Higher order anisotropies from hydrodynamical freeze-out models

Máté Csanád, Sándor Lökös, Boris Tomášik, Jakub Cimerman and Tamás Csörgő

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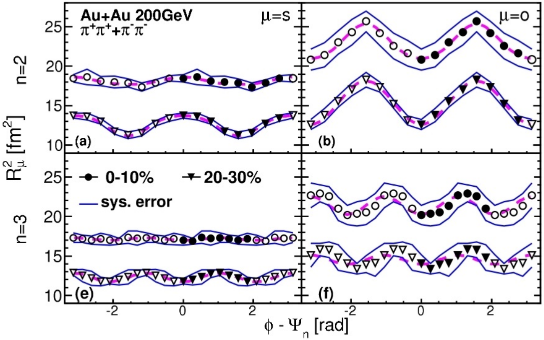
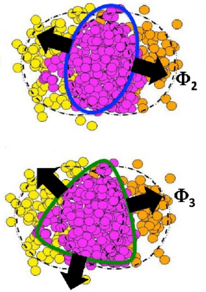
Introduction and motivation

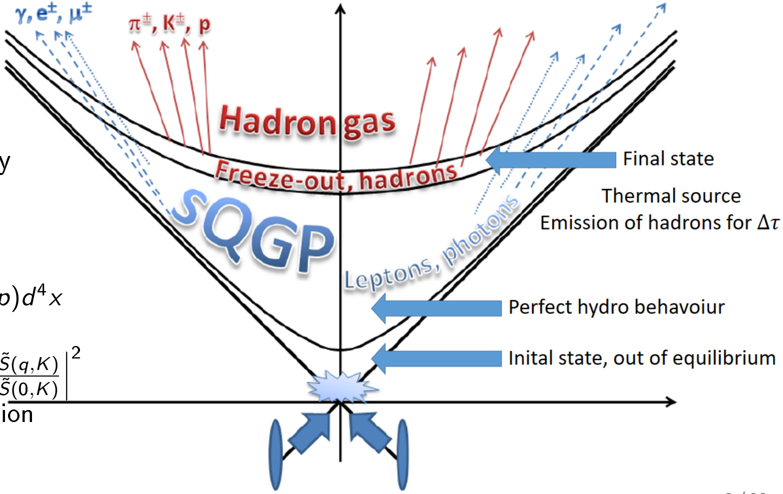
- sQGP behaves like perfect fluid → freeze-out hydro models or hydro solutions
- Finite number of nucleons → generalized space-time and the velocity field geometry
- Multipole in space-time solution: PRC90,054911 → higher order flows
- Azimuthally sensitive HBT have $\cos(n\phi)$ dependences → generalized velocity field needed
- These can be studied experimentally: NPA 904-905 (2013), PRL. 112 (2014) 22





Model building (recipe)

- Parametrization of the hypersurface (Cooper-Frye factor)
- Thermal distribution
- Propertime distribution at the freeze-out
- Quantity measures the geometry
- Some prefactors
- S(x,p) is the product of these
- Main measurable quantities:
  - Spectra: $N_1(p_t, \varphi) = \int S(x,p) d^4x$
  - Flows $\langle N_1(p_t, \varphi) \cos(n\varphi) \rangle$
  - Correlation $C_2(q, K) = 1 + \left| \frac{\tilde{S}(q,K)}{\tilde{S}(0,K)} \right|^2$
  - Azimuthally sensitive correlation e.g. $R^2_{\text{side}} = \langle r^2_{\text{side}} \rangle - \langle r_{\text{side}} \rangle^2$ where $r_{\text{side}} = r \sin(\alpha - \varphi)$



Two freeze-out model

	Buda-Lund model	Blast-wave model
Cooper-Frye factor	$p_\mu u^\mu$	$p_\mu u^\mu$
Propertime distribution	$\delta(\tau - \tau_0)$	$\frac{1}{\sqrt{2\pi}\Delta\tau} \exp\left[-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right]$
Thermal distribution	$\exp\left[-\frac{p_\mu u^\mu + \mu}{T}\right]$ where $\begin{aligned} T &= T_0/(1+a^2 s) \\ \frac{\mu}{T} &= \frac{\mu_0}{T_0} - bs \end{aligned}$	$\exp\left[-\frac{p_\mu u^\mu}{T}\right]$
Space-time geometry	s scale parameter	$\Theta\left(1 - \frac{r}{R(\varphi)}\right)$ box proflie

- Space-time anisotropy should put into the scale parameter or the box profile
- The velocity field anisotropy should put into the u^μ velocity field

Now put everything together!

The Buda-Lund model

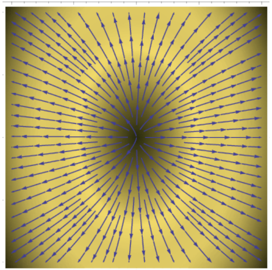
PRC54 (1996) 1390, EPJA (2016) 52: 311 and NPA742 (2004) 80-94

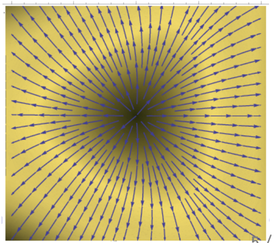
- The spatial asymmetry is described by the scaling variable
- General n-pole spatial asymmetry (elliptical case: $n = 2$):

$$s = \frac{r^2}{2R^2}\left(1 + \sum_n \epsilon_n \cos(n(\phi - \Psi_n))\right) + \frac{r_z^2}{2Z^2}$$

- Ψ_n is the angle of the n-th order reaction plane
- Derive the velocity field from a potential: $u_\mu = \gamma(1, \partial_x\Phi, \partial_y\Phi, \partial_z\Phi)$
- General n-pole asymmetrical potential (elliptical case: $n = 2$):

$$\Phi = H\frac{r^2}{2}\left(1 + \sum_n \chi_n \cos(n(\phi - \Psi_n))\right) + H_z\frac{r_z^2}{2}$$





General aspects Eur. Phys. J. A (2016) 52: 311

- There is multipole solution: Phys.Rev.C90(2014) based on HeavyIonPhys.A21:73(2004)
- In our case $u^\mu \partial_\mu s = 0$ can be fulfilled :
  - in $\mathcal{O}(\epsilon_n)$ and $\mathcal{O}(\chi_n)$ if $\dot{\epsilon}_n = -2\frac{\dot{R}}{R}\chi_n$ and $\frac{\dot{R}}{R} = H$
  - in any order with the same index $\frac{\dot{R}}{R} = H\left(1 + \frac{1}{2}\sum_{n=1}^{\infty} \epsilon_n \chi_n \left(1 + \frac{n^2}{4}\right)\right)$
- Or for any $k > 0$

$$\dot{\epsilon}_k = 2H\chi_k - 2\left(\frac{\dot{R}}{R} - H\right)\epsilon_k + H\sum_{n=1}^{\infty} \epsilon_n \chi_{n+k}\left(1 + \frac{n(n+k)}{4}\right) + H\sum_{\substack{n=1 \\ n\neq k}}^{\infty} \epsilon_n \chi_{|n-k|}\left(1 + \frac{n(n-k)}{4}\right)$$

The Blast-wave model

PRL (1979) 42, PRC (2004) 70, EPJ. A (2017) 53: 161

- Space anisotropy is characterized by Fourier series of the fireball radius

$$R(\theta) = R_0(1 - a_2\cos(2(\theta-\theta_2)) - a_3\cos(3(\theta-\theta_3)))$$

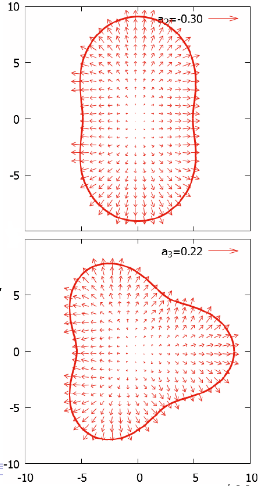
- The velocity field with $\rho = \rho\left(\frac{r}{R(\theta)}, \theta\right)$ and $\theta_b = \theta_b(r, \theta)$

$$u_\mu = (\cosh\eta_s\cosh\rho, \sinh\rho\cos\theta_b, \sinh\rho\sin\theta_b, \sinh\eta_s\cosh\rho)$$

- Flow anisotropy is characterized by Fourier series of the transverse rapidity

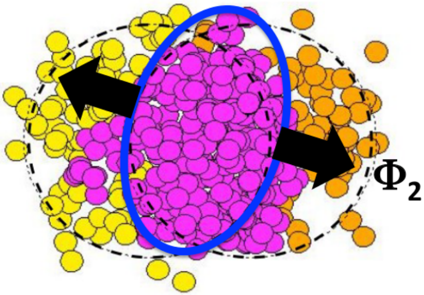
$$\rho\left(\frac{r}{R(\theta)}, \theta\right) = \frac{r}{R(\theta)}(1 + 2\rho_2\cos(2(\theta_b-\theta_2)) + 2\rho_3\cos(3(\theta_b-\theta_3)))$$

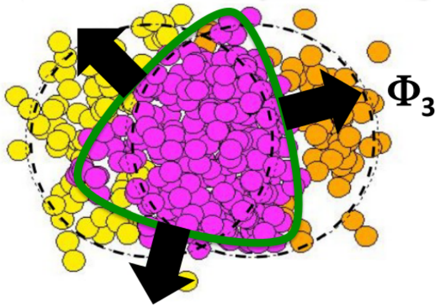
- θ_b is angle of the transverse velocity



Observables at freeze-out

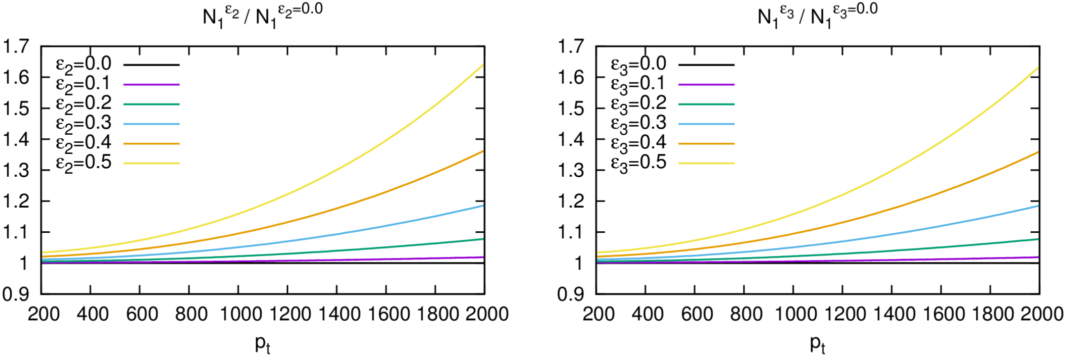
- Invariant transverse momentum distribution, flows, azimuthally sensitive HBT radii
- All asymmetries are investigated in their respective reaction plane
- Rotate the system to the second / third order plane and average on the angle of the third / second order plane
- The proper parameters can be set to zero to avoid the averaging





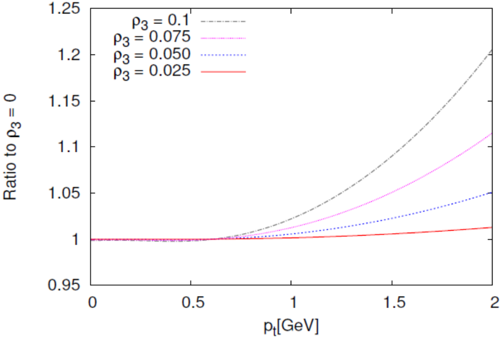
Invariant momentum distribution in Buda-Lund model

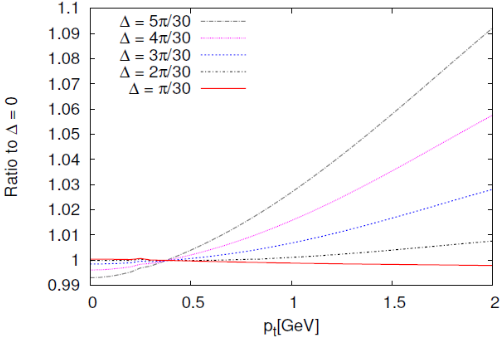
Significant change could be at high p_t, the log slope is not affected strongly



Invariant momentum distribution in Blast-wave model

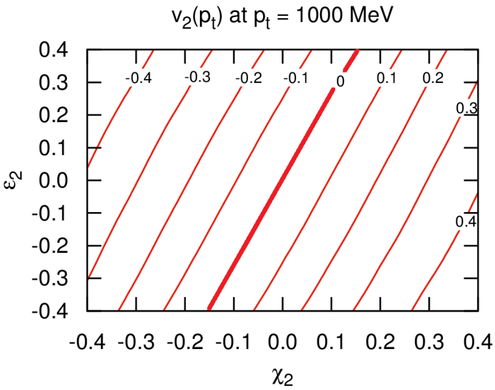
Flow anisotropy also influences azimuthally integrated spectrum a bit

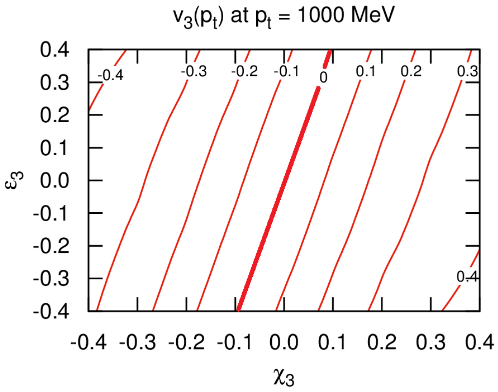




Ambiguity in determination of the flow

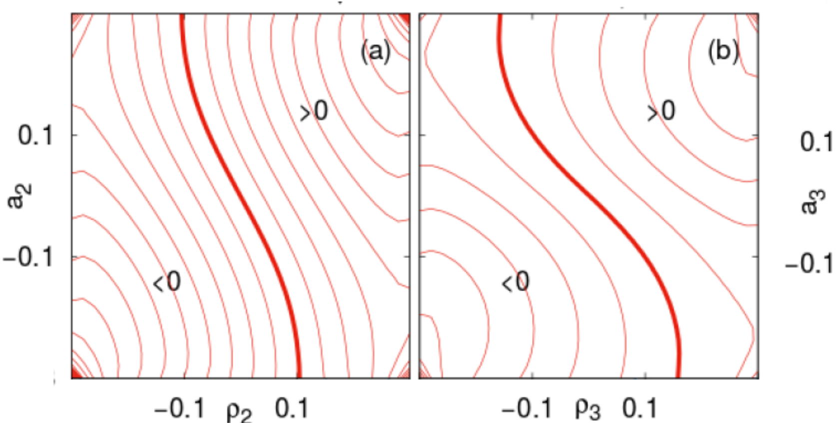
- The parameters affect the flows together
- We can get the same v_n with different combination
- In the Buda Lund model:





Ambiguity in determination of the flow

- The very same in the Blast-wave model
- Only the flow is not enough to determine the anisotropies
- $v_2(p_t)$ and $v_3(p_t)$ in the Blast-wave model at $p_t = 300$ MeV



HBT radii

- Calculate in the *out* − *side* − *long* system

$$R^2_{\text{out}} = \langle r^2_{\text{out}} \rangle - \langle r_{\text{out}} \rangle^2 \text{ and } R^2_{\text{side}} = \langle r^2_{\text{side}} \rangle - \langle r_{\text{side}} \rangle^2$$

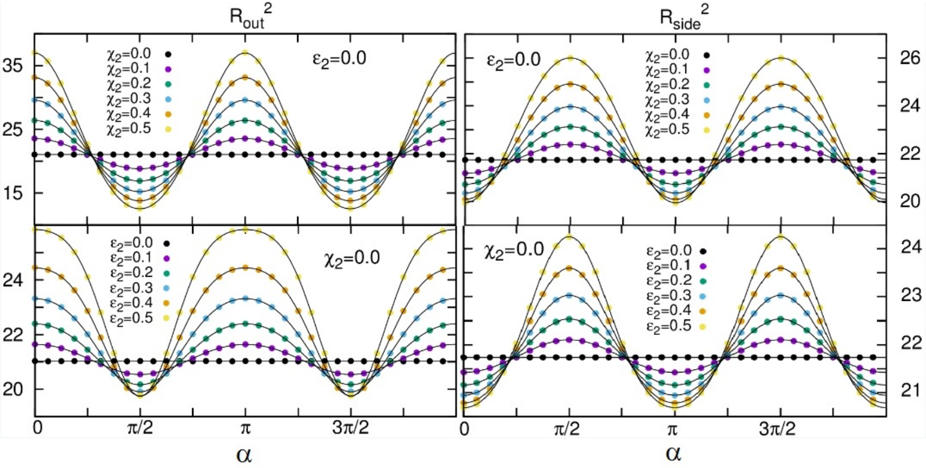
  where $r_{\text{out}} = r\cos(\phi - \alpha) - \beta_t t$ and $r_{\text{side}} = r\sin(\phi - \alpha)$

  → C. J. Plumberg, C. Shen, U. W. Heinz Phys.Rev. C88 (2013) 044914
- There can be higher order parts

  → B. Tomášik and U. A. Wiedemann, arXiv:hep-ph/0210250
- We use the following parametrisation in
  - elliptical case: $R^2_{\text{out}} = R^2_{\text{out},0} + R^2_{\text{out},2}\cos(2\alpha) + +R^2_{\text{out},4}\cos(4\alpha) + R^2_{\text{out},6}\cos(6\alpha)$
  - triangular case: $R^2_{\text{out}} = R^2_{\text{out},0} + R^2_{\text{out},3}\cos(3\alpha) + R^2_{\text{out},6}\cos(6\alpha) + R^2_{\text{out},9}\cos(9\alpha)$
- Similar to the R^2_{side}

Results of the parametrization – Second order case (Buda-Lund model)

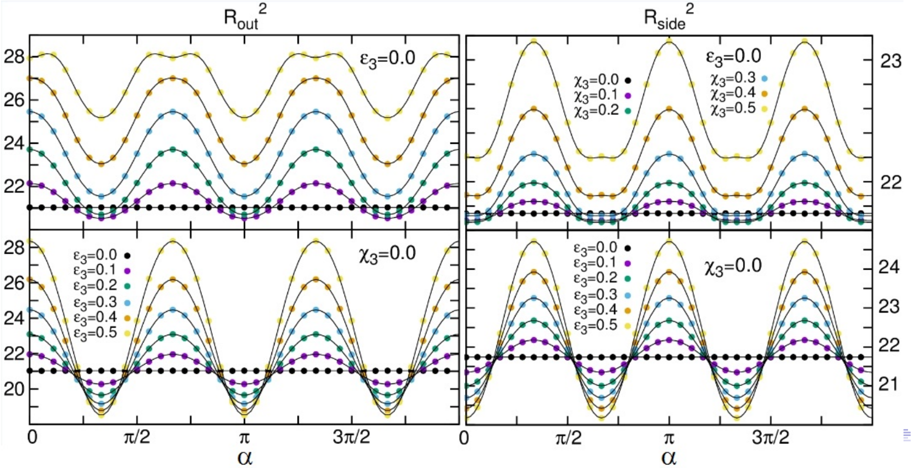
This case already have investigated: Eur.Phys.J.A37:111-119,2008
Mainly $\cos(2\phi)$ behaviour but higher order oscillations are also present



Results of the parametrization – Third order case (Buda-Lund model)

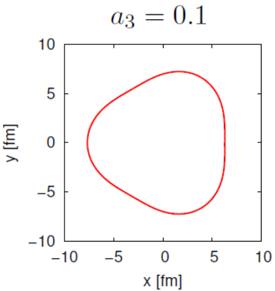
For details see: Eur. Phys. J. A (2016) 52: 311

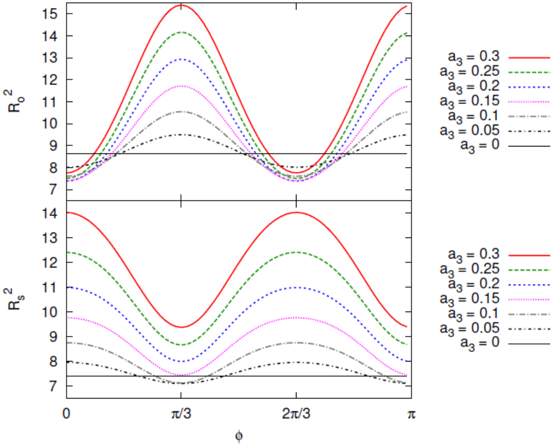
Mainly cos(3ϕ) behaviour but higher order oscillations are also present



Results of the parametrization with different *a* (Blast-wave model)

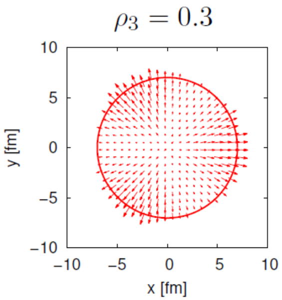
For details see: Eur. Phys. J. A (2017) 53: 161

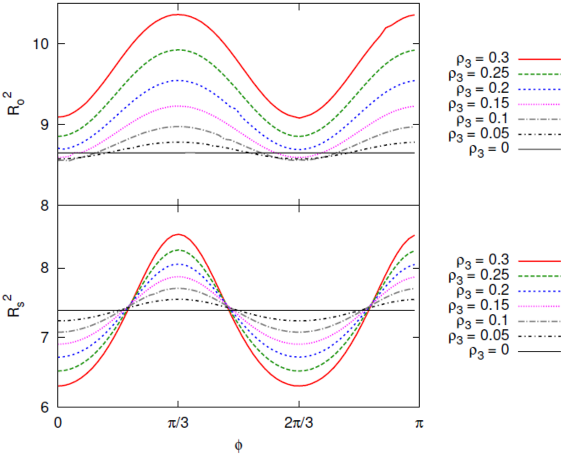




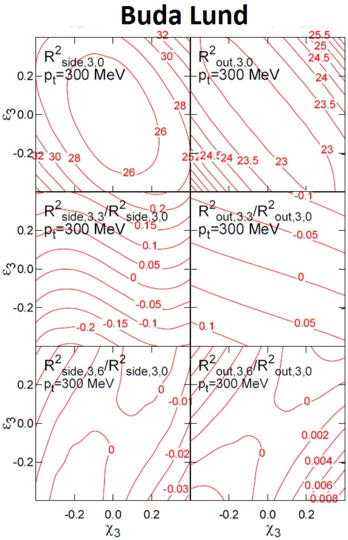
Results of the parametrization with different a (Blast-wave model)

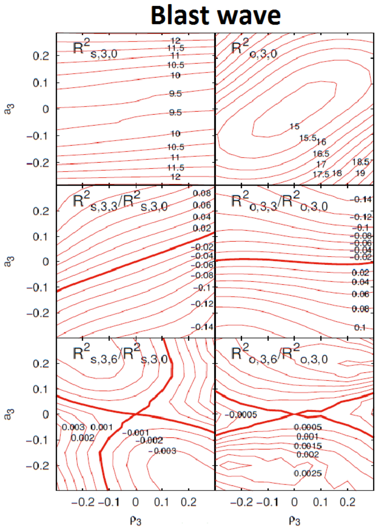
For details see: Eur. Phys. J. A (2017) 53: 161





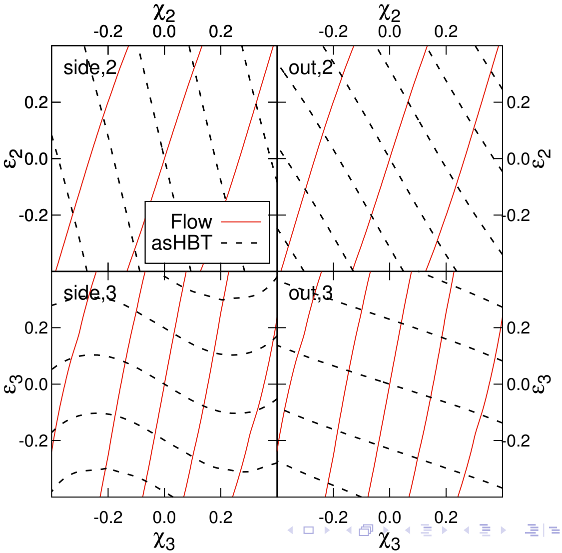
Mixing of the parameters in the Buda Lund and the Blast wave model model





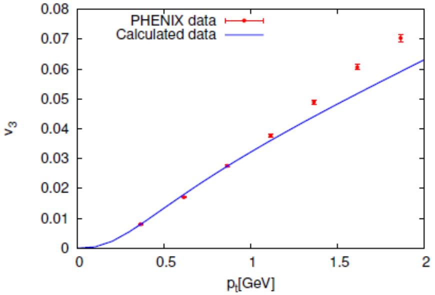
Disentangling

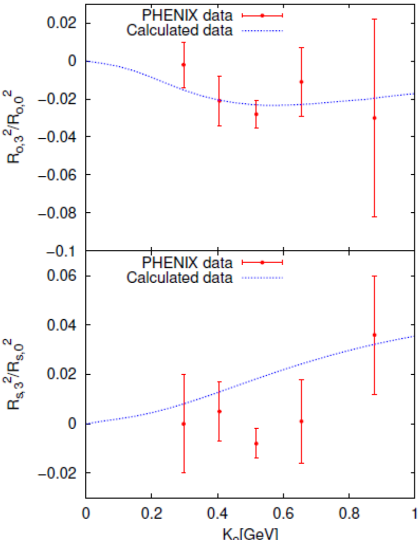
- How to disentangle the parameters of the flow and the asHBT radii?
- With a simultaneous measurements of these two observables
- After measuring v_n and one of the correlation radii $R^2_{i;n}$ we can combine contour plots and get exact parameters of both anisotropies



Qualitative illustration of the data analysis with the Blast-wave model

Momentum dependence of Fourier coefficients calculated from Blast-wave model compared with data from PHENIX.



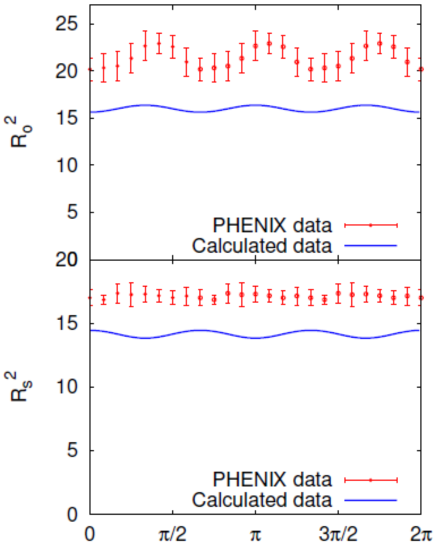


Qualitative illustration of the data analysis with the Blast wave model

Azimuthal dependence of HBT radii

Model parameters are $p_t = 530$ MeV, $\tau = 7.8$ fm/c, $\Delta\tau = 2.59$ fm/c, $R_0 = 11.4$ fm, $T = 98$ MeV, $\rho_0 = 0.98$

Difference between data and calculations shows that we can't use parameters from one experiment and use them to fit results from another experiment



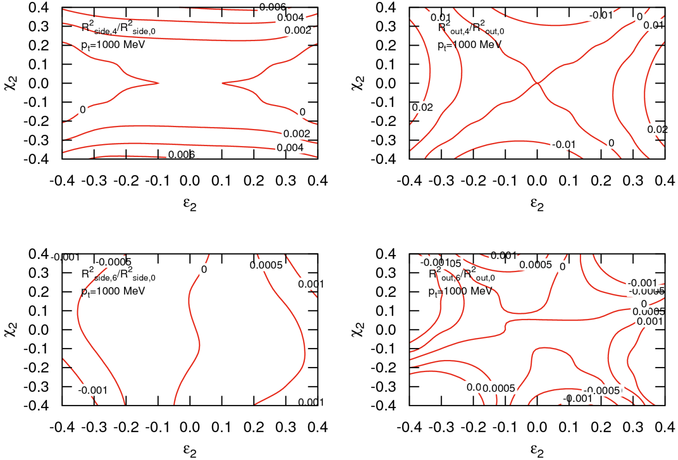
Conclusions

- The geometry and the velocity field are generalized in freeze-out models
- Higher order flows and azimuthally sensitive HBT radii can be derived
- Absolute value of the azimuthal HBT radii depend on asymmetries
- Higher order oscillation can be observed in HBT radii
- The spatial and velocity field anisotropies both influence v_n coefficients and HBT radii
- The asymmetry parameters can be disentangle from the flows and the amplitudes

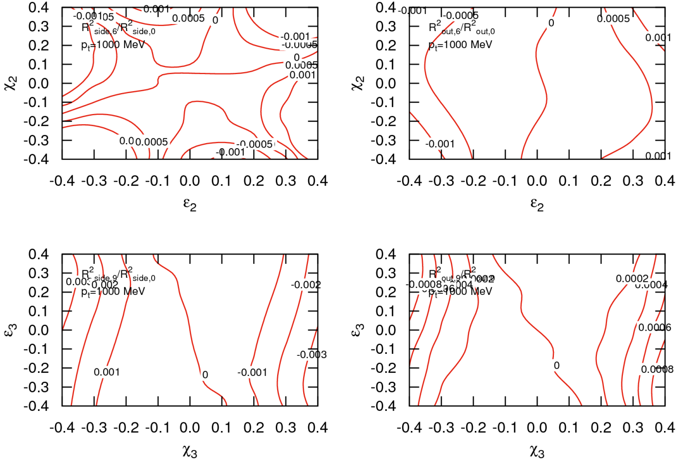
Thank you for your attention!

Backup slides – Higher order amplitudes

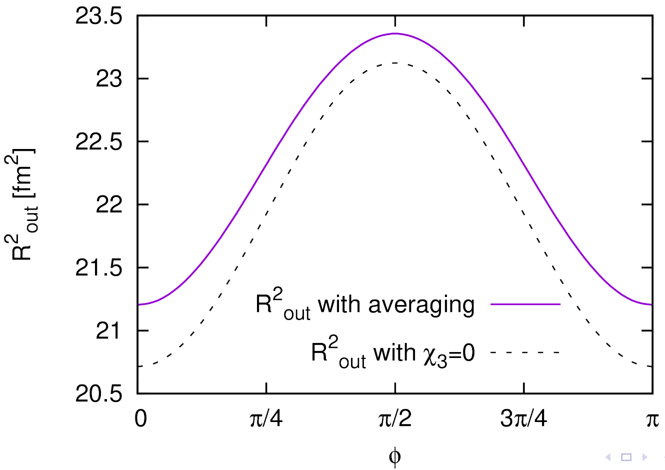
Second order:



Third order:

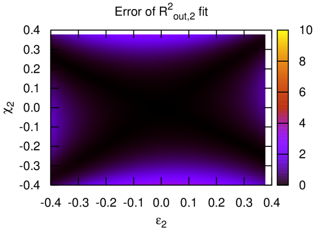

R2 side,6/R2 side,0
pt=1000 MeV
χ2
ε2
R2 out,6/R2 out,0
pt=1000 MeV
χ2
ε2
R2 side,9/R2 side,0
pt=1000 MeV
ε3
χ3
R2 out,9/R2 out,0
pt=1000 MeV
ε3
χ3
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
0.001
0.0005
0
-0.0005
-0.001
0.002
-0.002
-0.003
0.0002
0.0004
0.0006
0.0008

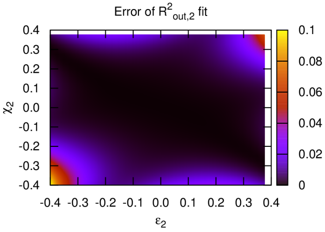
Averaging vs. set-to-zero

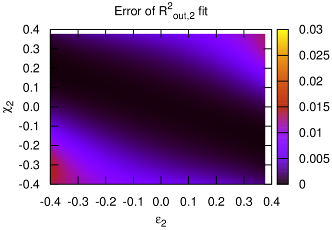


Backup slides – Square of residuals

An example: square of residuals of R^2_{out} with different parametrizations







Earlier results

Fits with elliptical Buda Lund model: Eur.Phys.J. A47 (2011) 58-66