

# NNLO $V + \text{jet}$

Alexander Huss

in collaboration with

A. Gehrmann–De Ridder, T. Gehrmann,  
E.W.N. Glover and T.A. Morgan

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## ZPW2017 – The second run of the LHC

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Zurich, January 10<sup>th</sup> 2017



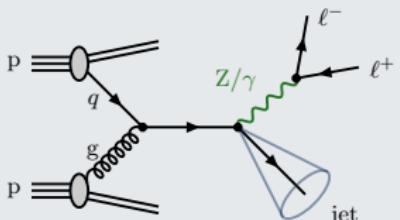
Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich



MC@NNLO

based on Phys. Rev. Lett. **117** (2016) 022001, JHEP **1607** (2016) 133, JHEP **1611** (2016) 094

# Motivation – Why Z + jet production?

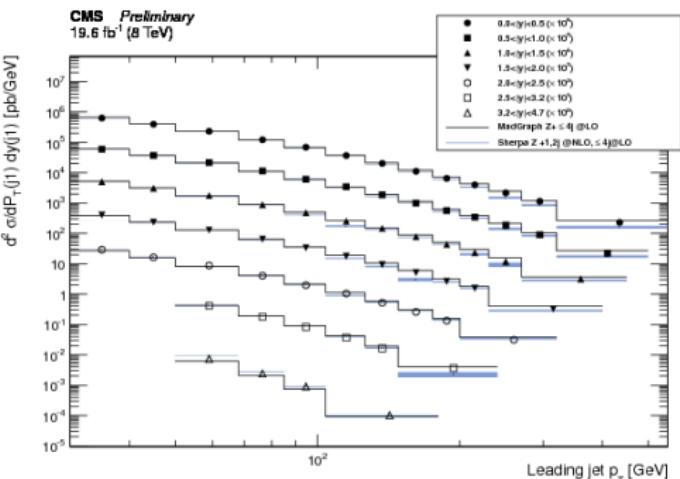


$$p\ p \rightarrow Z/\gamma^* + \text{jet} \rightarrow \ell^-\ell^+ + \text{jet} + X$$

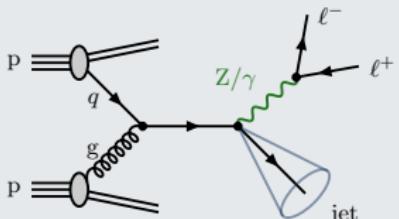
- ▶ large cross section
- ▶ clean leptonic signature
- +jet  $\leadsto$  sensitivity to  $\alpha_s$ , gluon PDF

- ▶ precision measurements
  - ↪ test pQCD
  - ↪ constrain PDFs (gluon)
- ▶ detector calibration
  - ↪ jet energy scale
- ▶ searches for BSM physics

high-precision predictions  
mandatory!



# Motivation – Why Z + jet production?



$$p\ p \rightarrow Z/\gamma^* + \text{jet} \rightarrow \ell^- \ell^+ + \text{jet} + X$$

- ▶ large cross section
- ▶ clean leptonic signature
- +jet  $\rightsquigarrow$  sensitivity to  $\alpha_s$ , gluon PDF

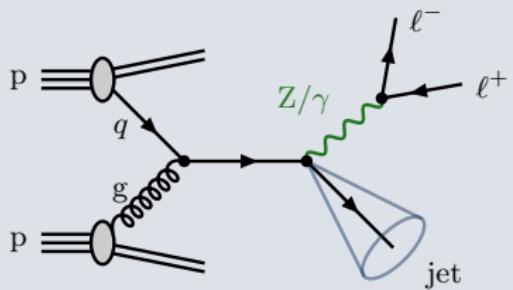
## Where do we stand?

- ▶ **NLO QCD** ..... [Ellis, Kunszt, Soper '92] [Giele, Glover, Kosower '93]
  - ▶ **NLO EW** ..... [Kühn, Kulesza, Pozzorini, Schulze '05] [Denner, Dittmaier, Kasprzik, Mück '11]
  - ▶ **NLO QCD+EW** (+multijet merging) ..... [Kallweit, Lindert, Maierhofer, Pozzorini, Schönherr '15]
  - ▶ **NNLO QCD** .....
    - ↪ Antenna subtraction (**this talk**) [Gehrmann-De Ridder, Gehrmann, Glover, AH, Morgan '15, '16]
    - ↪  $N$ -jettiness ..... [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello '15]  
[Boughezal, Liu, Petriello '16]
- $\rightsquigarrow$  validation (✓) & opportunity for benchmarks

# This talk

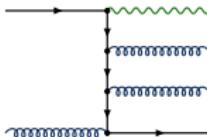
- ➊ Z + jet production @ NNLO
- ➋ Inclusive Z-boson production for  $p_T \neq 0$
- ➌ The  $\phi_\eta^*$  observable

# Z + jet production @ NNLO



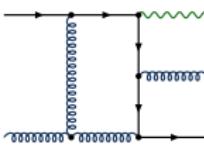
# Anatomy of an NNLO calculation

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} d\sigma_{\text{NNLO}}^{\text{RR}}$$



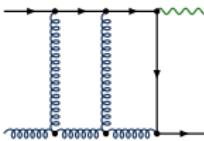
[Hagiwara, Zeppenfeld '89]  
[Berends, Giele, Kuijf '89]  
[Falck, Graudenz, Kramer '89]

$$+ \int_{\Phi_{Z+2}} d\sigma_{\text{NNLO}}^{\text{RV}}$$



[Glover, Miller '97]  
[Bern, Dixon, Kosower, Weinzierl '97]  
[Campbell, Glover, Miller '97]  
[Bern, Dixon, Kosower '98]

$$+ \int_{\Phi_{Z+1}} d\sigma_{\text{NNLO}}^{\text{VV}}$$

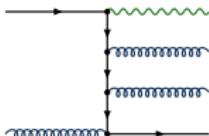


[Moch, Uwer, Weinzierl '02]  
[Garlandet al. '02]  
[Gehrmann, Tancredi '12]

Individual building blocks known for a while

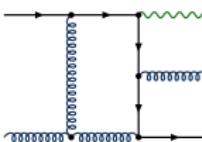
# Anatomy of an NNLO calculation

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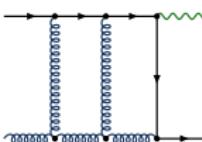
- ▶ single-unresolved
- ▶ double-unresolved

$$+ \int_{\Phi_{Z+2}} d\sigma_{\text{NNLO}}^{\text{RV}}$$



- ▶ single-unresolved
- ▶  $1/\epsilon^2, 1/\epsilon$

$$+ \int_{\Phi_{Z+1}} d\sigma_{\text{NNLO}}^{\text{VV}}$$



- ▶  $1/\epsilon^4, 1/\epsilon^3, 1/\epsilon^2, 1/\epsilon$

---

 $\sum$ 

**finite** (Kinoshita–Lee–Nauenberg & factorization)

Non-trivial cancellation of infrared singularities

# Anatomy of an NNLO calculation

$$\begin{aligned}\sigma_{\text{NNLO}} = & \int_{\Phi_{Z+3}} d\sigma_{\text{NNLO}}^{\text{RR}} \\ & + \int_{\Phi_{Z+2}} d\sigma_{\text{NNLO}}^{\text{RV}} \\ & + \int_{\Phi_{Z+1}} d\sigma_{\text{NNLO}}^{\text{VV}}\end{aligned}$$

## Different methods:

- ▶ Antenna subtraction ..... [Gehrmann–De Ridder, Gehrmann, Glover '05]
  - ▶ CoLoRful subtraction ..... [Del Duca, Somogyi, Trocsanyi '05]
  - ▶  $q_T$  subtraction ..... [Catani, Grazzini '07]
  - ▶ Sector-improved residue subtraction ..... [Czakon '10], [Boughezal, Melnikov, Petriello '11]
  - ▶  $N$ -jettiness subtraction ..... [Gaunt, Stahlhofen, Tackmann, Walsh '15]  
[Boughezal, Focke, Liu, Petriello '15]
- ...

---

 $\sum$  finite

Approaches: subtraction, slicing

# Z + jet @ NNLO using Antenna

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} \left( d\sigma_{\text{NNLO}}^{\text{RR}} - d\sigma_{\text{NNLO}}^{\text{S}} \right)$$

$$+ \int_{\Phi_{Z+2}} \left( d\sigma_{\text{NNLO}}^{\text{RV}} - d\sigma_{\text{NNLO}}^{\text{T}} \right)$$

$$+ \int_{\Phi_{Z+1}} \left( d\sigma_{\text{NNLO}}^{\text{VV}} - d\sigma_{\text{NNLO}}^{\text{U}} \right)$$

- $d\sigma_{\text{NNLO}}^{\text{S}}, d\sigma_{\text{NNLO}}^{\text{T}}$ :  
mimic  $d\sigma_{\text{NNLO}}^{\text{RR}}, d\sigma_{\text{NNLO}}^{\text{RV}}$   
in unresolved limits
- $d\sigma_{\text{NNLO}}^{\text{T}}, d\sigma_{\text{NNLO}}^{\text{U}}$ :  
analytic cancellation of  
poles in  $d\sigma_{\text{NNLO}}^{\text{RV}}, d\sigma_{\text{NNLO}}^{\text{VV}}$

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$\sum$       finite      -0

⇒ each line suitable for numerical evaluation in  $D = 4$

# Antenna subtraction – Checks of the calculation

## Analytic pole cancellation

- Poles  $(d\sigma^{RV} - d\sigma^T) = 0$
- Poles  $(d\sigma^{VV} - d\sigma^U) = 0$

DimReg:  $D = 4 - 2\epsilon$

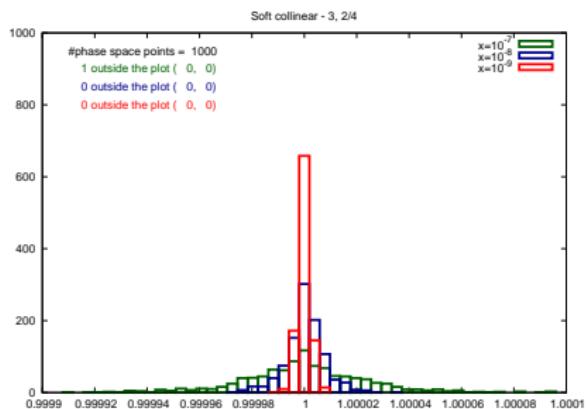
## Unresolved limits

- $d\sigma^S \rightarrow d\sigma^{RR}$  (single- & double-unresolved)
- $d\sigma^T \rightarrow d\sigma^{RV}$  (single-unresolved)

bin the ratio:  $d\sigma^S/d\sigma^{RR} \xrightarrow{\text{unresolved}} 1$

$q \bar{q} \rightarrow Z + g_3 \ g_4 \ g_5$  ( $g_3$  soft &  $g_4 \parallel \bar{q}$ )

```
09:26:35 ➤ ...maple/process/Z
$ form autoqgB1g2ZgtoqU.frm
FORM 4.1 (Mar 13 2014) 64-bits
#-
poles = 0;
6.58 sec out of 6.64 sec
```



(approach singular limit:  $x_i = 10^{-7}, 10^{-8}, 10^{-9}$ )



X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann,  
E.W.N. Glover, AH, T. Morgan, J. Niehues, J. Pires, D. Walker

## Common framework for NNLO corrections using Antenna Subtraction

### Processes:

- ▶ parton-level event generator
- ▶ based on antenna subtraction
- ▶ test & validation framework
- ▶ APPLfast-NNLO interface  
(Work in progress)
- ▶ ...
- ▶  $pp \rightarrow V \rightarrow \bar{\ell}\ell + 0, 1$  jets
- ▶  $pp \rightarrow H + 0, 1, 2$  jets
- ▶  $pp \rightarrow \text{dijets}$
- ▶  $ep \rightarrow 1, 2$  jets
- ▶  $e^+ e^- \rightarrow 3$  jets
- ▶ ...

# Z + jet: Fiducial cross section

## Calculational setup

[Gehrman-De Ridder, Gehrman, Glover, AH, Morgan '15]

- ▶ LHC @ 8 TeV [NNPDF2.3]:  $\alpha_s(M_Z) = 0.118$
- ▶ select resonant Z bosons:  $80 \text{ GeV} < m_{\ell\ell} < 100 \text{ GeV}$ ,  $|y^\ell| < 5$
- ▶ jets [anti- $k_T$  ( $R = 0.5$ )]:  $p_T^{\text{jet}} > 30 \text{ GeV}$ ,  $|y^{\text{jet}}| < 3$
- ▶ scale choice:  $\mu_F = \mu_R = M_Z \times [\frac{1}{2}, 1, 2]$

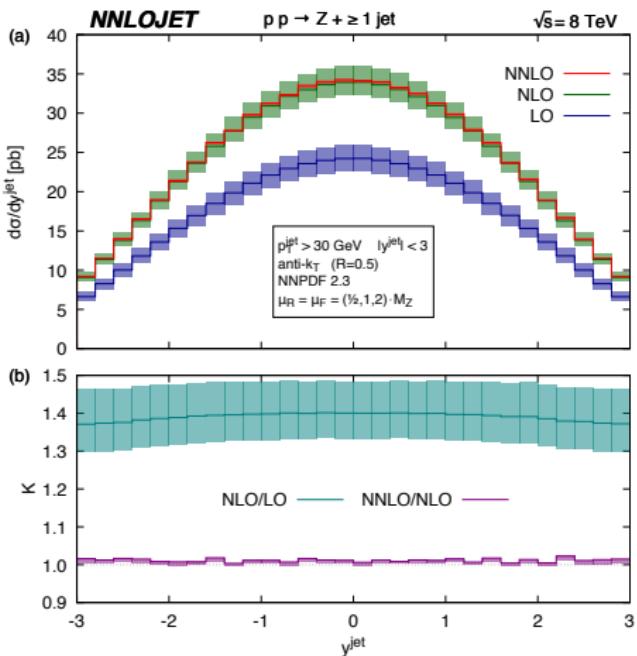
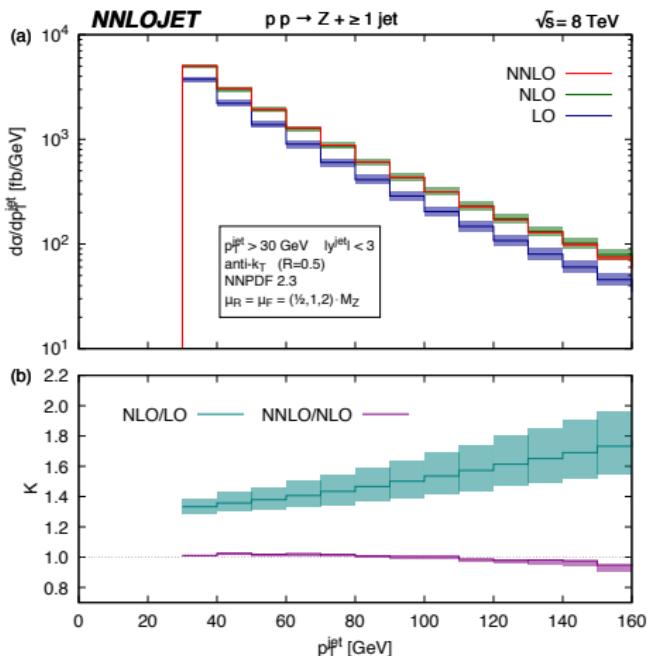
## Fiducial cross section

$$\begin{aligned}\sigma_{\text{LO}} &= 103.6^{+7.7}_{-7.5} \text{ pb} \\ \sigma_{\text{NLO}} &= 144.4^{+9.0}_{-7.2} \text{ pb} \\ \sigma_{\text{NNLO}} &= 145.8^{+0.0}_{-1.2} \text{ pb}\end{aligned}$$

The diagram shows three horizontal curly braces on the right side of the cross-section values. The top brace is labeled '+40%', the middle brace is labeled '+1%', and the bottom brace is labeled '+0.0%'.

starting from NLO, all partonic channels open up:  $qg$ ,  $q\bar{q}$ ,  $gg$ ,  $qq'$ , ...

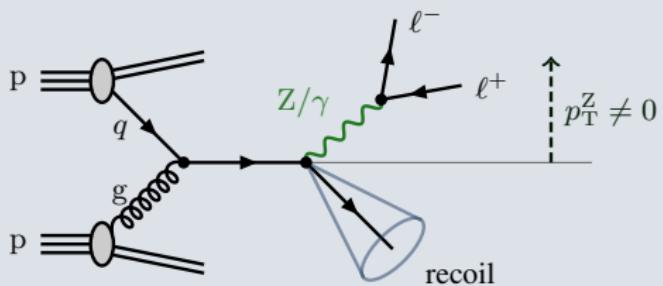
# Distributions: Leading jet



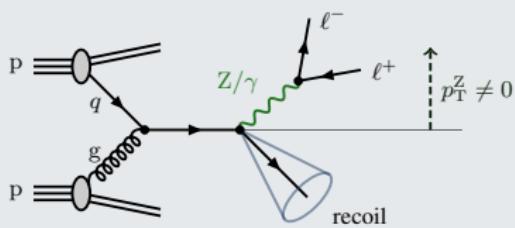
- **NLO corrections**  $\gtrsim 40\%$     ( $\delta_{\text{scale}}^{\text{NLO}} \sim \delta_{\text{scale}}^{\text{LO}}$ )
- **NNLO corrections**  $\lesssim 5\%$     (quite flat)
- significant reduction of scale uncertainty

$$K = \frac{d\sigma^{(N)\text{NLO}}(\mu)}{d\sigma^{(N)\text{LO}}(\mu = M_Z)}$$

# Inclusive Z-boson production for $p_T \neq 0$



# Motivation — Towards precision phenomenology

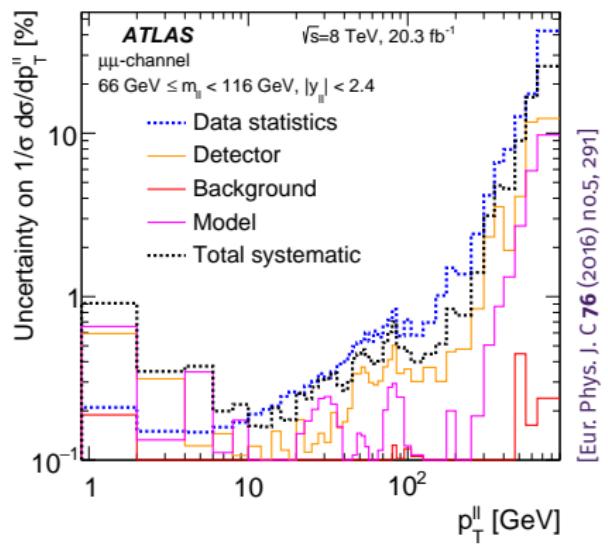


$$p\ p \rightarrow Z/\gamma^* + X \rightarrow \ell^-\ell^+ + X$$

► large cross section

► clean leptonic signature

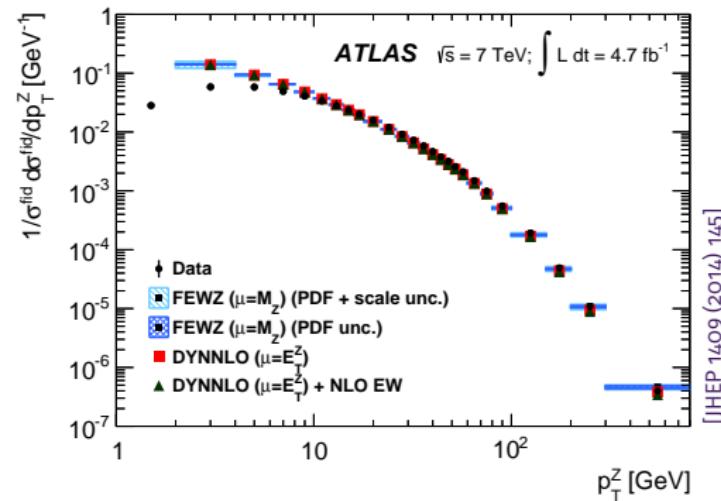
recoil  $\rightsquigarrow$  sensitivity to  $\alpha_s$ , gluon PDF



- fully inclusive w.r.t. QCD radiation
- only reconstruct  $\ell^+, \ell^-$
- $\rightsquigarrow$  sub-% accuracy!
- potential to constrain gluon PDFs
- $\rightsquigarrow$  NNLO needed!

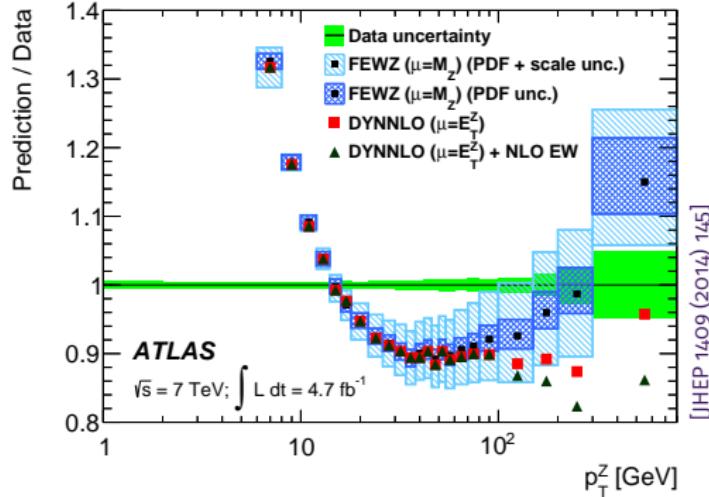
[Malik, Watt '14]

# Inclusive $p_T^Z$ at fixed order



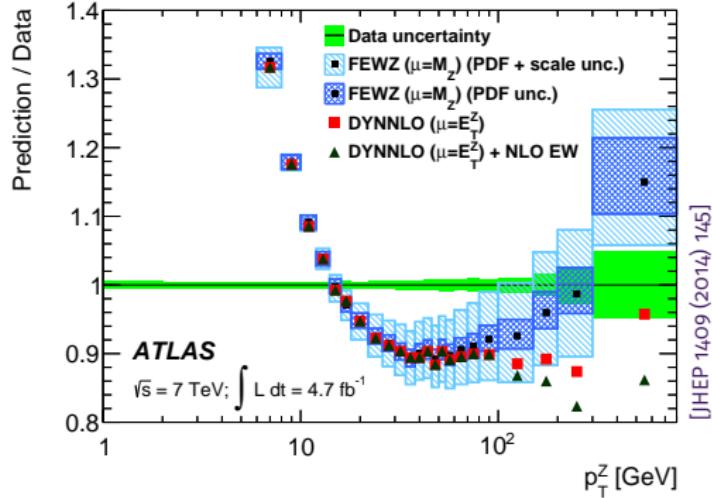
- ▶ low  $p_T^Z \lesssim 10 \text{ GeV}$ : resummation required
- ▶  $p_T^Z \gtrsim 20 \text{ GeV}$ : fixed-order prediction  $\sim 10\%$  below data!
- ▶ high  $p_T^Z \gtrsim 500 \text{ GeV}$ : EW corrections  $\sim -5 - 10\%$

# Inclusive $p_T^Z$ at fixed order

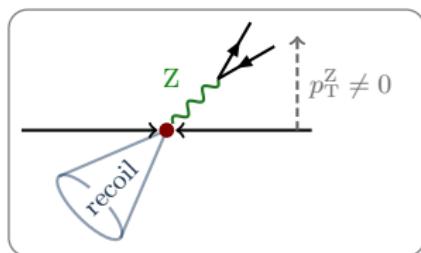


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# Inclusive $p_T^Z$ at fixed order

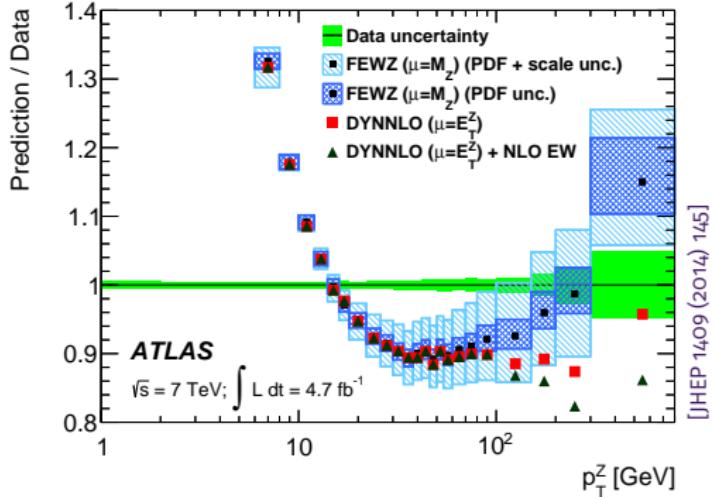


FEWZ  
DYNNOLO }  $Z + 0 \text{ jet} @ \text{NNLO}$

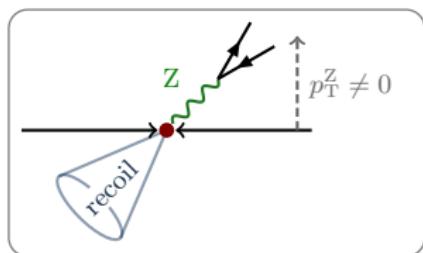


↪ Only NLO accurate  
in this distribution!

# Inclusive $p_T^Z$ at fixed order



FEWZ  
DYNNOLO }  $Z + 0 \text{ jet} @ \text{NNLO}$



Only NLO accurate  
in this distribution!

NNLO

$p_T^Z > p_{T,\text{cut}}^Z = 20 \text{ GeV}$

► requires hadronic recoil  
~~~  $Z + \geq 1\text{jet} @ \text{NNLO}$

# Inclusive $p_T^Z$ spectrum: Setup

## Calculational setup

- ▶ LHC @ 8 TeV
- ▶ PDF: NNPDF3.0     $\alpha_s(M_Z) = 0.118$
- ▶ jet cuts     $\longleftrightarrow$     fully inclusive w.r.t. QCD radiation
- ▶  $p_T^Z > 20 \text{ GeV}$
- ▶ scale choice (dynamical)

$$\mu_F = \mu_R = \sqrt{m_{\ell\ell}^2 + p_{T,Z}^2} \times [\tfrac{1}{2}, 1, 2]$$

### ATLAS setup

[arXiv:1512.02192]

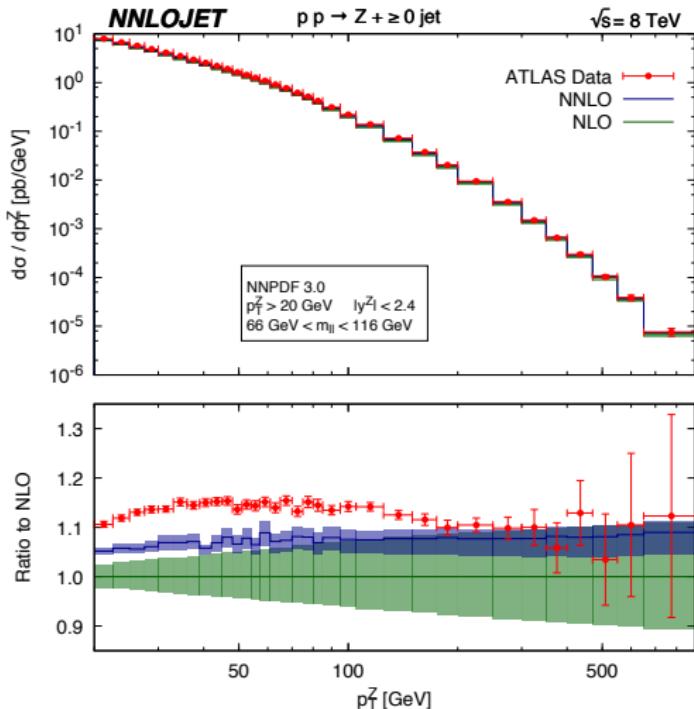
- ▶  $p_T^{\ell^\pm} > 20 \text{ GeV}, |y^{\ell^\pm}| < 2.4$
- ▶  $66 \text{ GeV} < m_{\ell\ell} < 116 \text{ GeV}$   
+ binning in  $y^Z$
- ▶  $|y^Z| < 2.4$  + binning in  $m_{\ell\ell}$

### CMS setup

[arXiv:1504.03511]

- ▶  $p_T^{\ell_1} > 25 \text{ GeV}, |y^{\ell_1}| < 2.1$
- ▶  $p_T^{\ell_2} > 10 \text{ GeV}, |y^{\ell_2}| < 2.4$
- ▶  $81 \text{ GeV} < m_{\ell\ell} < 101 \text{ GeV}$   
+ binning in  $y^Z$

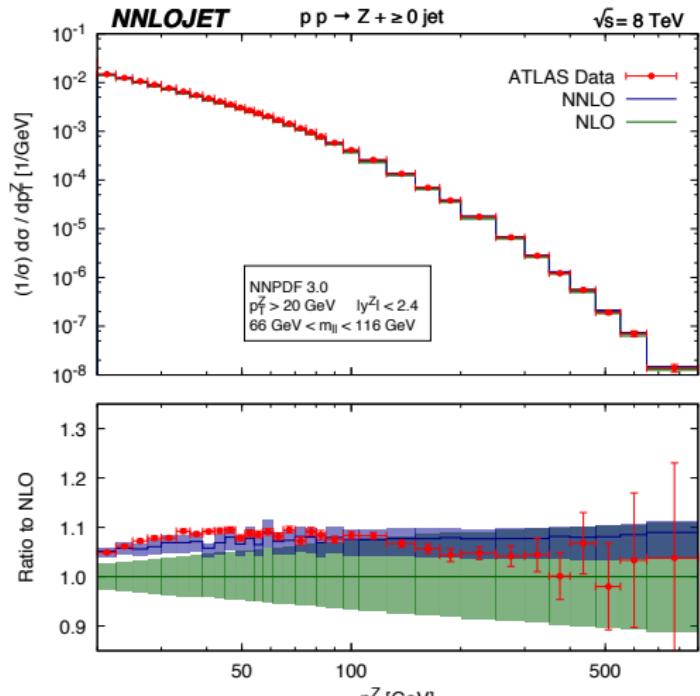
# Inclusive $p_T$ spectrum of $Z/\gamma^*$



- ▶ NNLO corrections
  - ↪ goes into right direction
- ▶ still undershoots data by  $\sim 5\%$
- ▶ Note: data does not include the  $\pm 2.8\%$  luminosity error!
  - ↪ use normalised distributions

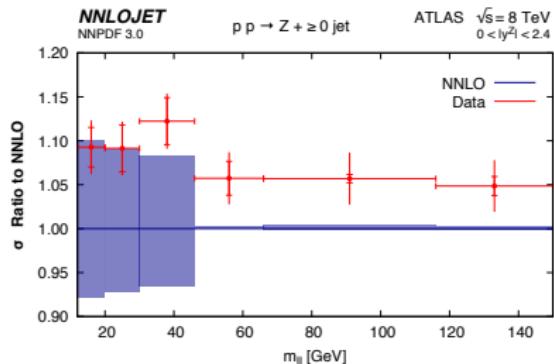
$$\frac{1}{\sigma} \cdot \frac{d\sigma}{dp_T^Z}$$

# Inclusive $p_T$ spectrum of $Z/\gamma^*$

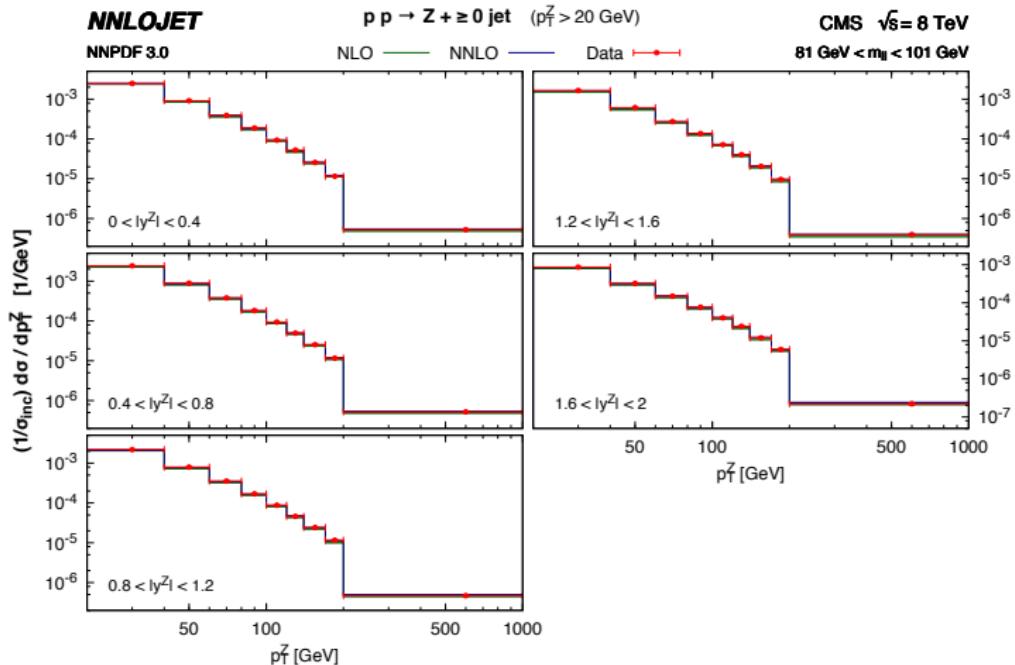


$$\frac{1}{\sigma} \cdot \frac{d\sigma}{dp_T^Z}$$

- uncorrelated scale variation  
↔ numerator & denominator
- significant improvement  
in **Data** vs. **Theory**

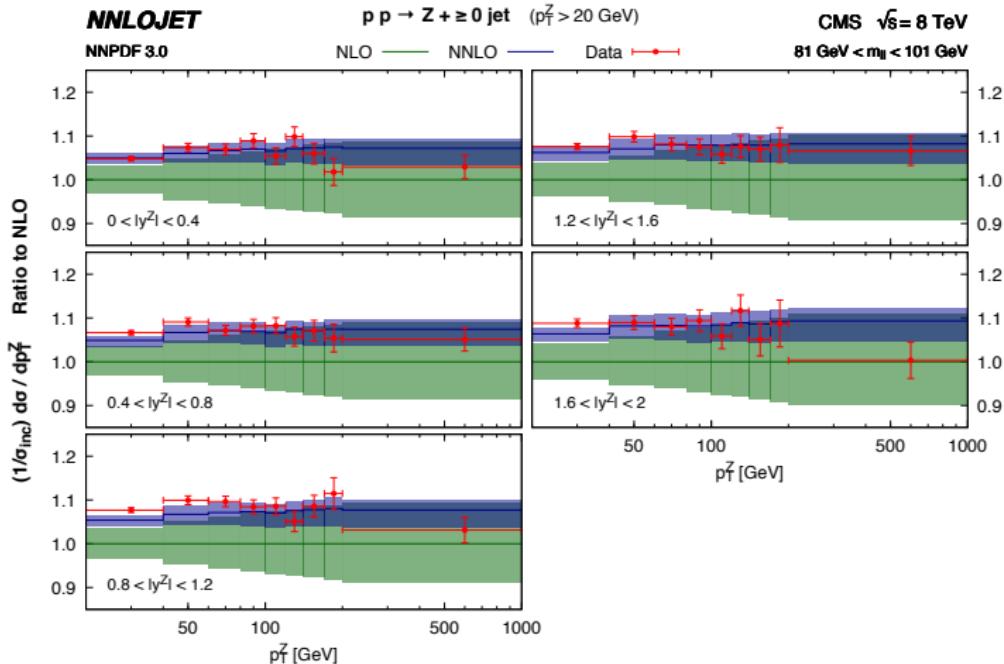


# Double-differential: $d\sigma/dp_T^Z$ binned in $y^Z$ — CMS



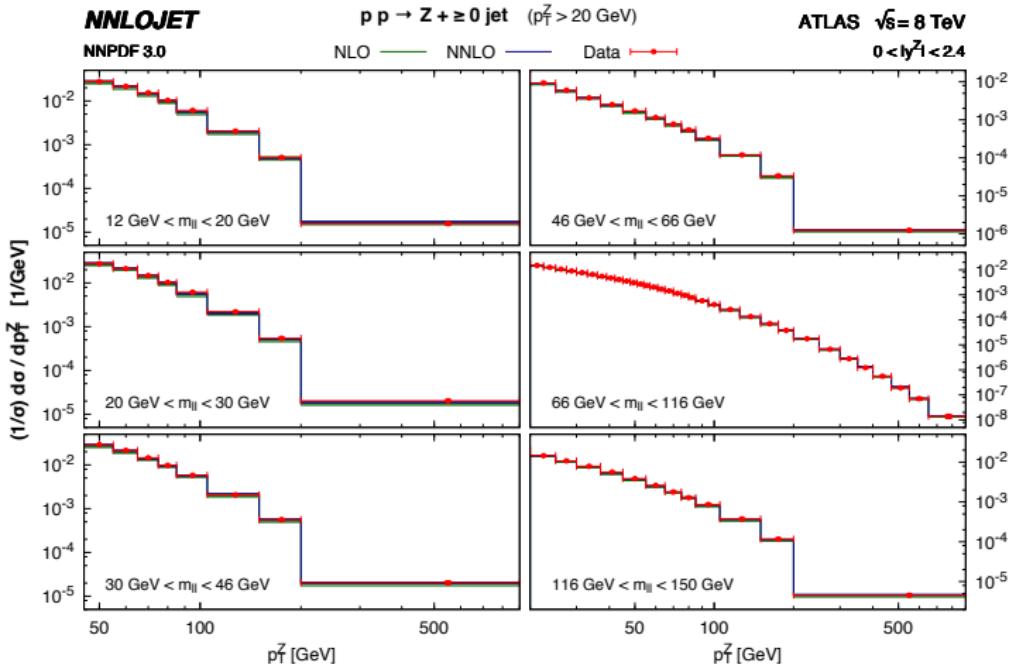
- $81 \text{ GeV} < m_{\ell\ell} < 101 \text{ GeV}$  (narrower mass window than ATLAS)
- 5 bins in  $y^Z$ :  $[0, 0.4]$   $[0.4, 0.8]$   $[0.8, 1.2]$   $[1.2, 1.6]$   $[1.6, 2]$

# Double-differential: $d\sigma/dp_T^Z$ binned in $y^Z$ — CMS



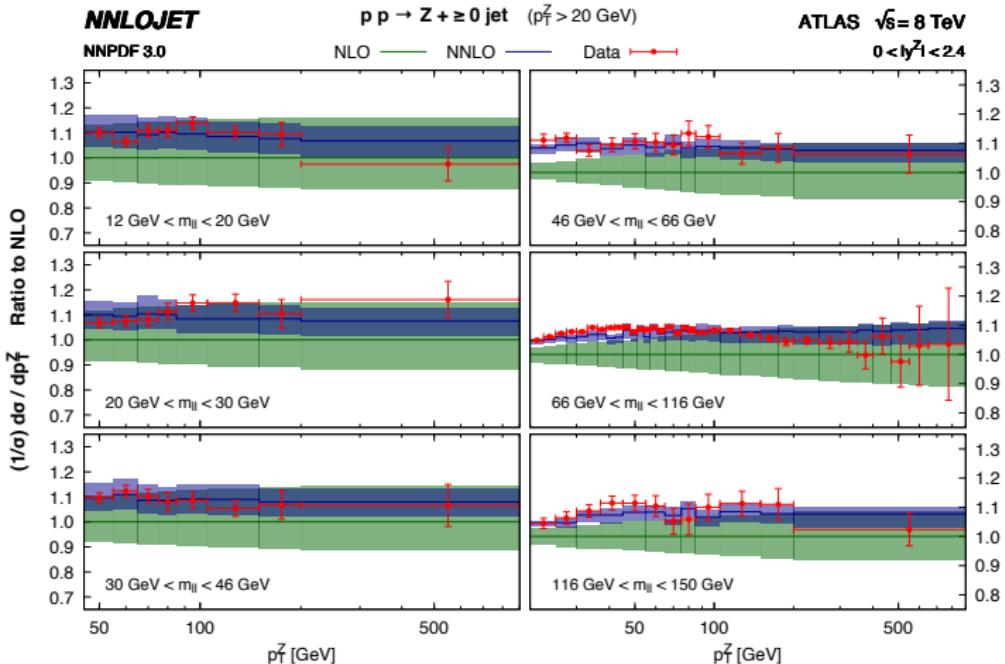
- ▶ NLO prediction systematically undershoots data, NNLO corrections  $\sim 5\text{--}10\%$
- ▶ improvement in theory vs. data comparison
- ▶ reduction of scale uncertainties

# Double-differential: $d\sigma/dp_T^Z$ binned in $m_{\ell\ell}$ — ATLAS



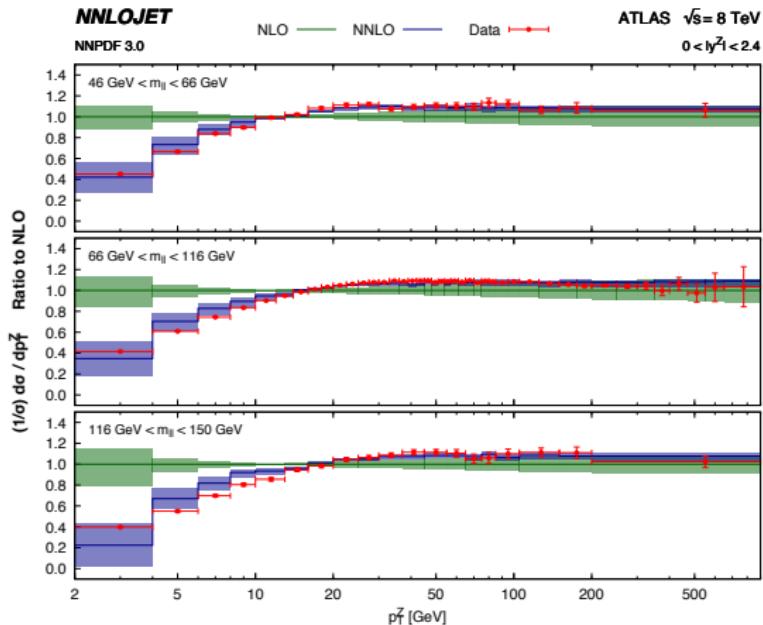
- $0 < |y^Z| < 2.4$
- 6 bins in  $m_{\ell\ell}$  [GeV]: [12, 20] [20, 30] [30, 46] [46, 66] [66, 116] [116, 150]

# Double-differential: $d\sigma/dp_T^Z$ binned in $m_{\ell\ell}$ — ATLAS



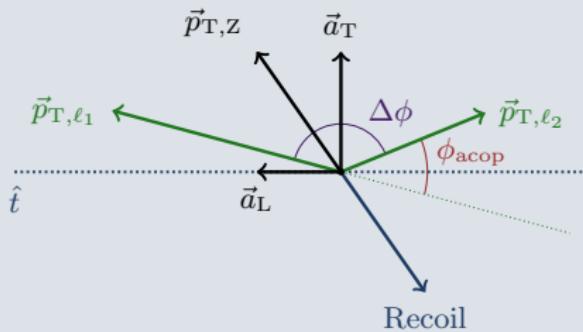
- significant tension between **NLO** prediction and data ( $m_{\ell\ell} \geq 46$  GeV)
- improvement in **theory** vs. **data** comparison
- reduction of scale uncertainties

# low- $p_T^Z$ region: $d\sigma/dp_T^Z$ binned in $m_{\ell\ell}$ — ATLAS

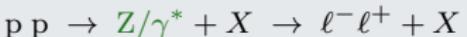
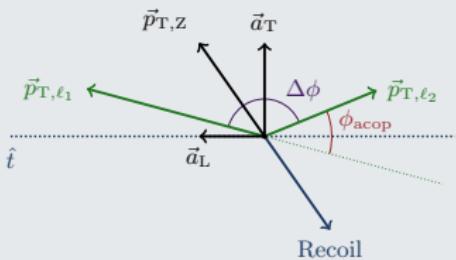


- ▶  $p_T^Z > 2 \text{ GeV}$  (first bin  $\in [0, 2] \text{ GeV}$ )
- ▶ **NLO:** bad description below  $p_T^Z \lesssim 40 \text{ GeV}$
- ▶ **NNLO:** shape well reproduced down to  $p_T^Z \sim 5 \text{ GeV}$
- ▶ deviation @ lower values: onset of large logarithmic corrections

# The $\phi_\eta^*$ observable



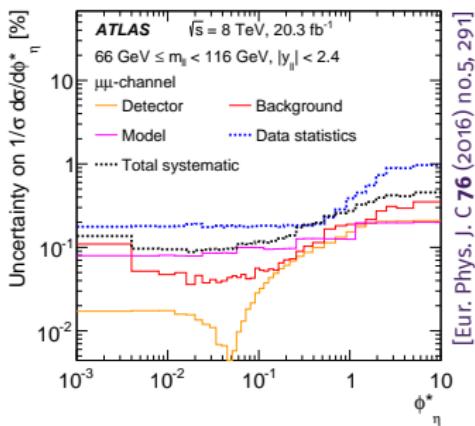
# Motivation — A precision probe in the low- $p_T^Z$ regime



- ▶ large cross section

- ▶ clean leptonic signature

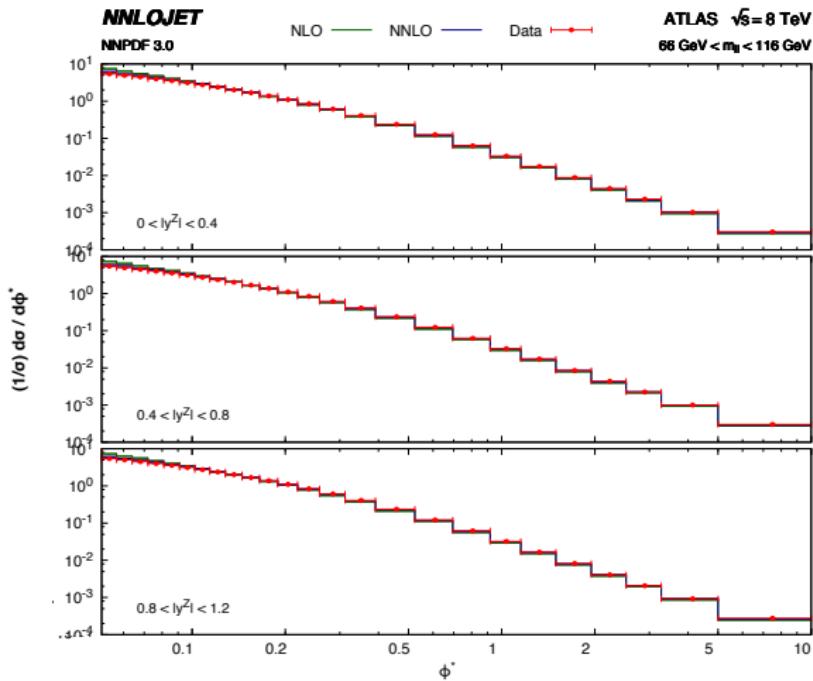
**recoil**  $\rightsquigarrow$  sensitivity to  $\alpha_s$ , gluon PDF



$$\phi_\eta^* = \tan\left(\frac{\phi_{\text{acop}}}{2}\right) \cdot \sin(\theta_\eta^*)$$

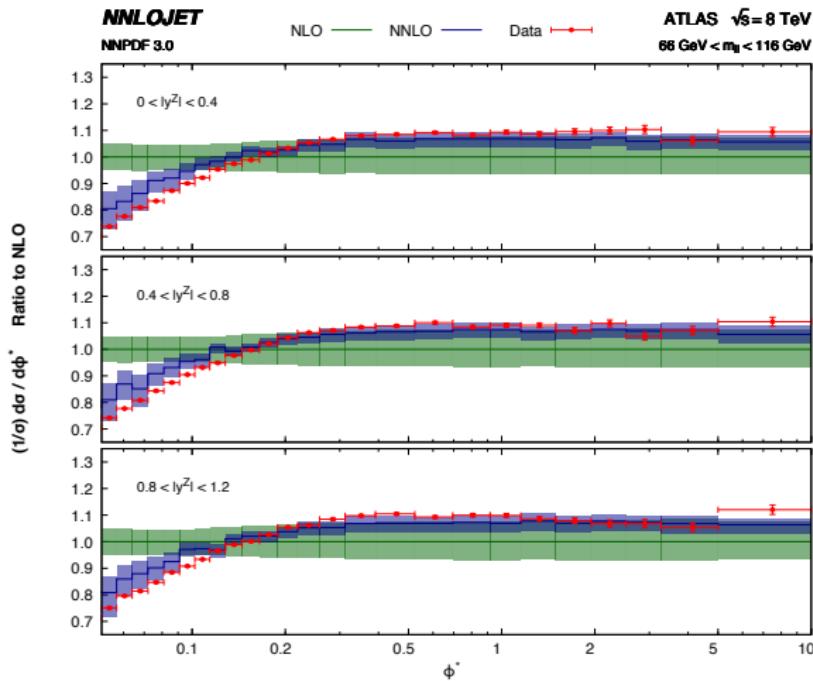
- ▶  $\phi_{\text{acop}} = \pi - \Delta\phi$ ,  $\cos(\theta_\eta^*) = \tanh[(\eta^\ell - \eta^{\ell^+})/2]$
- ▶ only depends on  $\ell^\pm$  directions (not energies)  
 $\rightsquigarrow$  better experimental resolution
- ▶ similar to  $p_T^Z$ :  $\phi_\eta^* > 0 \Rightarrow Z + \text{jets}$

$d\sigma/d\phi_\eta^*$  binned in  $y^Z$  — high  $\phi_\eta^*$  ( $> 0.051$ )



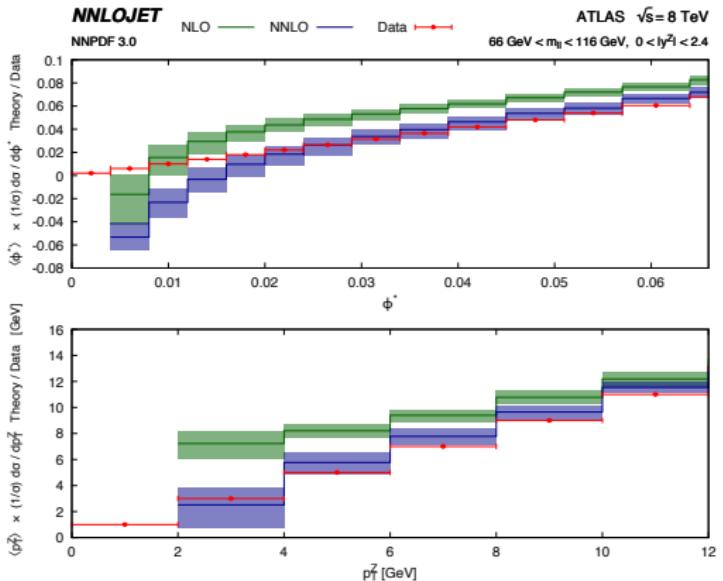
- $66 \text{ GeV} < m_{\ell\ell} < 116 \text{ GeV}$
- 6 bins in  $y^Z$ : [0, 0.4] [0.4, 0.8] [0.8, 1.2] [1.2, 1.6] [1.6, 2] [2, 2.4]

# $d\sigma/d\phi_\eta^*$ binned in $y^Z$ – high $\phi_\eta^*$ ( $> 0.051$ )



- ▶ high  $\phi_\eta^*$ : NNLO corrections  $\sim 10\%$ , low  $\phi_\eta^*$ : significant shape distortion
- ▶ improvement in **theory** vs. **data** comparison & reduction of scale uncertainties
- ▶ extends validity of fixed-order predictions before resummation takes over

# $d\sigma/d\phi_\eta^*$ vs. $d\sigma/dp_T^Z$ — small $\phi_\eta^*$



- relation between  $\phi_\eta^*$  &  $p_T^Z$ :  $\phi_\eta^* \approx \frac{p_T^Z}{2m_{\ell\ell}}$   $\rightsquigarrow$  align  $x$ -ranges
- **NLO**: does not describe data in plotting range
- **NNLO**: reliable down to  $\phi_\eta^* \sim 0.02 \leftrightarrow p_T^Z \sim 4 \text{ GeV}$

# Summary & Outlook

## Summary

- ▶ NNLO corrections to Z + jet production completed
- ▶ implementation in a flexible parton-level event generator **NNLOJET**
- ▶ related observables: inclusive  $p_T^Z$  spectrum,  $\phi_\eta^*$ 
  - ↪ improvement in the theory vs. data comparison
  - ↪ NNLO extends range of validity

## Outlook

- ▶ further studies:
  - ↪ angular coefficients, PDF fits, ...
- ▶ development on NNLOJET
  - ↪ interface to APPLfast-NNLO, more processes, performance, ...

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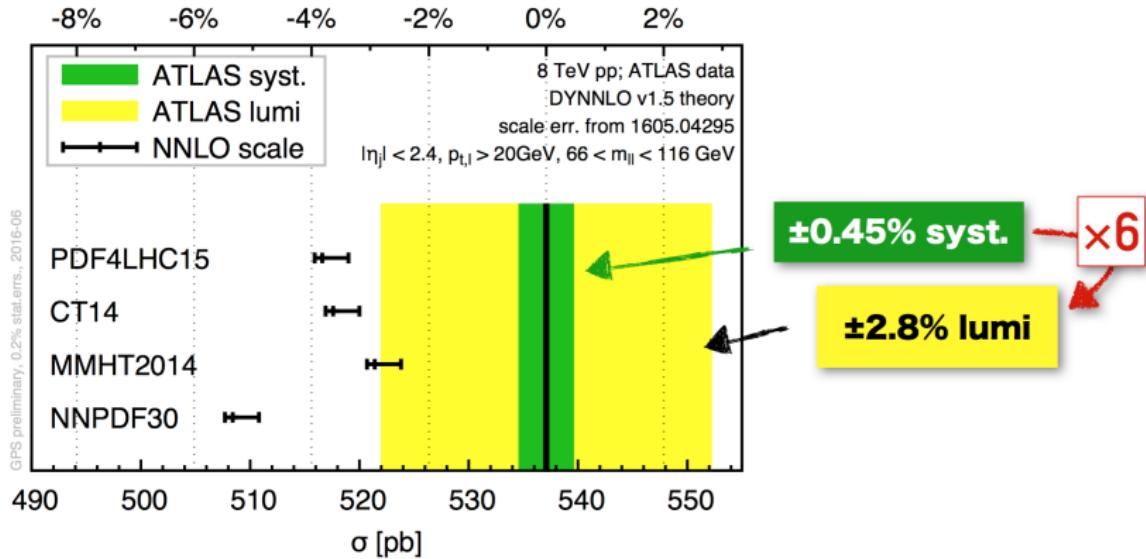
## Outlook

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Thank you

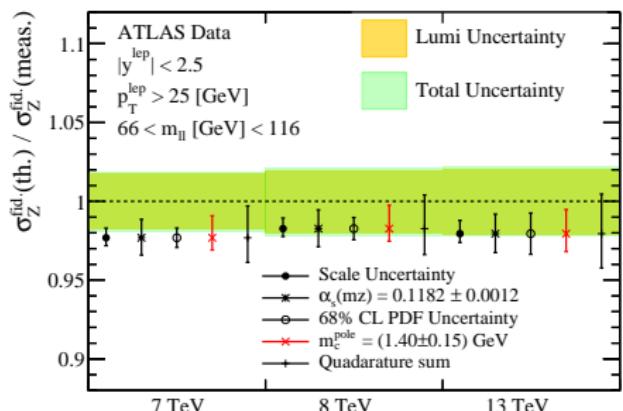
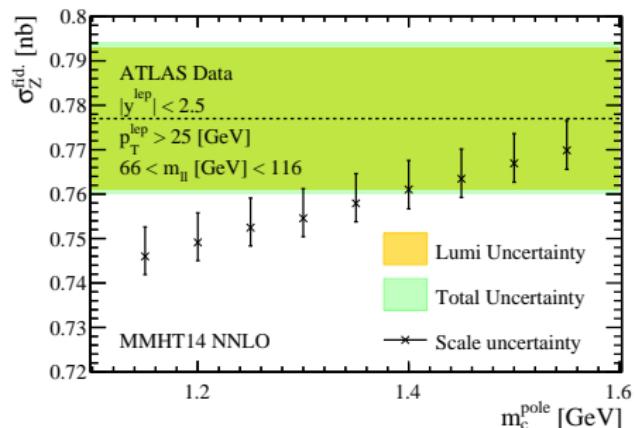
# Backup Slides

# The fiducial Drell-Yan cross section

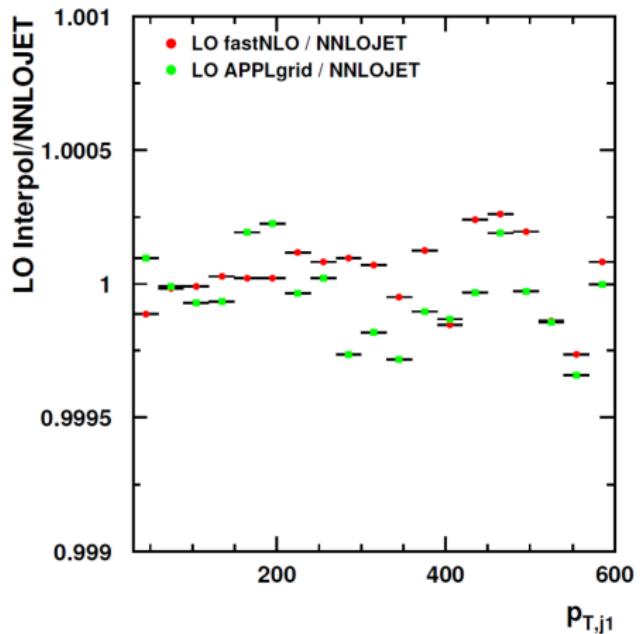


[talk by Gavin Salam, LHCP 2016]

# The fiducial Drell-Yan cross section — $m_c$ dep.



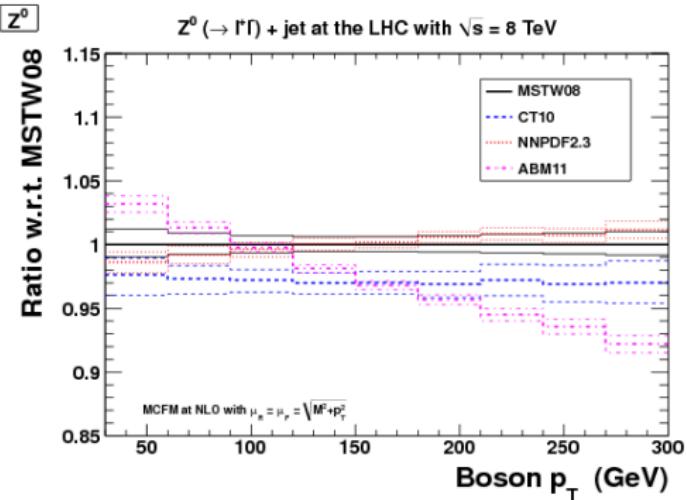
# APPLfast-NNLO



- ▶ single interface between NNLOJET
- + APPLgrid
- + fastNLO
- ▶ proof-of-concept implementation ✓
- ▶ closure test for all parts (VV, RV, RR) ✓
- ▶ todo: phase-space optimisation, merging of runs, performance, ...

[Britzger, Gwenlan, AH, Morgan, Rabbertz, Sutton]

# PDF constraints from $p_T^Z$

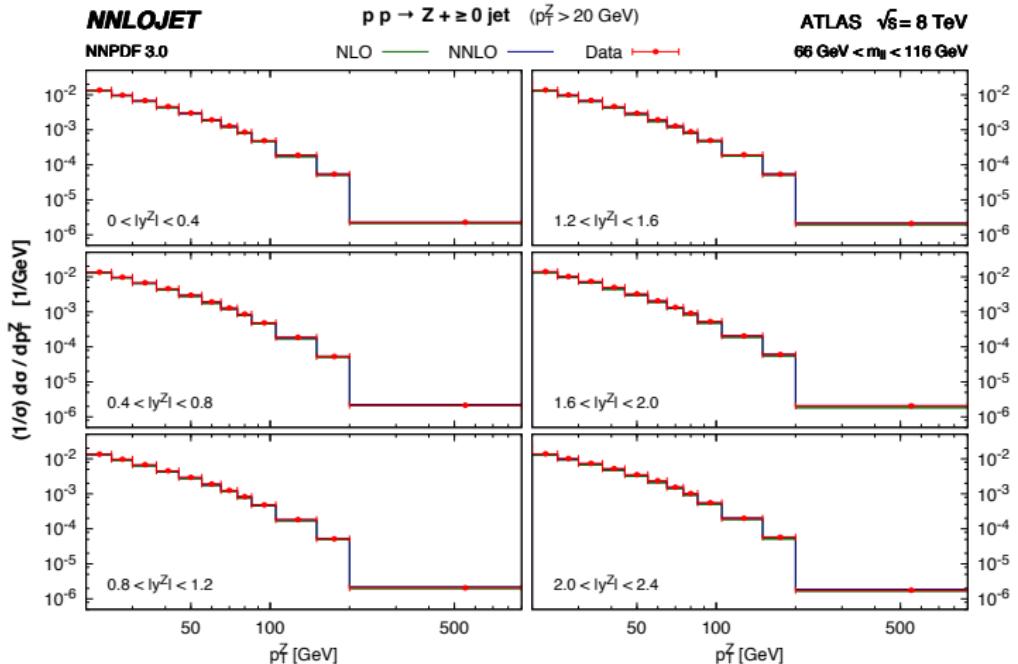


- ▶  $p_T^Z \gtrsim M_Z \rightsquigarrow$  fixed-order reliable
- ▶ left: only PDF uncertainties!  
(NLO scale uncertainty:  $\sim 10\%$ )
- ▶ potential to constrain gluon PDFs
- ⇒ NNLO calculation needed!

[Malik, Watt '14]

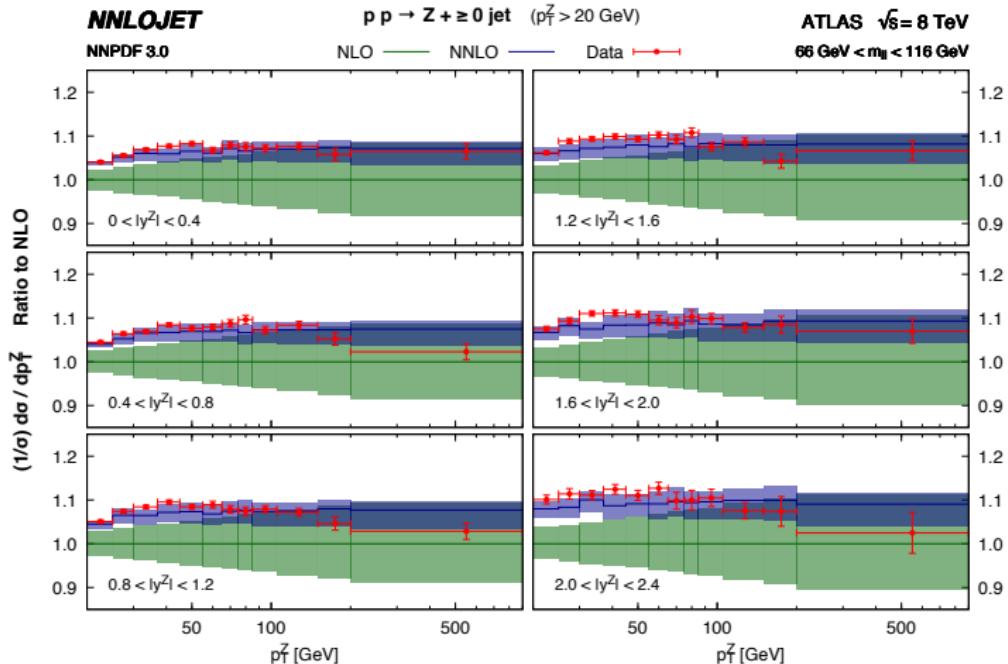
- ▶ repeat study at NNLO using newest generation of PDF sets
- ▶ work in progress: interface to APPLgrid, fastNLO
- ▶ tag flavour:  $Z + b(b) \leftrightarrow$  constrain b-quark PDFs

# Double-differential: $d\sigma/dp_T^Z$ binned in $y^Z$ — ATLAS



- ▶  $66 \text{ GeV} < m_{\ell\ell} < 116 \text{ GeV}$
- ▶ 6 bins in  $y^Z$ : [0, 0.4] [0.4, 0.8] [0.8, 1.2] [1.2, 1.6] [1.6, 2] [2, 2.4]

# Double-differential: $d\sigma/dp_T^Z$ binned in $y^Z$ — ATLAS



- improvement in **theory** vs. **data** comparison (+5–10% NNLO corrections)
- reduction of scale uncertainties

# The small $\phi_\eta^*$ region

- ▶ domain of large logarithms
  - ↪ breakdown of fixed-order predictions
- ▶ relation between  $p_T^Z$  &  $\phi_\eta^*$ 
  - ↪ compare leading-logarithmic behaviour

$$p_T^Z \left( \frac{1}{\sigma_0} \frac{d\sigma}{dp_T^Z} \right)_{LL} \sim -4C_F \frac{\alpha_s}{\pi} \ln \left( \frac{(p_T^Z)^2}{m_{\ell\ell}^2} \right) + 2C_F^2 \left( \frac{\alpha_s}{\pi} \right)^2 \ln^3 \left( \frac{(p_T^Z)^2}{m_{\ell\ell}^2} \right) + \dots$$

$$\phi_\eta^* \left( \frac{1}{\sigma_0} \frac{d\sigma}{d\phi_\eta^*} \right)_{LL} \sim -4C_F \frac{\alpha_s}{\pi} \ln (4(\phi_\eta^*)^2) + 2C_F^2 \left( \frac{\alpha_s}{\pi} \right)^2 \ln^3 (4(\phi_\eta^*)^2) + \dots$$

$$\Rightarrow \quad \phi_\eta^* \approx \frac{p_T^Z}{2m_{\ell\ell}}$$

# Z+jet: Channel breakdown

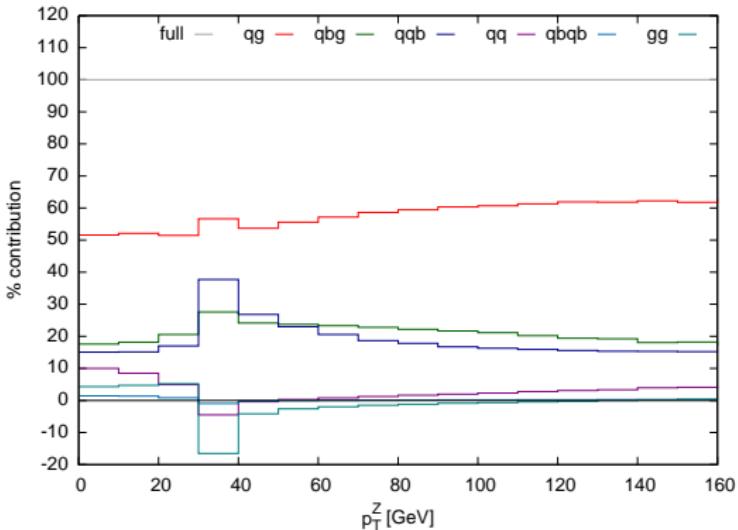
Variety of partonic channels contribute to Z+jet production

↪ what are the most important sub-processes?

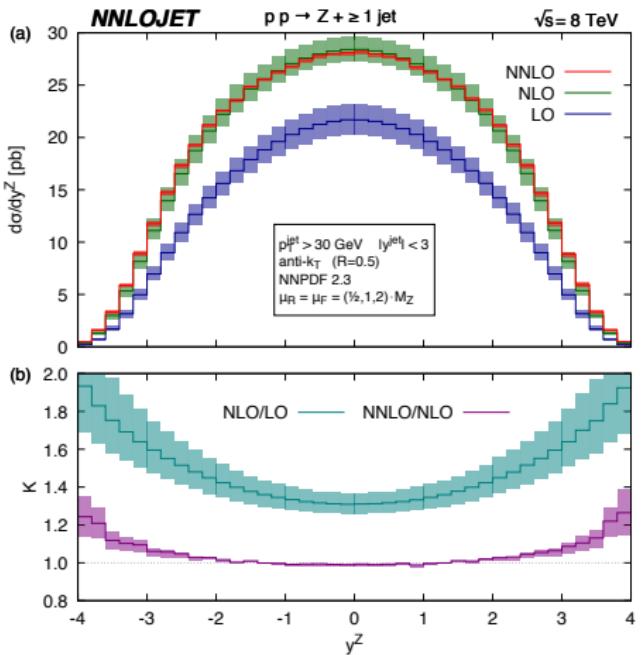
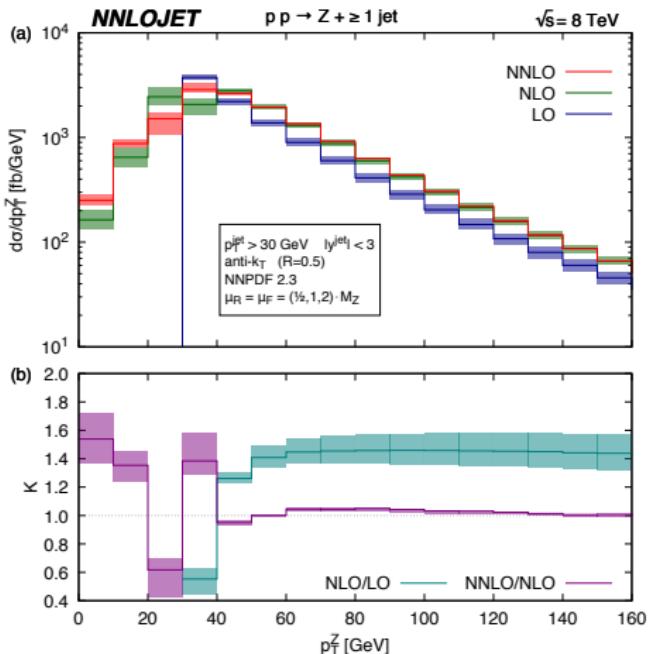
↪ channel breakdown @ NLO (default cuts & scale choice)

## Total cross section

| initial state    | contribution |
|------------------|--------------|
| $qg$             | 56%          |
| $q\bar{q}$       | 23%          |
| $\bar{q}g$       | 23%          |
| $gg$             | -3%          |
| $qq$             | 1%           |
| $\bar{q}\bar{q}$ | 0.1%         |



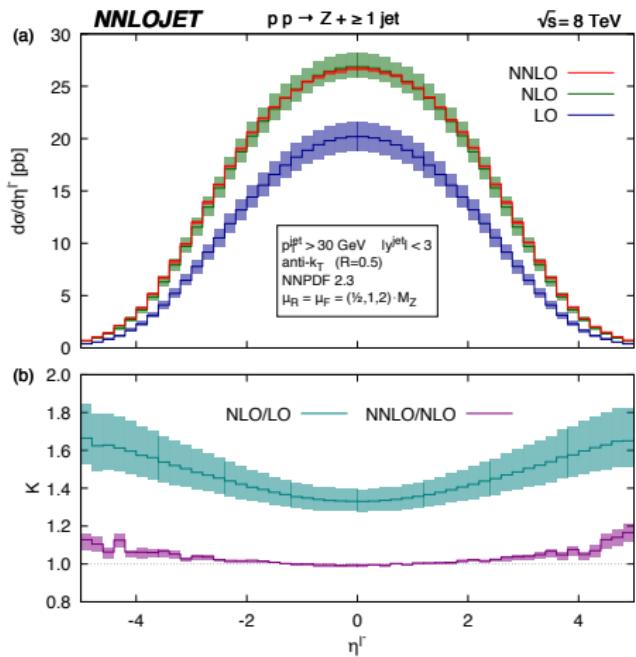
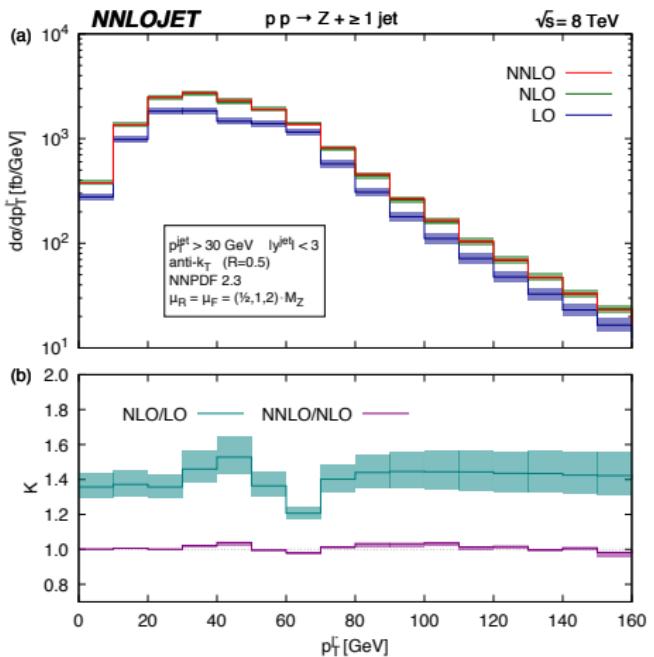
# Distributions: Z-boson



- ▶ “Sudakov shoulder” at  $p_T^Z \sim 30 \text{ GeV}$
- ▶ **NNLO** corrections not flat!
- ▶ significant reduction of scale uncertainty

$$K = \frac{d\sigma^{(N)\text{NLO}}(\mu)}{d\sigma^{(N)\text{LO}}(\mu = M_Z)}$$

# Distributions: lepton $\ell^-$



- significant reduction of scale uncertainty

$$K = \frac{d\sigma^{(N)\text{NLO}}(\mu)}{d\sigma^{(N)\text{LO}}(\mu = M_Z)}$$