Stochastic processes in space of quantum random joint events and equations for quantum transition probabilities

Alexander Biryukov

Samara University
General and Theoretical Physics Department

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TASK: To construct the equation for probabilities of quantum transitions on the basis of probability theory axioms.

HYPOTHESIS: The equation for probabilities of quantum process can be constructed on Kolmogorov's axioms if they are supplemented with a new axiom for joint events.

KOLMOGOROV'S AXIOMS:

INITIAL CONCEPTS:

Ω — a set of elementary events ω, space of elementary events.

$\mathfrak{F}$ — a set of subsets from Ω, set of casual events A, B, C, ...

AXIOM I:

$\mathfrak{F}$ — is algebra of sets (exist $A \cup B, A \cap B, A \setminus B, ...$, which belong $\mathfrak{F}$).

AXIOM II:

The probability of an event (a set measure) is entered $P(A) \geq 0$.

AXIOM III:

$P(\Omega) = 1$.

AXIOM IV:

not joint events if follows $A \cap B = 0, P(A + B) = P(A) + P(B)$.

Joint events in Kolmogorov's axiomatics

Two events S_1^k, S_2^k are joint, if $S_1^k \cap S_2^k \neq 0$, probability of association of events

$$P(S_1^k \cup S_2^k) = P(S_1^k) + P(S_2^k) - P(S_1^k \cap S_2^k)$$

Symmetric difference of two joint events

$$S_1^- = S_1^k \setminus (S_1^k \cap S_2^k), \quad , \quad S_2^- = S_2^k \setminus (S_1^k \cap S_2^k)$$

$$P(S_1^- \cup S_2^-) = P(S_1^k) + P(S_2^k) - 2P(S_1^k \cap S_2^k)$$

Symmetric difference of N joint events

$$P(\bigcup_{n=1}^{N} S_n^-) = \sum_{n=1}^{N} P(S_n^k) - 2 \sum_{n<m=1}^{N} P\big(S_n^k \cap S_m^k\big) +$$

$$+4 \sum_{n<m<k=1}^{N} P\big(S_n^k \cap S_m^k \cap S_k^k\big) + \ldots + (-2)^{N-1} P(S_1^k \cap S_2^k \cap \ldots \cap S_N^k).$$

In addition to Kolmogorov's axioms.
Let's define quantum joint events.

Two events S_1^q, S_2^q are quantum joint, if $S_1^q \cap S_2^q \neq 0$, probability of association of events

$$P(S_1^q \cup S_2^q) = P(S_1^q) + P(S_2^q) + P(S_1^q \cap S_2^q)$$

Symmetric sum of two quantum joint events is a postulate

$$S_1^+ = S_1^q \cup (S_1^q \cap S_2^q), \quad , \quad S_2^+ = S_2^q \cup (S_1^q \cap S_2^q)$$

$$P(S_1^+ \cup S_2^+) = P(S_1^q) + P(S_2^q) + 2P(S_1^q \cap S_2^q)$$

Symmetric association of N quantum events S_n^+ is sum:

$$P(\bigcup_{n=1}^{N} S_n^+) = \sum_{n=1}^{N} P(S_n^q) + 2 \sum_{n<m=1}^{N} P\big(S_n^q \cap S_m^q\big) -$$

$$-4 \sum_{n<m<k=1}^{N} P\big(S_n^q \cap S_m^q \cap S_k^q\big) + \ldots + (-2)^{N-1} P(S_1^q \cap S_2^q \cap \ldots \cap S_N^q).$$

The uniform equation for a symmetric difference and symmetric sums of random joint events

Let's designate joint events S_n^k, S_n^q one symbol S_n.
Let's designate events S_n^-, S_n^+ one symbol $\tilde{S}$.
These designations give the chance to write down the equations for $P(\bigcup_{n=1}^N S_n^-)$, $P(\bigcup_{n=1}^N S_n^+)$ in the form of one equation

$$P(\bigcup_{n=1}^{N} \tilde{S}_n) = \sum_{n=1}^{N} P(S_n) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_n \cap S_m) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_n \cap S_m \cap S_k) + \ldots + 2^{N-1} g_{12\ldots N} P(S_1 \cap S_2 \cap \ldots \cap S_N),$$

$$g_{1,2} = +1 \; or \; -1, \ldots$$

The equation for for probability of transition between the random not joint events

In space of random not joint events the stochastic proceeds process: At the moment time t_{in} the event $A_{in}(t_{in})$ is realized, at the moment $t > t_{in}$ there can be one of events $S_n, n = 1, 2, ...N$, at the moment $t_f > t_{in}$ the event $B_f(t_f)$ is realized. The probability of transition from an event $A_{in}(t_{in})$ to an event $B_f(t_f)$ is defined equation $P(B_f \mid A_{in}) = P(B_f \cap (\bigcup_{n=1}^N S_n) \cap A_{in}) = P(\bigcup_{n=1}^N (B_f \cap (S_n) \cap A_{in}))$

Events are not joint, therefore $$P(B_f \mid A_{in}) = \sum_{n=1}^{N} P(B_f \cap S_n \cap A_{in})$$

The equation it is possible to present in a look where probabilities of transition

$$P(B_f \mid A_{in}) = \sum_{n=1}^{N} P(B_f \mid S_n) P(S_n \mid A_{in})$$

This equation of chains of Markov. It is possible to write down in a look where it is designated $B_f \cap S_n \cap A_{in} = S_{fni}$

$$P(f \mid i) = \sum_{n=1}^{N} P(S_{fni})$$

The equation for probabilities of transition in space of the random joint events

The equation we will write down in a look

$$P(B_f \cap A_{in}) = P(B_f \cap (\bigcup_{n=1}^{N} \tilde{S}_n) \cap A_{in}) = P(\bigcup_{n=1}^{N} (B_f \cap \tilde{S}_n \cap A_{in})$$

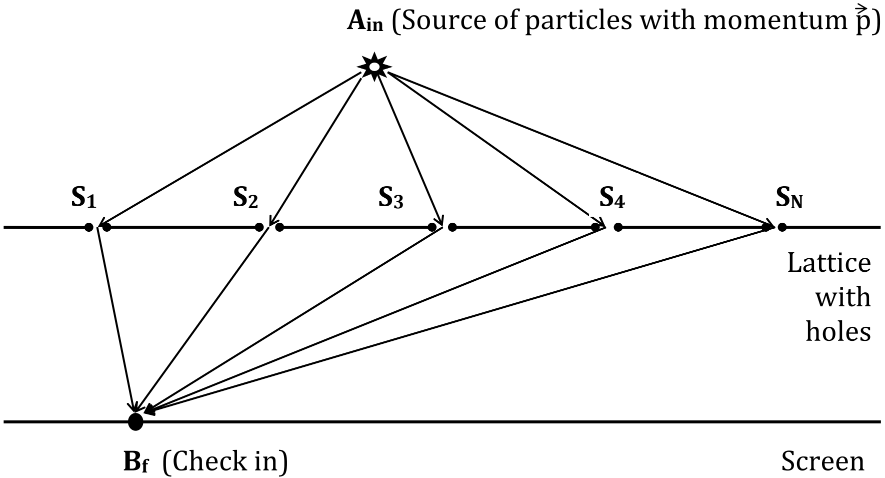
We will designate $B_f \cap \tilde{S}_n \cap A_{in} = \tilde{S}_{fni}$
It is possible to present the offered equation in a look

$$P(f \mid i) = P(\bigcup_{n=1}^{N} \tilde{S}_{fni}) = \sum_{n=1}^{N} P(S_{fni}) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_{fni} \cap S_{fm}) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots + 2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}),$$

$$g_{1,2} = +1 \; or \; -1$$

Interpretation of the stochastic process equation for joint events



Drawing 1. Particle diffraction

Diffraction of particles

$$\psi(\mathbf{r}_f, \mathbf{r}_i, t) = \sum_{n=1}^{N} \sqrt{\rho(S_{fni})} \exp[\frac{\imath}{\hbar} Et] \exp[\imath S_{fni}], \tag{1}$$

Where $\mathbf{r}_n$ - determines the coordinates of the center of the hole of the diffraction grating; $\mathbf{r}_f$— is the coordinate of the observation point on the screen; $\mathbf{r}_i$—coordinates of the particle source; $\sqrt{\rho(S_{fni})}$ -is the amplitude of the wave function; $S_{fni} = \hbar^{-1}\mathbf{pl}_{fni}$ — Dimensionless action of a particle along line passing through the points $\mathbf{r}_i, \mathbf{r}_n, \mathbf{r}_f$.
The probability density $\rho(\mathbf{r}_f, \mathbf{r}_i)$ to detect a particle at the screen point $\mathbf{r}_f$, is determined by the square of the wave Functions (1)

$$\rho(\mathbf{r}_f, \mathbf{r}_i) = \sum_{n=1}^{N} \rho(S_{fni}) + 2 \sum_{m<n=1}^{N} \sqrt{\rho(S_{fni})} \sqrt{\rho(S_{fmi})} \cos[S_{fni} - S_{fmiS_{fni}}]. \tag{2}$$

Probabilistic interpretation of particle diffraction

The probability of detecting a particle on a small area of the screen σ_f whose center has the coordinat $\mathbf{r}_f$: $P(\mathbf{r}_f, \sigma_f; \mathbf{r}_i) = \rho(\mathbf{r}_f, \mathbf{r}_i)\sigma_f$. We multiply the equation by σ_f. We represent the equation in the form

$$P(\mathbf{r}_f, \sigma_f; \mathbf{r}_i) = \sum_{n=1}^{N} P(\sigma_f; S_{fni}) + 2 \sum_{m<n=1}^{N} g(S_{fni}, S_{fmi}) P(S_{fni}, S_{fmi}), \tag{3}$$

where $P(\sigma_f; S_{fni}) = \rho(S_{fni})\sigma_f$ - probability of a particle falling onto the screen pad σ_f when it passed through the n th hole;

$$P(\sigma_f; S_{fni}, S_{fmi}) = \sqrt{P(\sigma_f; S_{fni})}\sqrt{P(\sigma_f; S_{fmi})}\, |\cos[S_{fni} - S_{fmi}]\,| \tag{4}$$

is probability of a particle falling onto the screen area σ_f, when it simultaneously passed through the n th and m -th holes (the probability of joint events).

$$g(S_{fni}, S_{fmi}) = \cos[S_{fni} - S_{fmi}]|\cos[S_{fni} - S_{fmi}]\,|^{-1} \tag{5}$$

is the function takes the value +1 or -1 depending on the action values S_{fni}, S_{fmi}.

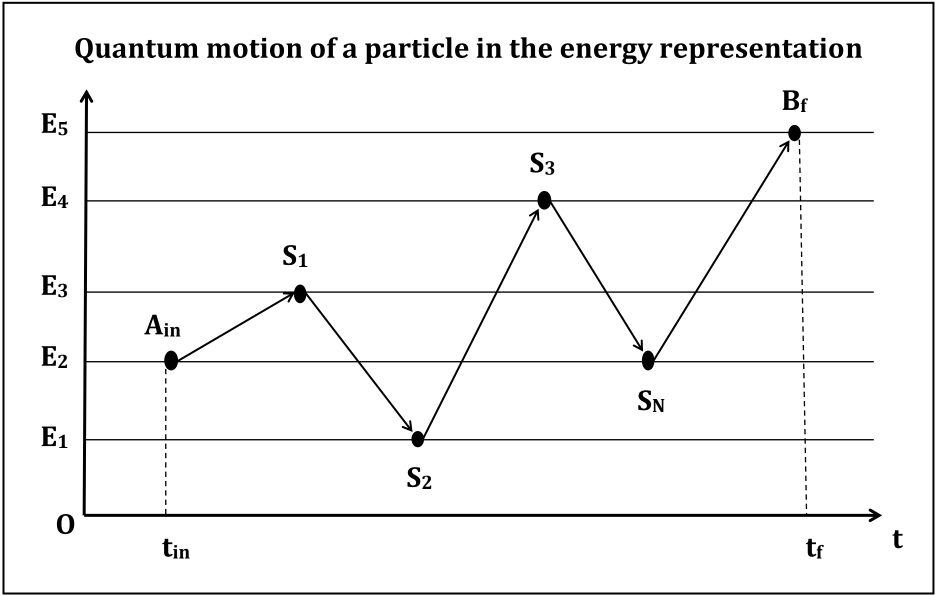
Probabilistic interpretation of particle diffraction and random joint events

In equation (4), we replace random variables by random events. The equation takes the form

$$P(B_f \mid A_i) = \sum_{n=1}^{N} P(S_{fni}) + 2 \sum_{m<n=1}^{N} g_{fnmi} P(S_{fni} \cap S_{fmi}), \tag{6}$$

where A_i – the event of the emission of a particle by a source; B_f –is the particle registration event on the screen site σ_f; $S_n, n = 1, 2, ..., N$— are the events of the particle passing through the n -th hole of the lattice; $S_{fni} = B_f \cap S_n \cap A_i$; $g_{fnmi} = g(S_{fni}, S_{fmi})$ –the function takes the values +1 or -1 depending on the relations between the events S_{fni} and S_{fmi}; $P(S_{fni} \cap S_{fmi})$ – is the probability of intersection of the joint events S_{fni} и S_{fmi} $(1 \geq P(S_{fni} \cap S_{fmi}) > 0$ for joint events and $P(S_{fni} \cap S_{fmi}) = 0$ for incompatible events.

Equation (6) coincides with the equation for stochastic processes in the space of joint events, if the events are only pairwise joint.


Quantum motion of a particle in the energy representation
E5
E4
E3
E2
E1
O
Bf
S3
S1
Ain
SN
S2
tin
tf
t

The equation of quantum motion of a particle in the energy representation

Equation of evolution of a statistical operator:

$$\hat{\rho}(t) = \hat{U}_D(t)\hat{\rho}(0)\hat{U}_D^+(t), \qquad (P5)$$

where $\hat{\rho}(t)$, $\hat{\rho}(0)$ — Statistical operators of the system, respectively, at time moment t and $t = 0$,

$$\hat{U}_D(t, t_0) = T\exp[-\frac{\imath}{\hbar}\int\limits_{t_0}^{t}\hat{V}_D(\tau)d\tau], \qquad (P6)$$

$$\hat{V}_D(\tau) = \exp[\frac{\imath}{\hbar}\hat{H}_{syst}\tau]\hat{V}_{inf}(\tau)\exp[-\frac{\imath}{\hbar}\hat{H}_{syst}\tau]. \qquad (P7)$$

In energy representation

$$\rho_{n_f m_f}(t) = \sum_{n_0, m_0}\langle n_f|\hat{U}_D(t)|n_0\rangle\rho_{n_0,m_0}\langle m_0|\hat{U}_D^+(t)|m_f\rangle, \qquad (P8)$$

where

$$\rho_{n_f m_f}(t) = \langle n_f|\hat{\rho}(t)|m_f\rangle, \qquad \rho_{n_0,m_0} = \langle n_0|\hat{\rho}(0)|m_0\rangle.$$

The probability of a quantum transition of a system can be represented as a sum along trajectories from a real functional:

$$P(n_f, t_f | n_{in}, t_{in}) = \sum_{n_1=m_1=1}^{N} P_f(n_1 = m_1) + \sum_{n_1>m_1=1}^{N} \int_0^1 \int_0^1 \int_0^1 \int_0^1 \times$$

$$\times P_f \cos[S[n_f; n_1, \xi_1; n_{in}, \xi_0] - S[n_f; m_1, \zeta_1; n_{in}, \zeta_0]] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1, \quad (52)$$

where

$$S[n_f; n_1, \xi_1; n_{in}, \xi_0] = S(n_f, t_f; n_1, t_1, \xi_1) + S(n_1, t_1; n_{in}, \xi_0);$$

$$t_f > t_1 > 0$$

$$S[n_k, t_k; n_{k-1}, t_{k-1}; \xi_{k-1}] = 2\pi(n_k - n_{k-1})\xi_{k-1} +$$

$$+\Omega^R_{n_k n_{k-1}}[\cos(2\pi(n_k - n_{k-1})\xi_{k-1} + (\Omega + \omega_{n_k, n_{k-1}})\frac{t_k + t_{k-1}}{2}) +$$

$$+\cos(2\pi(n_k - n_{k-1})\xi_{k-1} - (\Omega - \omega_{n_k, n_{k-1}})\frac{t_k + t_{k-1}}{2}))](t_k - t_{k-1}), \quad (54)$$

P_f is normalizing constant.

The equation is represented in the form

$$P(B_f) = \sum_{n=1}^{N} P(S_{fn}) + 2 \sum_{m<n=1}^{N} g_{fnm} P(S_{fn} \cap S_{fm}), \tag{10}$$

where

$$P(S_{fn} \cap S_{fj}) = \Big| \int_0^1 \int_0^1 \int_0^1 \int_0^1 P_f \cos[S[n_f; n_1, \xi_1; n_{in}, \xi_0] - S[n_f; m_1, \zeta_1; n_{in}, \zeta_0]] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 \Big|, \tag{57}$$

where

$$g(S_{fn}, S_{fm}) = \Big[\int_0^1 \int_0^1 \int_0^1 \int_0^1 \cos[S[n_f; n_1, \xi_1; n_{in}, \xi_0] S[n_f; m_1, \zeta_1; n_{in}, \zeta_0]] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 \Big] \times$$

$$\times \Big[\Big| \int_0^1 \int_0^1 \int_0^1 \int_0^1 \cos[S[n_f; n_1, \xi_1; n_{in}, \xi_0] S[n_f; m_1, \zeta_1; n_{in}, \zeta_0]] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 \Big| \Big]^{-1}, \tag{56}$$

$$g(S_{fn}, S_{fm}) = +1 \ \text{ or } \ -1$$

We will assume that S_{fm}, S_{fn} are random events

Results

1. The system of axioms of probability theory is supplemented by a new axiom on joint quantum random events.
2. In the new set of general events we have the equation for stochastic process

$$P(f \mid i) = \sum_{n=1}^{N} P(S_{fni}) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_{fni} \cap S_{fm}) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots + 2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}),$$

3. For the case when events in the process are only joint in pairs

$$P(S_{fni} \cap S_{fmi} \cap S_{fki}) = 0, \;\; P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}) = 0$$

The equation has the form $$P(B_f) = \sum_{n=1}^{N} P(S_n) + 2 \sum_{n>m=1}^{N} g(S_n, S_m) P(S_n \cap S_m)$$

This equation describes real quantum processes (particle diffraction, quantum transitions, and others).
4. There is a prospect for exploring a new space, a new equation.

Thanks for your attention!