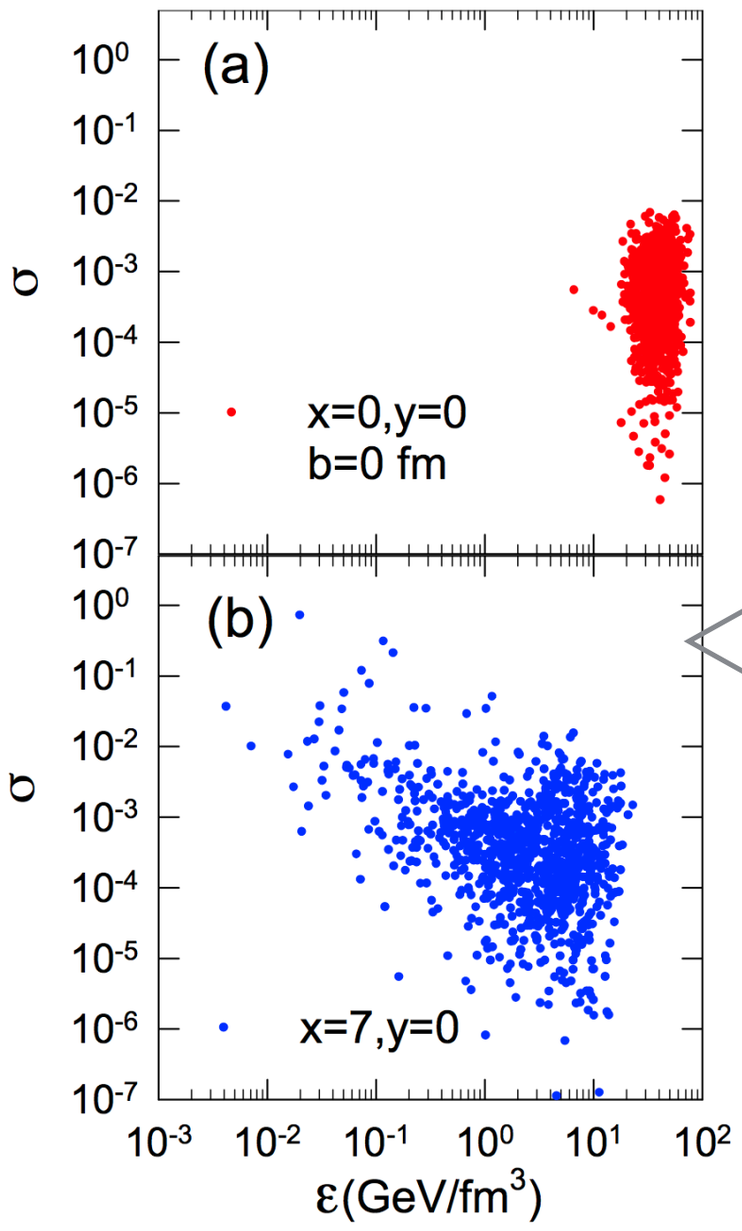
$$0.01 < \langle \sigma\,(x,y) \rangle < 10$$

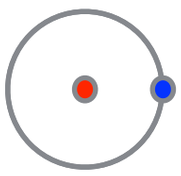


V Roy et al PRC, 92,064902 (2015)

Event-by-event distribution of sigma

Central, b=0 fm

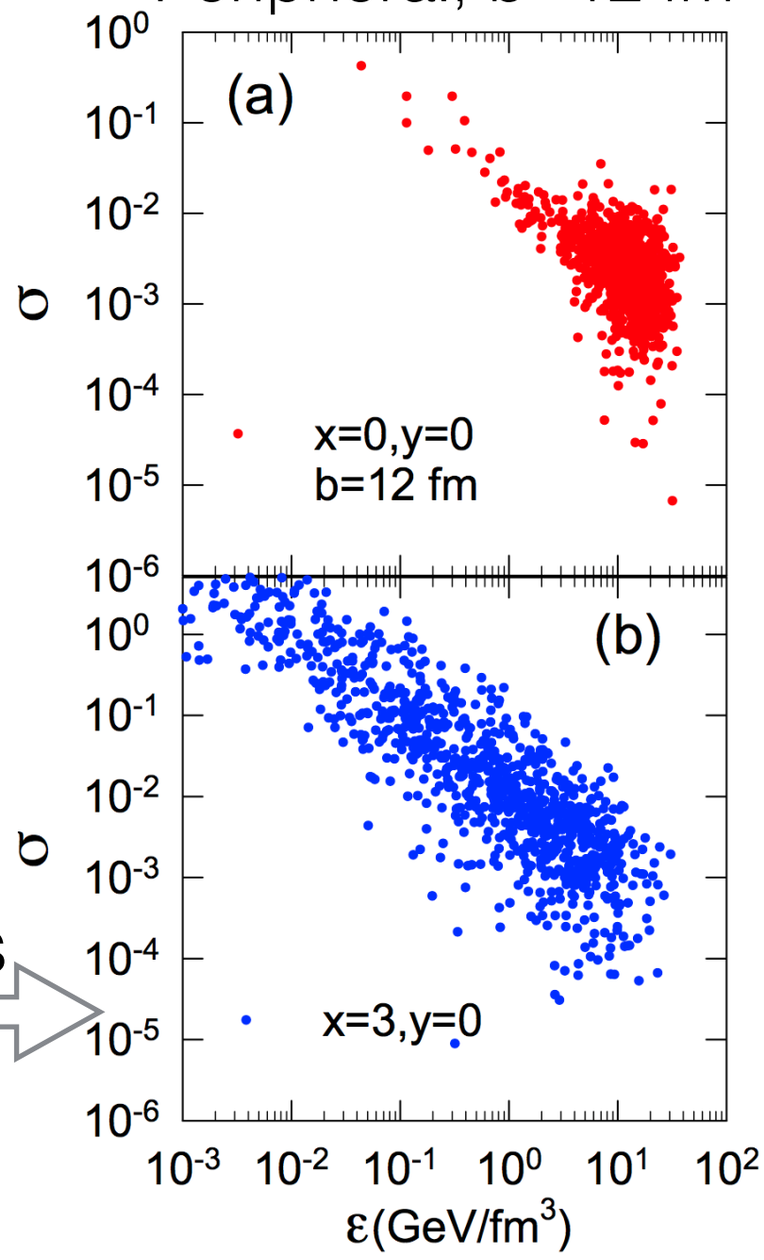


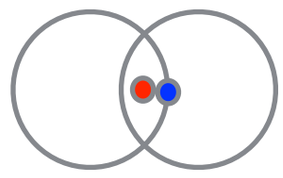


Central collisions
$\sigma \ll 1$

Peripheral collisions
$\sigma \sim 1$

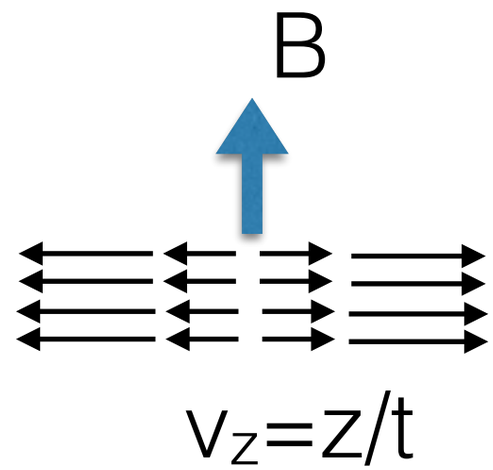
Peripheral, b=12 fm





Magneto-hydrodynamics in one dimension

Most simple possible model



- Bjorken flow,
- ideal fluid (zero shear, bulk viscosity and zero electrical resistivity),
- Transverse magnetic field,
- Constant speed of sound,
- temporal evolution of magnetic field:

  (a) “frozen in flux theorem” $B(\tau) = B_0 \frac{s}{s_0} = B_0 \frac{\tau_0}{\tau}$

  or

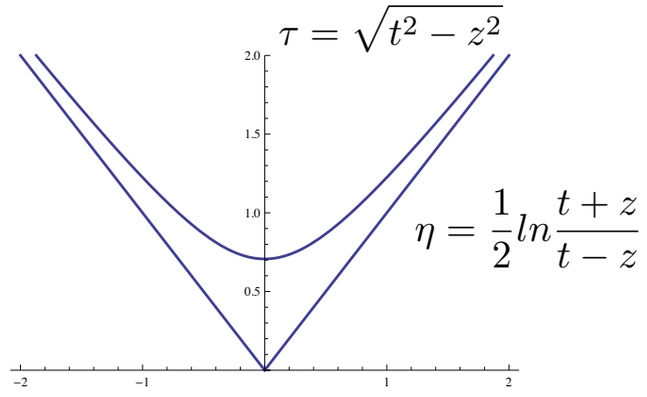
  (b) “Power law decay”. $B(\tau) = B_0 \left(\frac{\tau_0}{\tau} \right)^a$

Bjorken hydro in a nutshell

Energy-momentum tensor

$$T^{\mu\nu} = (e+p)\,u^\mu u^\nu - p g^{\mu\nu}$$

$$\partial_\nu T^{\mu\nu} = 0$$



$$u^\mu = (cosh\eta, 0, 0, sinh\eta)$$

$\|$ to four velocity

$$u_\mu \partial_\nu T^{\mu\nu} = 0$$

$$\partial_\tau e + \frac{e+p}{\tau} = 0$$

Evolution of energy density

$\perp$ to four velocity

$$(\eta_{\mu\nu} - u_\mu u_\nu)\partial_\alpha T^{\alpha\nu} = 0$$

$$\partial_\eta p = 0$$

Euler equation

Bjorken magneto-hydro in a nutshell

Energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{field}$$

$$T^{\mu\nu} = \left(e + p + B^2\right) u^\mu u^\nu - \left(p + \frac{B^2}{2}\right) g^{\mu\nu} - B^\mu B^\nu$$

$\parallel$ to four velocity

$$u_\mu \partial_\nu T^{\mu\nu} = 0$$

$$\partial_\tau \left(e + \frac{B^2}{2}\right) + \frac{e + p + B^2}{\tau} = 0$$

Evolution of energy density

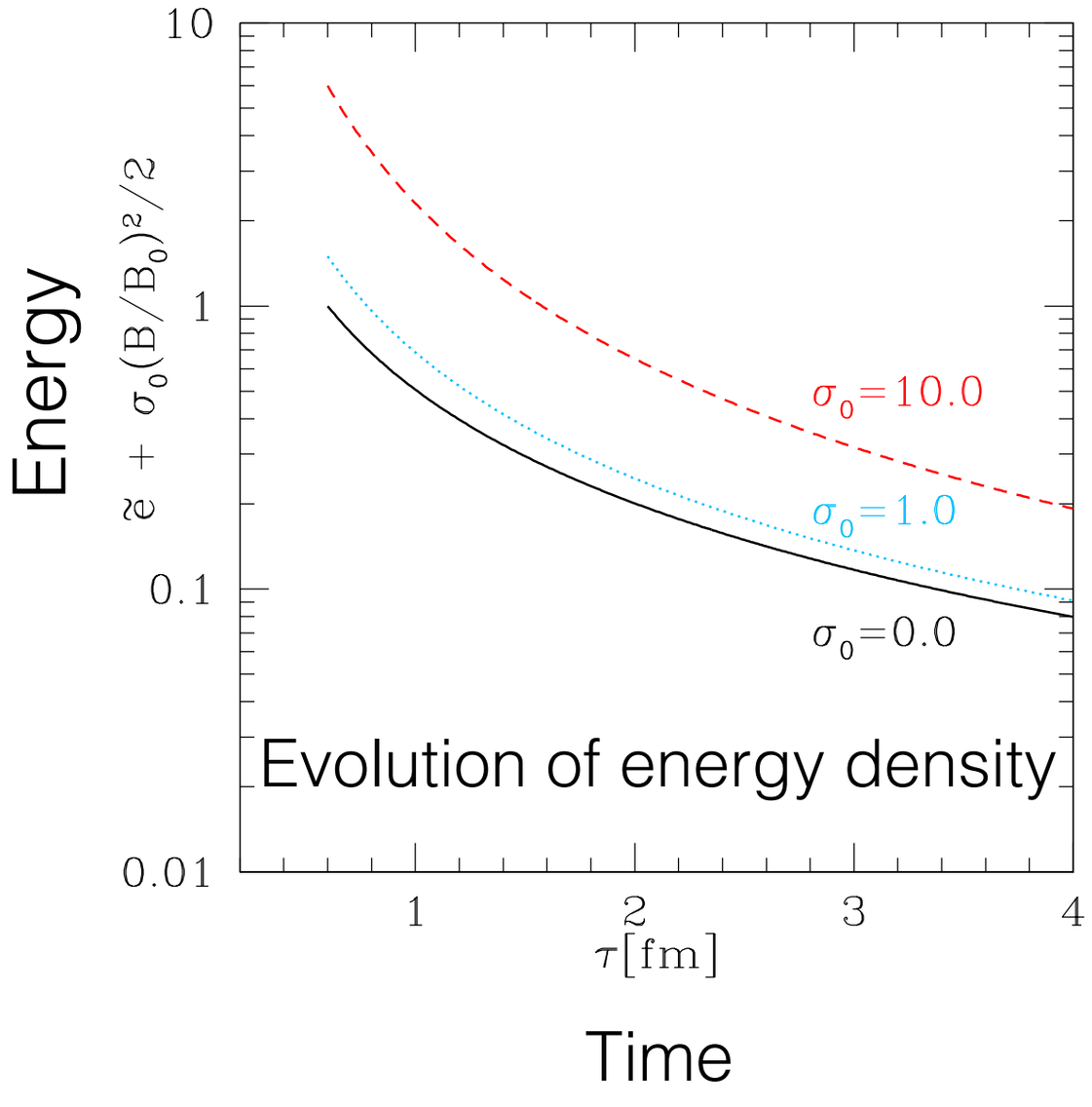
$\perp$ to four velocity

$$(\eta_{\mu\nu} - u_\mu u_\nu)\partial_\alpha T^{\alpha\nu} = 0$$

$$\frac{\partial}{\partial \eta}\left(p + \frac{1}{2}B^2\right) = 0$$

Euler equation

Magneto-hydrodynamics in one dimension: Frozen in flux



V Roy et al PLB, 750 , 45-52 (2015)

$$B(\tau) = B_0 \frac{s}{s_0} = B_0 \frac{\tau_0}{\tau}$$

$$\partial_\tau \left[\tilde{e} + \frac{\sigma_0}{2} \tilde{e}^{2/(1+c_s^2)} \right] + \frac{\tilde{e} + \tilde{p} + \sigma_0 \tilde{e}^{2/(1+c_s^2)}}{\tau} = 0$$

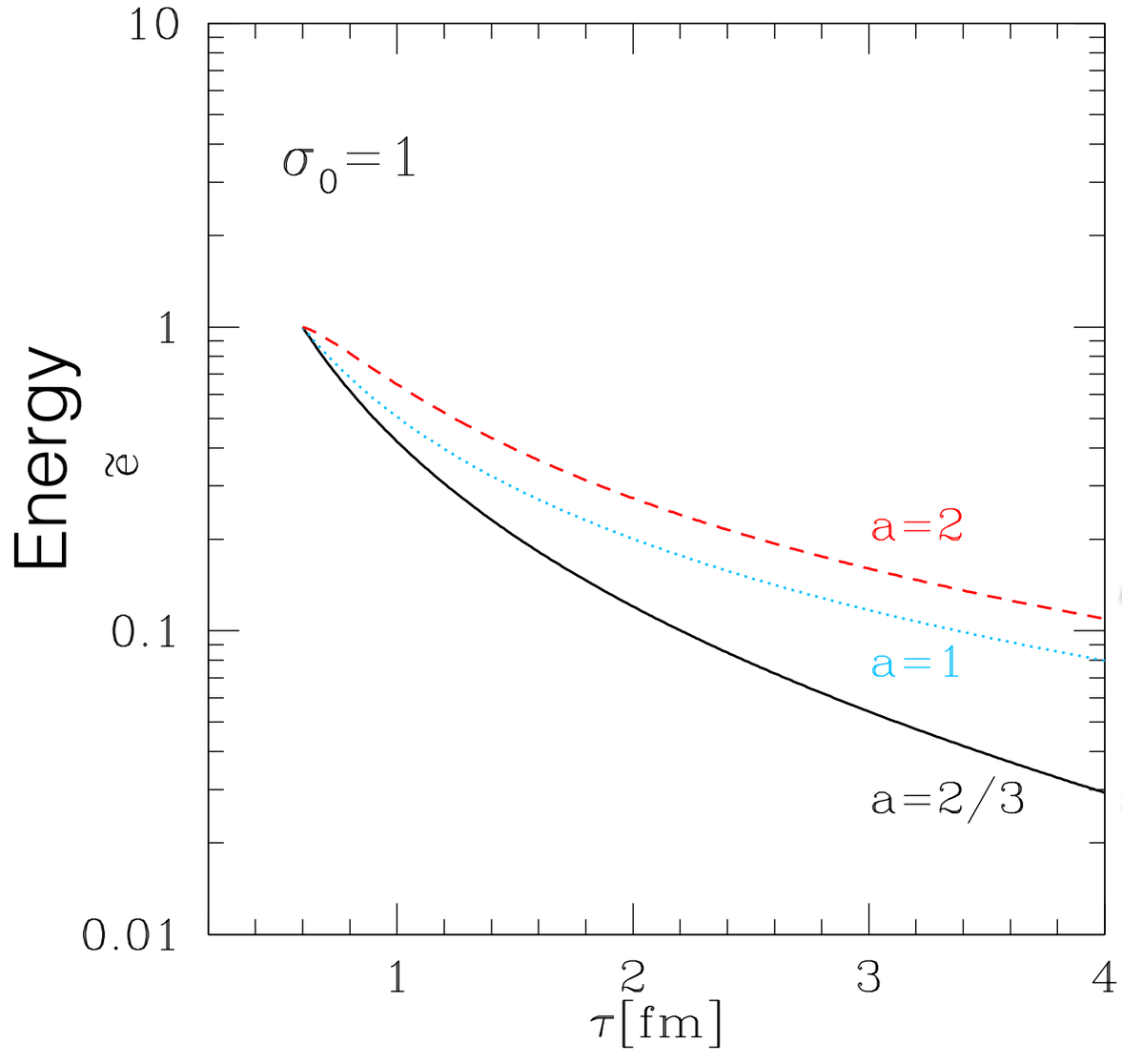
$$\tilde{e} = \frac{e}{e_0} \qquad \sigma_0 = \frac{B_0^2}{e_0}$$

$$\partial_\tau \, \tilde{e} = -(1 + c_s^2) \frac{\tilde{e}}{\tau}$$

Same as Bjorken expansion without mag field !!

Solution : $\tilde{e} = \left(\frac{\tau_0}{\tau} \right)^{1+c_s^2}$

Power law decay of magnetic fields



V Roy et al PLB, 750 , 45-52 (2015)

$$B(\tau) = B_0 \left(\frac{\tau_0}{\tau}\right)^a$$

$a > 1$ Quickly decaying B

Fluid gain energy

$a = 1$ Frozen flux, 0 resistivity

Same as without B

$a < 1$ Slowly decaying B

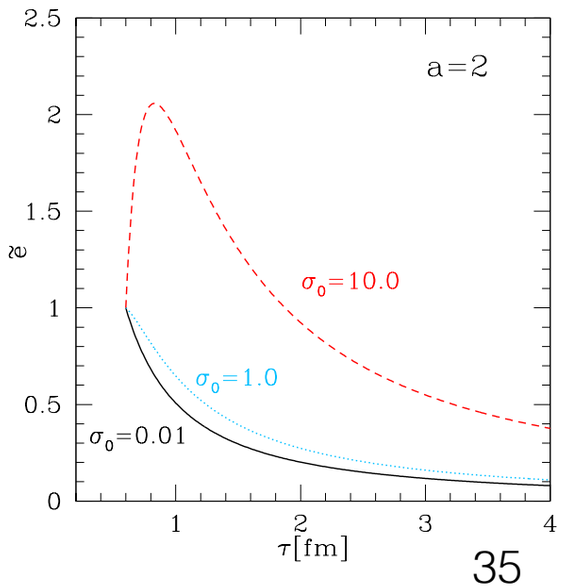
Fluid losses energy

Solution for power law decay of magnetic field

$$\partial_\tau \left[\tilde{e} + \frac{\sigma_0}{2} \left(\frac{\tau_0}{\tau} \right)^{2a} \right] + \left(1 + c_s^2 \right) \frac{\tilde{e}}{\tau} + \frac{\sigma_0}{\tau} \left(\frac{\tau_0}{\tau} \right)^{2a} = 0$$

The solution is

$$\tilde{e}(\tau) = \left(\frac{\tau_0}{\tau} \right)^{1+c_s^2} + \sigma_0 \frac{1-a}{1+c_s^2-2a} \left[\left(\frac{\tau_0}{\tau} \right)^{1+c_s^2} - \left(\frac{\tau_0}{\tau} \right)^{2a} \right]$$



Equations of magneto-hydro

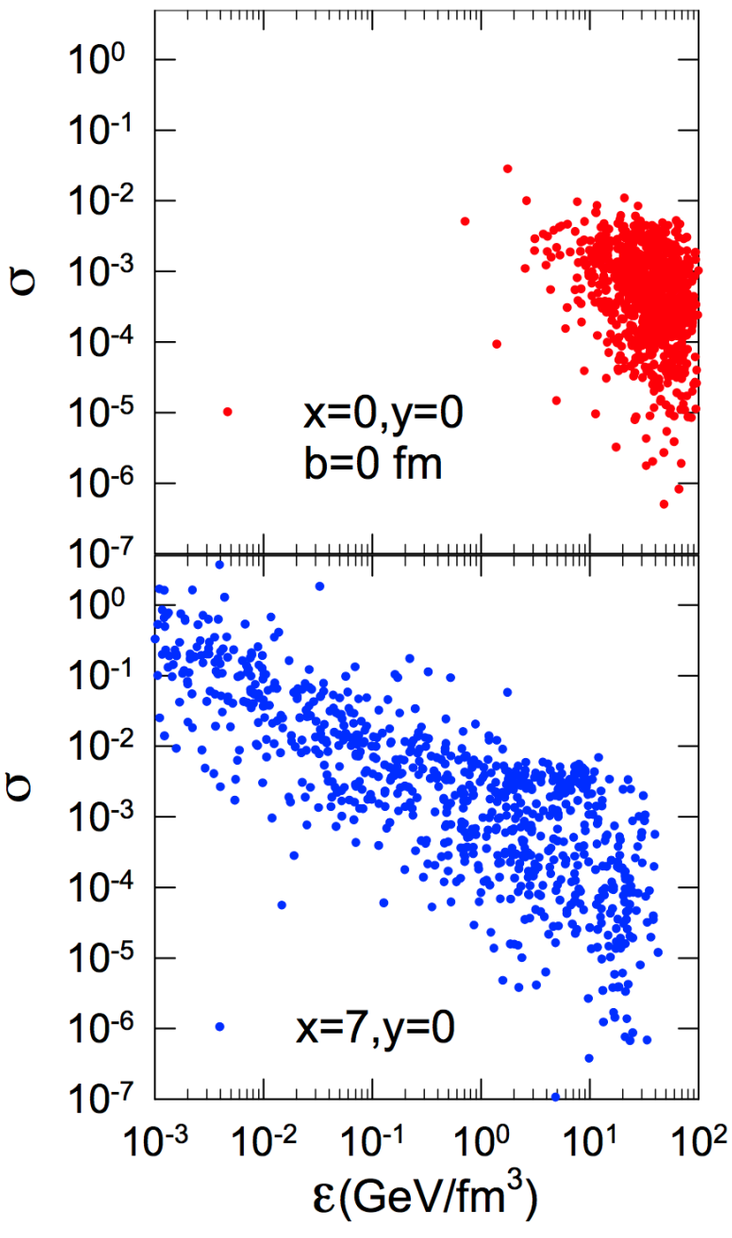
Current density : $$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right),$$

Magnetic field : $$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\sigma\mu}\left(\nabla^2 \mathbf{B} - \mu\epsilon\frac{\partial^2 \mathbf{B}}{\partial t^2}\right)$$

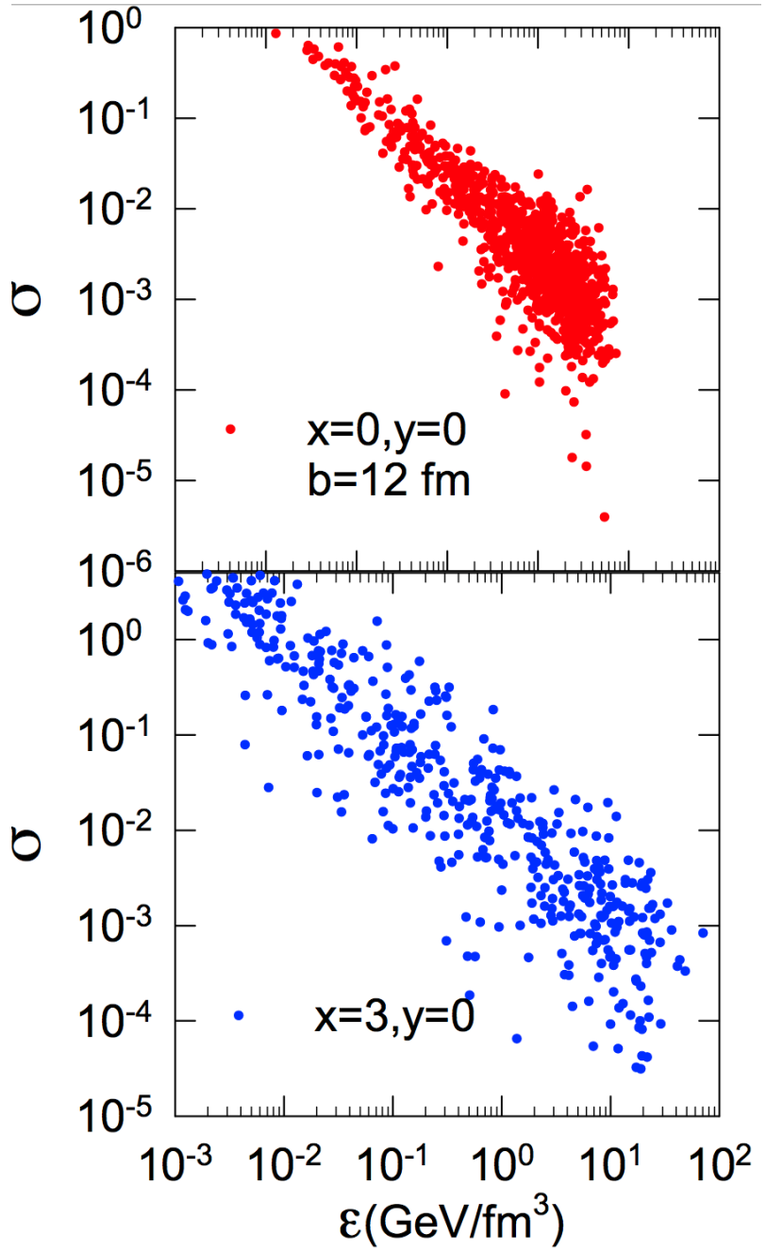
Electric Field : $$\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{v}}{\partial t} \times \mathbf{B} = \mathbf{v} \times (\nabla \times \mathbf{E}) + \frac{1}{\sigma\mu}\left(\nabla^2 \mathbf{E} - \mu\epsilon\frac{\partial^2 \mathbf{E}}{\partial t^2}\right)$$

Magnetic Reynolds Number $$R_m \equiv LU\sigma\mu,$$

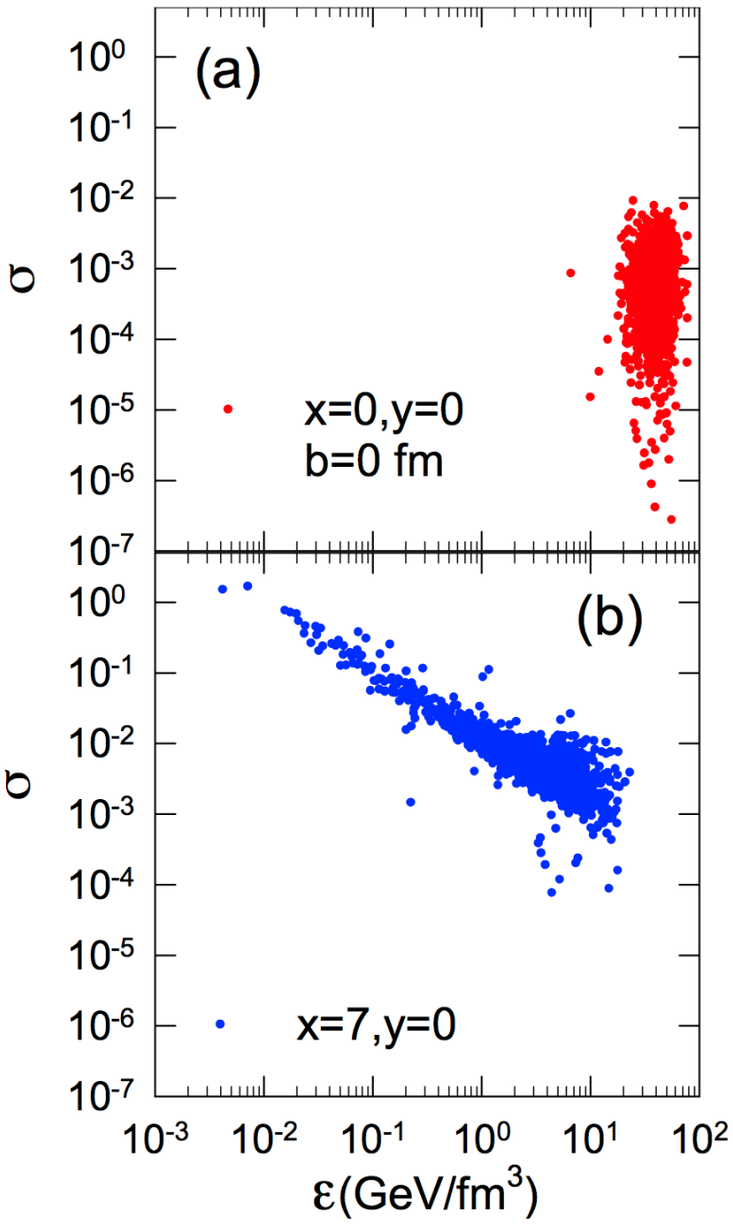
E-by-e sigma for different gaussian smearing

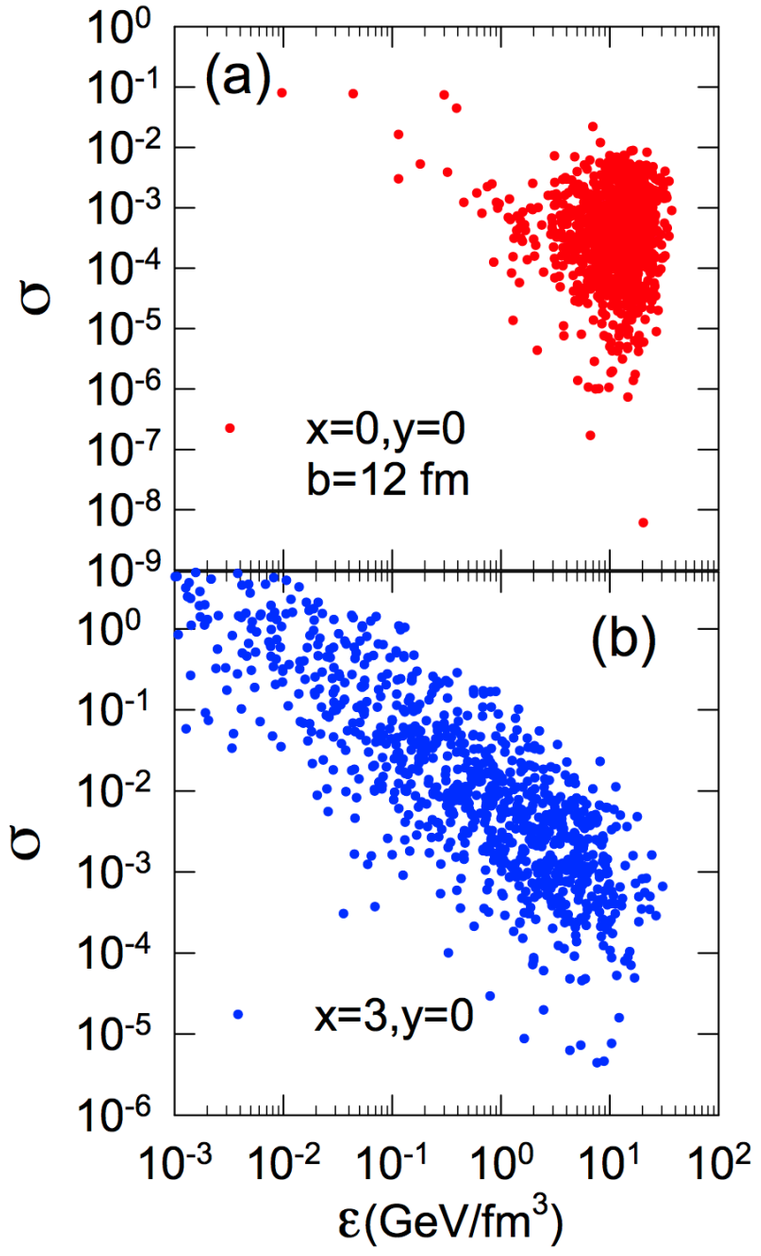


sigma_g=0.25



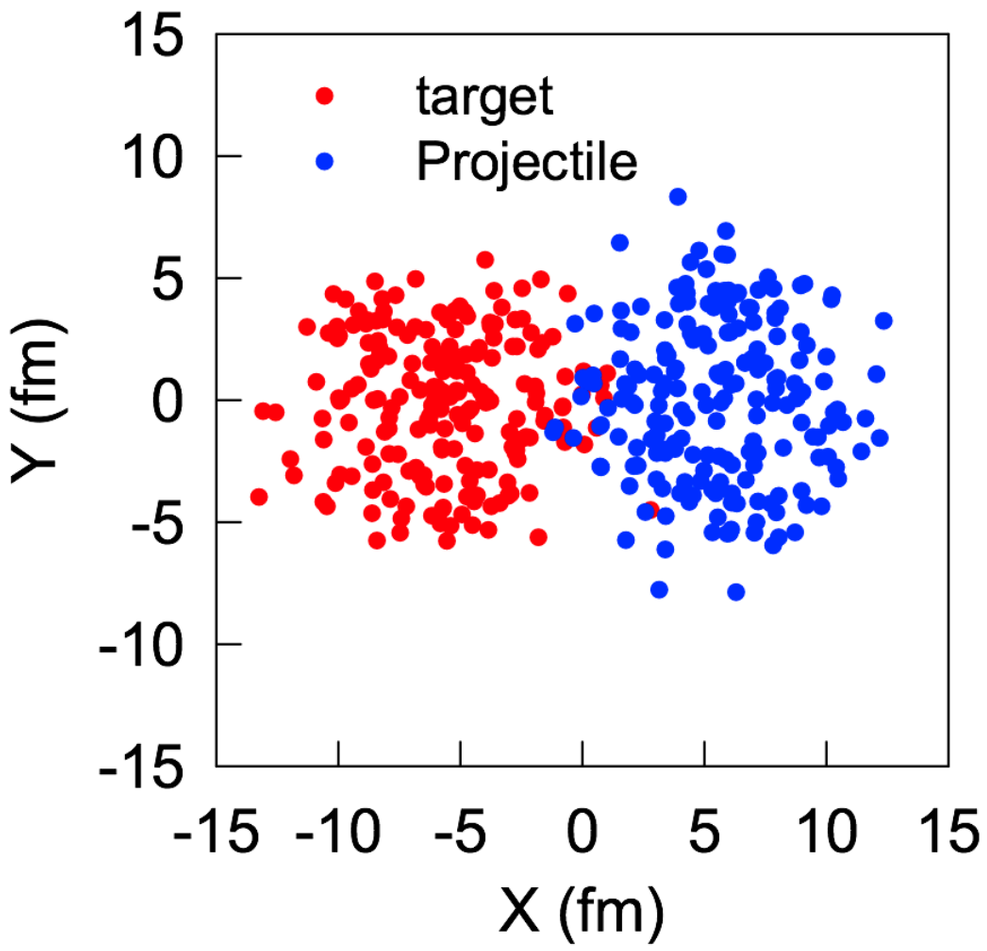
Event-by-event sigma for electric field





E-by-e calculation of fields

Monte-Carlo Glauber
Au+Au



In reality the nucleon's position fluctuates

Total field = $\sum_{proton} \vec{B}_{proton}$

Consequence:

- $\vec{B} \neq 0$ in central collision
- non uniform E/B