Effective interactions and prospects for a resolution of the fundamental cosmological problems in quantum gravity

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A possible effective interaction in the quantum gravity is considered. The compensation equation for a spontaneous generation of this interaction is shown to have a non-trivial solution. Would be consequences of a possible existence of effective interactions in the gravity theory are discussed. An example of running gravitational coupling is presented, which corresponds to a description of effects, which nowadays are prescribed to a dark mater and to a dark energy.

Introduction

An effective three-graviton interaction

Due to well-known problems of the dark matter and the dark energy numerous possibilities of modified gravity are considered. This approach assumes existence of new effective interactions of the gravitational field in addition to the fundamental Einstein-Hilbert Lagrangian.

In the present talk we would discuss a possibility of anomalous gravitation interaction in terms of non-perturbative effects of the Einstein-Hilbert gravity. For the purpose we rely on the approach induced by N.N. Bogoliubov compensation principle

N.N. Bogoliubov, ZhETF, v. 34 pp. 58,66,73 (1958);
N.N. Bogoliubov, Soviet Phys.-Uspekhi, v. 67 p. 236 (1959);
N. N. Bogoliubov, Physica Suppl. (Amsterdam), v. 26, p. 1 (1960).

In works

B. A. Arbuzov, M. K. Volkov and I. V. Zaitsev, Int. J. Mod.Phys. A, v. 21 p. 5721 (2006),
B. A. Arbuzov, Eur. Phys. J., v. C61 p. 51 (2009),
B. A. Arbuzov and I. V. Zaitsev, Phys. Rev., v. D85 : 093001 (2012) et al.

the method was applied to gauge theories of the Stanard Model.

Namely, this approach was applied to studies of a spontaneous generation of effective non-local interactions in renormalizable gauge theories. For general review see book

B. A. Arbuzov, Non-perturbative Effective Interactions in the Standard Model*, De Gruyter, Berlin, 2014.*

In particular we deal with an application of the approach to the electro-weak interaction and a possibility of spontaneous generation of effective anomalous three-boson interaction of the form

$$\frac{g\,\lambda}{3!\,M_W^2}\,F\,\epsilon_{abc}\,W^a_{\mu\nu}\,W^b_{\nu\rho}\,W^c_{\rho\mu}\,; \qquad (1)$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon_{abc} W^b_\mu W^c_\nu\,.$$

where $g \simeq 0.65$ is the electro-weak coupling. Here $F(p_i)$ is a form-factor, which guarantees effective interaction (1) acting in a limited region of the momentum space. This form-factor is uniquely defined by the compensation equation of Bogoliubov approach. The approach gives unique results for physical parameters, so we have none adjusting parameter in the scheme. Would-be existence of effective interaction (1) leads to important non-perturbative effects in the electro-weak interaction. Note, that interaction (1) was considered for long time on phenomenological grounds
K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B, 282 (1987), p. 253.

We would take interaction (1) as a leading hint for choosing of an effective interaction in the gravity theory. Considering links between vector non-abelian gauge theories and the gravity theory, one easily see that gauge field $W^a_{\mu\nu}$ plays the same role as Riemann curvature tensor $R^m_{n\mu\nu}$. Thus the anomalous interaction which is strictly analogous to interaction (1) is the following

$$L^G_{eff} = \frac{G}{2!} F \sqrt{-g}\, \epsilon^{nbdc} R_{mn\mu\nu} R_{ab\lambda\rho} R_{cd\sigma\delta} \times$$
$$g^{ma} g^{\nu\lambda} g^{\rho\sigma} g^{\mu\delta};\; R_{mn\mu\nu} = g_{ms} R^s_{n\mu\nu};$$
$$R^s_{n\mu\nu} = \frac{\partial\Gamma^s_{n\nu}}{\partial x_\mu} - \frac{\partial\Gamma^s_{n\mu}}{\partial x_\nu} + \Gamma^s_{r\mu}\Gamma^r_{n\nu} - \Gamma^s_{r\nu}\Gamma^r_{n\mu};$$
$$\Gamma^i_{kl} = \frac{1}{2} g^{im}\left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}\right). \quad (2)$$

Here F is again some form-factor to be defined by a compensation equation. We introduce this equation in the first approximation in what follows. We define the Lorentz structure of the anomalous three-graviton vertex, being quite lengthy, by calculations with application of FORM. We also use the standard Feynmann rules for the quantum gravity
B. DeWitt, Phys. Rev., 162, 1239 (1967);
L.D. Faddeev and V.N. Popov, Sov. Phys. Usp., 16, 777 (1974).
It is important to emphasize, that wouldbe interaction (2) violates both the spatial P-invariance and the temporal T-invariance.

Thus it might influence qualitative features of the Universe evolution. In particular, a violation of the T-invariance is necessary for an origin of the baryon asymmetry of the Universe
A.D. Sakharov, JETP, 51*, 1059 (1980).*

Now let us turn to the compensation equation, which firstly answers the most important question, if interaction (2) can be spontaneously generated, and secondly, in case of affirmative answer to the first question, provides form-factor $F(p_i)$.

We start with the standard Einstein-Hilbert Lagrangian and expression L_{SM}*, describing gauge interactions of the Standard Model*

$$L_0 = \frac{1}{8\,\pi\,\kappa^2}\sqrt{-g}\,R + L_{SM}\,. \qquad (3)$$

Then we apply to expression (2) the add-subtract procedure, which in details is explained in (?)

$$L = L_0' + L_{int}'\,; \quad L_0' = L_0 - L_{eff}^G\,; \qquad (4)$$

$$L_{int}' = L_{eff}^G\,. \qquad (5)$$

Now let us formulate the compensation equation. We are to demand, that considering the theory with Lagrangian L_0' (4), all contributions to three-graviton connected vertices, corresponding to Lorentz structure (2) are summed up to zero. That is the undesirable interaction part in the wouldbe free Lagrangian (4) is compensated. Then we are rested with interaction (2) only in the proper place (5).

In a diagram form this demand in the first approximation is presented in Figure 1.

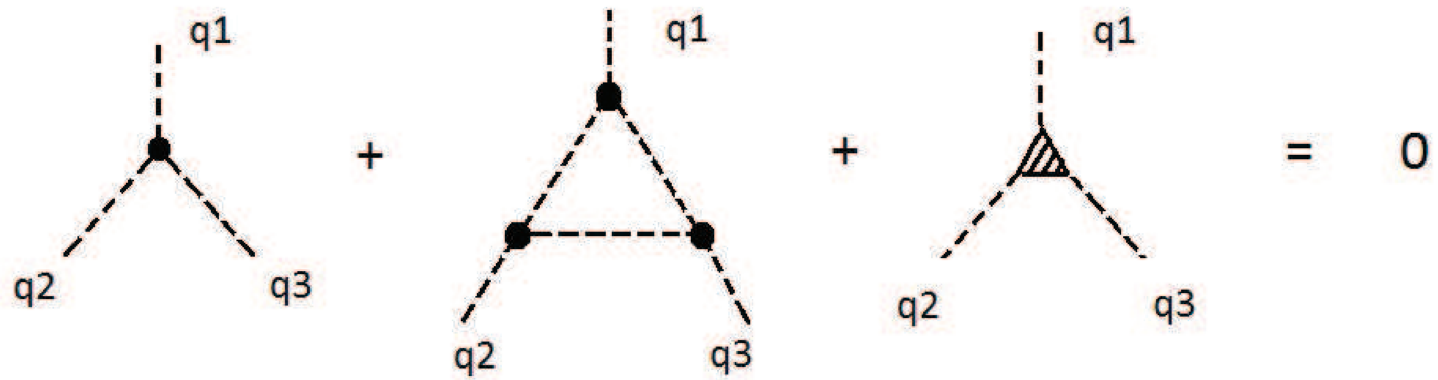

Figure 1: A diagram representation of the compensation equation in the first approximation. Dotted lines correspond to gravitons, a black spot represents interaction (2), the striped triangle represents a contribution of the Standard Model diagrams.

The corresponding integral equation with integrations in the Euclid momentum space is obtained also by FORM calculations and turns to be the following

$$F(x) = F_{0G} + \frac{3G^2}{16\pi^2}\Big(-\frac{1}{x^3}\int_0^x y^7 F(y)\,dy + \frac{4}{x^2}\int_0^x y^6 F(y)\,dy - \frac{3}{x}\int_0^x y^5 F(y)\,dy + 8\int_0^x y^4 F(y)\,dy + 18\int_x^\infty y^4 F(y)\,dy - 25x\int_x^\infty y^3 F(y)\,dy + 24x^2\int_x^\infty y^2 F(y)\,dy - 11x^3\int_x^\infty y F(y)\,dy + 2x^4\int_x^\infty F(y)\,dy\Big); \quad x = p^2. \tag{6}$$

where F_{0G} means an inhomogeneous part of the equation, which in Figure 1 is denoted by the striped triangle.

Assuming $F_{0G} = Const$, by successive differentiations of equation (6) we obtain a linear differential equation for F(x), in which new variable z is introduced

$$\left[\left(z\frac{d}{dz}+\frac{3}{5}\right)\left(z\frac{d}{dz}+\frac{2}{5}\right)\left(z\frac{d}{dz}+\frac{1}{5}\right)\left(z\frac{d}{dz}\right)\times \left(z\frac{d}{dz}-\frac{1}{5}\right)\left(z\frac{d}{dz}-\frac{2}{5}\right)\left(z\frac{d}{dz}-\frac{3}{5}\right)\left(z\frac{d}{dz}-\frac{4}{5}\right)+ z\left(z\frac{d}{dz}+\frac{14}{15}\right)\right]F(z) = 0; \quad z = \frac{81\, G^2\, x^5}{15625\, \pi^2}. \qquad (7)$$

The differential equation is equivalent to equation (6) with boundary conditions. We have the following solution with condition $F(0) = 1$

$$F(z) = \frac{6\,\Gamma\left(\frac{1}{15}\right)}{125\,\Gamma\left(\frac{4}{5}\right)} G_{18}^{50}\left(z|^{1/15}_{0,1/5,2/5,3/5,4/5,-3/5-2/5,-1/5}\right); \quad (8)$$

where $G_{pq}^{mn}\left(z\,|^{a_1,\dots,a_p}_{b_1,\dots,b_q}\right)$ is a Meijer function.

On the other hand, assuming $F_{0G} = 0$, we may calculate $F(0)$ from equation (6), that gives

$$F(0) = \frac{18}{5}. \qquad (9)$$

However, form-factor $F(z)$ has to be unity at zero. So there is evidently additional contribution to $F(0)$, that is

$$F_{0G} \neq 0. \qquad (10)$$

This contribution might be given by diagrams including matter fields, for example, by those being presented in Figure 2. First of all we would draw attention to presence of W exchange in Figure 2. The contribution is provided by the the T-odd and P-odd part of fundamental fermions interaction with weak bosons $W^{\pm}$.

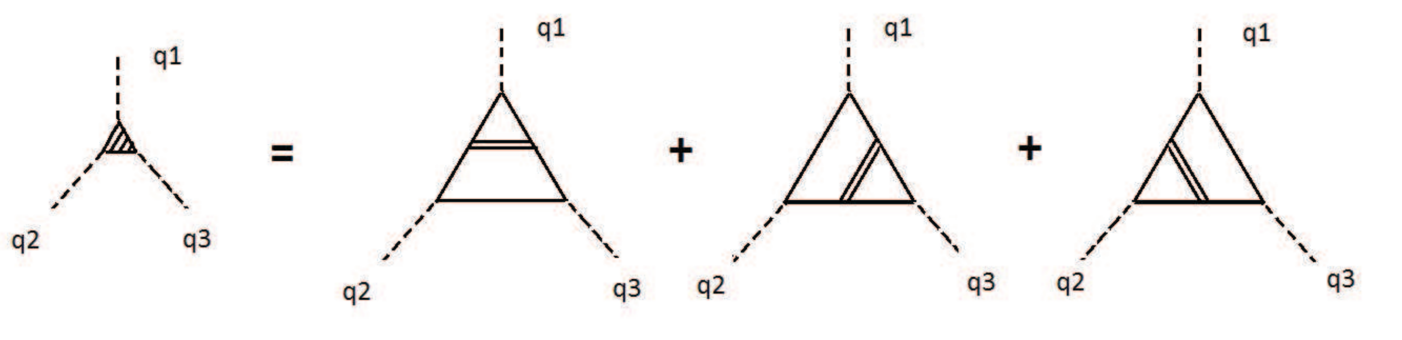

Figure 2: Diagrams, describing the first approximation for the Standard Model contribution to three-graviton vertex (2). Simple lines correspond to matter fermions (quarks** etc.**, double lines correspond to weak bosons W.

The interaction of W with quarks and leptons contains γ_5 matrix and so corresponding traces inevitably contains antisymmetric tensor $\epsilon_{\alpha\beta\gamma\delta}$, which is present in interaction (2). The vertex of a graviton interaction with a spinor field, is the following

$$V(\mu,\nu,p_1,p_2) = \kappa(\gamma_\mu(p_1+p_2)_\nu + \gamma_\nu(p_1+p_2)_\mu);$$

where p_1 is the momentum of the incoming spinor, p_2 is the same of the outgoing one. κ is connected with the Planck mass

$$\kappa = \frac{1}{M_{Pl}}; \tag{11}$$

The T-odd part of the quarks weak interaction is the following

$$-\frac{\imath g A\lambda^3\bar{\eta}}{\sqrt{2}}\Big(\bar{u}\gamma_\mu(1+\gamma_5)b + \bar{t}\gamma_\mu(1+\gamma_5)d\Big)W^\mu + h.c.;$$

where (?)

$$\lambda = 0.22537 \pm 0.00061;\ \bar{\eta} = 0.353 \pm 0.013;$$
$$A = 0.814^{+0.023}_{-0.024}. \tag{12}$$

We readily estimate, that diagrams in Figure 2 with quark loops give the following contribution to the inhomogeneous part of the equation

$$F_{0G} \,=\, -\,\frac{3\,g^2\kappa^3\,A\,\lambda^3\bar{\eta}}{64\,\pi^4\,G\,M_W^2}\,\ln\,\frac{M_W\,m_b}{m_u\,m_d}; \qquad (13)$$

where $g \simeq 0.65$ *is the electro-weak gauge constant. As for lepton loops, we have no full information yet on corresponding mixing parameters, and thus we use for estimates only (13), the more so, as a quark loop has additional color factor 3. From the main equation (6) we have the following condition*

$$F(0)\,+\,F_{0G}\,=1\,. \qquad (14)$$

Expression (13) has to be equal to

$$F_{0G}\,=1\,-\,F(0)\,=\,-3.003\,. \qquad (15)$$

Then with previous relations (6, 15) we obtain the following estimate for the coupling constant of the effective interaction (2) G. In doing this we have to bear in mind, that integral equation (6) is divided by coupling constant G due to the overall procedure for searches for non-trivial solutions of compensation equations. Thus we have

$$G \simeq \frac{g^2 \kappa^3 A \lambda^3 \overline{\eta}}{64\, \pi^4\, M_W^2} \ln \frac{M_W m_b}{m_u m_d} \simeq 3.2 \cdot 10^{-67} GeV^{-5}. \quad (16)$$

As we have already mentioned, for the moment we can not substitute reliable values for the average neutrino mass m_ν and mixing parameters in analogous to (16) lepton expression. We may only safely assert, that m_ν is not zero due to existence of the effect of neutrino oscillations. In any case it may not be more than $3\, eV$.

In view of this we have taken for the estimate just quarks, as particles giving contribution to coupling constant G, the more so, as in the quark loops we have the additional color factor 3. It is evident, that massless particles, namely photons and gluons, do not give contribution due to parity conservation of their interactions. To obtain more definite connection between two parameters G and κ one needs perform difficult calculations, which will be done elsewhere. However our estimate (16) allows us to consider qualitatively effects of the interaction (2).

With physical mass of W and bearing in mind relation (11), where Planck mass $M_{Pl} \simeq 1.22 \cdot 10^{19}\ GeV$ is very large, we understand, that possible value (16) is essentially larger, than seemingly natural value $G_{Pl}\ \sim\ \kappa^5$.

The interaction (2) due to a presence of the antisymmetric tensor $\epsilon_{\alpha\beta\gamma\delta}$ gives no contribution to spherically symmetric problems of gravitation (Schwartzschield solution, Friedmann solution** etc.**). However it could manifest itself in problems without spherical symmetry in a rotating system (**e.g.** spiral galaxy). The considerable enhancement of possible value G in comparison to natural value G_{Pl} by the following factor

$$\frac{G}{G_{Pl}} = \frac{g^2\, A\, \lambda^3\, \overline{\eta}\, M_{Pl}^2}{64\, \pi^4\, M_W^2}\, \ln\, \frac{M_W\, m_b}{m_u\, m_d} \simeq 8.7 \cdot 10^{28}\,; \quad (17)$$

is quite remarkable and presumably may lead to observable effects. Let us remind, that effective interaction (2) is P and T non-invariant.

A model for a running gravity coupling

We have considered above wouldbe properties of the quantum gravity. The theory itself contains dimensional coupling constant

$$\beta = \frac{\kappa^2}{4\,\pi}. \qquad (18)$$

In a conventional quantum field theory such quantity corresponds to a running coupling, for example, $\alpha_s(Q^2)$ in QCD. Thus one should expect, that quantity (18) is also a running one. However, the quantum gravity theory is non-renormalizable. It means that we have no regular method to obtain an expression for the running coupling.

So an application of a non-perturbative approach is necessary. As a matter of fact, in gauge theories of the Standard Model contributions of the non-perturbative nature may be present also. We may refer just to the strong coupling $\alpha_s(Q^2)$, in which the well known non-physical Landau singularity appears in perturbative calculations. It is a general belief, that non-perturbative contributions somehow eliminate this singularity. In particular, in work
B.A. Arbuzov and I.V. Zaitsev, Int. J. Mod. Phys. A, 28: 1350127 (2013).
we have shown, that the singularity is excluded due to the spontaneous generation of effective three-gluon interaction being analogous to (1).

In any case, a consideration of possible properties of running gravitational coupling (18) deserves an attention. In the present section we consider a model, which illustrates the problem.

It is very important to have some hints on the scale of the possible non-perturbative effects in our problem. Here the example, which is considered in the previous Section may be instructive.

First of all, let us consider relation (17), which leads to the following estimate of characteristic length l_{eff} *for effective interaction (2)*

$$l_{eff} \simeq l_{Pl}\left(\frac{G}{G_{Pl}}\right)^{\frac{1}{5}} \simeq 6.1 \cdot 10^5 \cdot l_{Pl} \simeq 10^{-27}\, cm; \quad (19)$$

that is an essential enhancement in comparison to the Planck one.

However effective interaction (2) is not the only possible one. Maybe there are also other effective interactions, which have even larger characteristic lengths. For example, we see, that in expression (16) both maximal mass parameter $M_{Pl}\,(\kappa = 1/M_{Pl})$ and masses of fundamental particles are present. Neutrino mass m_ν is the minimal one known. So we estimate a maximal possible scale of the length dimension for non-perturbative effects as follows

$$l_0 = \frac{M_{Pl}}{m_\nu^2} = 2.711 \cdot 10^{20} m = 8.78\, kpc; \qquad (20)$$

where we have used for the neutrino mass its upper bound $m(\nu_e) \leq 3\, eV$. We see, that this estimate gives a kpc scale, which is appropriate to a size of a galaxy.

Let us assume, that running Newton gravity coupling G_N, which is proportional to coupling β (18) depends on a distance in the following way

$$G_N(r) = G_{N0}F(r);\ F(r) = \frac{1}{3}G^{30}_{14}\left(\frac{r}{r_0}\Big|^{1}_{0,2,4,-2}\right) + 7\,G^{21}_{14}\left(\frac{r}{r_0}\Big|^{1}_{2,4,0,-2}\right); \tag{21}$$

where r_0 is of the order the of magnitude of l_0 (20) and G_{N0} is just the well-known Newton constant. We use Meijer functions in the model, because in similar problems we encounter these functions. The Fourier transform of a Meijer function is again a Meijer function.

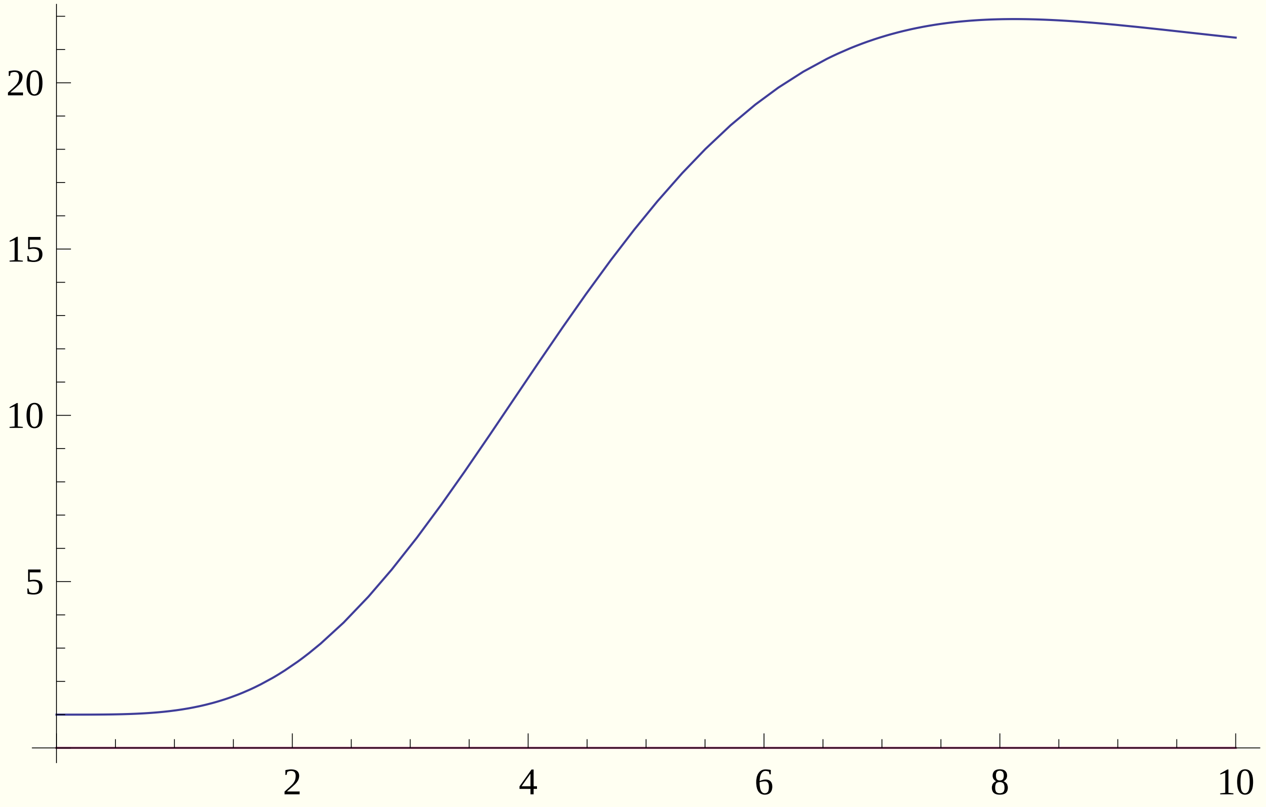

Figure 3: Behavior of $F(x^2)$, $x = \sqrt{r/r_0}$.

The gravity is not a logarithmic theory, as e.g. QCD, but a power-mode theory. We choose the coefficients in (21) so that for $r \to 0$ $G_N \to G_{N0}$, and for $r \to \infty$ $G_N \to 21\, G_{N0}$. The last asymptotic corresponds to the accelerated expansion of the Universe, which usually is prescribed to a dark energy.

The Fourier transform, corresponding to expression (21), in a dependence on variable $y = 16\, k^2 r_0^2$, where k is a momentum, is the following (?)

$$\bar{F}(k^2) = \frac{4}{3\,\pi}\Big(G_{6\,2}^{1\,4}\Big(y\,\Big|\,{1, -\frac{1}{2}, -1, -\frac{3}{2}, \frac{3}{2}, 2 \atop \frac{1}{2}, 0}\Big) + 21\, G_{6\,2}^{2\,3}\Big(y\Big|\,{-\frac{1}{2}, -1, -\frac{3}{2}, 1, \frac{3}{2}, 2 \atop \frac{1}{2}, 0}\Big)\Big). \qquad (22)$$

Function (22) is shown in Figure 4.

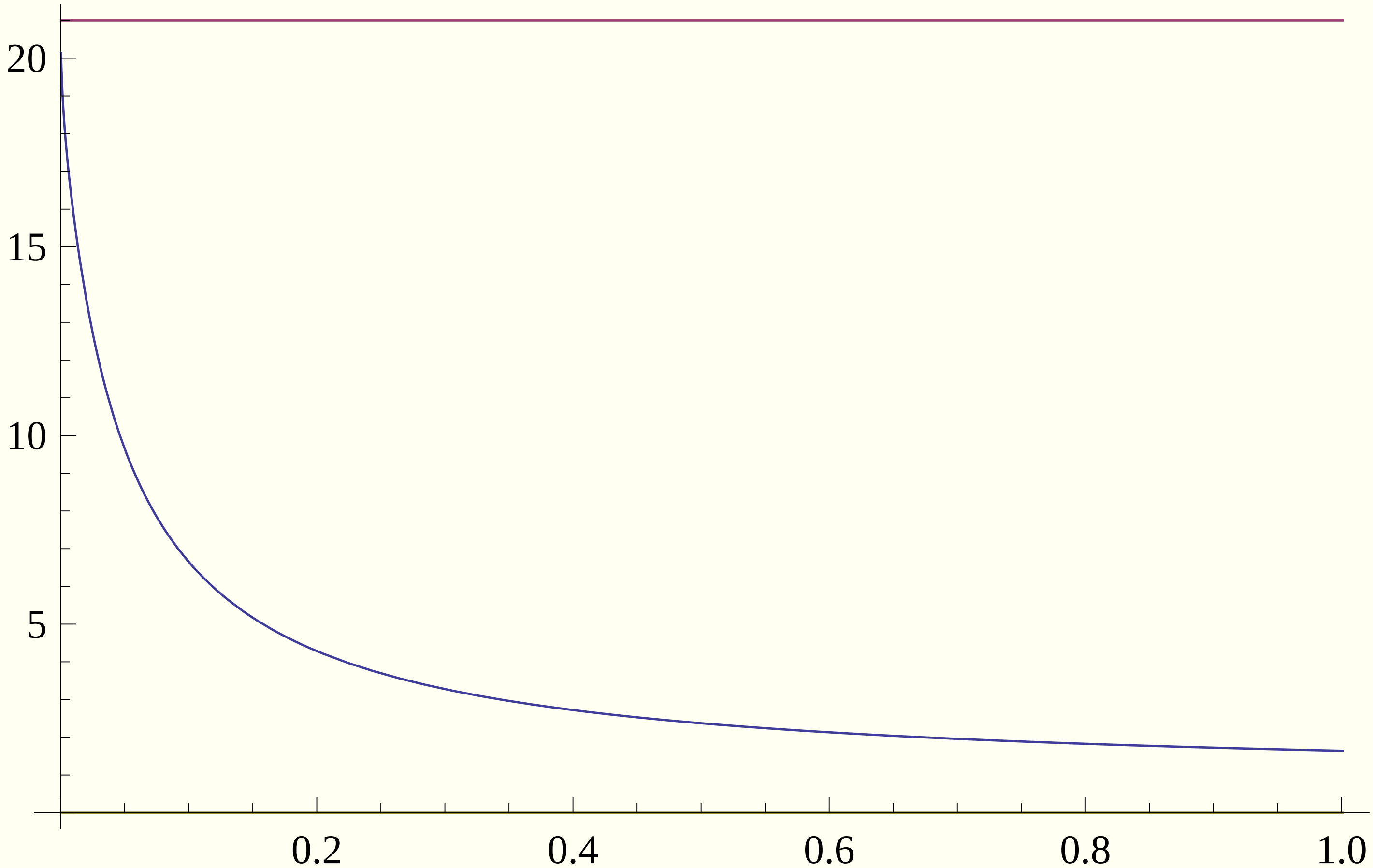

Figure 4: The behavior of $G_N(16\,k^2 r_0^2)/G_{N0}$. The upper line corresponds to the absence of the momentum dependence: $G_N = G_N(0) = 21\,G_{N0}$.

The momentum dependence resembles the situation with the asymptotic freedom in QCD.

The main difference consists in a dependence at the infinity. In QCD the strong coupling decreases

$$\alpha_s(k^2)_{k^2\to\infty} \simeq \frac{4\,\pi}{\beta_0\,\ln\frac{k^2}{\Lambda^2}}\,; \qquad (23)$$

while in the case under the consideration the gravity coupling tends to the Newton constant

$$\lim\ G_N(k^2)_{k^2\to\infty} = G_{N0}. \qquad (24)$$

Now we apply representation (21) to rotation curves of galaxies.

Let us choose $r_0 = 5\,kpc$. We take for a rotation curve of a flat disk galaxy the following expression

$$V(r) = V_0\sqrt{2M_G(I_0(y)K_0(y) - I_1(y)K_1(y))F(r)};$$
$$y = \frac{r}{R_G}; \quad V_0 = 207.4\,\frac{km}{s}; \tag{25}$$

where M_G is a galaxy mass in $10^{10}\,M(Sun)$, R_G is its radius in kpc and r is a distance in a rotation curve in kpc as well.

Then we substitute a galaxy mass and its radius to obtain a corresponding rotation curve. We have taken five galaxies for examples. Black spots in figures denote observation data.

Upper curves in Figures correspond to representation (25). All figures show how velocities in km/s depend on distances in kpc. Lower curves in Figures correspond to $F(r) = 1$, i.e. without additional interaction. The examples show, that representation (21) essentially improves the agreement with observational data.

Galaxy UGS 2885, $M_G = 28.8$, $R_G = 10$, ***Figure 5.***
Galaxy NGC 6674, $M_G = 21.0$, $R_G = 4.58$, ***Figure 6.***
Galaxy NGC 801, $M_G = 15.4$, $R_G = 5.5$, ***Figure 7.***
Galaxy NGC 3521, $M_G = 9.0$, $R_G = 3.0$, ***Figure 8.***
Galaxy NGC 2683, $M_G = 5.8$, $R_G = 2.4$, ***Figure 9.***

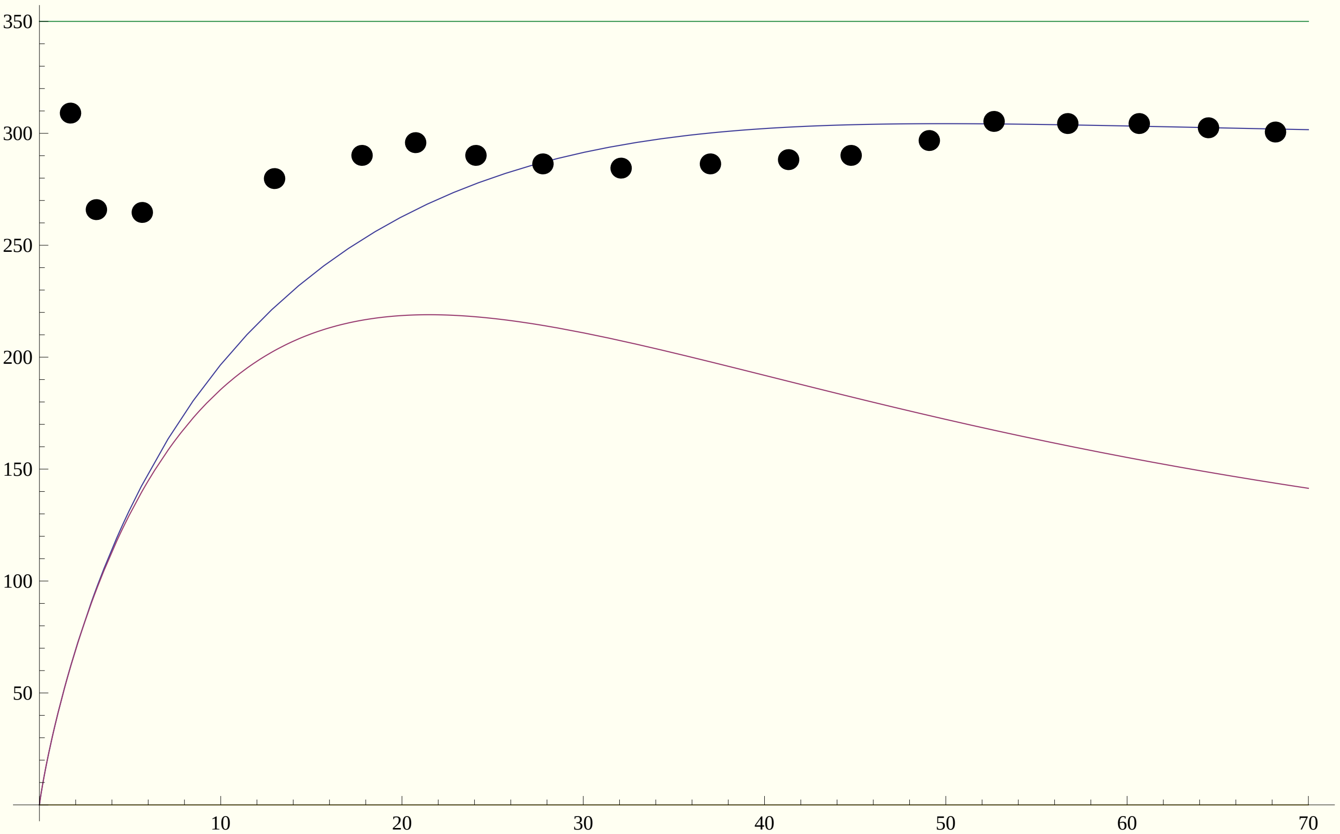

Figure 5: Rotation curve for Galaxy UGS 2885.

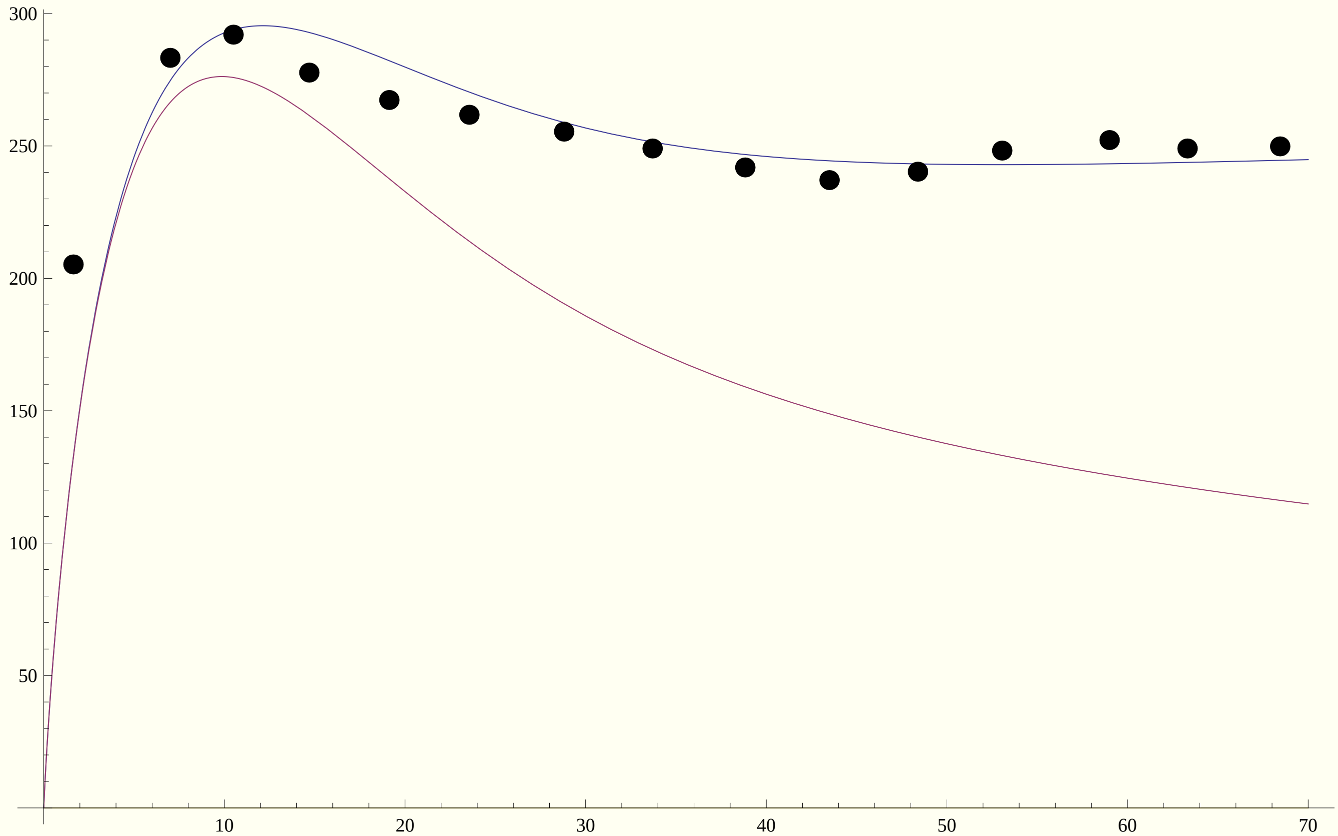

Figure 6: Rotation curve for Galaxy NGC 6674.

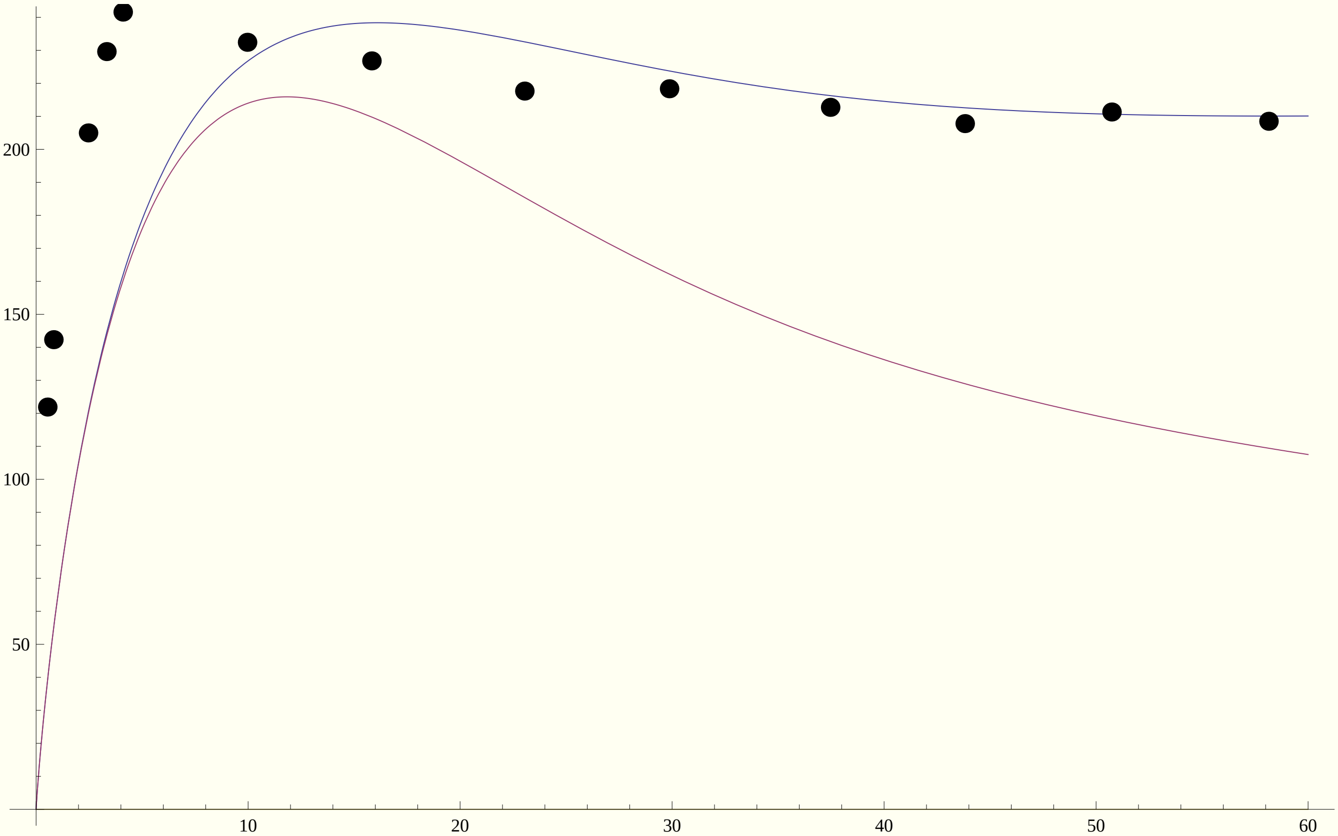

Figure 7: Rotation curve for Galaxy NGC 801.

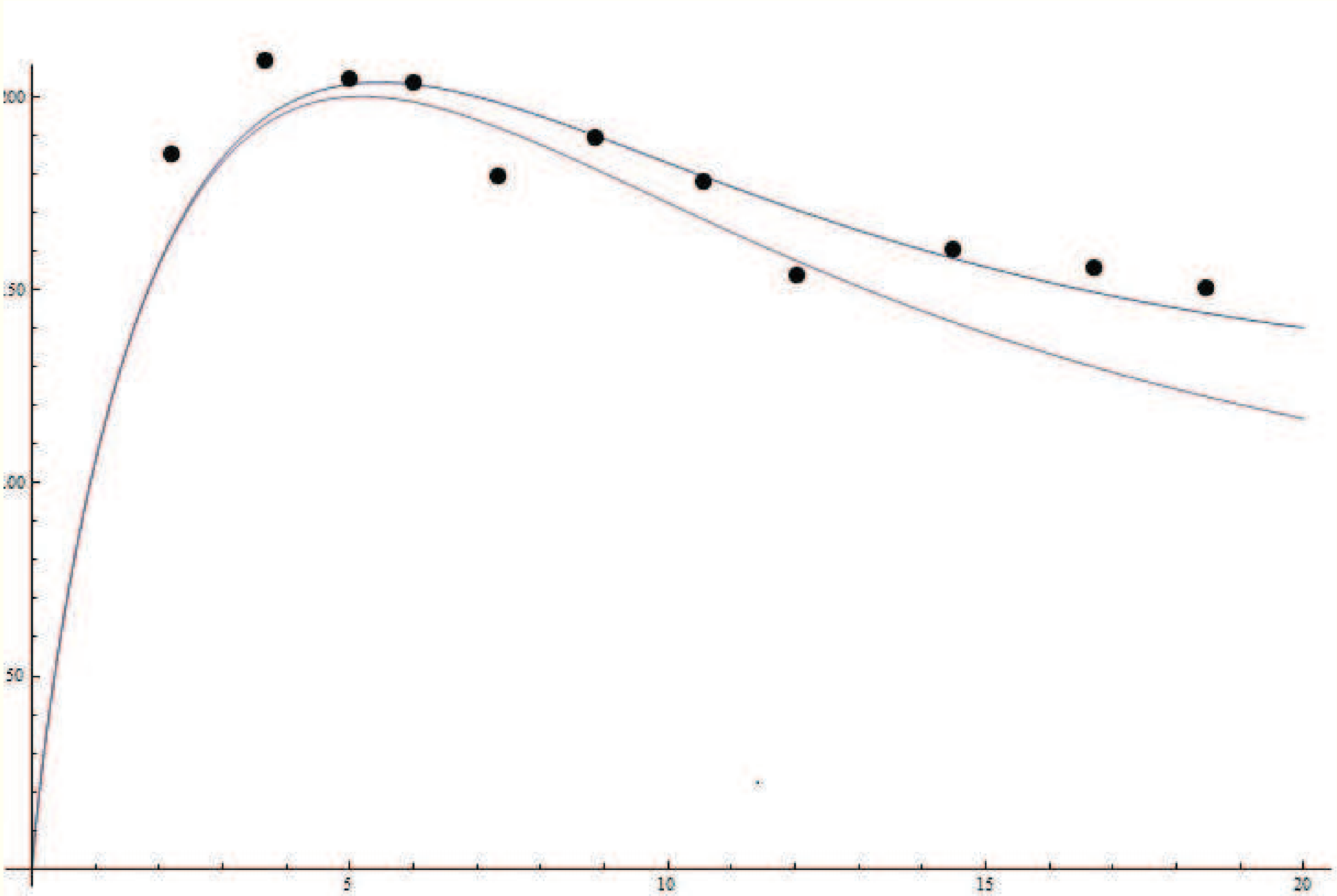

Figure 8: Rotation curve for Galaxy NGC 3521.

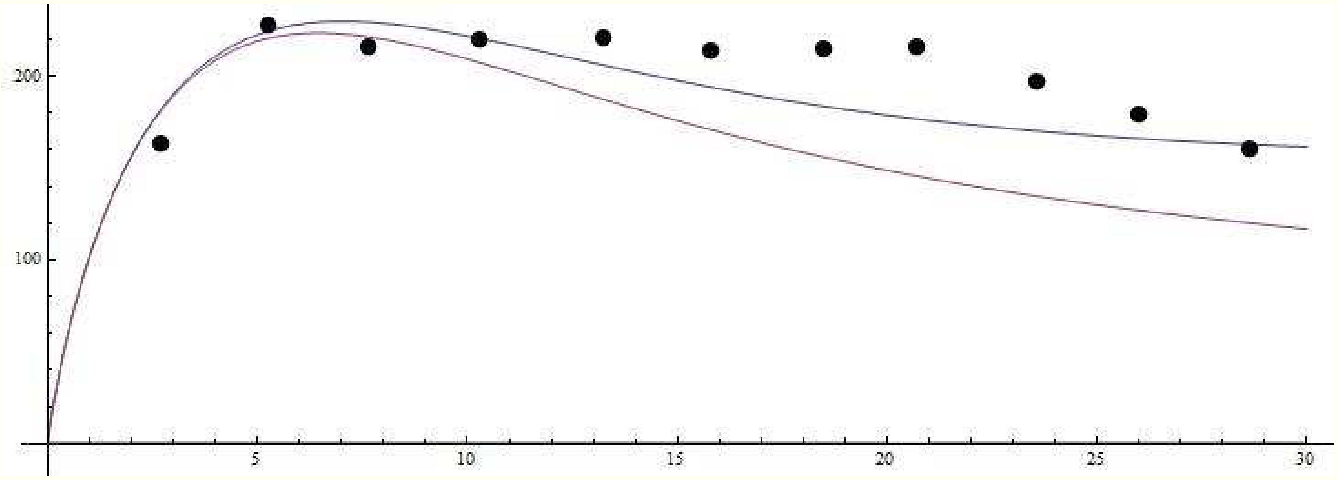

Figure 9: Rotation curve for Galaxy NGC 2683.

Conclusion

The results are published in B.A. Arbuzov and I.V. Zaitsev, Theor. Math. Phys. 191 (2), 635-640 (2017). Both this work and the present talk deal with a possibility of a presence of effective interactions in the gravity theory, the fundamental General Relativity. What are benefits of such possibility?

1. A wouldbe appearance of the P-odd and T-odd effective interaction of gravitons with a new scale.

2. The model for dark effects, which with an assumption on a special scale could pretend to a description of the real situation in the astrophysics.

As for the last item, it is for the moment a hypothesis. It just show a possibility how things might be, in case searches for a dark matter and a proper understanding of the dark energy would fail.

Note, that numerous proposals for a physical content of the dark substances also have a hypothetical status yet.

There are three regions in this model, in which properties of the gravitation turn to be quite different. In the first region $r \ll 10\,kpc$ the gravity coupling with the high accuracy equals the Newton one G_{N0}. In the region with $r \simeq 10\,kpc$ this coupling increases, that leads to effects in the rotation curves, which usually are prescribed to a dark matter. Finally, in the region with $r \gg 10\,kpc$ G_N is 21 times more, than the Newton coupling G_{N0}, that might serve as an explanation of the accelerated expansion of the Universe, which is nowadays usually prescribed to the dark energy.

Thus, in case of such dependence of the gravity on a distance being realized, the presumptive effects of the dark matter and the dark energy are simultaneously described. Of course the present results are qualitative, a further specification is necessary for a more accurate comparison with data.

The purpose of the work is just to demonstrate a wouldbe tool to deal with effects in the gravity physics, which relies on close similarity of the gravity and vector non-abelian fields.

Thanks for the attention