# New solutions of fireball hydrodynamics with shear and bulk viscosity

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# Outline

Context, motivation

Exact accelerating solution for e/p =1

Generalization for e/p = k

A new solution with bulk viscosity

Adding shear viscosity

Effect on observables

Summary and conclusion

# Context

**Renowned exact solutions**

- **Landau-Khalatnikov solution: dn/dy ~ Gaussian**
- **Hwa solution (1974) – Bjorken: solution + $\varepsilon_0$ (1983)**
- **Chiu, Sudarshan and Wang: plateaux**

**Revival of interest:** **Buda-Lund model + exact solutions,**
**Biró, Karpenko+Sinyukov, Pratt (2007),**
**Bialas+Janik+Peschanski, Borsch+Zhdanov (2007)**
**CsT+Csanád+Nagy (2007-2008)**
**CsT+Csernai+Hama+Kodama (2004)**
**Gubser**
**Hatta, Noronha, Xiao, ....**

**New simple solutions → Evaluation of measurables**

**Rapidity distribution → Advanced initial energy density**

**HBT radii → Advanced life-time estimation**

# Goal

**Need for solutions that are:**

- **explicit**
- **simple**
- **accelerating**
- **relativistic**
- **realistic / compatible with the data:**
  - **lattice QCD EoS**
  - **ellipsoidal symmetry (spectra, $v_2$, $v_4$, HBT)**
  - **finite dn/dy**
- **Generelization of a class that satisfies each of these criteria**
  - **but not simultaneously**

**M.I. Nagy, T. Cs., M. Csanád,** arXiv:0709.3677v1 , PRC77:024908 (2008)

**T. Cs, M. I. Nagy, M. Csanád,** arXiv:nucl-th/0605070v4, PLB (2008)

**M. Csanád, M. I. Nagy, T. Cs,** arXiv:0710.0327v3 [nucl-th] EPJ A (2008)

**Dedicated to Bolyai, Lobachevski and Gauss**

# Perfect fluid hydrodynamics

**Energy-momentum tensor:**

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} \qquad w = \varepsilon + p$$

$$\partial_\nu T^{\mu\nu} = 0$$

**Relativistic Euler equation:**

$$w u^\nu \partial_\nu u^\mu = \left(g^{\mu\rho} - u^\mu u^\rho\right) \partial_\rho p$$

**Energy conservation:**

$$w \partial_\mu u^\mu = -u^\mu \partial_\mu \varepsilon$$

**Charge conservation**

$$\sum \mu_i \partial_\mu \left(n_i u^\mu\right) = 0$$

**Consequence is entropy conservation:**

$$\partial_\mu \left(\sigma u^\mu\right) = 0.$$

# Self-similar, ellipsoidal solutions

**Publication (for example):**

T. Cs, L.P.Csernai, Y. Hama, T. Kodama, Heavy Ion Phys. A 21 (2004) 73

**3D spherically symmetric HUBBLE flow:**

$$u^\mu = \frac{x^\mu}{\tau}$$

**No acceleration:**

$$u^\mu \partial_\mu u_\nu = 0.$$

**Define a scaling variable for self-similarly expanding ellipsoids:**

$$s = \frac{r_x^2}{\dot{X}_0^2 t^2} + \frac{r_y^2}{\dot{Y}_0^2 t^2} + \frac{r_z^2}{\dot{Z}_0^2 t^2}$$

**EoS: (massive) ideal gas**

$$\epsilon = mn + \kappa p\,, \qquad p = nT.$$

$$\varepsilon_Q = m_Q n_Q + \lambda_\varepsilon n_Q T + B, \qquad p_Q = \lambda_p n_Q T - B,$$

$$n(t,\mathbf{r}) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s) \qquad T(t,\mathbf{r}) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\mathcal{V}(s)} \quad \Rightarrow \quad p(t,\mathbf{r}) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+3/\kappa}$$

**Scaling function $\mathcal{V}(s)$ can be chosen freely.**

**Shear and bulk viscous corrections in NR limit: known analytically.**

# New, simple, exact solutions

$$v = \tanh \lambda\eta,$$
$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh \frac{\eta}{2}\right)^{-(d-1)\phi_\lambda}$$

Possible cases (one row of the table is one solution):

| Case | $\lambda$ | $d$ | $\kappa$ | $\phi_\lambda$ |
|---|---|---|---|---|
| a.) | 2 | $\in \mathbb{R}$ | $d$ | 0 |
| b.) | $\frac{1}{2}$ | $\in \mathbb{R}$ | 1 | $\frac{\kappa+1}{\kappa}$ |
| c.) | $\frac{3}{2}$ | $\in \mathbb{R}$ | $\frac{4d-1}{3}$ | $\frac{\kappa+1}{\kappa}$ |
| d.) | 1 | $\in \mathbb{R}$ | $\in \mathbb{R}$ | 0 |
| e.) | $\in \mathbb{R}$ | 1 | 1 | 0 |

M. I. Nagy, T.Cs., M. Csanád:
arXiv:0709.3677v1

→ New, accelerating, d dimension

↘ d dimensional with p=p(τ,η)

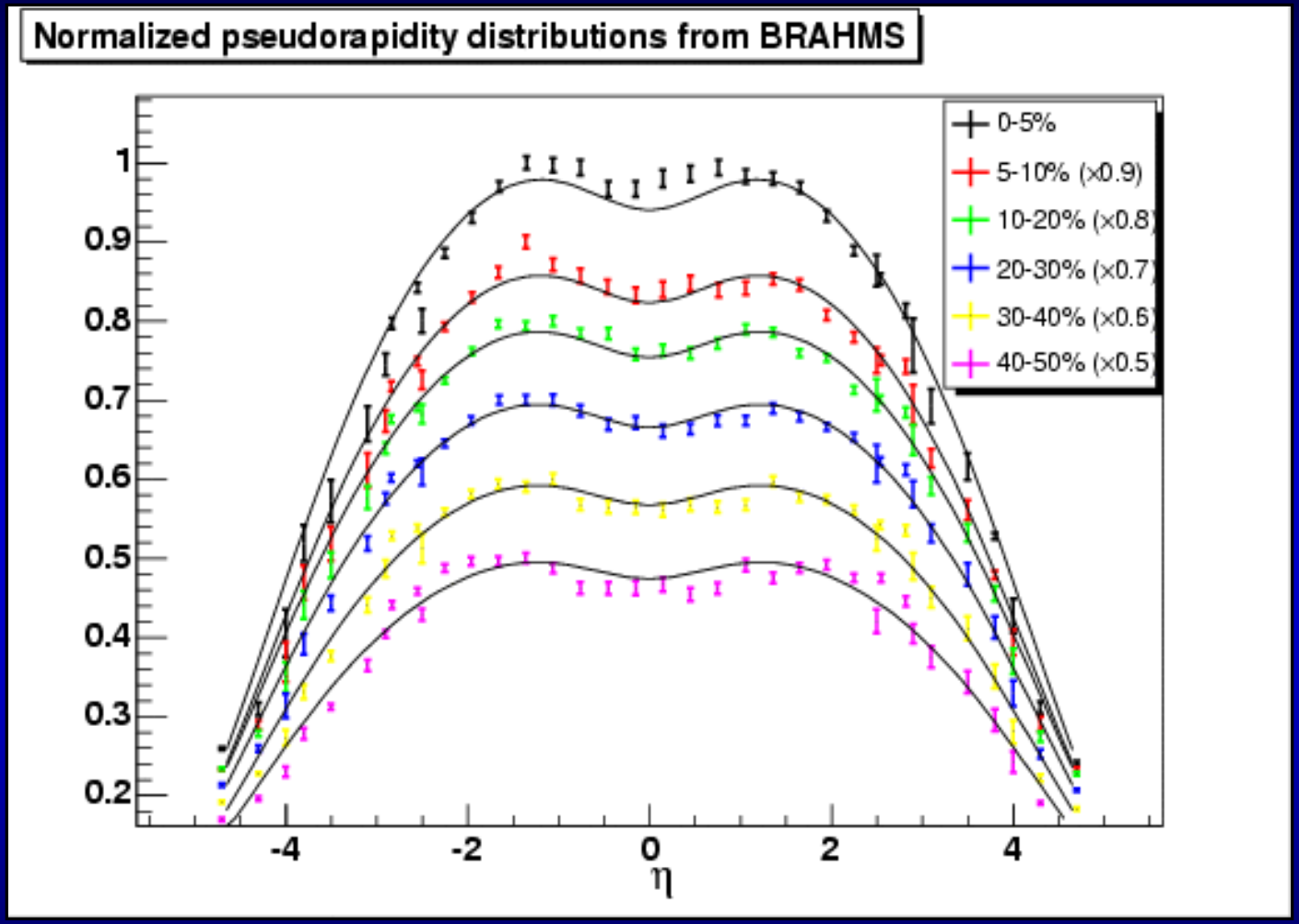
↗ (thanks T. S. Biró)

→ Hwa-Bjorken, Buda-Lund type

→ κ = e/p= 1, but general velocity profile

If $\kappa = d = 1$ , general solution is obtained, for ARBITRARY initial conditions. It is STABLE !

# Pseudorapidity distributions




BRAHMS data fitted with the analytic formula of

Additionally: $y \rightarrow \eta$ transformation



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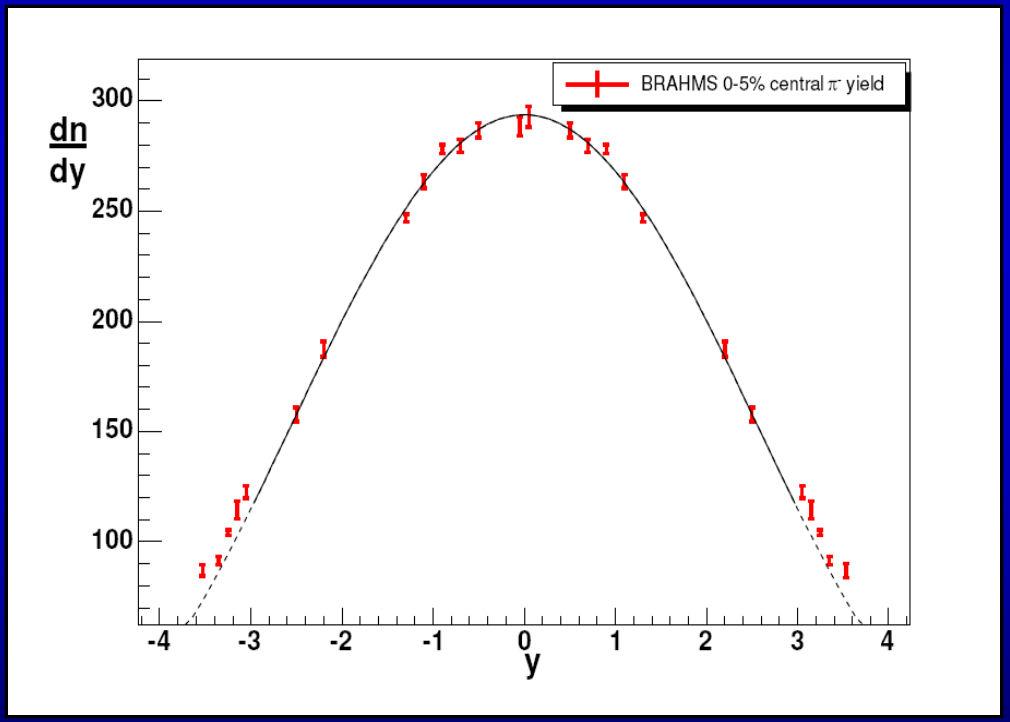
# BRAHMS rapidity distribution

$$\frac{\mathrm{d}n}{\mathrm{d}y} \approx \left.\frac{\mathrm{d}n}{\mathrm{d}y}\right|_{y=0} \cosh^{\pm\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) e^{-\frac{m}{T_f}\left[\cosh^{\alpha}\left(\frac{y}{\alpha}\right)-1\right]},$$




$$\lambda = \frac{\alpha - 1}{\alpha - 2}.$$

| | |
|---|---|
| $\alpha$ | $7.4 \pm 0.13$ |
| $\frac{\mathrm{d}n}{\mathrm{d}y}\|_{y=0}$ | $294 \pm 1$ |
| $\chi^2$/NDF | 30.6/14 |
| CL | 0.6% |
| $T_f$ (MeV) | 200 (fixed) |
| $m$ (MeV) | 140 (fixed) |
| $\lambda$ | $1.18 \pm 0.01$ (derived) |

**BRAHMS dn/dy data fitted with the analytic formula**



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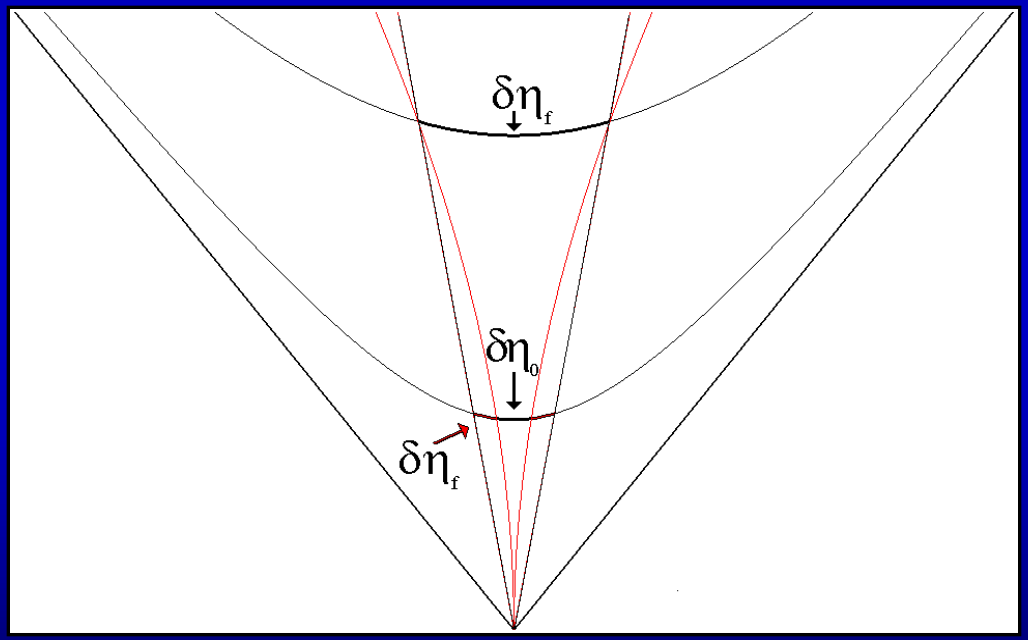
# Advanced energy density estimate

**Fit result:** $\lambda > 1$

**Flows accelerate:** ⟺ **do work**

⟺ **initial energy density is higher than Bjorken's**




$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}$$

**Work and acceleration.**

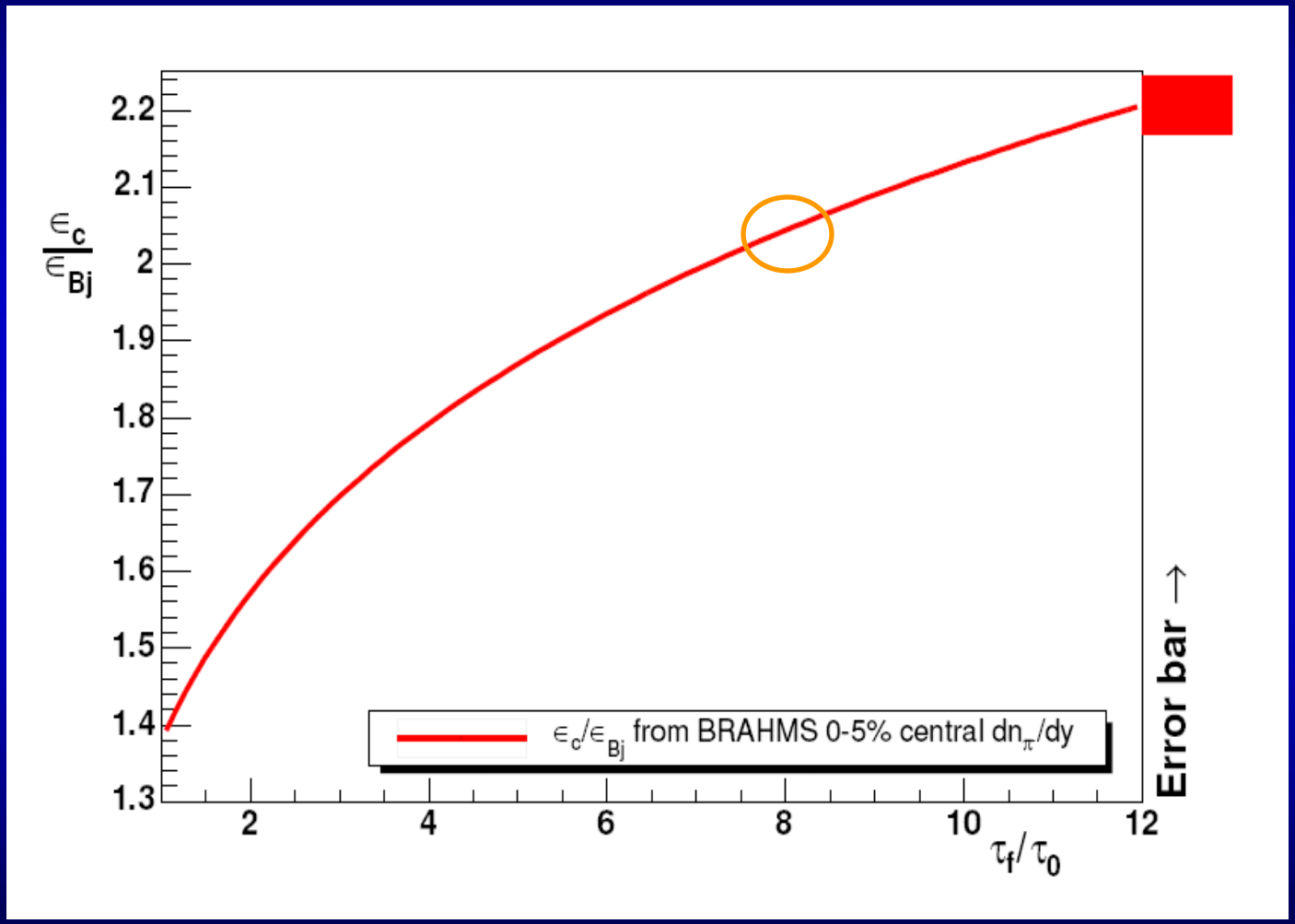
**FYI:**

$$\varepsilon_{Bj} = \frac{\langle m_t \rangle}{(R^2\pi)\tau_0}\frac{dn}{dy}$$

**For $\lambda > 1$ (accelerating) flows, both factors > 1**

# Advanced energy density estimate

**Correction depends on timescales, dependence is:**




**With a typical $\tau_f/\tau_0$ of ~8-10, one gets a correction factor of 2!**

# Conjecture: EoS dependence of $\varepsilon_0$

Four constraints

1) $\varepsilon_{Bj}$ is independent of EoS ($\lambda = 1$ case)

2) $c_s^2 = 1$ case is solved for any $\lambda > 0.5$

$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}$$

Corrections due to respect these limits.

3) $c_s^2$ dependence of $\varepsilon(\tau)$ is known in NR limit

4) Numerical hydro results, e.g. K. Morita, arXiv:nucl-th/0611093v2

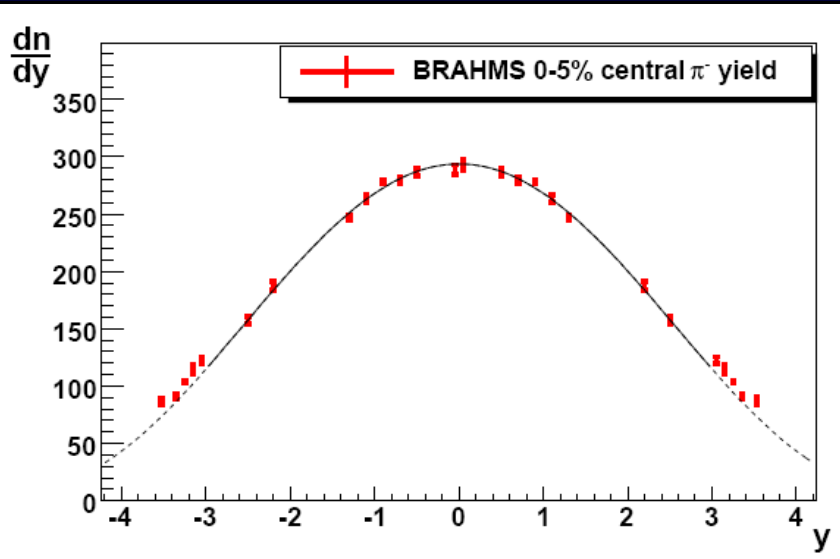
Conjectured formula – given by the principle of Occam's razor:

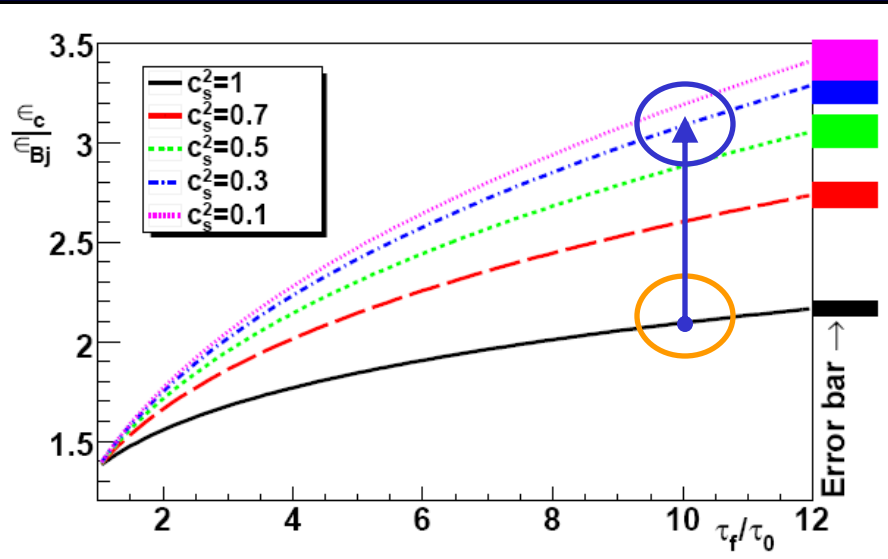
$$\frac{\varepsilon_{c_s^2}}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)(1-c_s^2)}$$

Using $\lambda = 1.18$, $c_s = 0.35$, $\tau_f/\tau_0 = 10$, we get $\varepsilon_{cs}/\varepsilon_{Bj} = 2.9$

$\varepsilon_0 = 14.5$ GeV/fm$^3$ in 200 GeV, 0-5 % Au+Au at RHIC

# Conjectured EoS dependence of $\varepsilon_0$




$$\frac{\varepsilon_{c_s^2}}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)(1-c_s^2)}$$

Using $\lambda = 1.18$, and $\tau_f/\tau_0 = 10$ as before

and $c_s = 0.35$, [PHENIX, arXiv:nucl-ex/0608033v1 ] we get $\varepsilon_{cs}/\varepsilon_{Bj} = 2.9$

$\varepsilon_0 = 14.5$ GeV/fm$^3$ in 200 GeV, 0-5 % Au+Au at RHIC



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# Advanced life-time estimate

**Life-time estimation: for Hwa-Bjorken type of flows**

$$R_{long} = \sqrt{\frac{T_f}{m_t}}\tau_{Bj} \quad \Rightarrow \quad \tau_{Bj} = \sqrt{\frac{m_t}{T_f}}R_{long}.$$

**Makhlin & Sinyukov, Z. Phys. C 39, 69 (1988)**

**Underestimates lifetime (Renk, CsT, Wiedemann, Pratt, ...)**

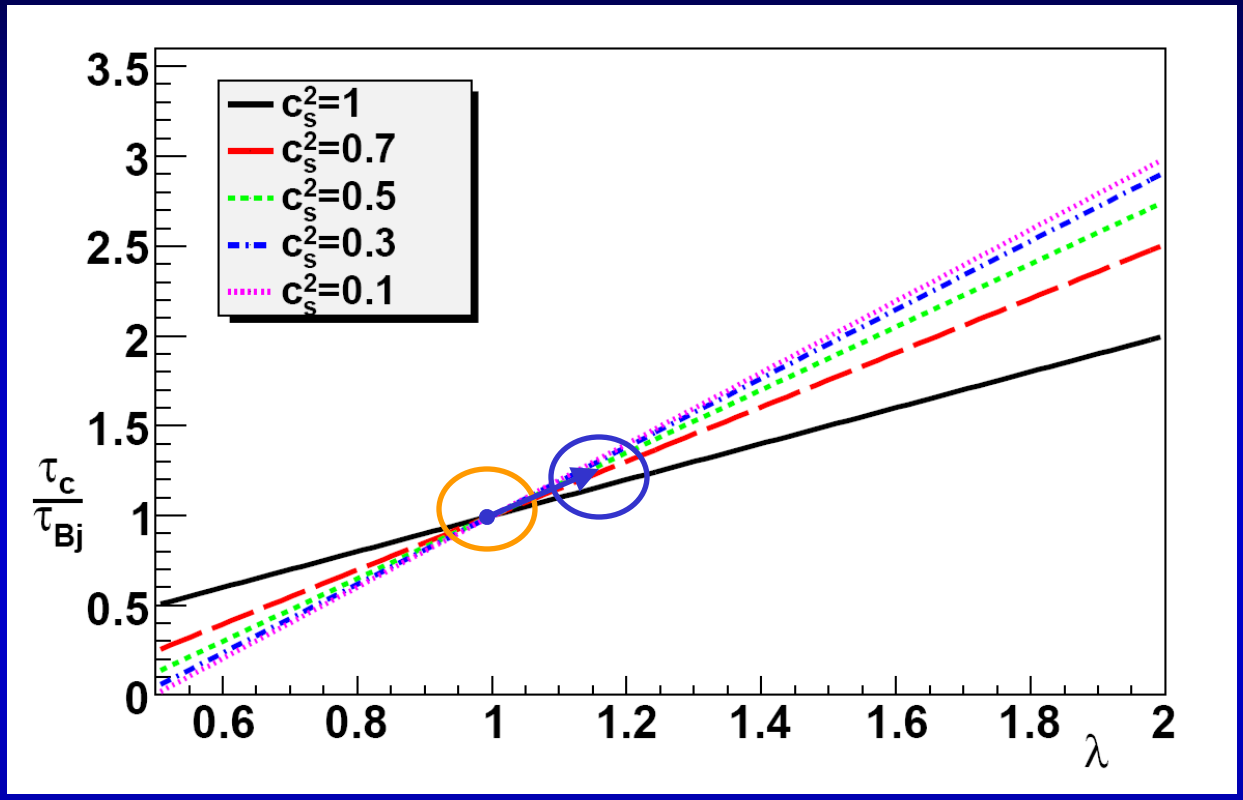
**Advanced life-time estimate:**

**width of dn/dy related to acceleration and work**

$$R_{\rm long} = \sqrt{\frac{T_f}{m_t}\frac{\tau_{\rm c}}{\lambda}} \quad \Rightarrow \quad \tau_{\rm c} = \lambda\tau_{Bj}.$$

**At RHIC energies: correction is about +20%**

# Conjectured EoS dependence of $\tau_c$




$$R_{\rm long} = \sqrt{\frac{T_f}{m_t}} \frac{\tau_{\rm c}}{\lambda + (\lambda - 1)(1 - c_s^2)} \quad \Rightarrow \quad \tau_{\rm c_s} = [\lambda + (\lambda - 1)(1 - c_s^2)]\tau_{Bj}.$$

Using $\lambda = 1.18$, and $c_s = 0.35$, we get $\tau_{cs}/\tau_{Bj} = 1.36$

in 200 GeV, 0-5 % Au+Au at RHIC



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# New, exact solutions for e/p > 1

$$\frac{1-\kappa\tanh^2(\Omega-\eta_s)}{1-\tanh^2(\Omega-\eta_s)}\Omega' = \lambda,$$

$$\Rightarrow \quad \Omega' = \lambda\frac{1-\tanh^2(\Omega-\eta_s)}{1-\kappa\tanh^2(\Omega-\eta_s)},$$

$$\Rightarrow \quad \eta_s = -H + \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}}\mathrm{Arctan}\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}}\tanh(H)\right).$$

**Possible cases (one row of the table is one solution):**

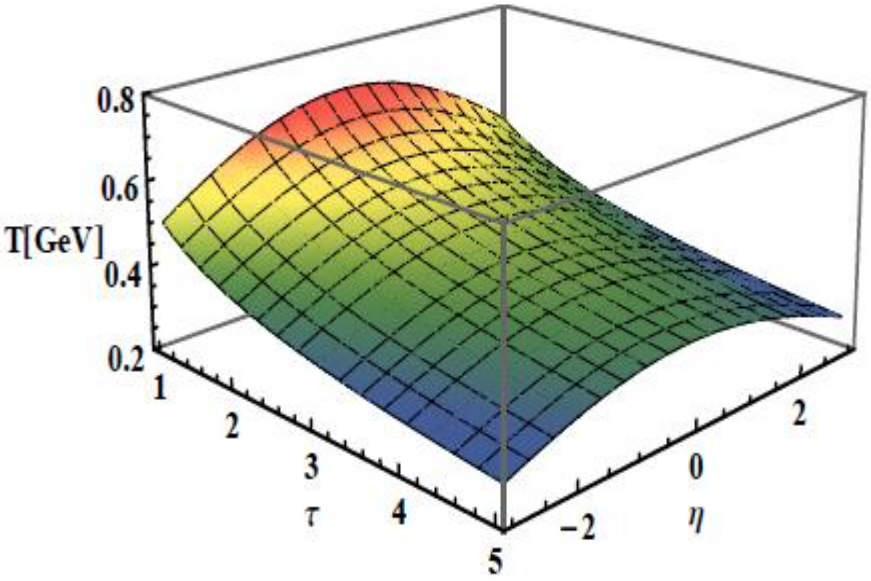
$$H = \Omega - \eta_s \ll 1,\ \kappa > 1,\ \lambda > 1,$$

$$T = T_i\left(\frac{\widetilde{\tau}_0}{\tau}\right)^{\frac{\lambda}{\kappa}}\exp\left[-\frac{\lambda(\lambda-1)(\kappa-1)}{2\kappa}\eta_s^2\right],$$

$$H = \Omega - \eta_s \ll 1,$$

$$\mathrm{Arctan}\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}}\tanh(H)\right) \simeq \sqrt{\frac{\kappa-\lambda}{\lambda-1}}H,$$

$$\Rightarrow \quad H = (\lambda-1)\eta_s, \quad v = \tanh((\lambda-1)\eta_s).$$



# New, exact solutions: bulk viscosity

$$T^{\mu\nu} = eu^\mu u^\nu - p\Delta^{\mu\nu} + \Pi^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu},$$

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi,$$

$$e = \kappa p, \qquad p = nT,$$

$$\partial_\mu(N^\mu) = 0.$$

$$De = -(e + p + \Pi)\theta + \sigma_{\mu\nu}\pi^{\mu\nu},$$

$$(e + p + \Pi)Du^\alpha = \nabla^\alpha(p + \Pi) - \Delta^\alpha_\nu u_\mu D\pi^{\mu\nu} - \Delta^\alpha_\nu \nabla_\mu \pi^{\mu\nu},$$

$$S = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2},$$

$$u^\mu = \frac{x^\mu}{\tau} = \gamma\left(1, \frac{\dot{X}}{X}r_x, \frac{\dot{Y}}{Y}r_y, \frac{\dot{Z}}{Z}r_z\right),$$

$$\theta = \partial_\mu u^\mu = \frac{d}{\tau},$$

$$D = u^\mu\partial_\mu = \frac{\partial}{\partial\tau}.$$

$$\tau = \sqrt{t^2 - \boldsymbol{r}^2}$$

$$\eta_s = \frac{1}{2}\ln\left(\frac{t+r_z}{t-r_z}\right)$$

$$\sigma_{\mu\nu}\pi^{\mu\nu} = 0,$$

$\zeta/s$ $\eta/s$

# New, exact solution with bulk viscosity

$$u^\mu = \frac{x^\mu}{\tau} = \gamma \left(1, \frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z\right),$$

$$\frac{de}{d\tau} + \frac{d(e+p)}{\tau} - \zeta(\frac{d}{\tau})^2 = 0.$$

$$e = e_0 \left(\frac{\tau_0}{\tau}\right)^{\left(1+\frac{1}{\kappa}\right)d} + \zeta \frac{d}{\tau} \frac{d}{\left(1+\frac{1}{\kappa}\right)d - 1},$$

$\zeta/s$ $\eta/s$

$$\mathbf{v} = \frac{\mathbf{r}}{t} \quad \text{or} \quad u^\mu = \frac{x^\mu}{\tau},$$

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s),$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3\left(1+\frac{1}{\kappa}\right)} + \frac{\zeta}{\kappa} \frac{3}{\tau} \frac{3}{\left(1+\frac{1}{\kappa}\right)3 - 1},$$

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{3}{\kappa}} + \frac{\zeta}{\kappa n_0} \frac{3}{\tau} \left(\frac{\tau}{\tau_0}\right)^3 \frac{3}{\left(1+\frac{1}{\kappa}\right)3 - 1} \mathcal{T}(s),$$

$$\theta = \partial_\mu u^\mu = \frac{d}{\tau},$$

$$D = u^\mu \partial_\mu = \frac{\partial}{\partial \tau}.$$

$\tau = \sqrt{t^2 - \boldsymbol{r}^2}$

$\eta_s = \frac{1}{2} \ln \left(\frac{t+r_z}{t-r_z}\right)$



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# New viscous solution in 1+3 dim

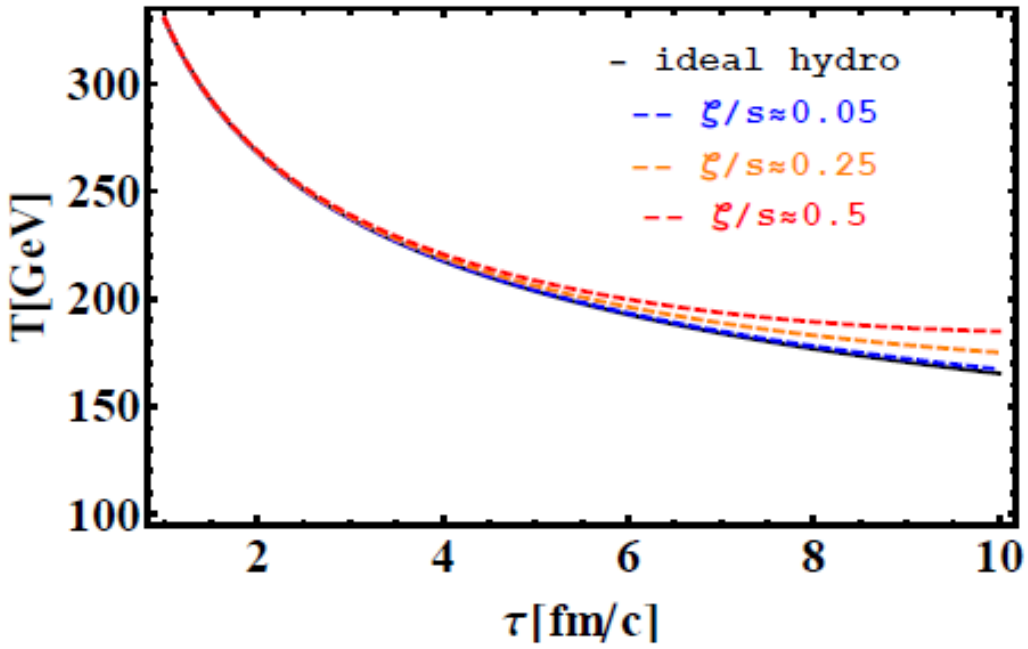



**Fig. 1**: The proper time $\tau$ and the ratio $\zeta/s$ of temperature for the new temperature solution for given primary initial condition $T_0 = 330$ MeV [], $\tau_0 = 1$ fm/c for RHIC, based on Eq.(21). (color online).

**Bulk viscosity important at late stage, heats up**

**Shear viscosity effects cancel for asymptotically Hubble flows**

# Conclusions

Explicit solutions of a very difficult problem

New estimates of initial energy density

New exact solution

for arbitrary EOS with const e/p

after 10 years, finally

For asymptotically Hubble flows

shear effects cancel at late time

bulk viscosity heats up matter

A lot to do ...

more general EoS

less symmetry, ellipsoidal solutions

Rotating viscous solutions

# Thank you for your attention!

**Questions?**

# Backup slides

# Some general remarks

**Hydrodynamics=**

**Initial conditions ⊗ dynamical equations ⊗ freeze-out conditions**

**Exact solution = formulas solve hydro without approximation**

**Parametric solution = shape parameters introduced,**
**time dependence given by ordinary coupled diff. eqs.**

**Hydro inspired parameterization**
**= shape parameters determined only at the freeze-out**
**their time dependence is not considered**

**Report on new class of exact, parametric solution of relativistic hydro**

**M.I. Nagy, T. Cs., M. Csanád,** arXiv:0709.3677v1 , PRC77:024908 (2008)

T. Cs, M. I. Nagy, M. Csanád, arXiv:nucl-th/0605070v4, PLB (2008)

M. Csanád, M. I. Nagy, T. Cs, arXiv:0710.0327v3 [nucl-th] EPJ A (2008)

# High temperature superfluidity at RHIC!

**All "realistic" hydrodynamic calculations for RHIC fluids to date have assumed zero viscosity**

$\eta = 0 \rightarrow$ **perfect fluid**

**a conjectured quantum limit:**

$$\eta \geq \frac{\hbar}{4\pi}(Entropy\ Density) \equiv \frac{\hbar}{4\pi} s$$

P. Kovtun, D.T. Son, A.O. Starinets, hep-th/04

**How "ordinary" fluids compare to this limit?**

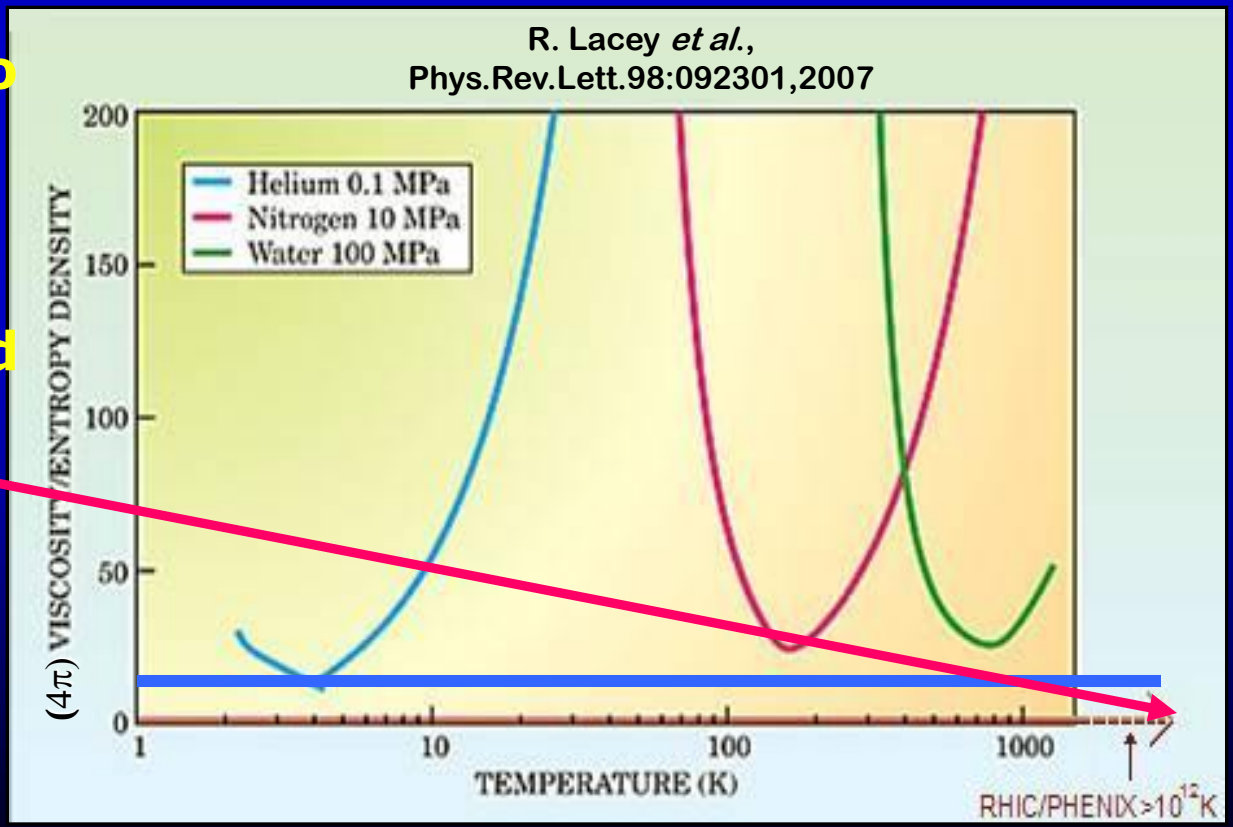
$(4\pi)\,\eta/s > 10$

**RHIC's perfect fluid**

$(4\pi)\,\eta/s \sim 1$ !

**T > 2 Terakelvin**

**The hottest & most perfect fluid ever made...**



# How Perfect is Perfect? Measure η/s !

## Damping (flow, fluctuations, heavy quark motion) ~ η/s

FLOW: *Has the QCD Critical Point Been Signaled by Observations at RHIC?,* R. Lacey *et al.*, Phys.Rev.Lett.98:092301,2007 (nucl-ex/0609025)

$$\frac{\eta}{s} = (1.1 \pm 0.2 \pm 1.2)\frac{1}{4\pi}$$

*The Centrality dependence of Elliptic flow, the Hydrodynamic Limit, and the Viscosity of Hot QCD*, H.-J. Drescher *et al.*, (arXiv:0704.3553)

$$\frac{\eta}{s} = (1.9 - 2.5)\frac{1}{4\pi}$$

FLUCTUATIONS: *Measuring Shear Viscosity Using Transverse Momentum Correlations in Relativistic Nuclear Collisions*, S. Gavin and M. Abdel-Aziz, Phys.Rev.Lett.97:162302,2006 (nucl-th/0606061)

$$\frac{\eta}{s} = (1.0 - 3.8)\frac{1}{4\pi}$$

DRAG, FLOW: *Energy Loss and Flow of Heavy Quarks in Au+Au Collisions at $\sqrt{s_{NN}}$ = 200 GeV* (PHENIX Collaboration), A. Adare *et al.*, Phys.Rev.Lett.98:172301,2007 (nucl-ex/0611018)

$$\frac{\eta}{s} = (1.3 - 2.0)\frac{1}{4\pi}$$

C H A R M !

# Landau-Khalatnikov solution

**Publications:**

**L.D. Landau, Izv. Acad. Nauk SSSR 81 (1953) 51**

**I.M. Khalatnikov, Zhur. Eksp.Teor.Fiz. 27 (1954) 529**

**L.D.Landau and S.Z.Belenkij, Usp. Fiz. Nauk 56 (1955) 309**

**Implicit 1D solution with approx. Gaussian rapidity distribution**

**Basic relations:** $u^0 = \cosh\Omega(t,x), u^1 = \sinh\Omega(t,x)$

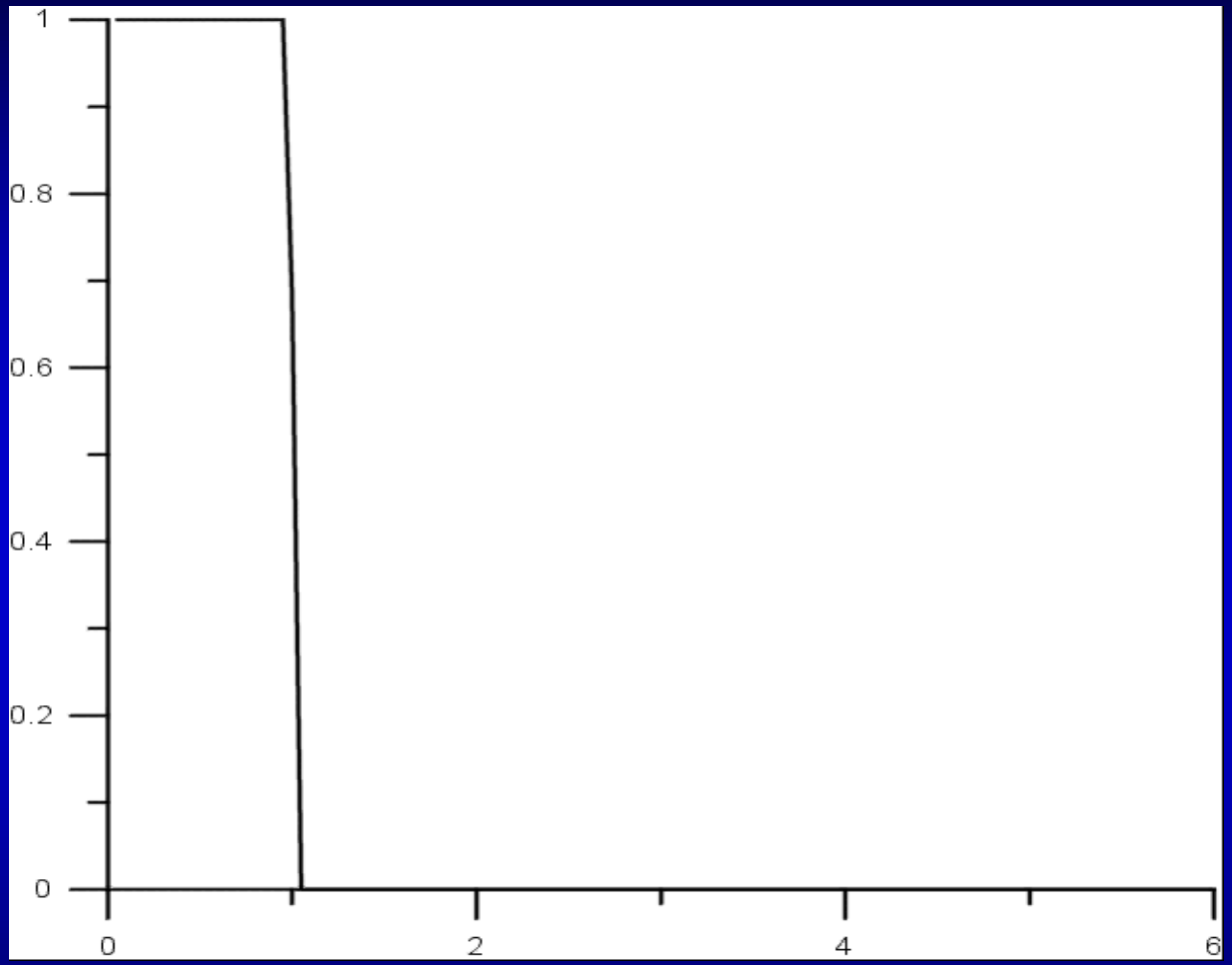
**Unknown variables:** $T(t,x),\ \Omega(t,x)$

**Auxiliary function:** $\chi(\Omega,T)$

$$t = \frac{\partial\chi}{\partial T}\cosh\Omega - \frac{1}{T}\frac{\partial\chi}{\partial\Omega}\sinh\Omega, \quad x = \frac{\partial\chi}{\partial T}\sinh\Omega - \frac{1}{T}\frac{\partial\chi}{\partial\Omega}\cosh\Omega$$

**Expression of** $\chi(\Omega,T)$ **is a true „tour de force”**

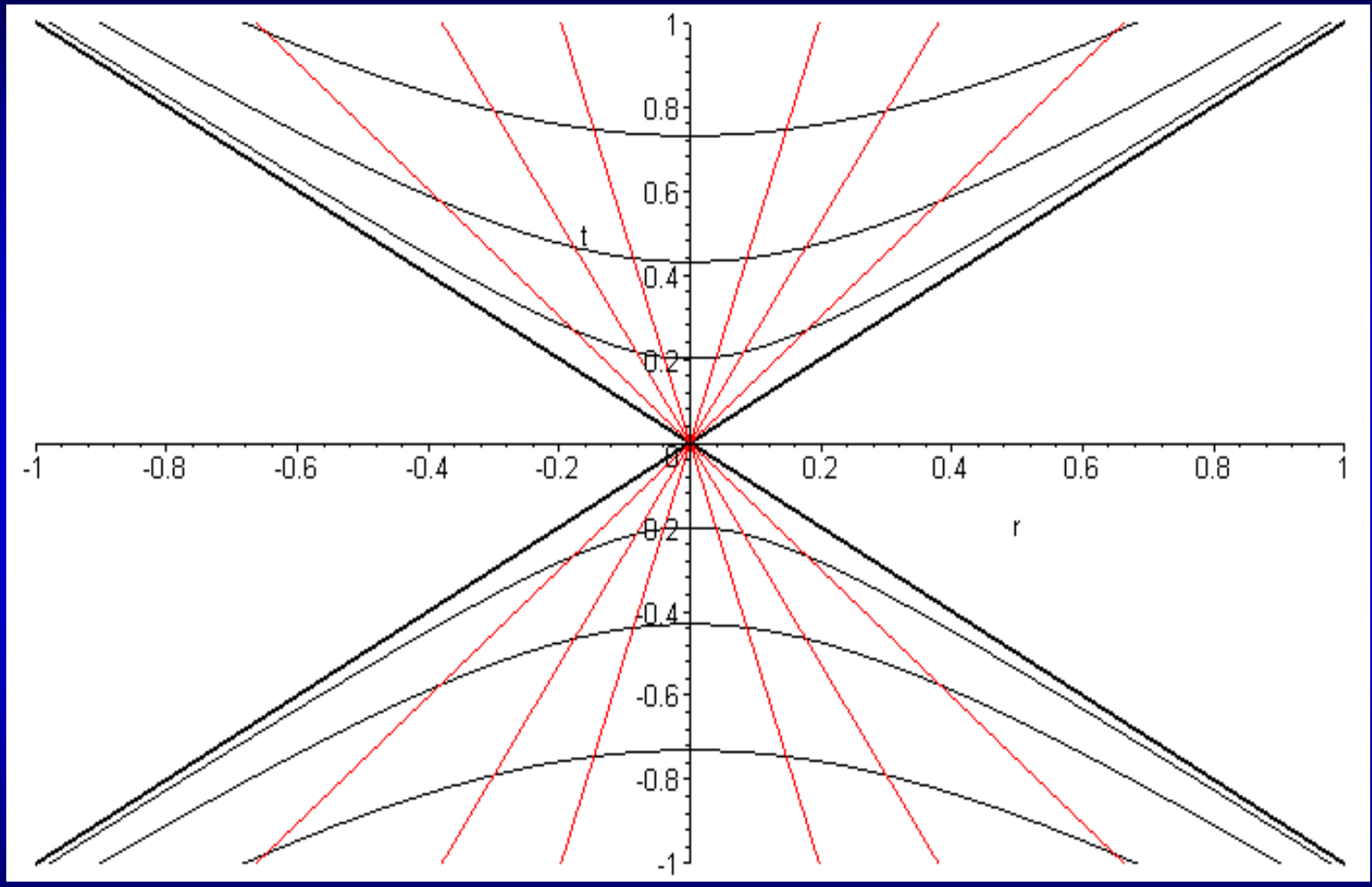
# Landau-Khalatnikov solution




Temperature distribution (animation courtesy of T. Kodama)

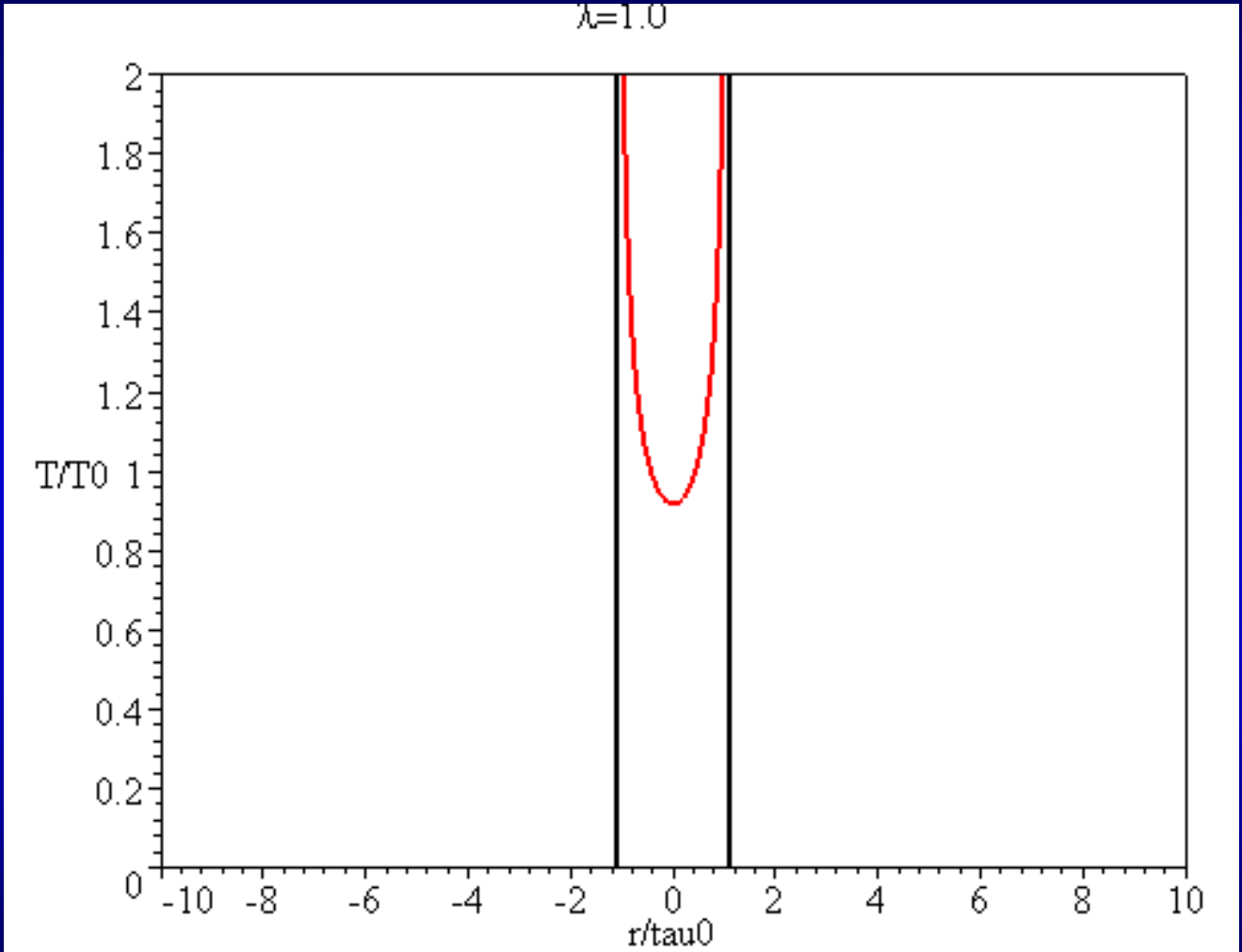
„Tour de force” implicit solution: t=t(T,v), r=r(T,v)

# Hwa-Bjorken solution




The Hwa-Bjorken solution / Rindler coordinates

# Hwa-Bjorken solution




The Hwa-Bjorken solution / Temperature evolution

# Bialas-Janik-Peschanski solution

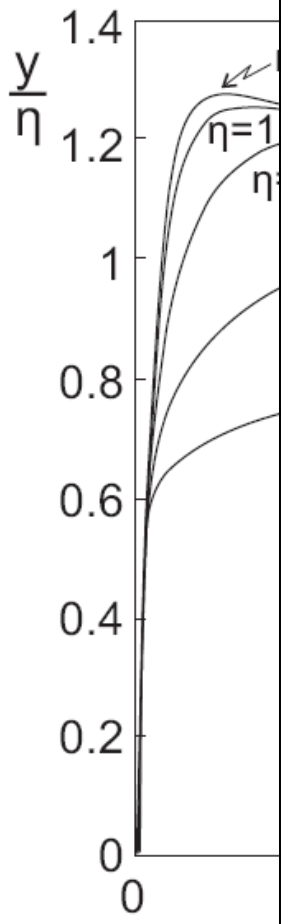
**Publications:**

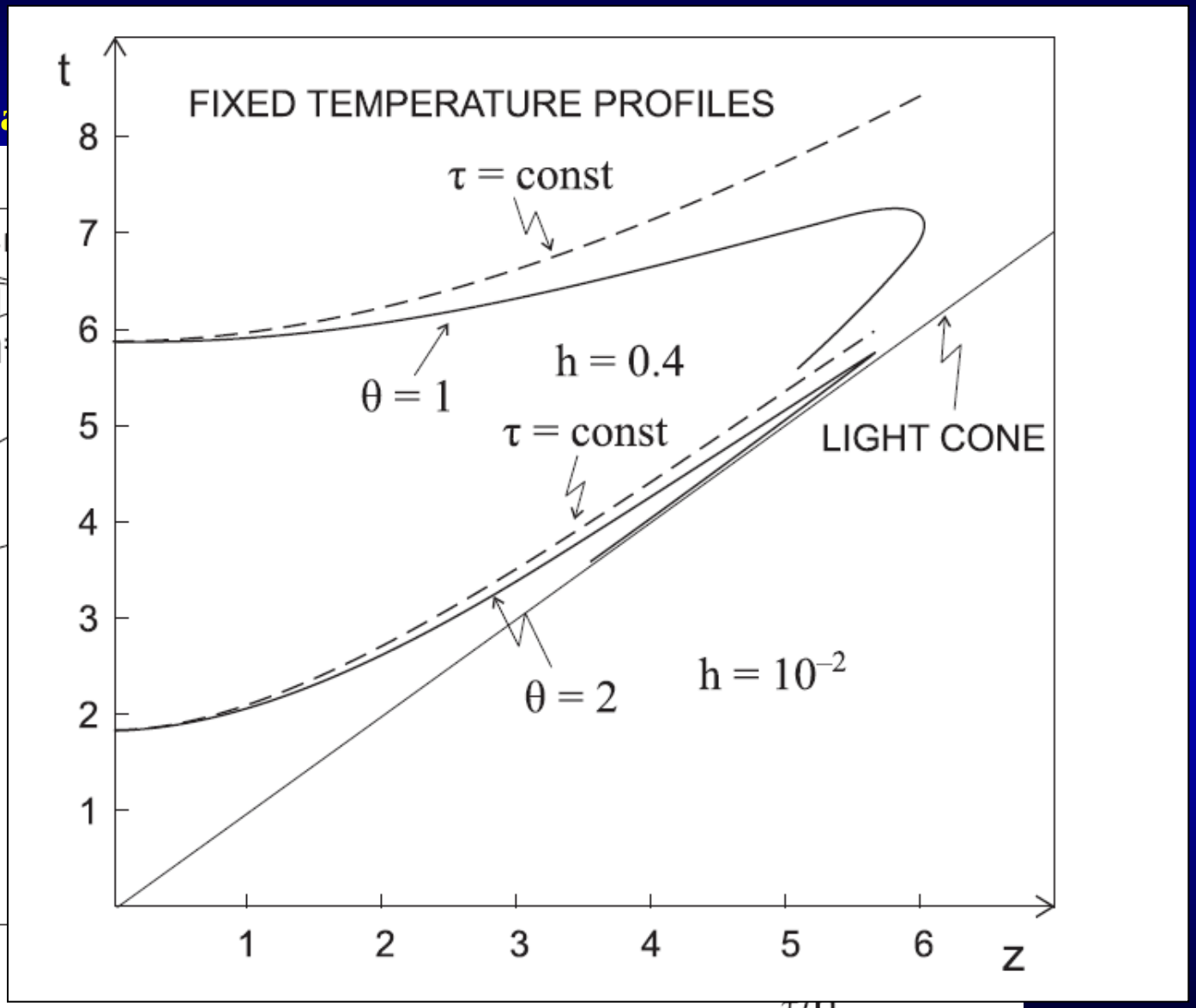
A. Bialas, R. Ja

**Ac**

**Ge**

$2y =$



# Hwa-Bjorken solution

**Publications:**

**R.C. Hwa, Phys. Rev. D10, 2260 (1974)**

**J.D. Bjorken, Phys. Rev. D27, 40(1983)**

**Accelerationless, expanding 1D simple boost-invariant solution**

$$t = \tau \cosh \eta, \quad r = \tau \sinh \eta$$

**Rindler coordinates:**

$$\eta = \operatorname{arctanh} \frac{r}{t}, \tau = \sqrt{t^2 - r^2} = \sqrt{x_\mu x^\mu}$$

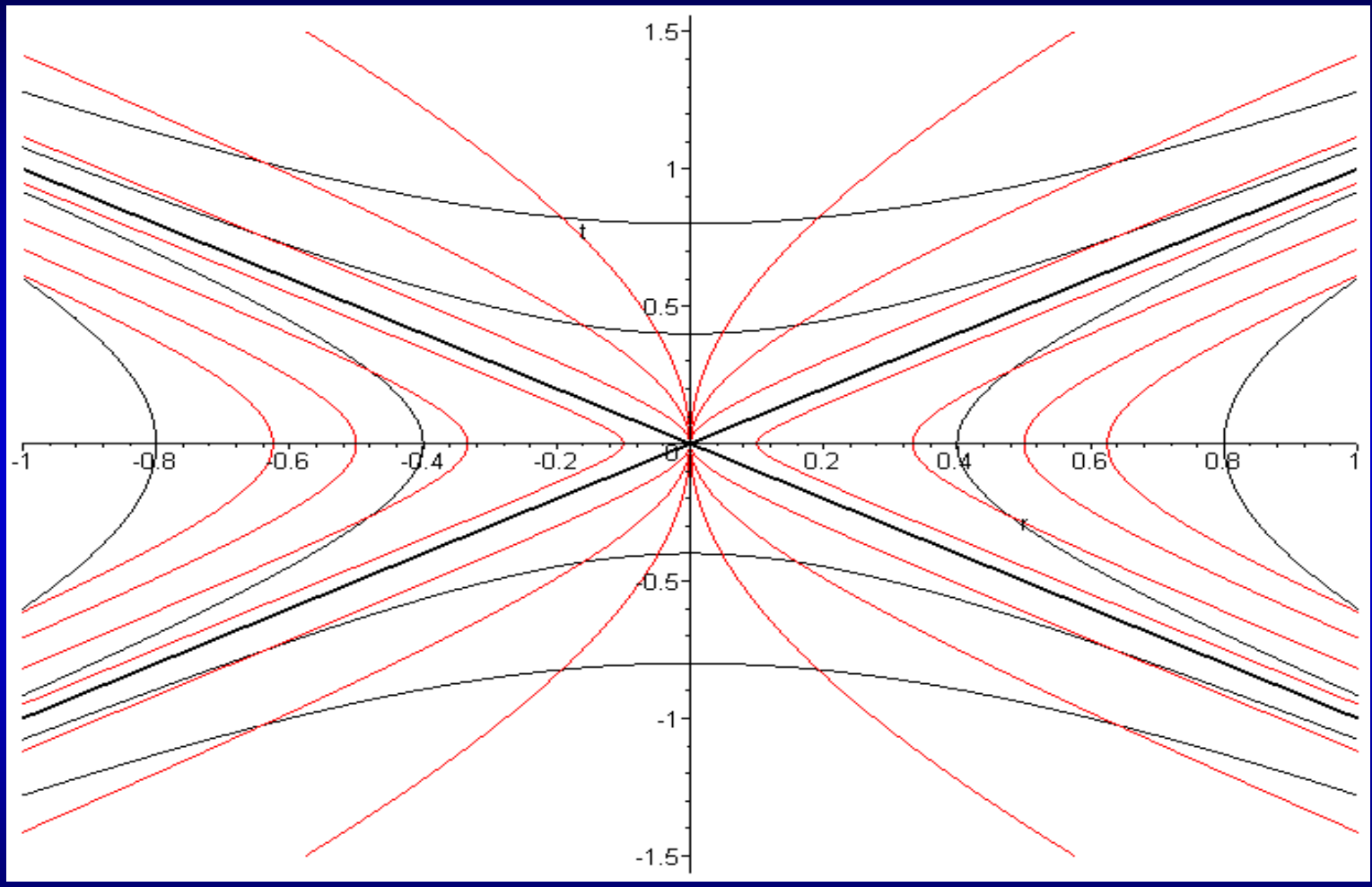
**Boost-invariance (valid for asymptotically high energies):**

$$\varepsilon = \varepsilon(\tau), \; p = p(\tau), \; s = s(\tau), \; u^\mu = \frac{x^\mu}{\tau}, \; v = \frac{z}{t} \Rightarrow \frac{dv}{dt} = 0$$

$$\partial_\mu (s u^\mu) = 0 \qquad \frac{s}{s_0} = \frac{\tau_0}{\tau}$$

$T = T(\tau)$ depends on EoS, e.g. $\varepsilon = \kappa p, \; \kappa = D, \; D = 1 \Rightarrow T(\tau) = T_0 \frac{\tau_0}{\tau}$

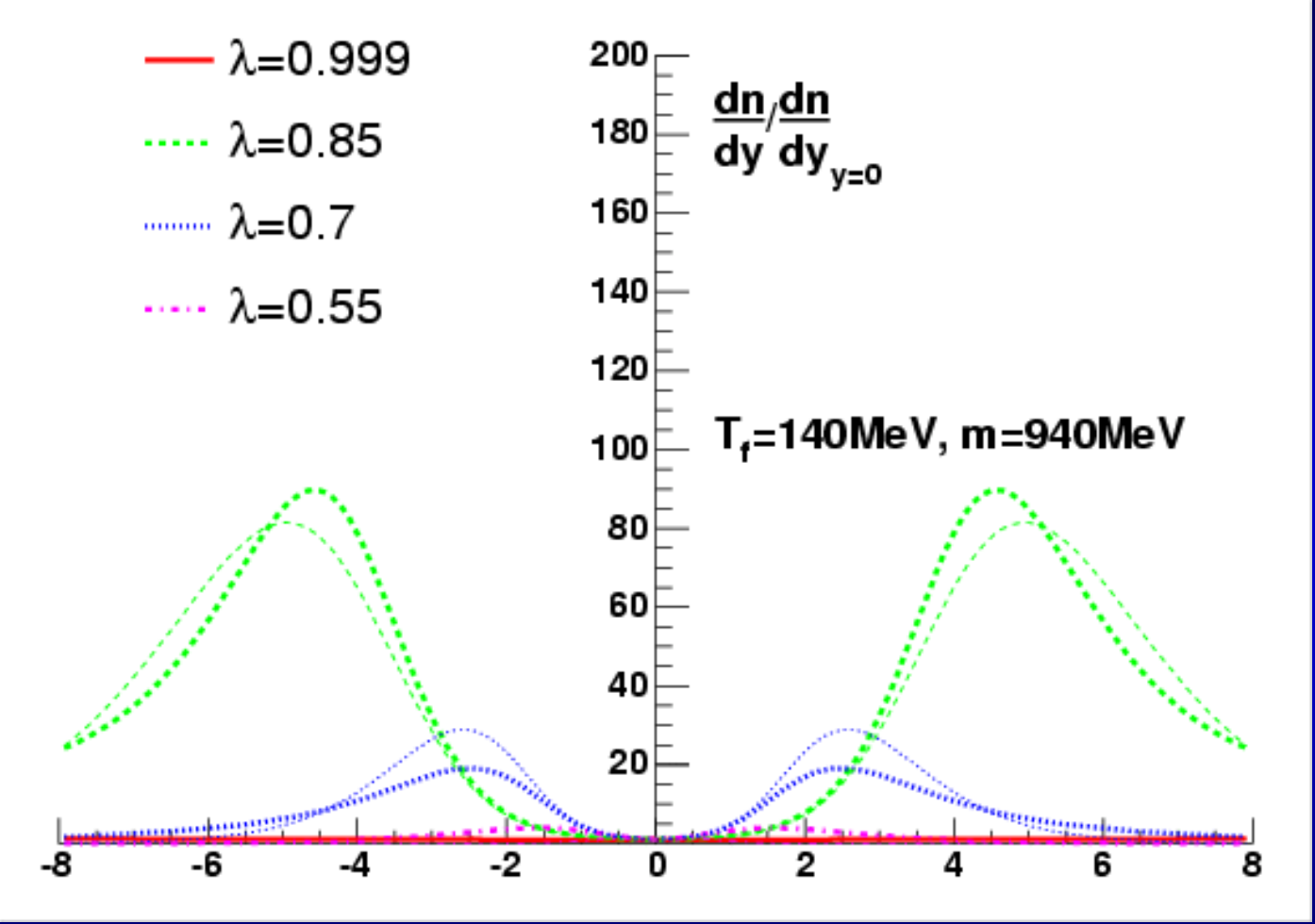
# New simple solutions in 1+d dim




The fluid lines (red) and the pseudo-orthogonal freeze-out surface (black)

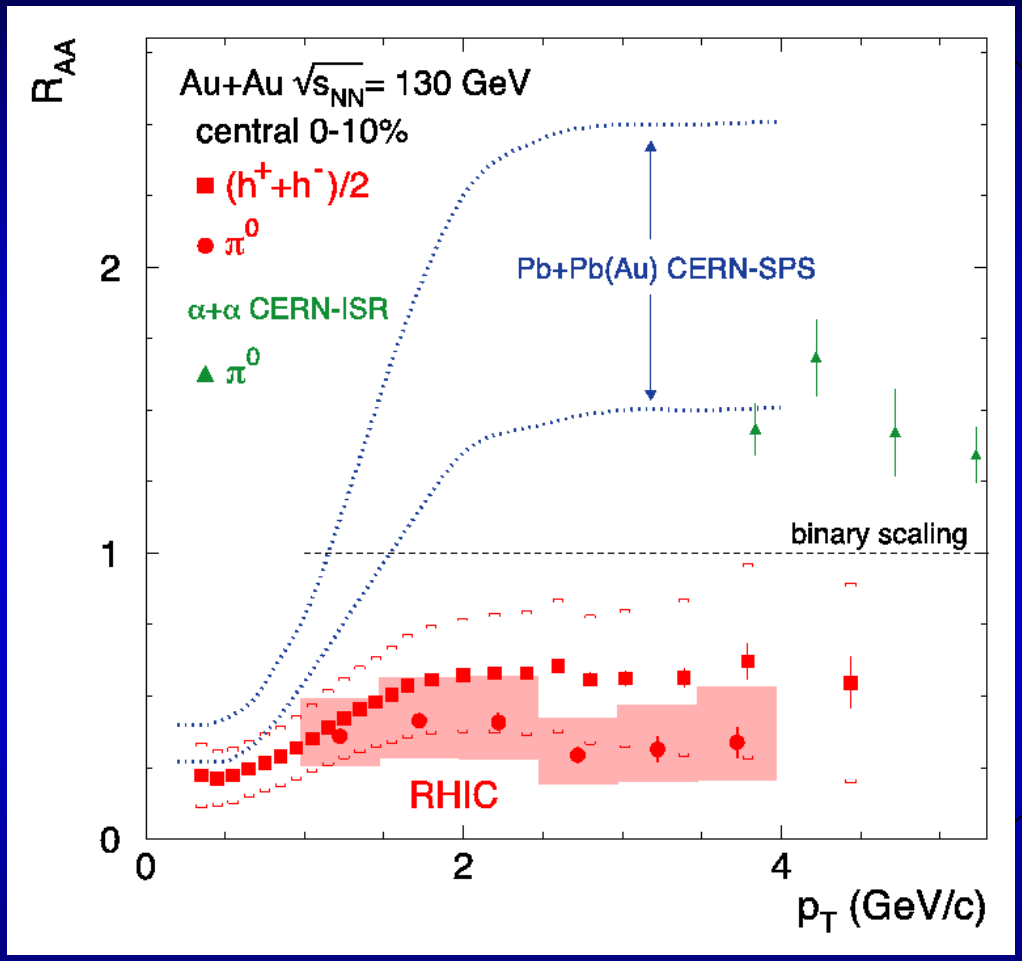
$A = const.$

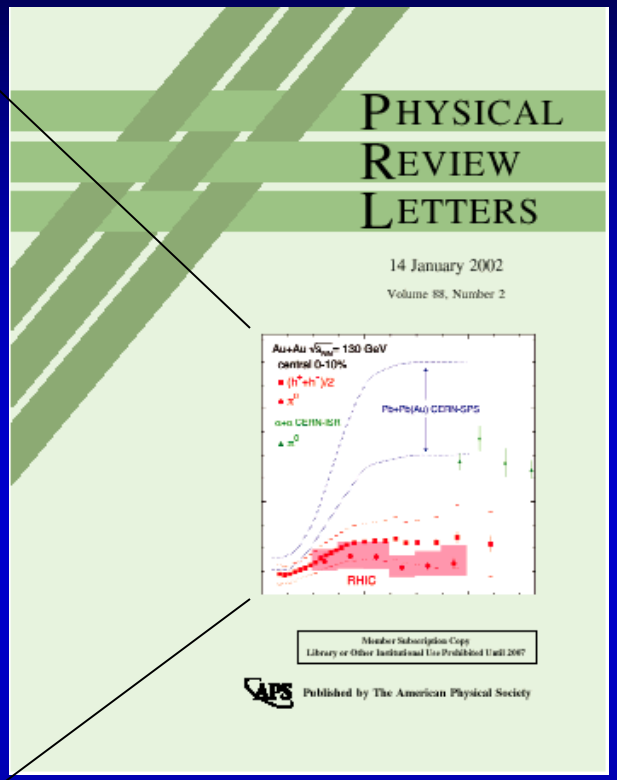
# Rapidity distribution




Rapidity distribution from the 1+1 dimensional solution, for $\lambda \cdot < 1$

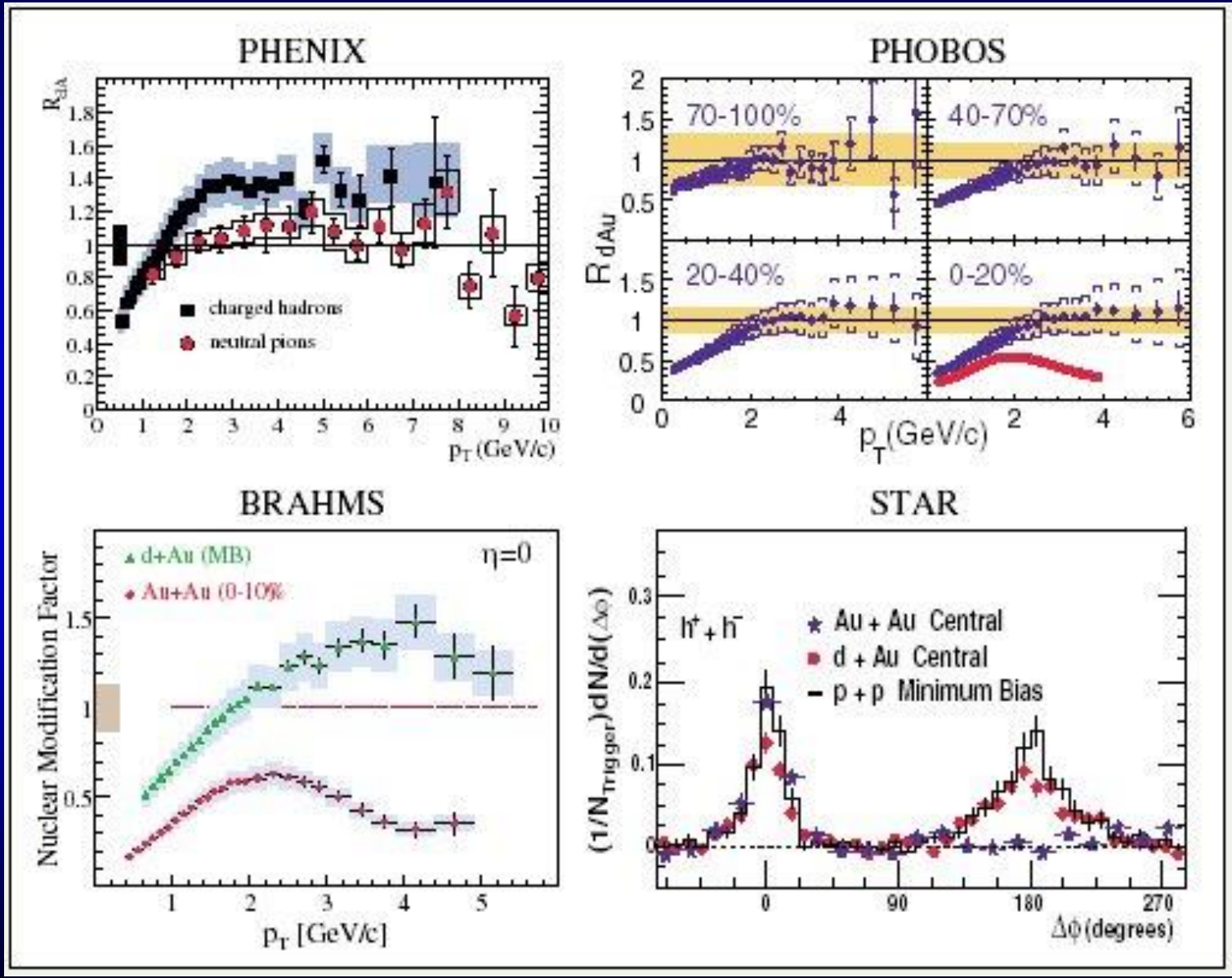
# 1st milestone: new phenomena




**Suppression of high $p_t$ particle production in Au+Au collisions at RHIC**

# 2nd milestone: new form of matter



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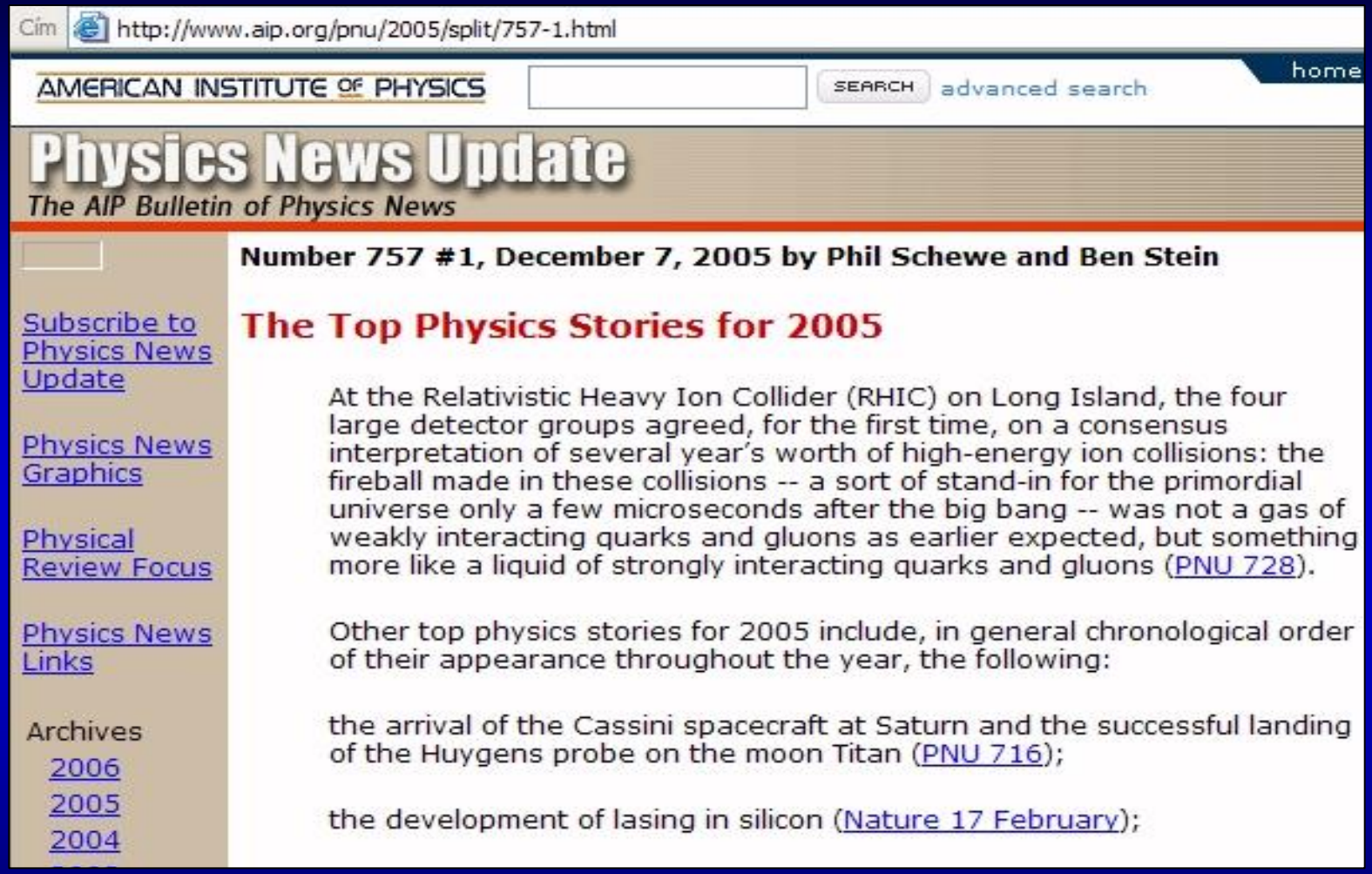
# 3rd milestone: Top Physics Story 2005



Cim http://www.aip.org/pnu/2005/split/757-1.html

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**Number 757 #1, December 7, 2005 by Phil Schewe and Ben Stein**

## The Top Physics Stories for 2005

At the Relativistic Heavy Ion Collider (RHIC) on Long Island, the four large detector groups agreed, for the first time, on a consensus interpretation of several year's worth of high-energy ion collisions: the fireball made in these collisions -- a sort of stand-in for the primordial universe only a few microseconds after the big bang -- was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons (PNU 728).

Other top physics stories for 2005 include, in general chronological order of their appearance throughout the year, the following:

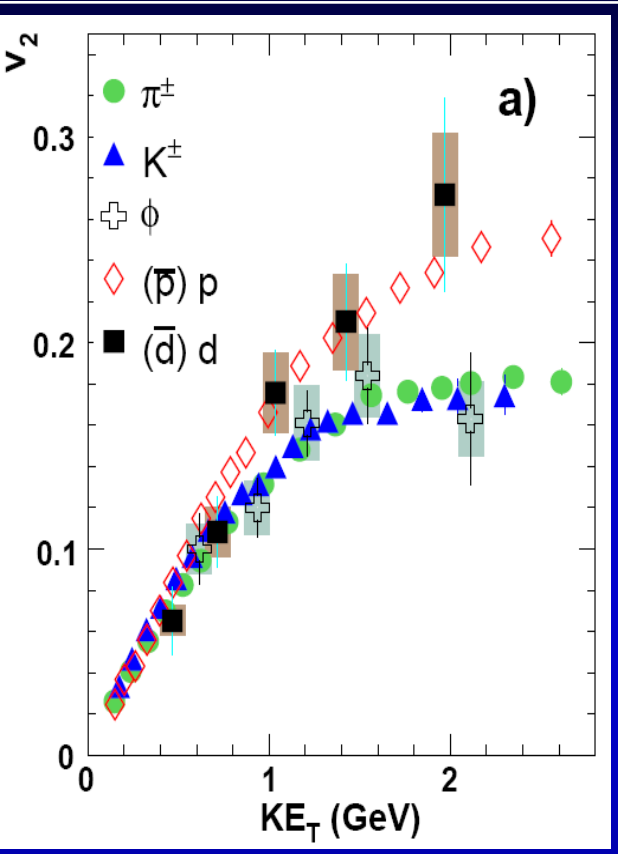
the arrival of the Cassini spacecraft at Saturn and the successful landing of the Huygens probe on the moon Titan (PNU 716);

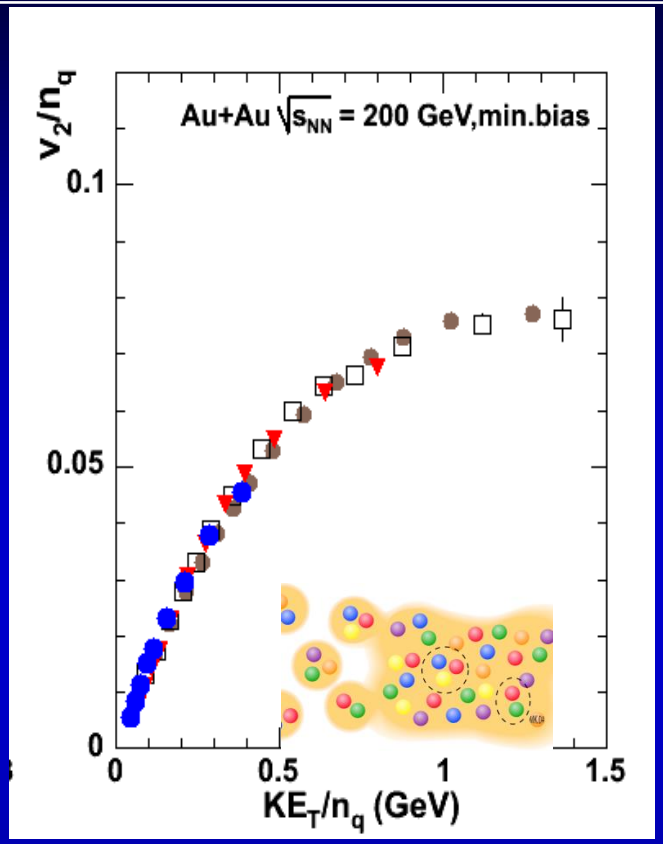
the development of lasing in silicon (Nature 17 February);

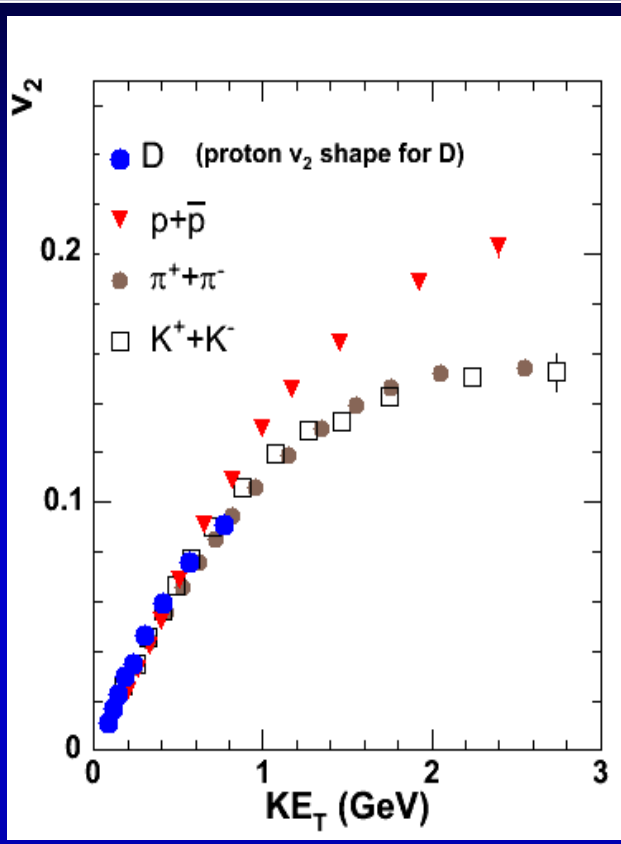
**http://arxiv.org/abs/nucl-ex/0410003**

**PHENIX White Paper: second most cited in nucl-ex during 2006**

# 4th Milestone: A fluid of quarks




$v_2$ *for the φ follows that of other mesons*

$$v_2^{hadron}(KE_T^{hadron}) \approx n v_2^{quark}(KE_T^{quark})$$
$$KE_T^{hadron} \approx n KE_T^{quark}$$

$v_2$ *for the D follows that of other mesons*

***Strange and even charm quarks participate in the flow***



M. Csanád, T. Csörgő, ZF. Jiang

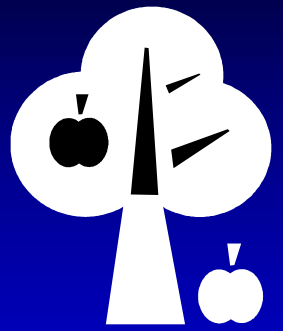
# Predictions of the Buda-Lund model

Hydro predicts scaling (even viscous)

What does a scaling mean?

See Hubble's law – or Newtonian gravity:

$$t = \sqrt{2gh}$$



Cannot predict acceleration or height

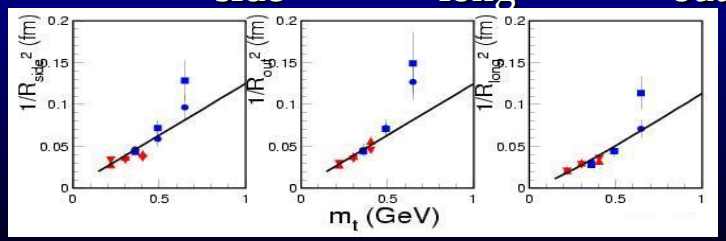
Collective, thermal behavior →

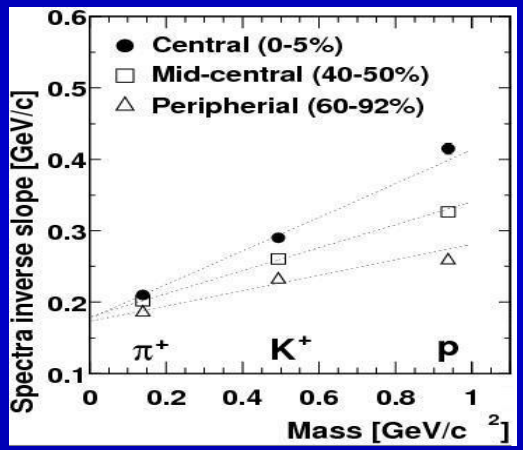
Loss of information

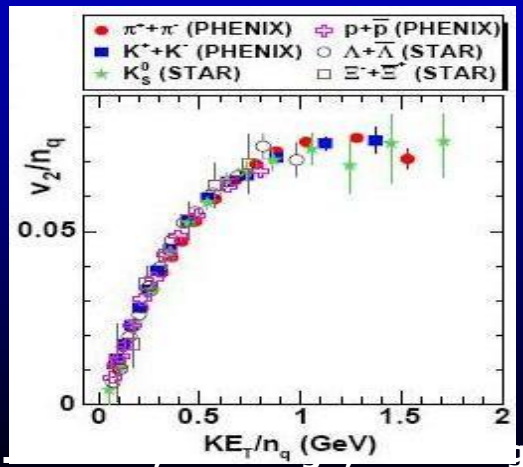
Spectra slopes: $T_{\text{eff}} = T_0 + m u_t^2$

Elliptic flow: $v_2 = \dfrac{I_1(w)}{I_0(w)}, w \sim KE_T$

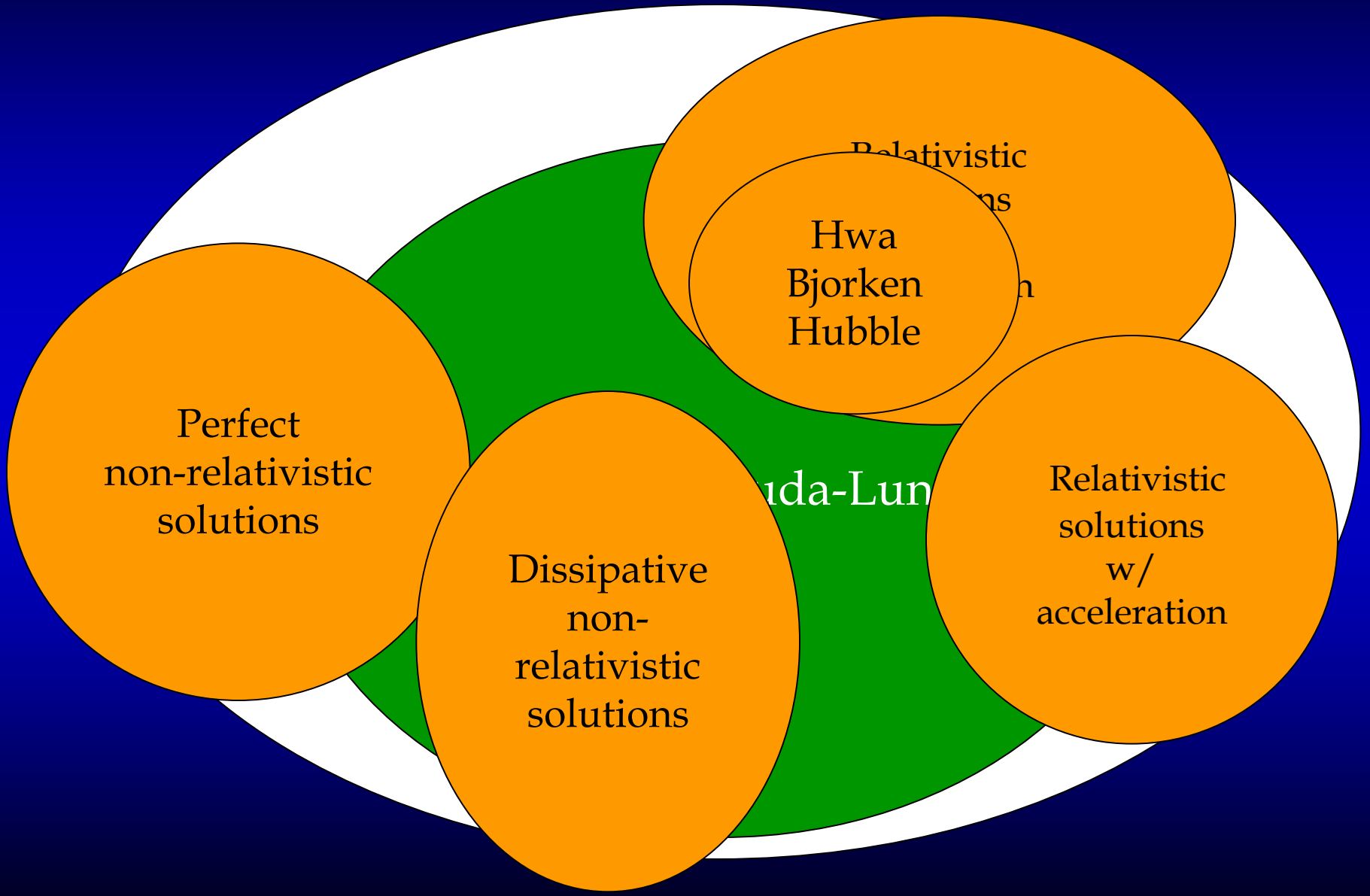
HBT radii: $R_{\text{side}}^2 \approx R_{\text{long}}^2 \approx R_{\text{out}}^2 \sim \dfrac{1}{m_t}$



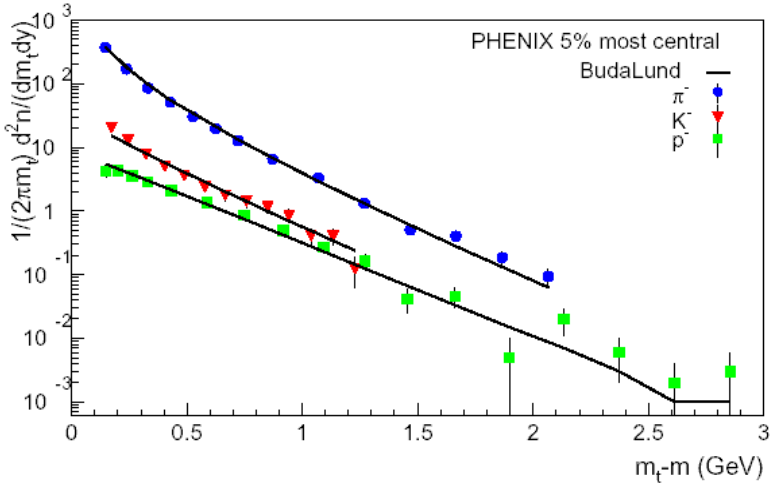



# What does the data tell us

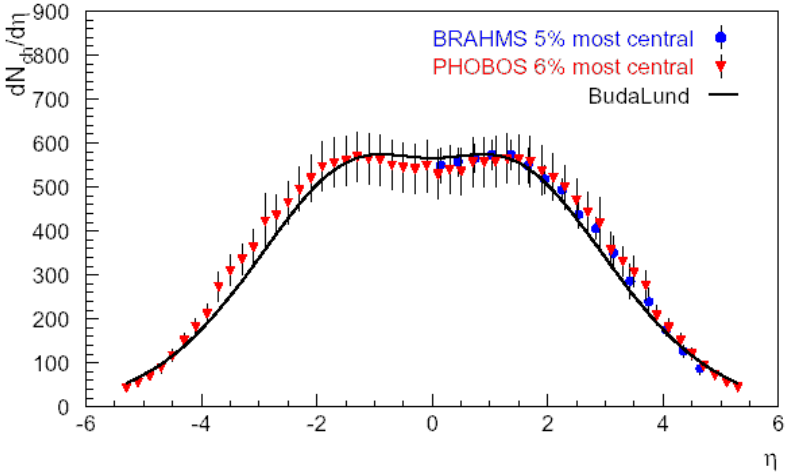


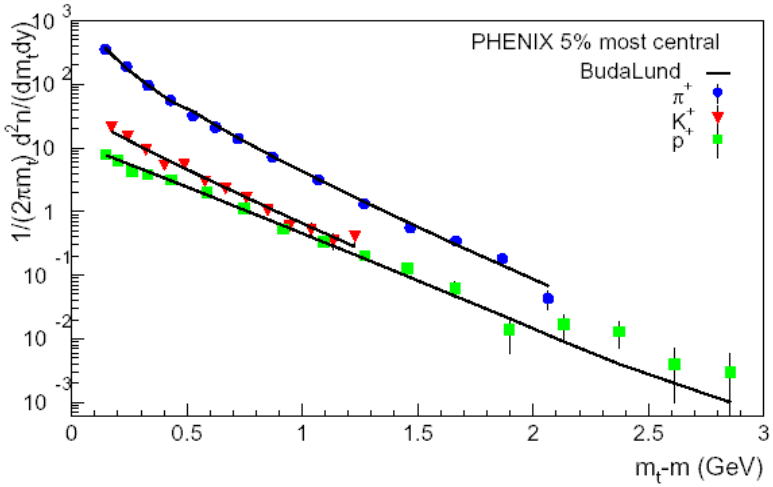
# BudaLund fits to 130 GeV RHIC data

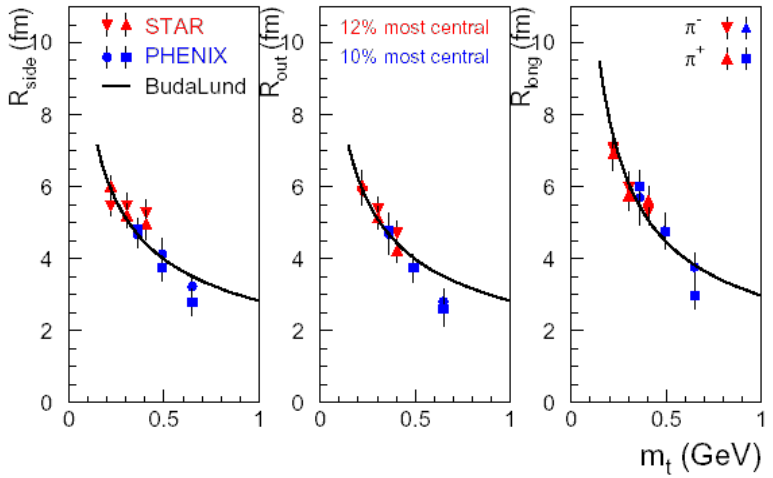
BudaLund hydro fits to 130 AGeV Au+Au




BudaLund hydro fits to 130 AGeV Au+Au



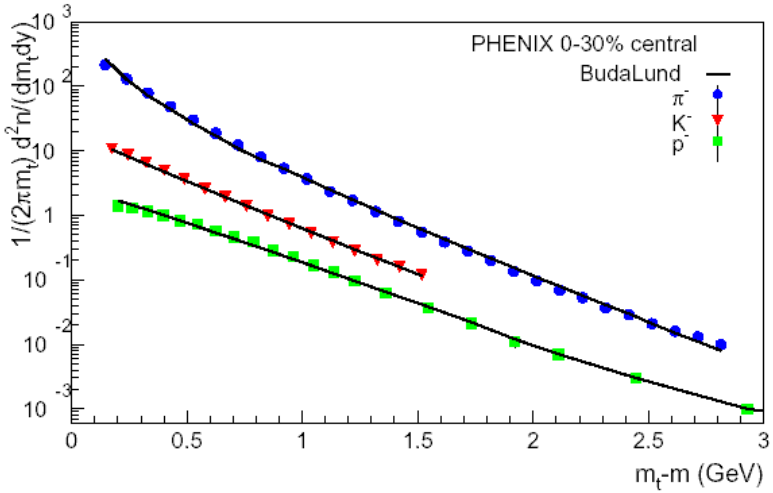



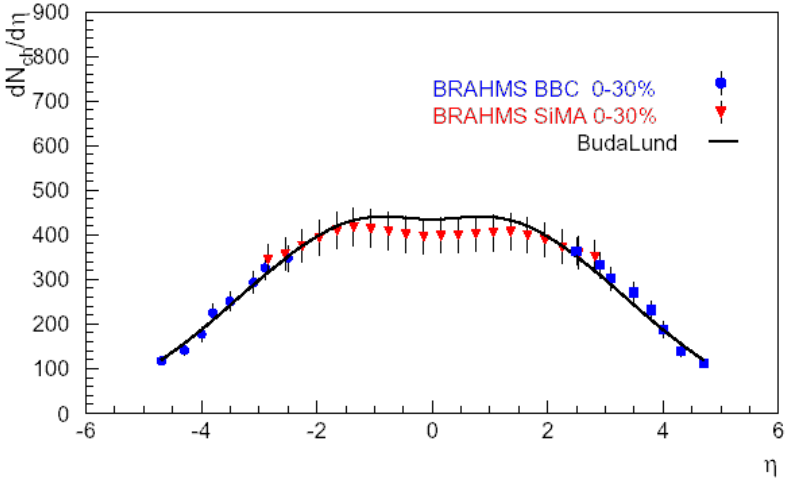

**M. Csanád, T. Csörgő, B. Lörstad, A. Ster, nucl-th/0311102, ISMD03**

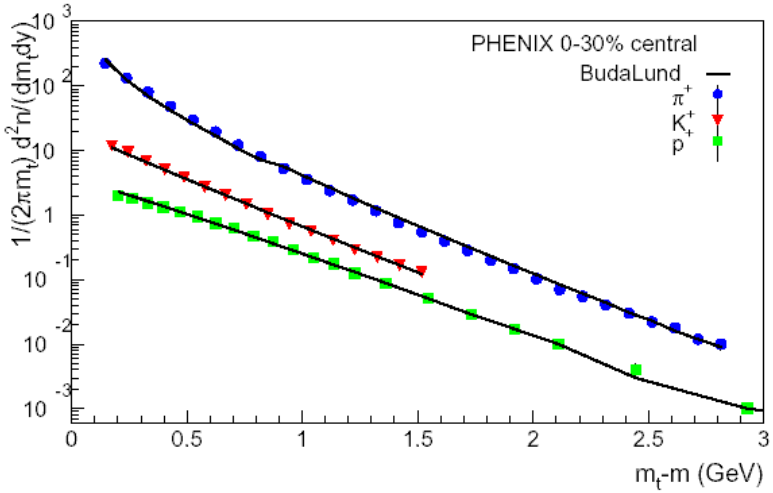
# BudaLund fits to 200 GeV RHIC data

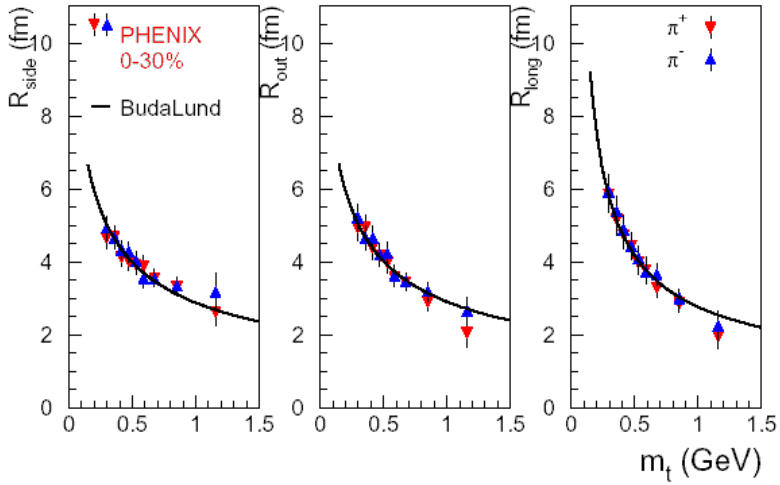
BudaLund v1.5 fits to 200 AGeV Au+Au




BudaLund v1.5 fits to 200 AGeV Au+Au

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