New solutions of fireball hydrodynamics with shear and bulk viscosity

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Outline

Context, motivation

Exact accelerating solution for e/p =1

Generalization for e/p = k

A new solution with bulk viscosity

Adding shear viscosity

Effect on observables

Summary and conclusion

M. Csanád, T. Csörgő, ZF. Jiang

Context

Renowned exact solutions

- **Landau-Khalatnikov solution: dn/dy ~ Gaussian**
- **Hwa solution (1974) – Bjorken: solution + ε_0 (1983)**
- **Chiu, Sudarshan and Wang: plateaux**

Revival of interest: **Buda-Lund model + exact solutions,**
Biró, Karpenko+Sinyukov, Pratt (2007),
Bialas+Janik+Peschanski, Borsch+Zhdanov (2007)
CsT+Csanád+Nagy (2007-2008)
CsT+Csernai+Hama+Kodama (2004)
Gubser
Hatta, Noronha, Xiao, ...

New simple solutions → Evaluation of measurables

Rapidity distribution → Advanced initial energy density

HBT radii → Advanced life-time estimation

Goal

Need for solutions that are:

- **explicit**
- **simple**
- **accelerating**
- **relativistic**
- **realistic / compatible with the data:**
  - **lattice QCD EoS**
  - **ellipsoidal symmetry (spectra, v_2, v_4, HBT)**
  - **finite dn/dy**
- **Generelization of a class that satisfies each of these criteria**
  - **but not simultaneously**

M.I. Nagy, T. Cs., M. Csanád, arXiv:0709.3677v1 , PRC77:024908 (2008)

T. Cs, M. I. Nagy, M. Csanád, arXiv:nucl-th/0605070v4, PLB (2008)

M. Csanád, M. I. Nagy, T. Cs, arXiv:0710.0327v3 [nucl-th] EPJ A (2008)

Dedicated to Bolyai, Lobachevski and Gauss

Perfect fluid hydrodynamics

Energy-momentum tensor:

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} \qquad w = \varepsilon + p$$

$$\partial_\nu T^{\mu\nu} = 0$$

Relativistic Euler equation:

$$w u^\nu \partial_\nu u^\mu = \left(g^{\mu\rho} - u^\mu u^\rho\right) \partial_\rho p$$

Energy conservation:

$$w \partial_\mu u^\mu = -u^\mu \partial_\mu \varepsilon$$

Charge conservation

$$\sum \mu_i \partial_\mu \left(n_i u^\mu\right) = 0$$

Consequence is entropy conservation:

$$\partial_\mu \left(\sigma u^\mu\right) = 0.$$

Self-similar, ellipsoidal solutions

Publication (for example):

T. Cs, L.P.Csernai, Y. Hama, T. Kodama, Heavy Ion Phys. A 21 (2004) 73

3D spherically symmetric HUBBLE flow:

$$u^\mu = \frac{x^\mu}{\tau}$$

No acceleration:

$$u^\mu \partial_\mu u_\nu = 0.$$

Define a scaling variable for self-similarly expanding ellipsoids:

$$s = \frac{r_x^2}{\dot{X}_0^2 t^2} + \frac{r_y^2}{\dot{Y}_0^2 t^2} + \frac{r_z^2}{\dot{Z}_0^2 t^2}$$

EoS: (massive) ideal gas

$$\epsilon = mn + \kappa p\,, \qquad p = nT.$$

$$\varepsilon_Q = m_Q n_Q + \lambda_\varepsilon n_Q T + B, \qquad p_Q = \lambda_p n_Q T - B,$$

$$n(t,\mathbf{r}) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s) \qquad T(t,\mathbf{r}) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\mathcal{V}(s)} \quad \Rightarrow \quad p(t,\mathbf{r}) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+3/\kappa}$$

Scaling function $\mathcal{V}(s)$ can be chosen freely.

Shear and bulk viscous corrections in NR limit: known analytically.

New, simple, exact solutions

$$v = \tanh \lambda\eta,$$
$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh \frac{\eta}{2}\right)^{-(d-1)\phi_\lambda}$$

Possible cases (one row of the table is one solution):

Case	λ	d	κ	ϕ_λ
a.)	2	$\in \mathbb{R}$	d	0
b.)	$\frac{1}{2}$	$\in \mathbb{R}$	1	$\frac{\kappa+1}{\kappa}$
c.)	$\frac{3}{2}$	$\in \mathbb{R}$	$\frac{4d-1}{3}$	$\frac{\kappa+1}{\kappa}$
d.)	1	$\in \mathbb{R}$	$\in \mathbb{R}$	0
e.)	$\in \mathbb{R}$	1	1	0

M. I. Nagy, T.Cs., M. Csanád:
arXiv:0709.3677v1

→ New, accelerating, d dimension

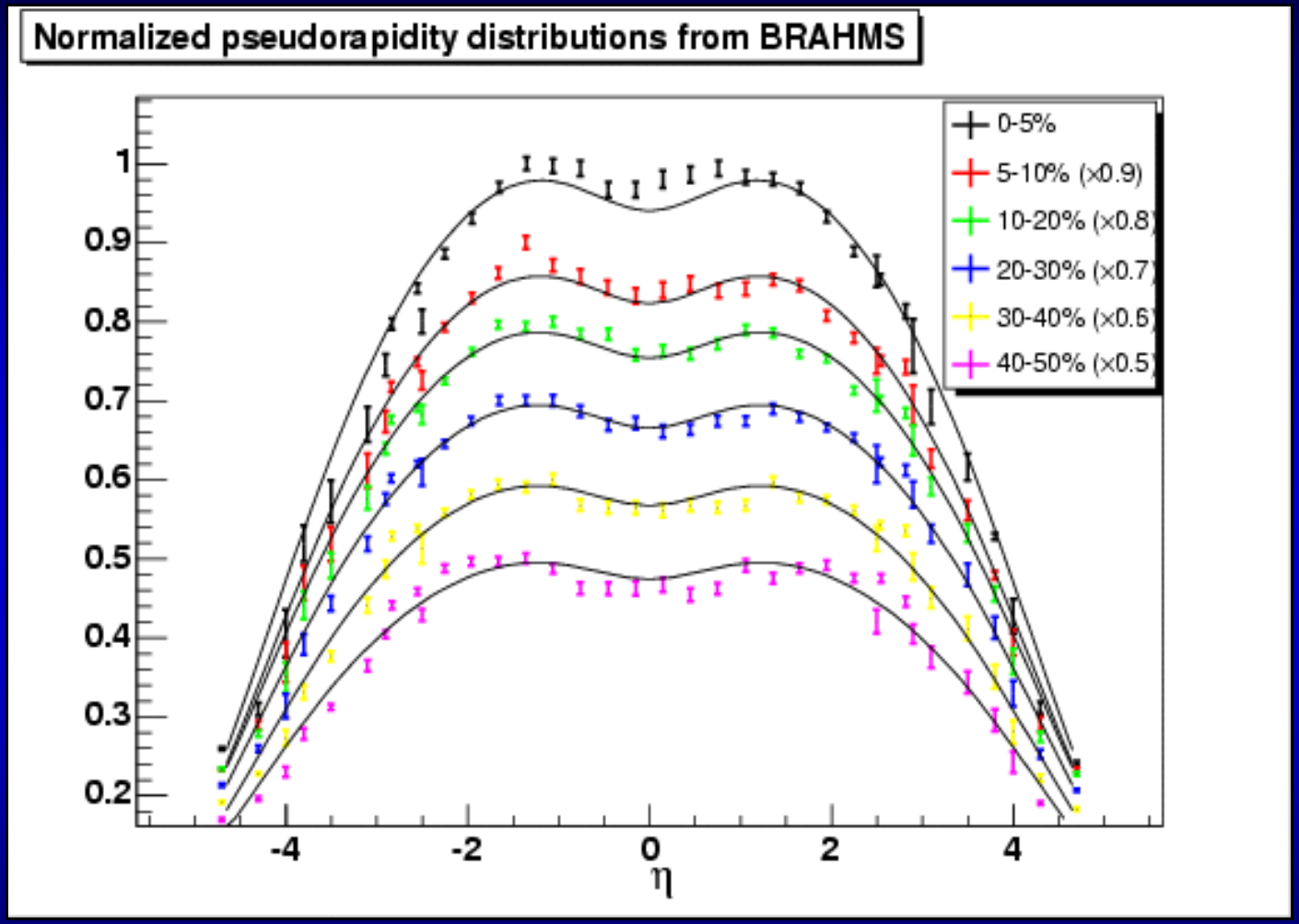
↘ d dimensional with p=p(τ,η)
↗ (thanks T. S. Biró)

→ Hwa-Bjorken, Buda-Lund type

→ κ = e/p= 1, but general velocity profile

If κ = d = 1 , general solution is obtained, for ARBITRARY initial conditions. It is STABLE !

Pseudorapidity distributions



BRAHMS data fitted with the analytic formula of

Additionally: $y \rightarrow \eta$ transformation

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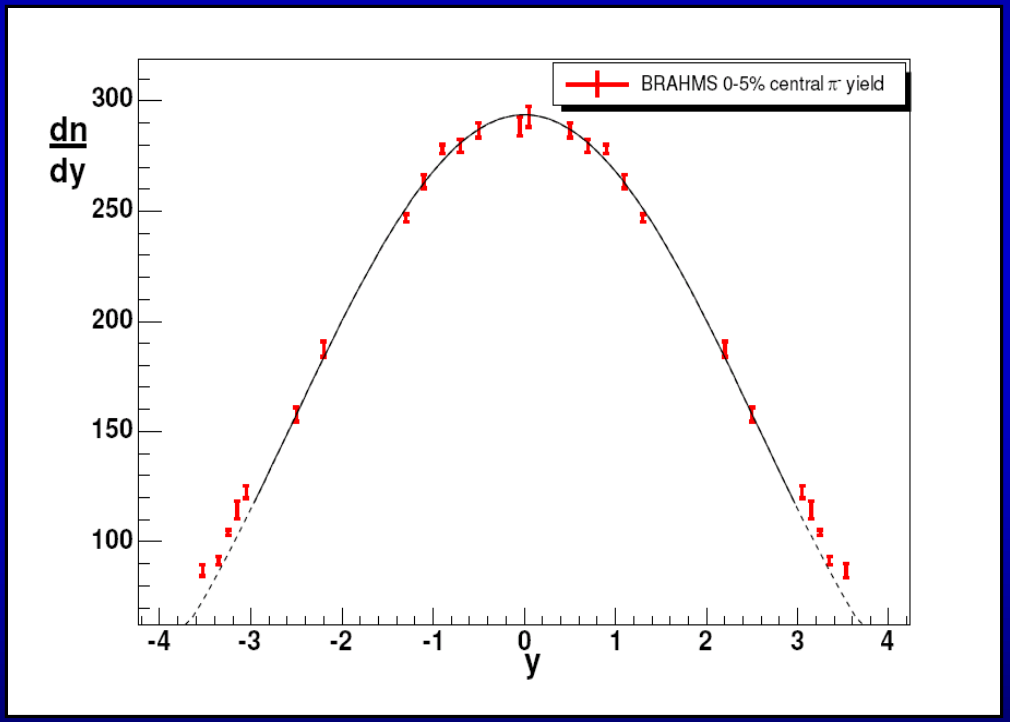
BRAHMS rapidity distribution

$$\frac{\mathrm{d}n}{\mathrm{d}y} \approx \left.\frac{\mathrm{d}n}{\mathrm{d}y}\right|_{y=0} \cosh^{\pm\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) e^{-\frac{m}{T_f}\left[\cosh^{\alpha}\left(\frac{y}{\alpha}\right)-1\right]},$$



$$\lambda = \frac{\alpha - 1}{\alpha - 2}.$$

α	7.4 ± 0.13
$\frac{\mathrm{d}n}{\mathrm{d}y}\|_{y=0}$	294 ± 1
χ^2/NDF	30.6/14
CL	0.6%
T_f (MeV)	200 (fixed)
m (MeV)	140 (fixed)
λ	1.18 ± 0.01 (derived)

BRAHMS dn/dy data fitted with the analytic formula

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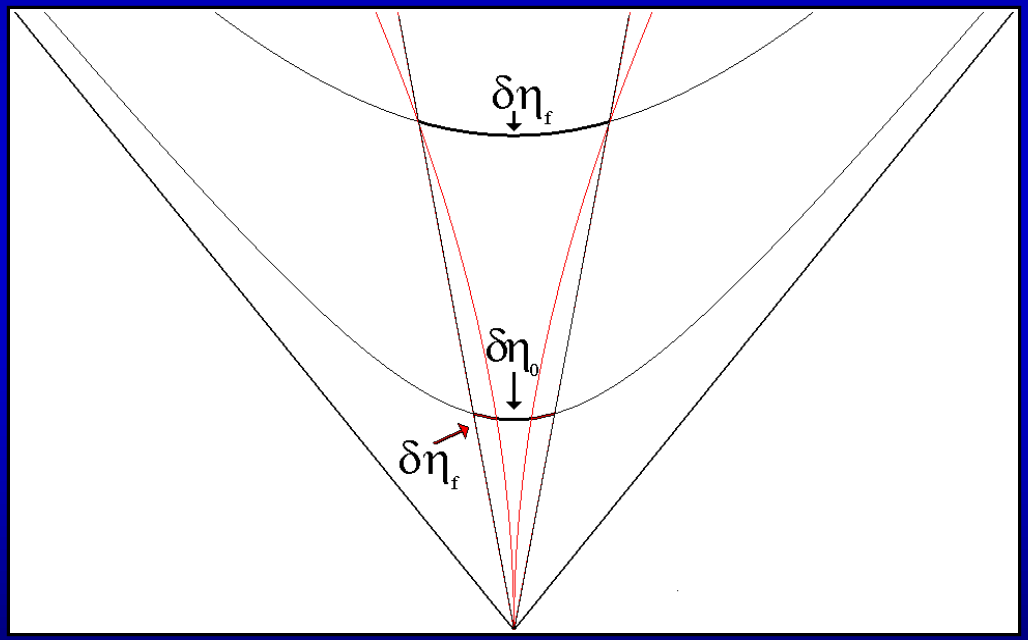
Advanced energy density estimate

Fit result: $\lambda > 1$

Flows accelerate: ⟺ **do work**

⟺ **initial energy density is higher than Bjorken's**



$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}$$

Work and acceleration.

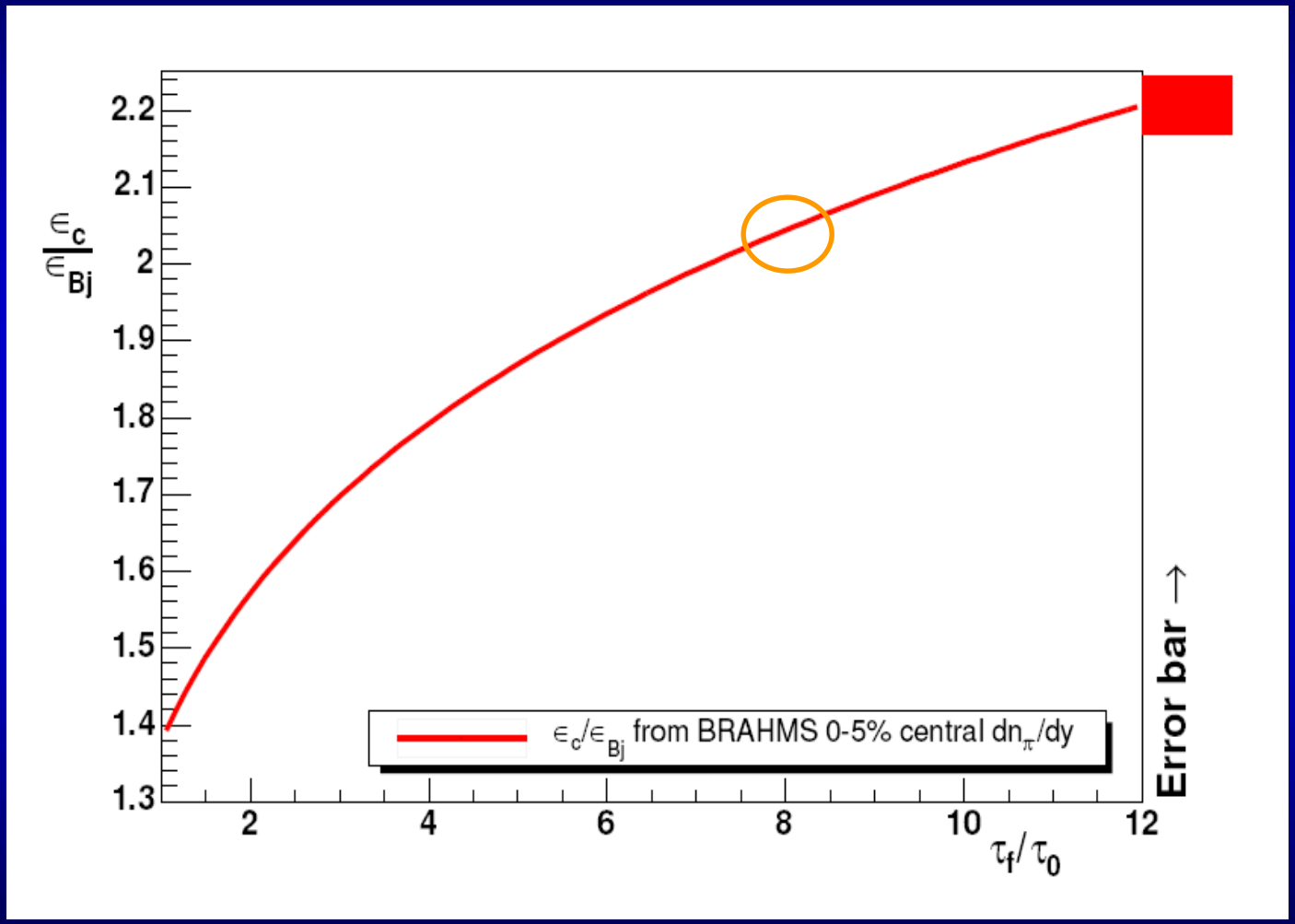
FYI:

$$\varepsilon_{Bj} = \frac{\langle m_t \rangle}{(R^2\pi)\tau_0}\frac{dn}{dy}$$

For $\lambda > 1$ (accelerating) flows, both factors > 1

Advanced energy density estimate

Correction depends on timescales, dependence is:



With a typical τ_f/τ_0 of ~8-10, one gets a correction factor of 2!

Conjecture: EoS dependence of ε_0

Four constraints

1) ε_{Bj} is independent of EoS ($\lambda = 1$ case)

2) $c_s^2 = 1$ case is solved for any $\lambda > 0.5$

$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}$$

Corrections due to respect these limits.

3) c_s^2 dependence of $\varepsilon(\tau)$ is known in NR limit

4) Numerical hydro results, e.g. K. Morita, arXiv:nucl-th/0611093v2

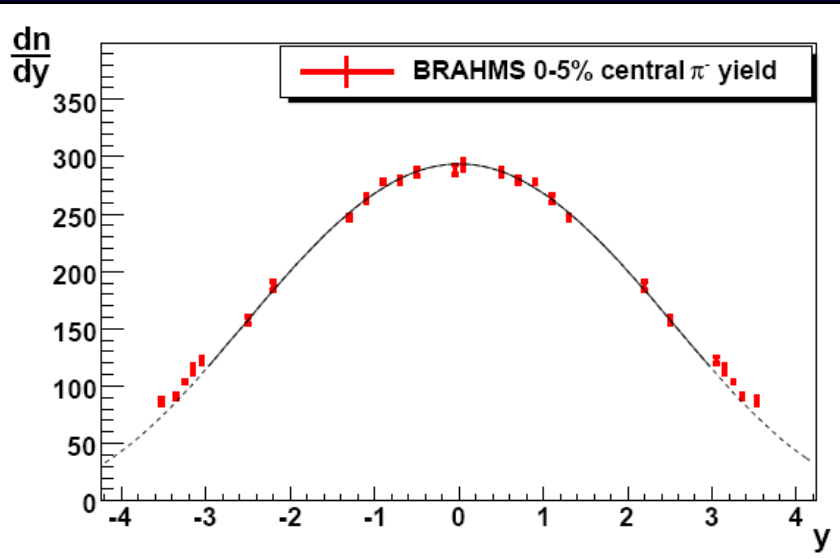
Conjectured formula – given by the principle of Occam's razor:

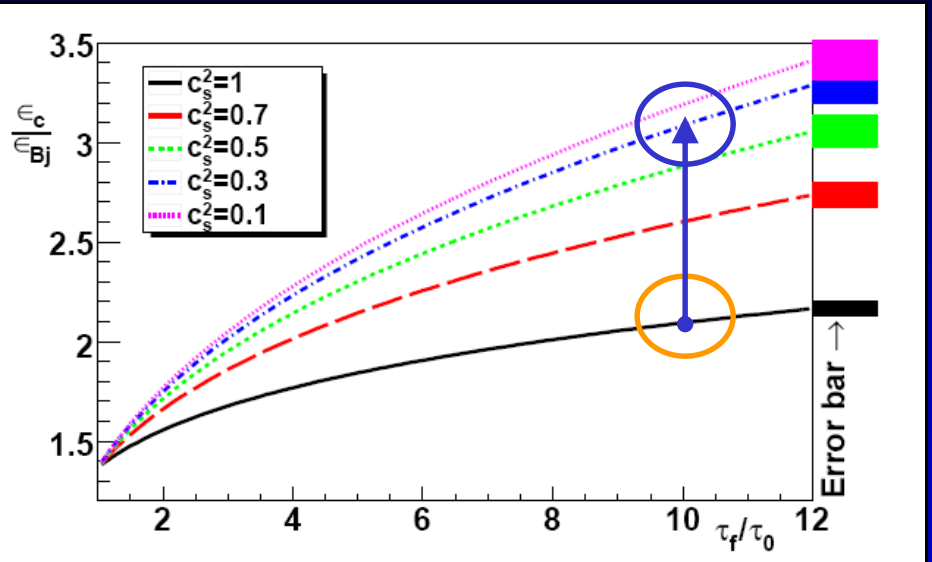
$$\frac{\varepsilon_{c_s^2}}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)(1-c_s^2)}$$

Using $\lambda = 1.18$, $c_s = 0.35$, $\tau_f/\tau_0 = 10$, we get $\varepsilon_{cs}/\varepsilon_{Bj} = 2.9$

$\varepsilon_0 = 14.5$ GeV/fm^3 in 200 GeV, 0-5 % Au+Au at RHIC

Conjectured EoS dependence of ε_0





$$\frac{\varepsilon_{c_s^2}}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)(1-c_s^2)}$$

Using $\lambda = 1.18$, and $\tau_f/\tau_0 = 10$ as before

and $c_s = 0.35$, [PHENIX, arXiv:nucl-ex/0608033v1] we get $\varepsilon_{cs}/\varepsilon_{Bj} = 2.9$

$\varepsilon_0 = 14.5$ GeV/fm^3 in 200 GeV, 0-5 % Au+Au at RHIC

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Advanced life-time estimate

Life-time estimation: for Hwa-Bjorken type of flows

$$R_{long} = \sqrt{\frac{T_f}{m_t}}\tau_{Bj} \quad \Rightarrow \quad \tau_{Bj} = \sqrt{\frac{m_t}{T_f}}R_{long}.$$

Makhlin & Sinyukov, Z. Phys. C 39, 69 (1988)

Underestimates lifetime (Renk, CsT, Wiedemann, Pratt, ...)

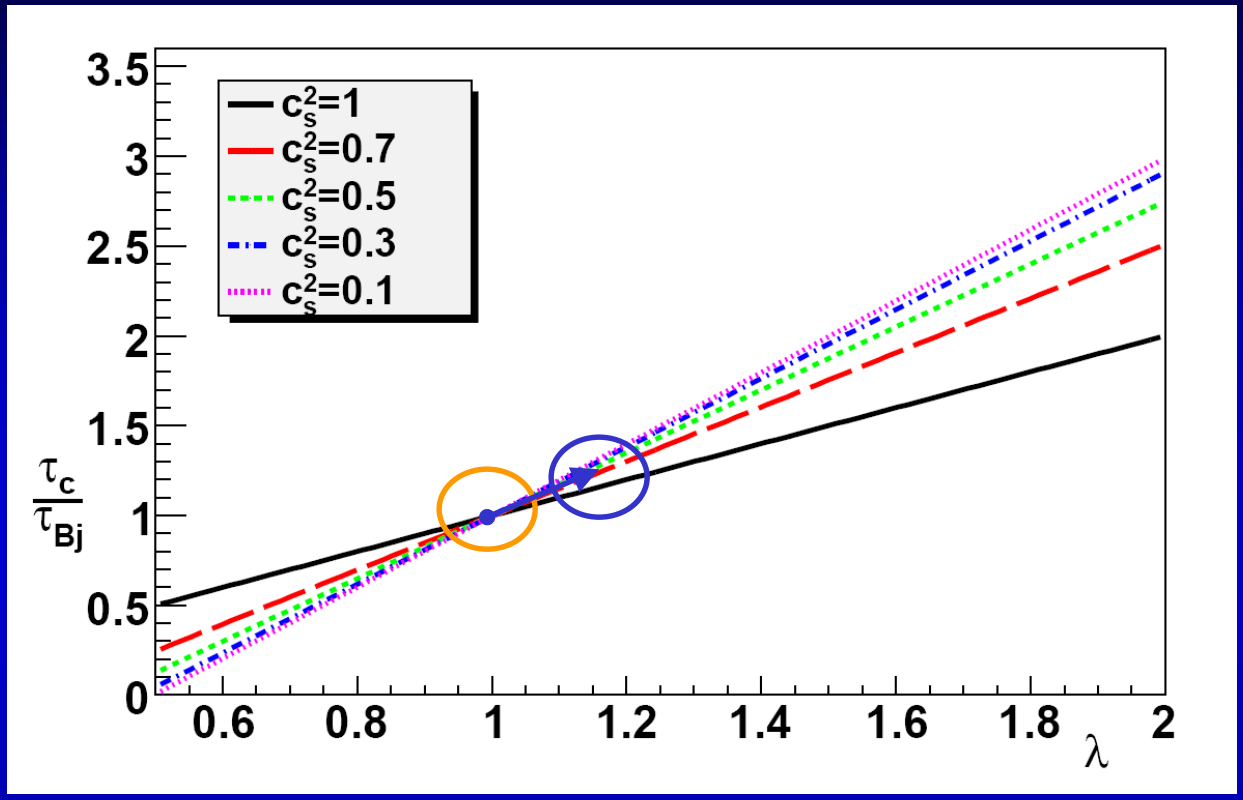
Advanced life-time estimate:

width of dn/dy related to acceleration and work

$$R_{\rm long} = \sqrt{\frac{T_f}{m_t}\frac{\tau_{\rm c}}{\lambda}} \quad \Rightarrow \quad \tau_{\rm c} = \lambda\tau_{Bj}.$$

At RHIC energies: correction is about +20%

Conjectured EoS dependence of τ_c



$$R_{\rm long} = \sqrt{\frac{T_f}{m_t}} \frac{\tau_{\rm c}}{\lambda + (\lambda - 1)(1 - c_s^2)} \quad \Rightarrow \quad \tau_{\rm c_s} = [\lambda + (\lambda - 1)(1 - c_s^2)]\tau_{Bj}.$$

Using $\lambda = 1.18$, and $c_s = 0.35$, we get $\tau_{cs}/\tau_{Bj} = 1.36$

in 200 GeV, 0-5 % Au+Au at RHIC

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New, exact solutions for e/p > 1

$$\frac{1-\kappa\tanh^2(\Omega-\eta_s)}{1-\tanh^2(\Omega-\eta_s)}\Omega' = \lambda,$$

$$\Rightarrow \quad \Omega' = \lambda\frac{1-\tanh^2(\Omega-\eta_s)}{1-\kappa\tanh^2(\Omega-\eta_s)},$$

$$\Rightarrow \quad \eta_s = -H + \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}}\mathrm{Arctan}\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}}\tanh(H)\right).$$

Possible cases (one row of the table is one solution):

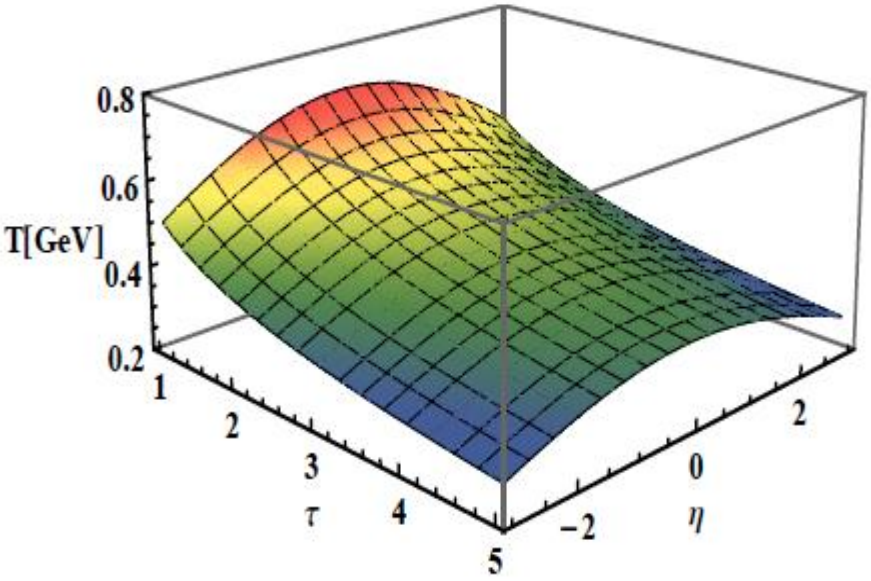
$$H = \Omega - \eta_s \ll 1,\ \kappa > 1,\ \lambda > 1,$$

$$T = T_i\left(\frac{\widetilde{\tau}_0}{\tau}\right)^{\frac{\lambda}{\kappa}}\exp\left[-\frac{\lambda(\lambda-1)(\kappa-1)}{2\kappa}\eta_s^2\right],$$

$$H = \Omega - \eta_s \ll 1,$$

$$\mathrm{Arctan}\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}}\tanh(H)\right) \simeq \sqrt{\frac{\kappa-\lambda}{\lambda-1}}H,$$

$$\Rightarrow \quad H = (\lambda-1)\eta_s, \quad v = \tanh((\lambda-1)\eta_s).$$



New, exact solutions: bulk viscosity

$$T^{\mu\nu} = eu^\mu u^\nu - p\Delta^{\mu\nu} + \Pi^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu},$$

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi,$$

$$e = \kappa p, \qquad p = nT,$$

$$\partial_\mu(N^\mu) = 0.$$

$$De = -(e + p + \Pi)\theta + \sigma_{\mu\nu}\pi^{\mu\nu},$$

$$(e + p + \Pi)Du^\alpha = \nabla^\alpha(p + \Pi) - \Delta^\alpha_\nu u_\mu D\pi^{\mu\nu} - \Delta^\alpha_\nu \nabla_\mu \pi^{\mu\nu},$$

$$S = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2},$$

$$u^\mu = \frac{x^\mu}{\tau} = \gamma\left(1, \frac{\dot{X}}{X}r_x, \frac{\dot{Y}}{Y}r_y, \frac{\dot{Z}}{Z}r_z\right),$$

$$\theta = \partial_\mu u^\mu = \frac{d}{\tau},$$

$$D = u^\mu\partial_\mu = \frac{\partial}{\partial\tau}.$$

$$\tau = \sqrt{t^2 - \boldsymbol{r}^2}$$

$$\eta_s = \frac{1}{2}\ln\left(\frac{t+r_z}{t-r_z}\right)$$

$$\sigma_{\mu\nu}\pi^{\mu\nu} = 0,$$

ζ/s η/s

New, exact solution with bulk viscosity

$$u^\mu = \frac{x^\mu}{\tau} = \gamma \left(1, \frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z\right),$$

$$\frac{de}{d\tau} + \frac{d(e+p)}{\tau} - \zeta\left(\frac{d}{\tau}\right)^2 = 0.$$

$$e = e_0 \left(\frac{\tau_0}{\tau}\right)^{\left(1+\frac{1}{\kappa}\right)d} + \zeta \frac{d}{\tau} \frac{d}{\left(1+\frac{1}{\kappa}\right)d - 1},$$

ζ/s η/s

$$\mathbf{v} = \frac{\mathbf{r}}{t} \quad \text{or} \quad u^\mu = \frac{x^\mu}{\tau},$$

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s),$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3\left(1+\frac{1}{\kappa}\right)} + \frac{\zeta}{\kappa} \frac{3}{\tau} \frac{3}{\left(1+\frac{1}{\kappa}\right)3 - 1},$$

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{3}{\kappa}} + \frac{\zeta}{\kappa n_0} \frac{3}{\tau} \left(\frac{\tau}{\tau_0}\right)^3 \frac{3}{\left(1+\frac{1}{\kappa}\right)3 - 1} \mathcal{T}(s),$$

$$\theta = \partial_\mu u^\mu = \frac{d}{\tau},$$

$$D = u^\mu \partial_\mu = \frac{\partial}{\partial \tau}.$$

$\tau = \sqrt{t^2 - \boldsymbol{r}^2}$

$\eta_s = \frac{1}{2} \ln\left(\frac{t+r_z}{t-r_z}\right)$

New viscous solution in 1+3 dim

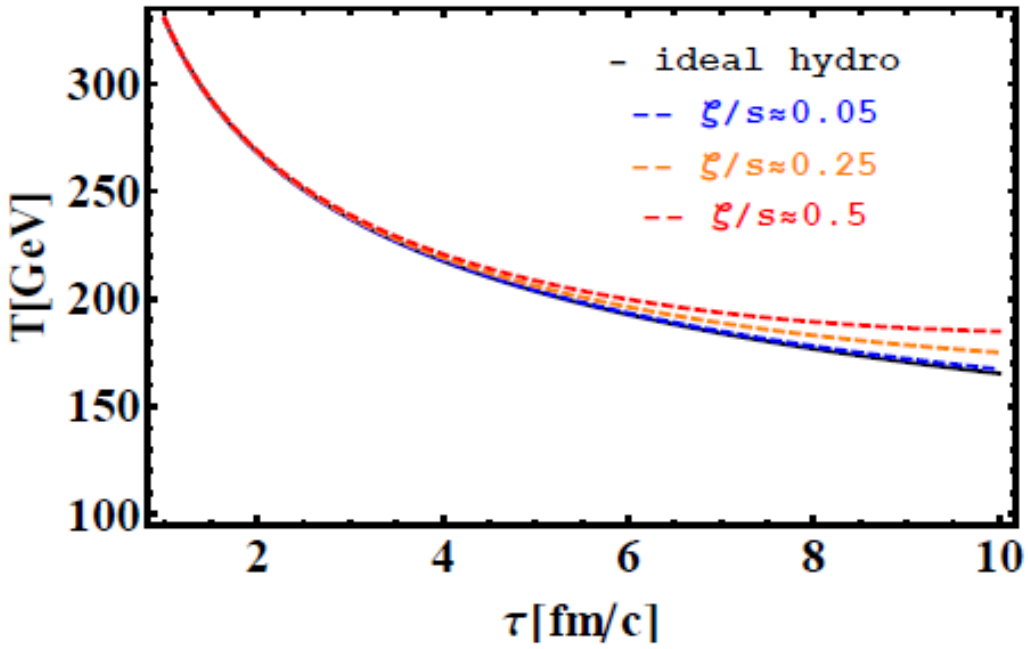


Fig. 1: The proper time τ and the ratio ζ/s of temperature for the new temperature solution for given primary initial condition $T_0 = 330$ MeV [], $\tau_0 = 1$ fm/c for RHIC, based on Eq.(21). (color online).

Bulk viscosity important at late stage, heats up

Shear viscosity effects cancel for asymptotically Hubble flows

Conclusions

Explicit solutions of a very difficult problem

New estimates of initial energy density

New exact solution

for arbitrary EOS with const e/p

after 10 years, finally

For asymptotically Hubble flows

shear effects cancel at late time

bulk viscosity heats up matter

A lot to do ...

more general EoS

less symmetry, ellipsoidal solutions

Rotating viscous solutions

Thank you for your attention!

Questions?

Backup slides

Some general remarks

Hydrodynamics=

Initial conditions ⊗ dynamical equations ⊗ freeze-out conditions

Exact solution = formulas solve hydro without approximation

Parametric solution = shape parameters introduced,
time dependence given by ordinary coupled diff. eqs.

Hydro inspired parameterization
= shape parameters determined only at the freeze-out
their time dependence is not considered

Report on new class of exact, parametric solution of relativistic hydro

M.I. Nagy, T. Cs., M. Csanád, arXiv:0709.3677v1 , PRC77:024908 (2008)

T. Cs, M. I. Nagy, M. Csanád, arXiv:nucl-th/0605070v4, PLB (2008)

M. Csanád, M. I. Nagy, T. Cs, arXiv:0710.0327v3 [nucl-th] EPJ A (2008)

High temperature superfluidity at RHIC!

All "realistic" hydrodynamic calculations for RHIC fluids to date have assumed zero viscosity

$\eta = 0 \rightarrow$ **perfect fluid**

a conjectured quantum limit:

$$\eta \geq \frac{\hbar}{4\pi}(Entropy\ Density) \equiv \frac{\hbar}{4\pi} s$$

P. Kovtun, D.T. Son, A.O. Starinets, hep-th/04

How "ordinary" fluids compare to this limit?

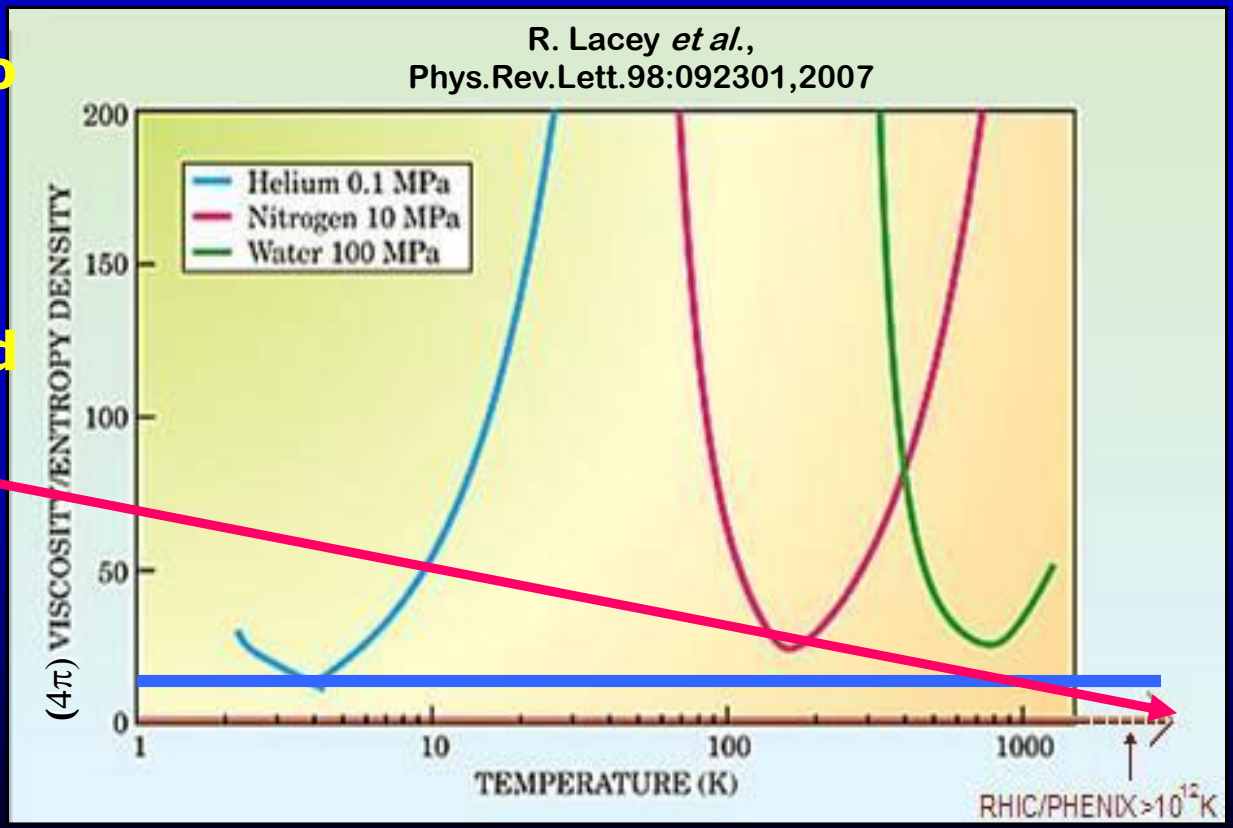
$(4\pi)\,\eta/s > 10$

RHIC's perfect fluid

$(4\pi)\,\eta/s \sim 1$!

T > 2 Terakelvin

The hottest & most perfect fluid ever made...



How Perfect is Perfect? Measure η/s !

Damping (flow, fluctuations, heavy quark motion) ~ η/s

FLOW: *Has the QCD Critical Point Been Signaled by Observations at RHIC?,* R. Lacey *et al.*, Phys.Rev.Lett.98:092301,2007 (nucl-ex/0609025)

$$\frac{\eta}{s} = (1.1 \pm 0.2 \pm 1.2)\frac{1}{4\pi}$$

The Centrality dependence of Elliptic flow, the Hydrodynamic Limit, and the Viscosity of Hot QCD, H.-J. Drescher *et al.*, (arXiv:0704.3553)

$$\frac{\eta}{s} = (1.9 - 2.5)\frac{1}{4\pi}$$

FLUCTUATIONS: *Measuring Shear Viscosity Using Transverse Momentum Correlations in Relativistic Nuclear Collisions*, S. Gavin and M. Abdel-Aziz, Phys.Rev.Lett.97:162302,2006 (nucl-th/0606061)

$$\frac{\eta}{s} = (1.0 - 3.8)\frac{1}{4\pi}$$

DRAG, FLOW: *Energy Loss and Flow of Heavy Quarks in Au+Au Collisions at $\sqrt{s_{NN}}$ = 200 GeV* (PHENIX Collaboration), A. Adare *et al.*, Phys.Rev.Lett.98:172301,2007 (nucl-ex/0611018)

$$\frac{\eta}{s} = (1.3 - 2.0)\frac{1}{4\pi}$$

CHARM!

Landau-Khalatnikov solution

Publications:

L.D. Landau, Izv. Acad. Nauk SSSR 81 (1953) 51

I.M. Khalatnikov, Zhur. Eksp.Teor.Fiz. 27 (1954) 529

L.D.Landau and S.Z.Belenkij, Usp. Fiz. Nauk 56 (1955) 309

Implicit 1D solution with approx. Gaussian rapidity distribution

Basic relations: $u^0 = \cosh\Omega(t,x), u^1 = \sinh\Omega(t,x)$

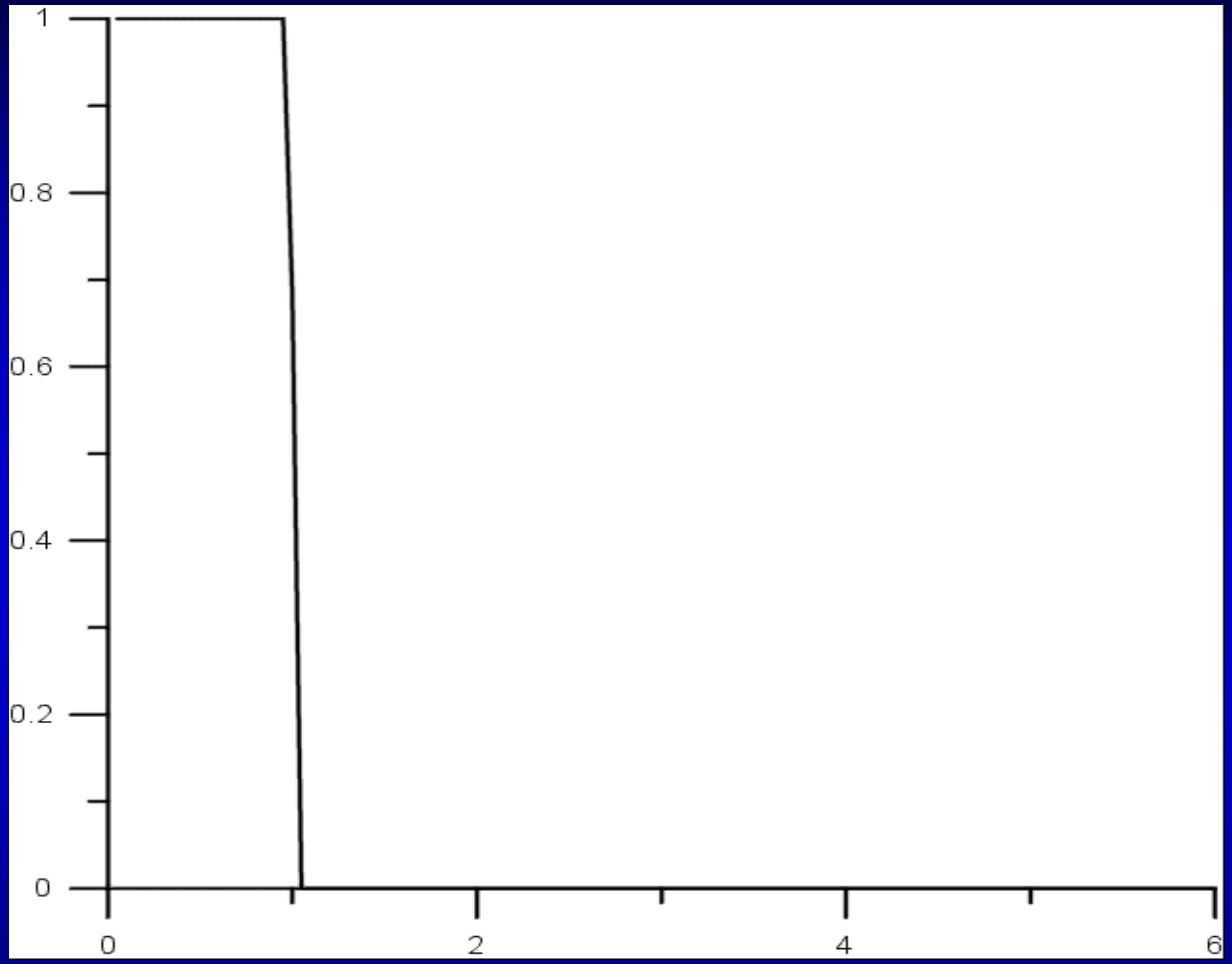
Unknown variables: $T(t,x),\ \Omega(t,x)$

Auxiliary function: $\chi(\Omega,T)$

$$t = \frac{\partial\chi}{\partial T}\cosh\Omega - \frac{1}{T}\frac{\partial\chi}{\partial\Omega}\sinh\Omega, \quad x = \frac{\partial\chi}{\partial T}\sinh\Omega - \frac{1}{T}\frac{\partial\chi}{\partial\Omega}\cosh\Omega$$

Expression of $\chi(\Omega,T)$ **is a true „tour de force”**

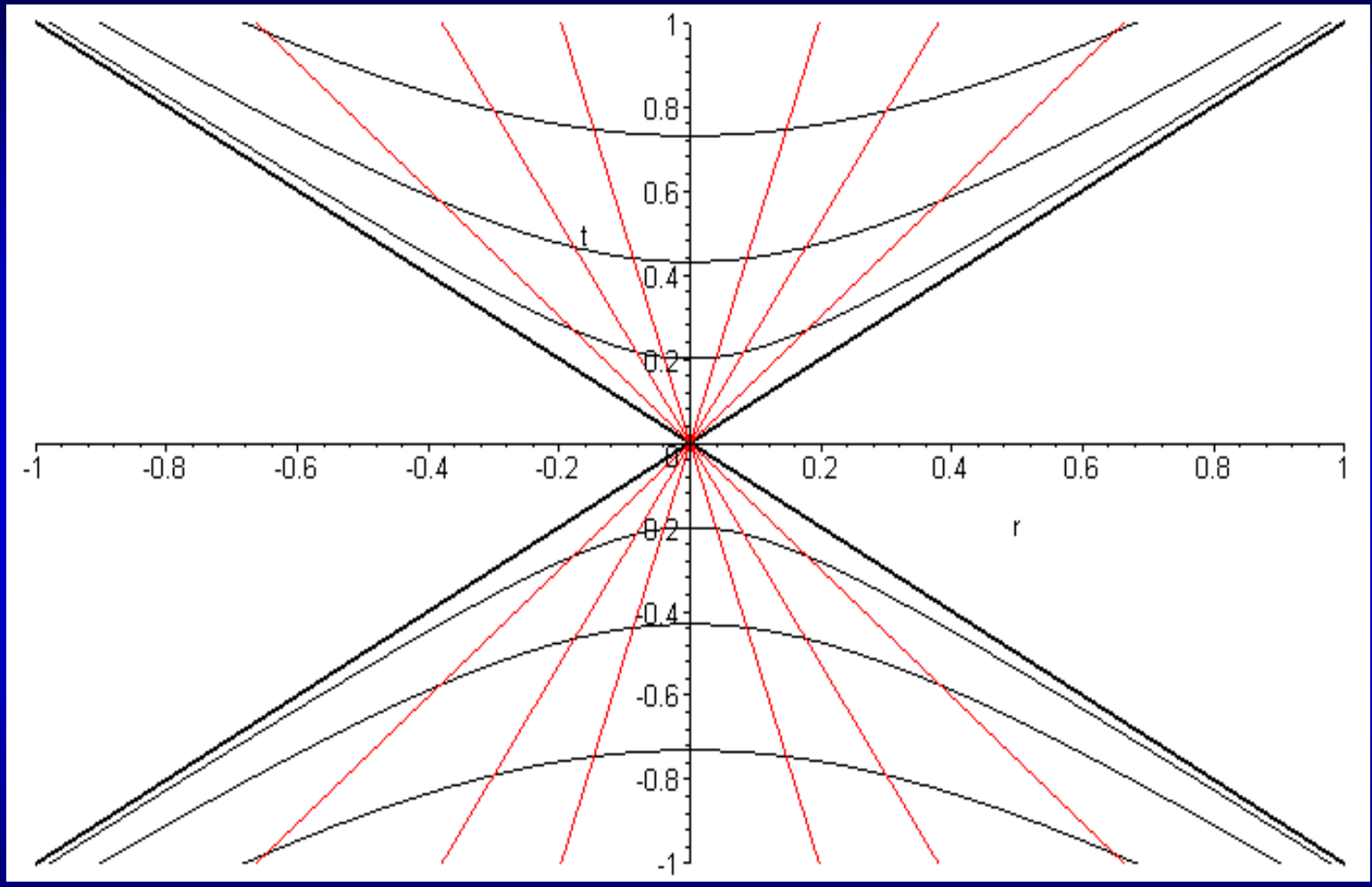
Landau-Khalatnikov solution



Temperature distribution (animation courtesy of T. Kodama)

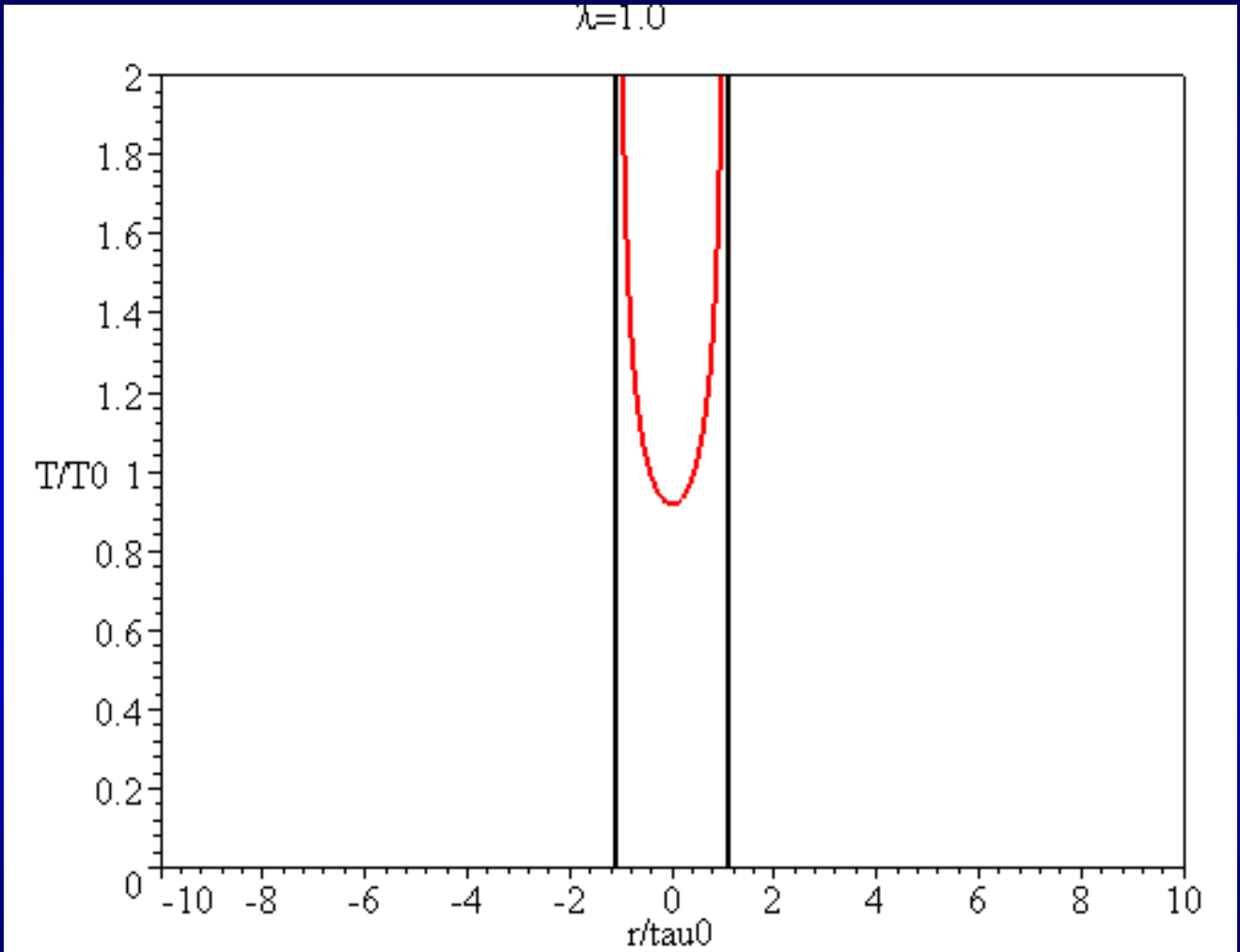
„Tour de force” implicit solution: t=t(T,v), r=r(T,v)

Hwa-Bjorken solution



The Hwa-Bjorken solution / Rindler coordinates

Hwa-Bjorken solution



The Hwa-Bjorken solution / Temperature evolution

Bialas-Janik-Peschanski solution

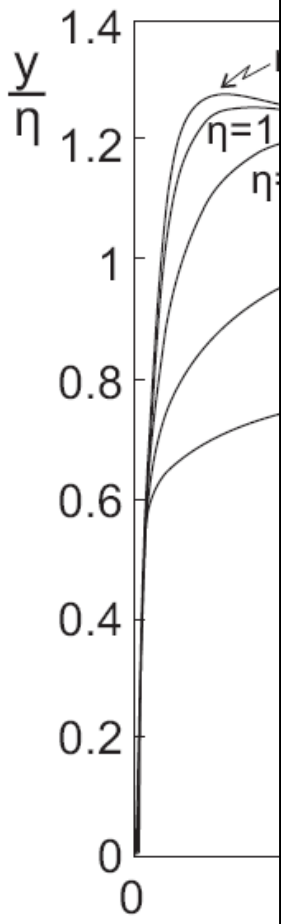
Publications:

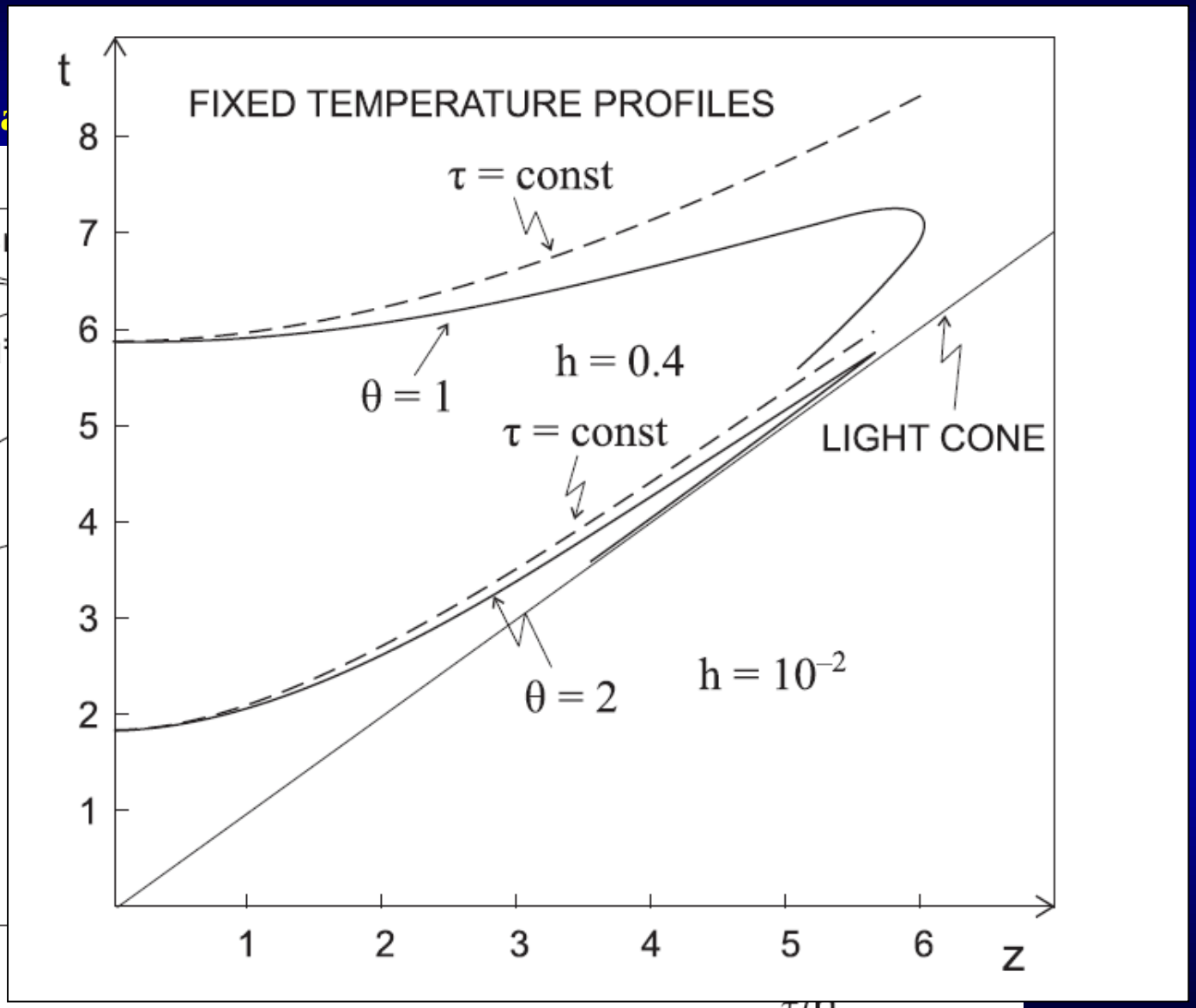
A. Bialas, R. Ja

Ac

Ge

$2y =$





Hwa-Bjorken solution

Publications:

R.C. Hwa, Phys. Rev. D10, 2260 (1974)

J.D. Bjorken, Phys. Rev. D27, 40(1983)

Accelerationless, expanding 1D simple boost-invariant solution

$$t = \tau \cosh \eta, \quad r = \tau \sinh \eta$$

Rindler coordinates:

$$\eta = \operatorname{arctanh} \frac{r}{t}, \tau = \sqrt{t^2 - r^2} = \sqrt{x_\mu x^\mu}$$

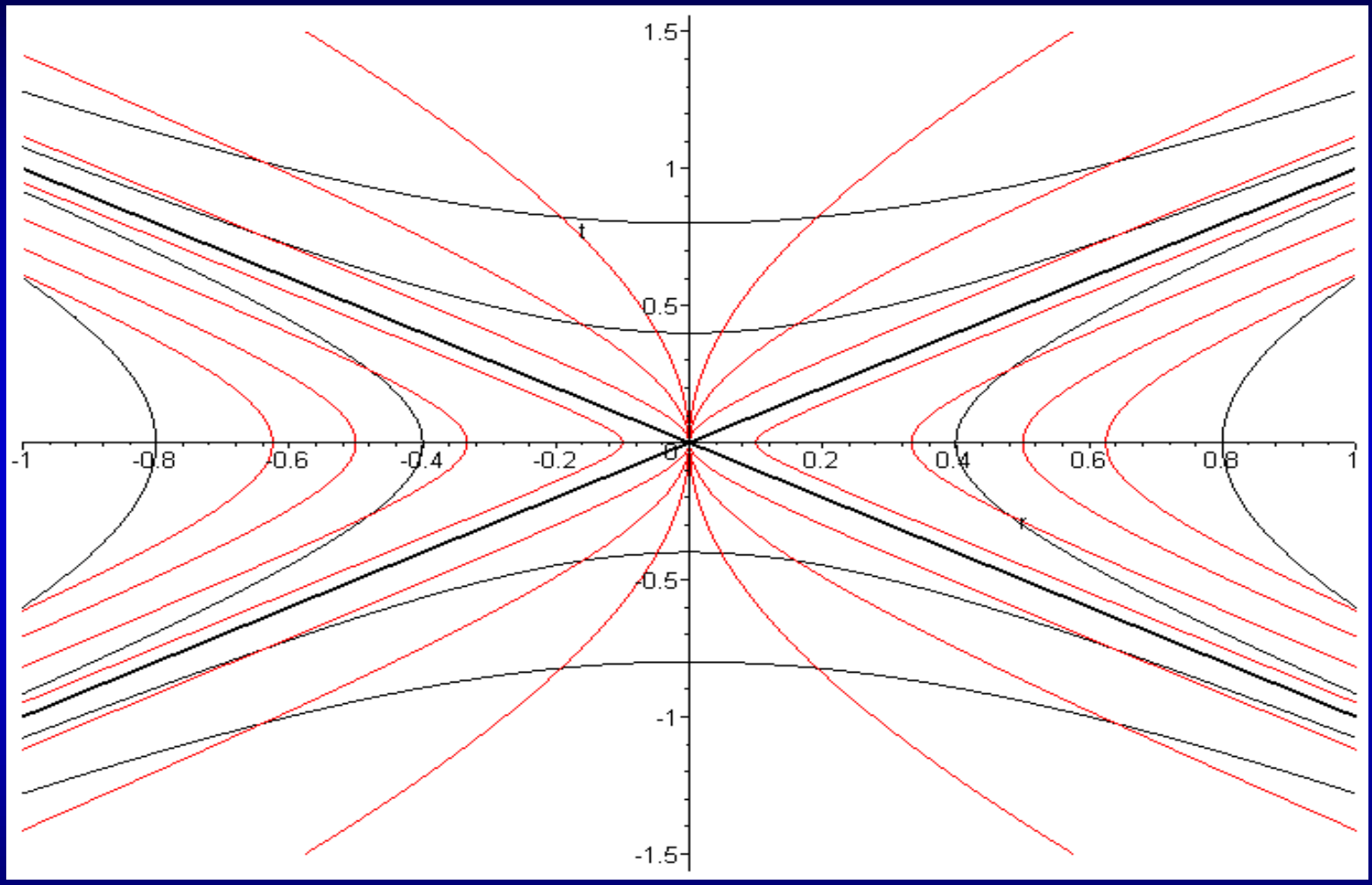
Boost-invariance (valid for asymptotically high energies):

$$\varepsilon = \varepsilon(\tau), \; p = p(\tau), \; s = s(\tau), \; u^\mu = \frac{x^\mu}{\tau}, \; v = \frac{z}{t} \Rightarrow \frac{dv}{dt} = 0$$

$$\partial_\mu (s u^\mu) = 0 \qquad \frac{s}{s_0} = \frac{\tau_0}{\tau}$$

$T = T(\tau)$ depends on EoS, e.g. $\varepsilon = \kappa p, \; \kappa = D, \; D = 1 \Rightarrow T(\tau) = T_0 \frac{\tau_0}{\tau}$

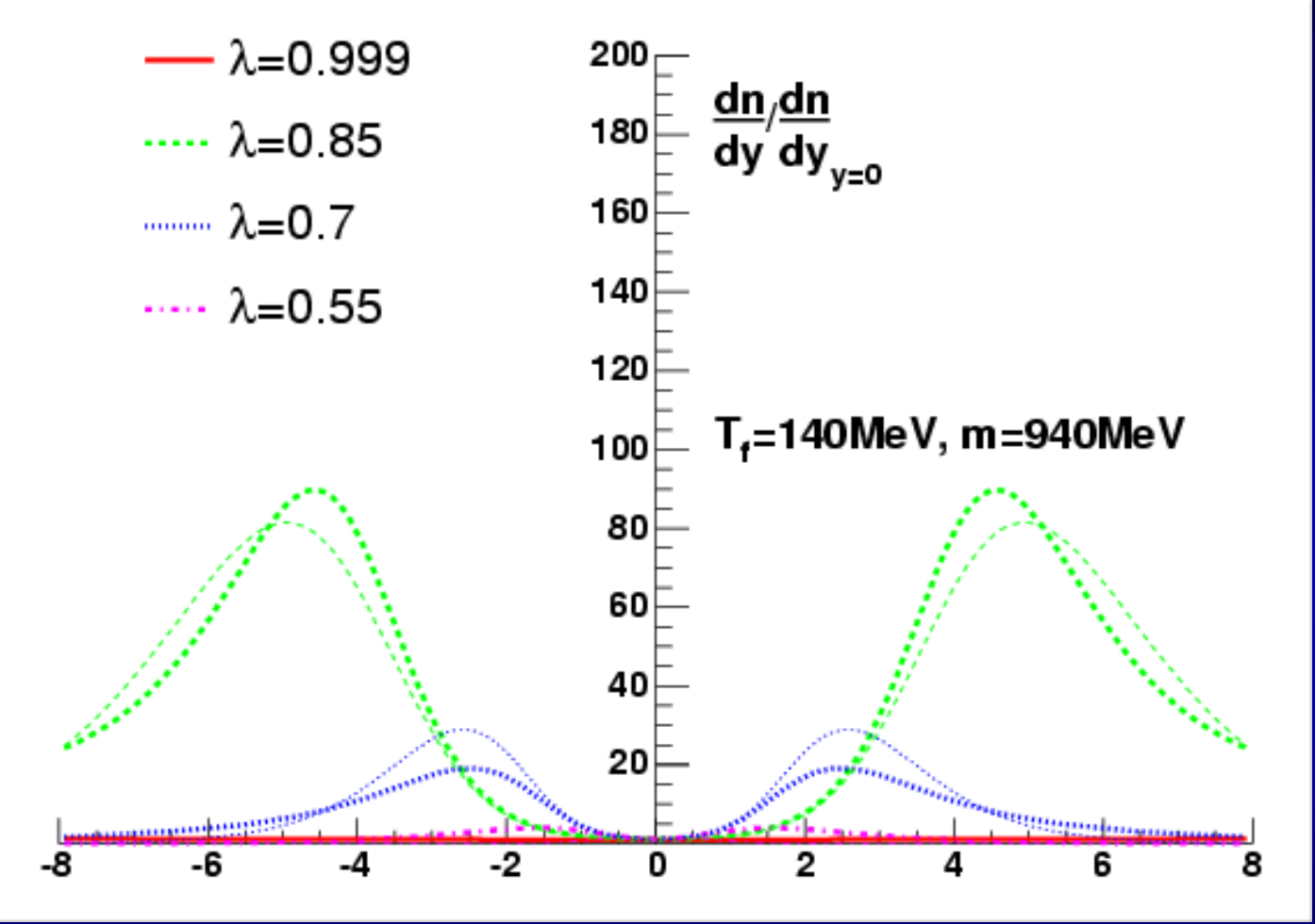
New simple solutions in 1+d dim



The fluid lines (red) and the pseudo-orthogonal freeze-out surface (black)

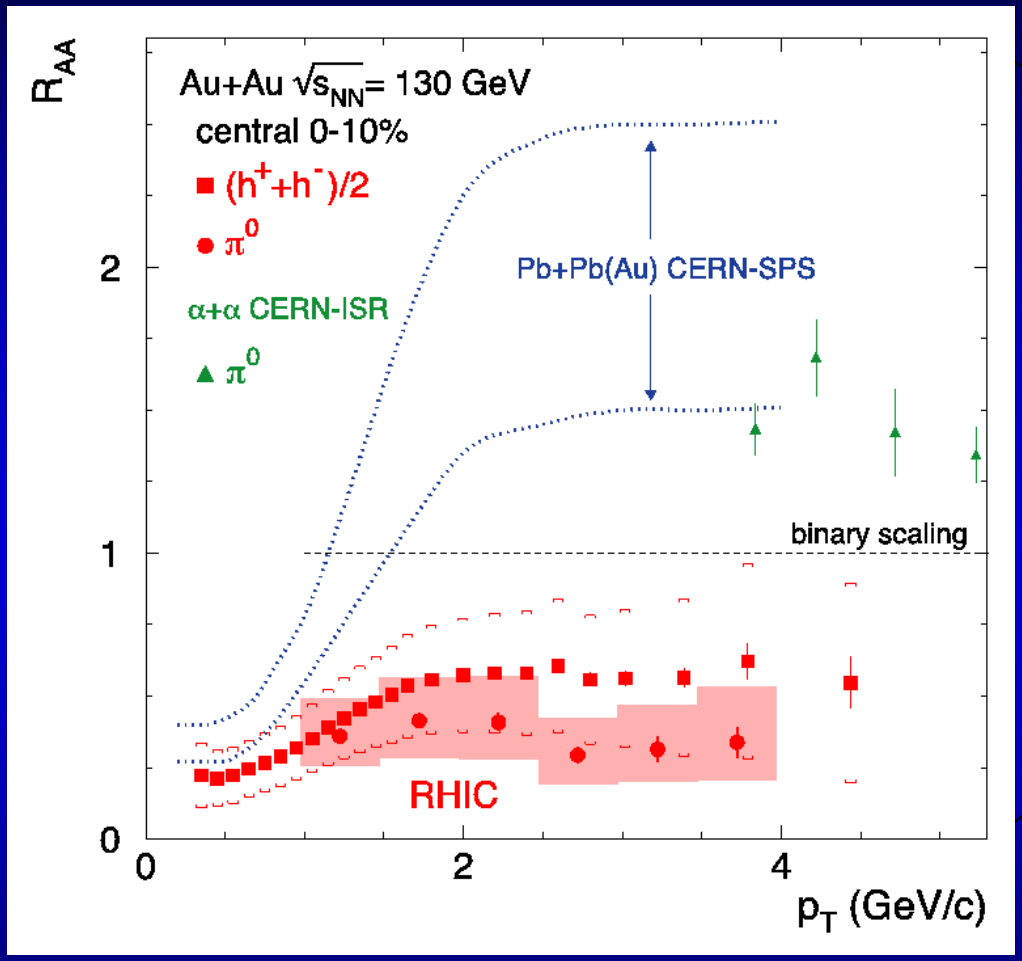
$A = const.$

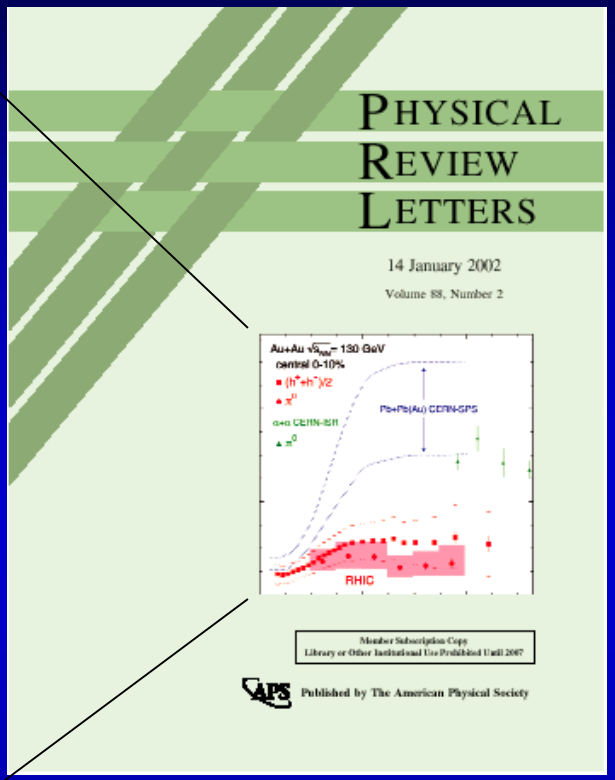
Rapidity distribution



Rapidity distribution from the 1+1 dimensional solution, for $\lambda \cdot < 1$

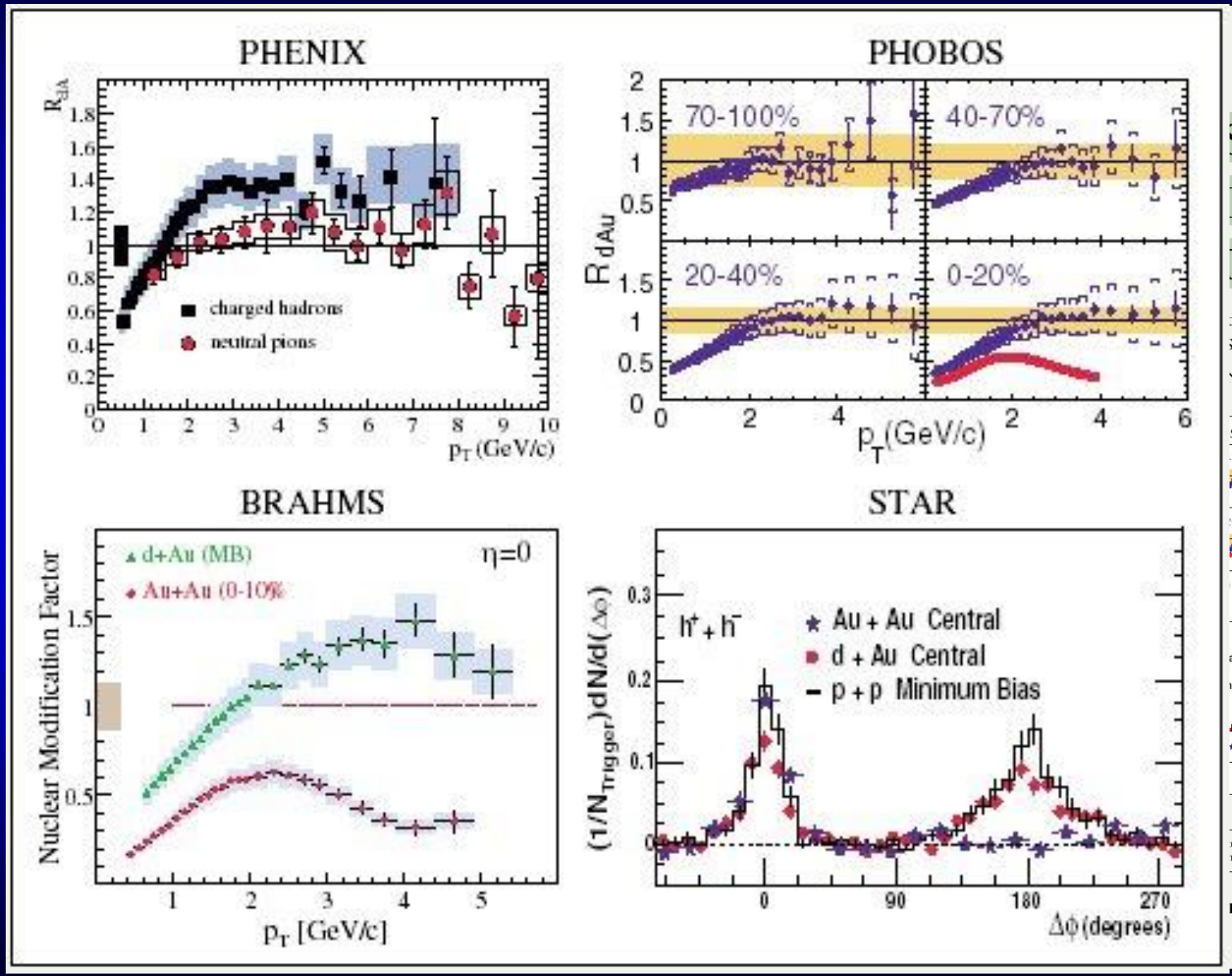
1st milestone: new phenomena

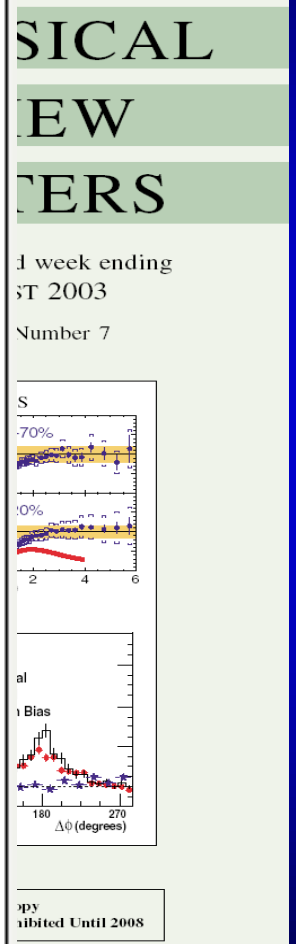




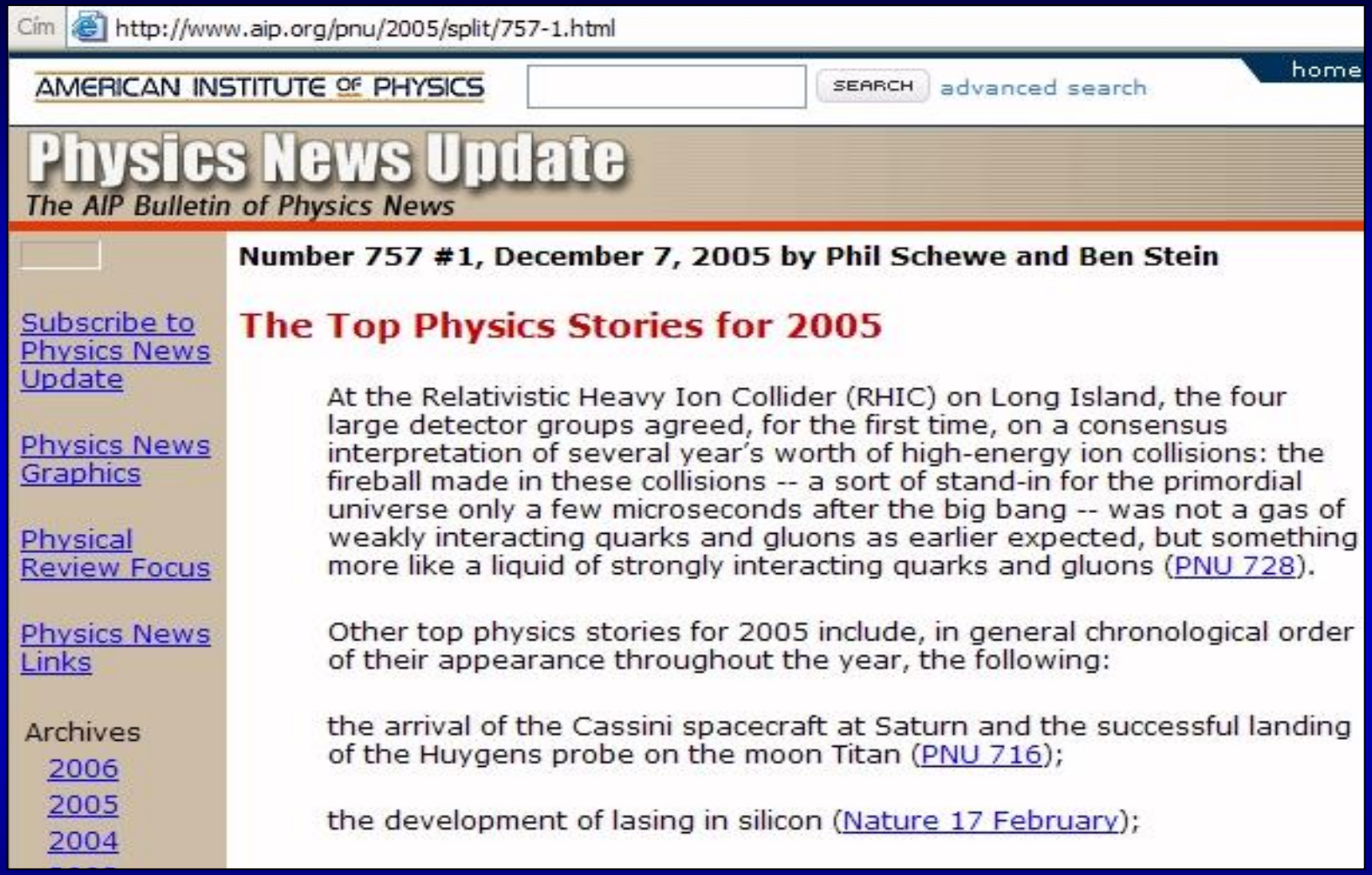
Suppression of high p_t particle production in Au+Au collisions at RHIC

2nd milestone: new form of matter





3rd milestone: Top Physics Story 2005



Cim http://www.aip.org/pnu/2005/split/757-1.html

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Number 757 #1, December 7, 2005 by Phil Schewe and Ben Stein

The Top Physics Stories for 2005

At the Relativistic Heavy Ion Collider (RHIC) on Long Island, the four large detector groups agreed, for the first time, on a consensus interpretation of several year's worth of high-energy ion collisions: the fireball made in these collisions -- a sort of stand-in for the primordial universe only a few microseconds after the big bang -- was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons (PNU 728).

Other top physics stories for 2005 include, in general chronological order of their appearance throughout the year, the following:

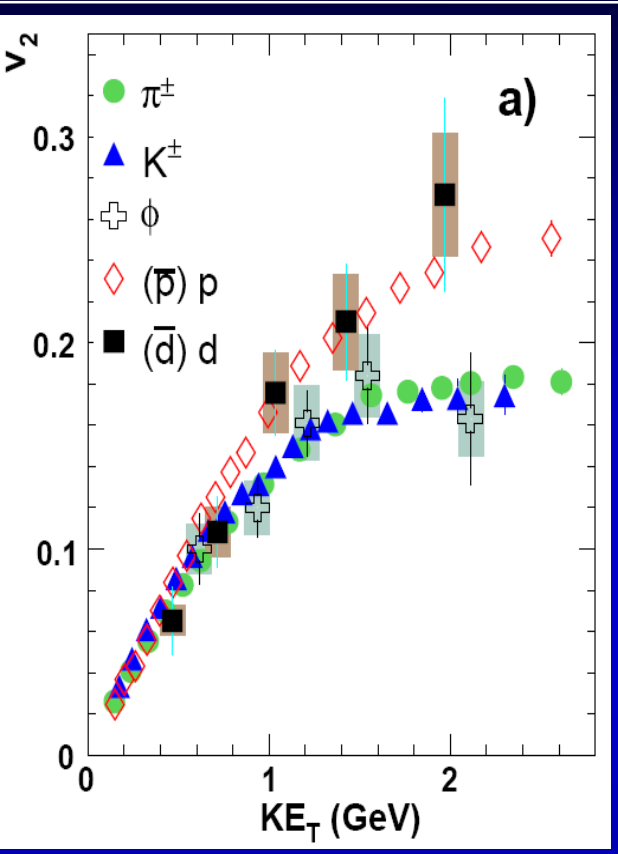
the arrival of the Cassini spacecraft at Saturn and the successful landing of the Huygens probe on the moon Titan (PNU 716);

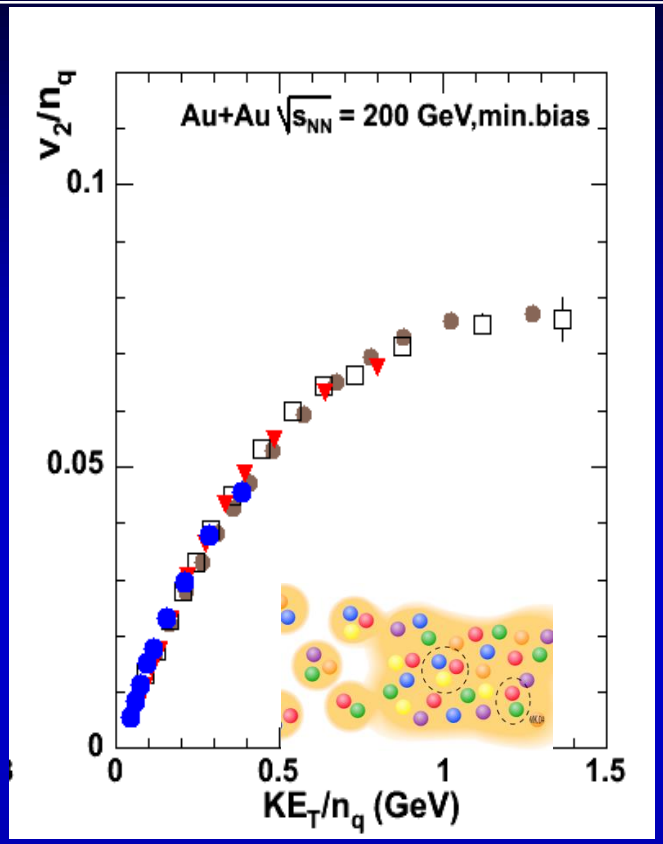
the development of lasing in silicon (Nature 17 February);

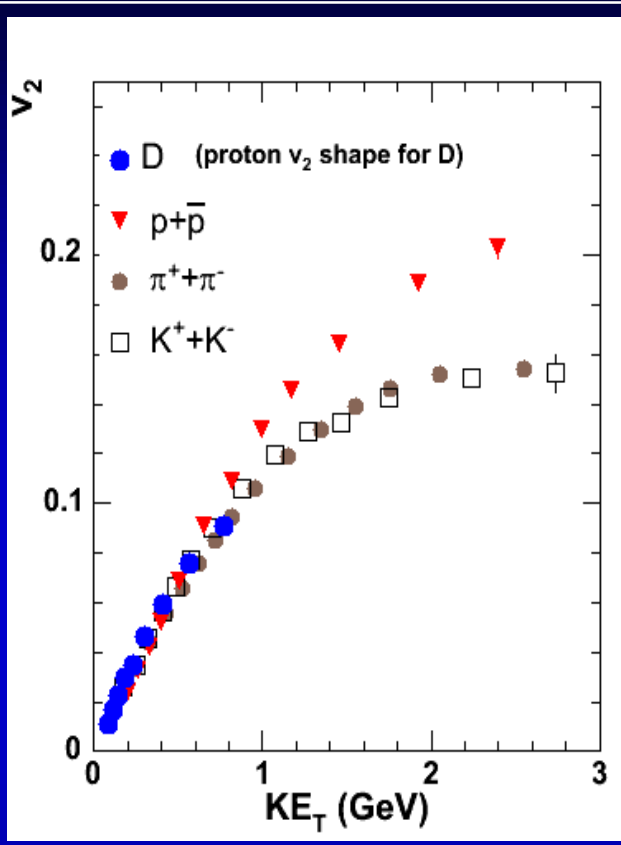
http://arxiv.org/abs/nucl-ex/0410003

PHENIX White Paper: second most cited in nucl-ex during 2006

4th Milestone: A fluid of quarks







v_2 *for the φ follows that of other mesons*

$$v_2^{hadron}(KE_T^{hadron}) \approx n v_2^{quark}(KE_T^{quark})$$
$$KE_T^{hadron} \approx n KE_T^{quark}$$

v_2 *for the D follows that of other mesons*

Strange and even charm quarks participate in the flow

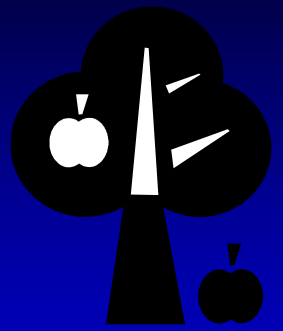
M. Csanád, T. Csörgő, ZF. Jiang

Predictions of the Buda-Lund model

Hydro predicts scaling (even viscous)

What does a scaling mean?

See Hubble's law – or Newtonian gravity: $t = \sqrt{2gh}$



Cannot predict acceleration or height

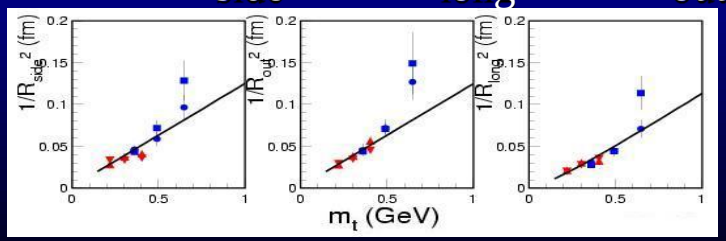
Collective, thermal behavior $\rightarrow$

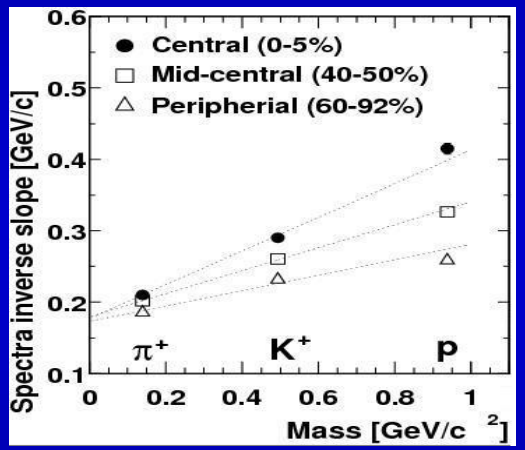
Loss of information

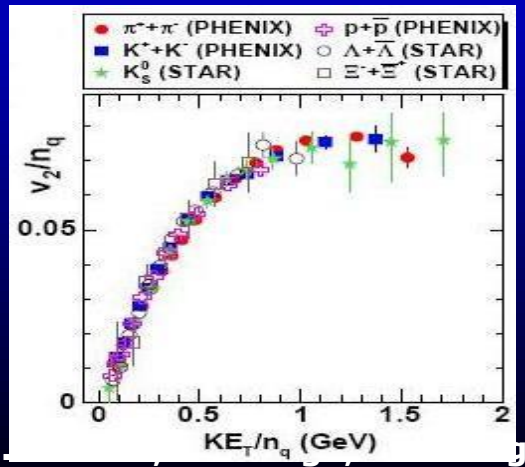
Spectra slopes: $T_{\text{eff}} = T_0 + m u_t^{\,2}$

Elliptic flow: $v_2 = \dfrac{I_1(w)}{I_0(w)}, w \sim KE_T$

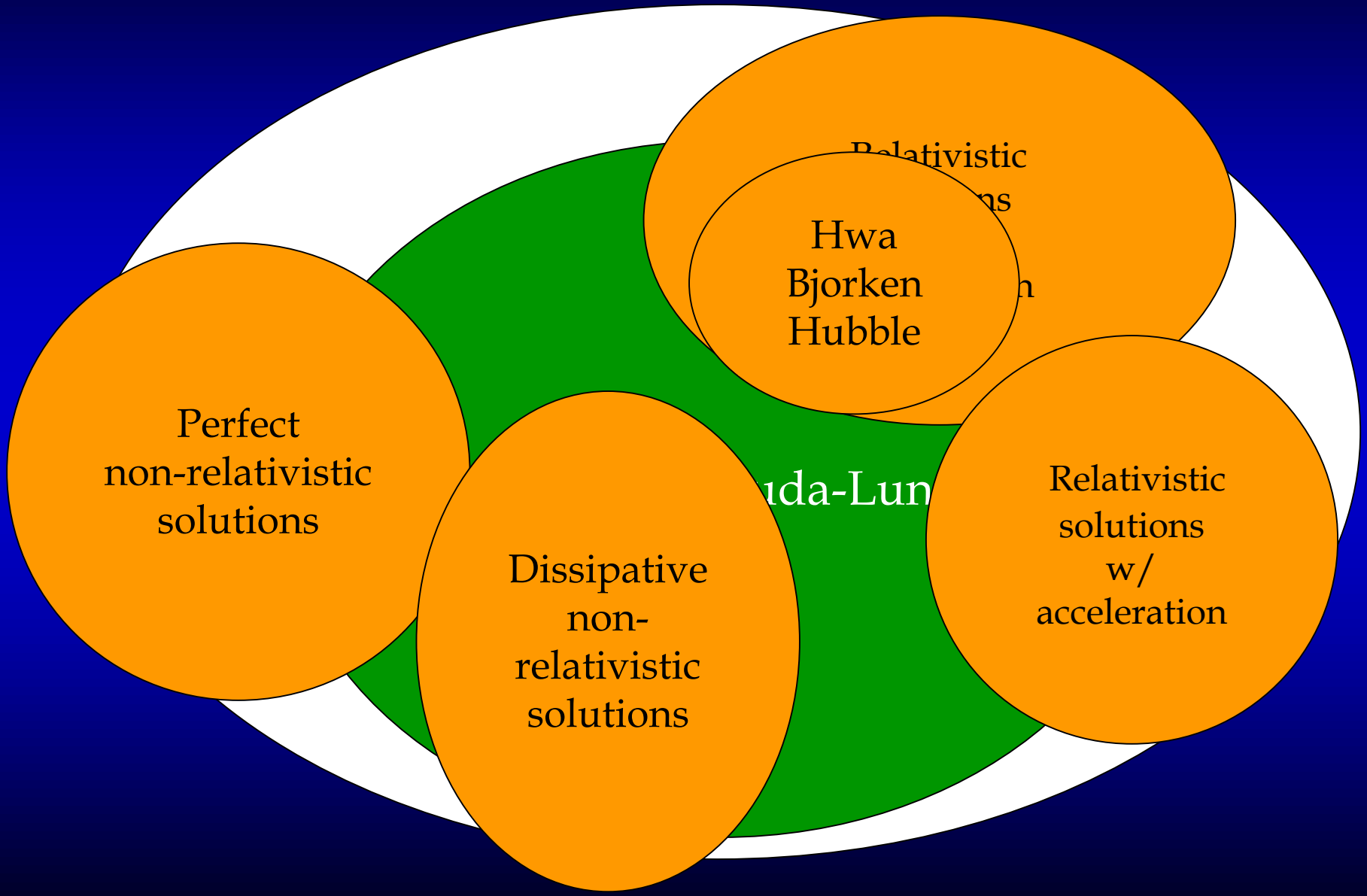
HBT radii: $R_{\text{side}}^{\,2} \approx R_{\text{long}}^{\,2} \approx R_{\text{out}}^{\,2} \sim \dfrac{1}{m_t}$





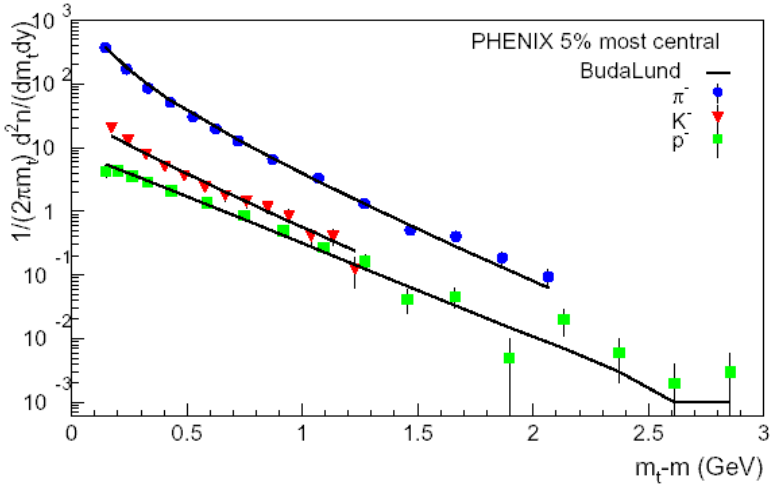


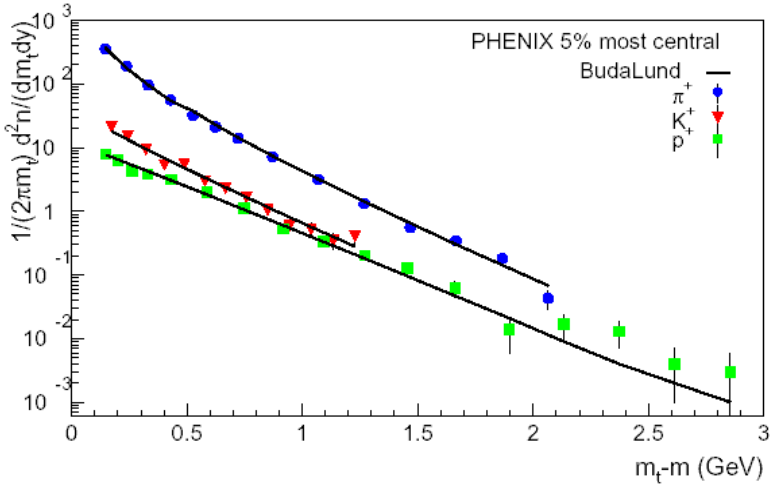
What does the data tell us



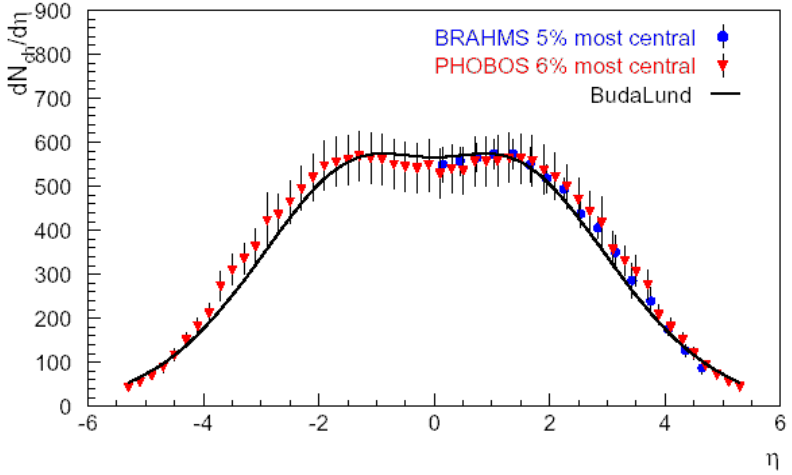
BudaLund fits to 130 GeV RHIC data

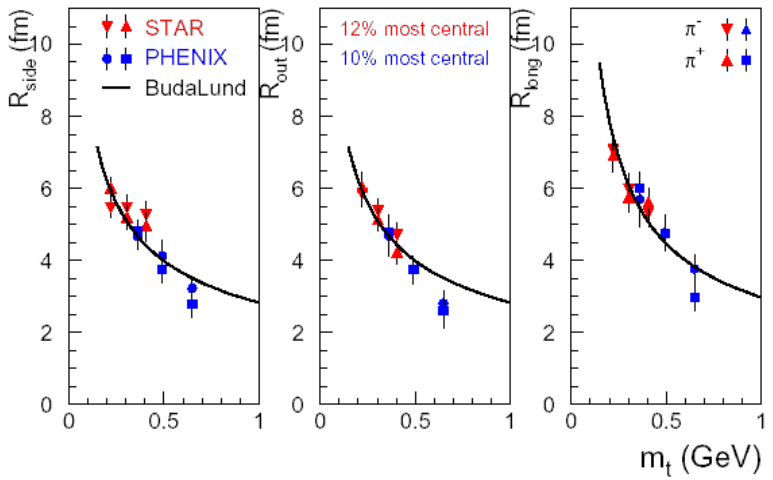
BudaLund hydro fits to 130 AGeV Au+Au





BudaLund hydro fits to 130 AGeV Au+Au

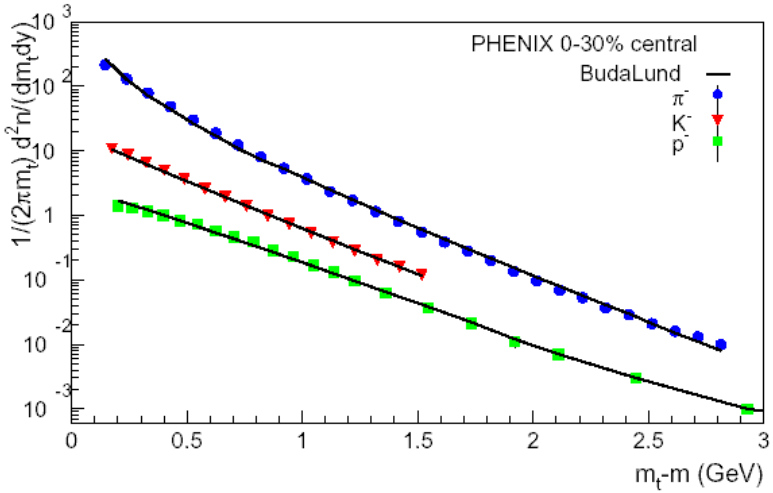




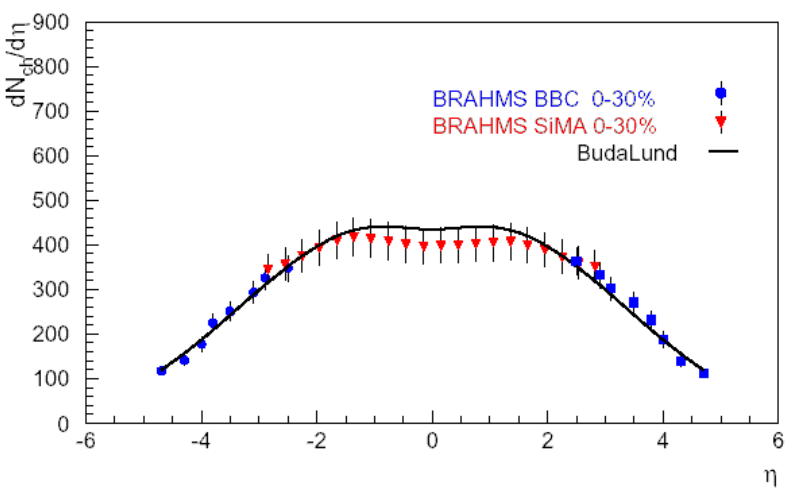
M. Csanád, T. Csörgő, B. Lörstad, A. Ster, nucl-th/0311102, ISMD03

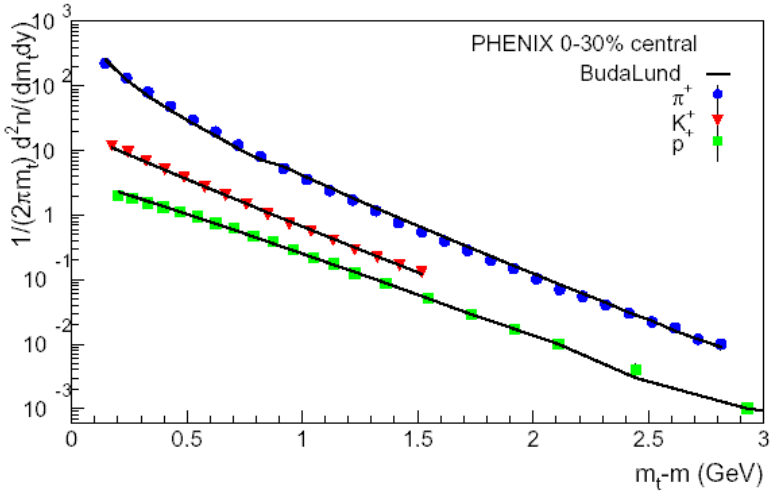
BudaLund fits to 200 GeV RHIC data

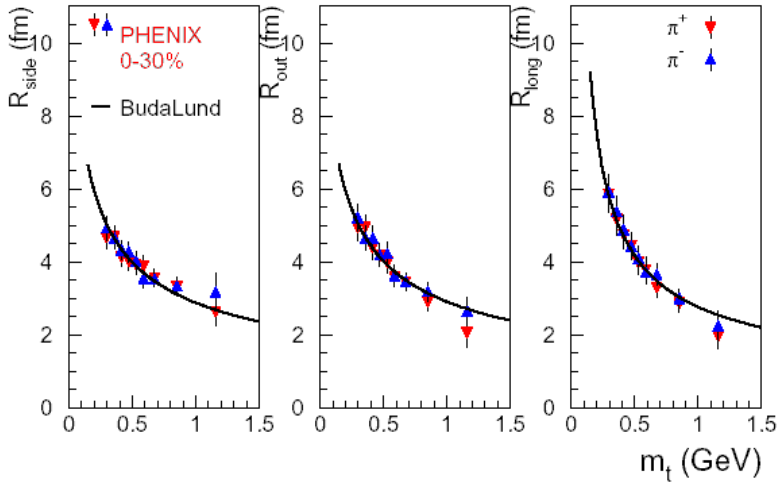
BudaLund v1.5 fits to 200 AGeV Au+Au



BudaLund v1.5 fits to 200 AGeV Au+Au







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