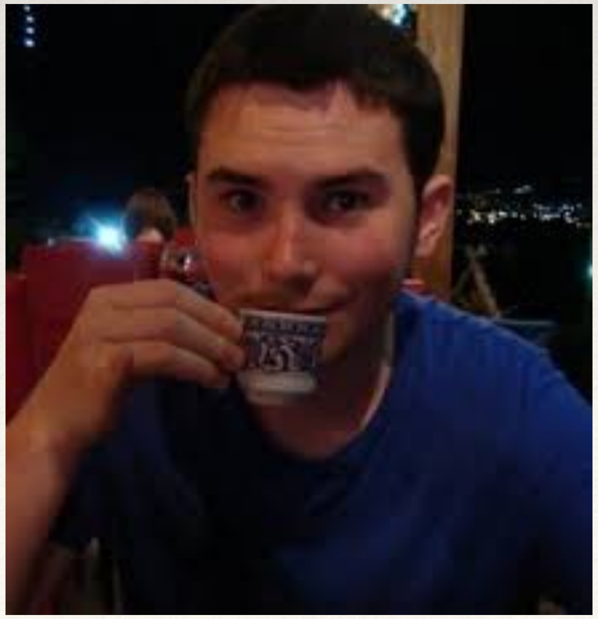
Zurich, *January 2017*

MCFM: approaching the third decade

Keith Ellis,
IPPP, Durham

- Color singlet production at NNLO in MCFM, Boughezal et al, 1605.08011
- Direct photon production at next-to-next-to-leading order, Campbell, RKE, Williams, 1612.04333
- Predictions for diphoton production at the LHC through NNLO in QCD, Campbell, RKE, Li, Williams, 1603.02663



MCFM

- MCFM is a parton level Monte Carlo program, containing many (~200) low multiplicity processes involving W,Z,H,t, and jets calculated at NLO.
- NLO is the first serious approximation in QCD.
- Perturbation theory is appropriate because is small (~10%) at high energy.
- Because of the nature of renormalization group improved perturbation theory, we do not achieve 1% accuracy at NLO, (even leaving aside PDF, EW, parameter uncertainties).
- Drive to extend to NNLO to move towards percent-level precision.

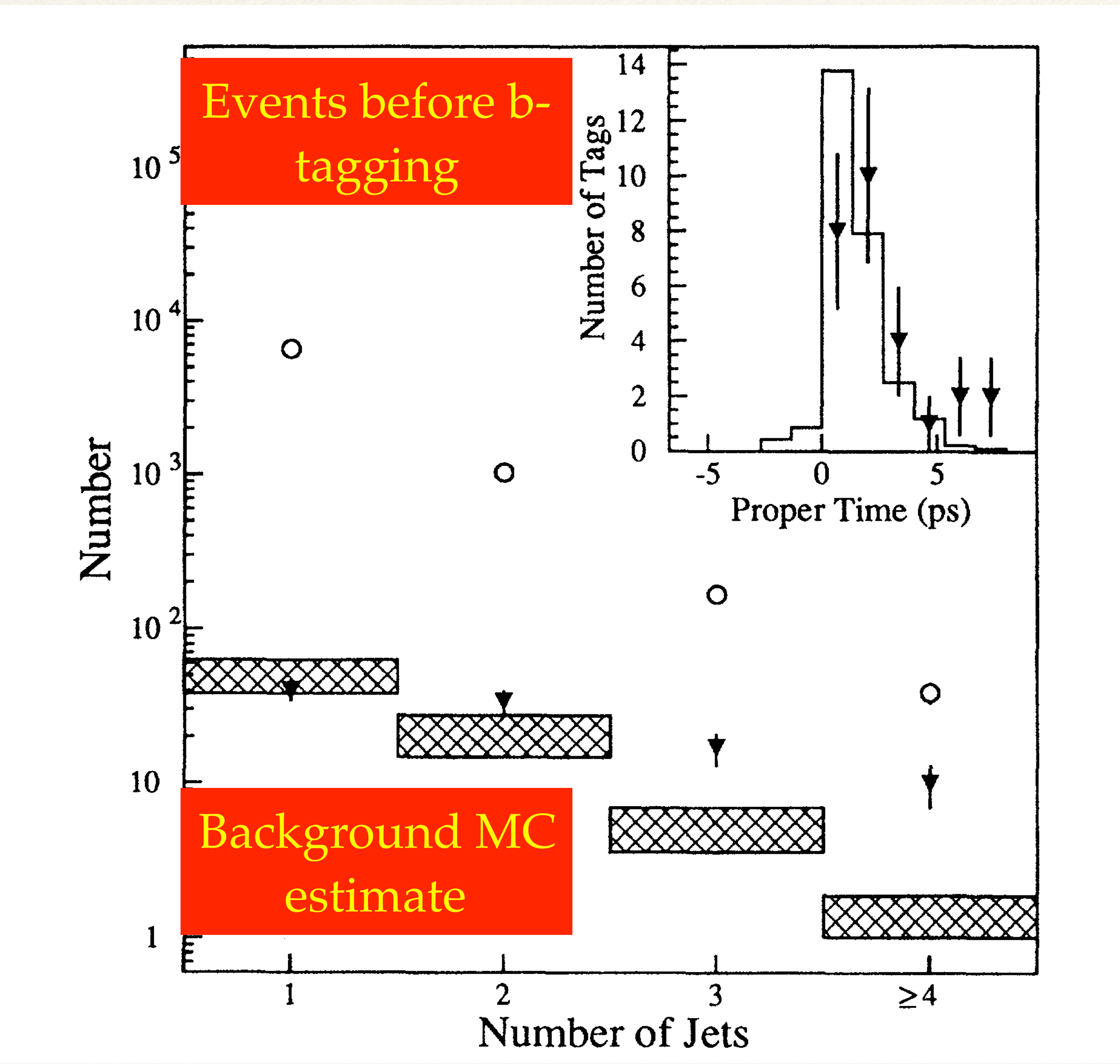
```
      function a6f(st,j1,j2,j3,j4,j5,j6,za,zb)
      implicit none
      include 'types.f'
      complex(dp):: a6f
************************************************************
*     Author: R.K. Ellis
*     July, 1998.
************************************************************

c---Atreepm is the amplitude for
c---q-(-p4)+Q-(-p2)+l-(-p5) ---> q+(p1)+Q+(p3)+l+(p6)
c---All outgoing particles are right-handed
      include 'constants.f'
      include 'nf.f'
      include 'mxpart.f'
      include 'cplx.h'
      include 'zprods_decl.f'
      include 'sprods_com.f'
      include 'epinv.f'
      include 'scale.f'
      integer:: j1,j2,j3,j4,j5,j6
      complex(dp):: atree,virt,Lnrat
      character*2 st
      virt=epinv+Lnrat(musq,-s(j2,j3))+2._dp
      a6f=atree(st,j1,j2,j3,j4,j5,j6,za,zb)*virt
      return
      end
```

The role of precision in discovery

Top quark at the Tevatron

CDF, PRL74, 1995



Berends et al , PLB224, 1989

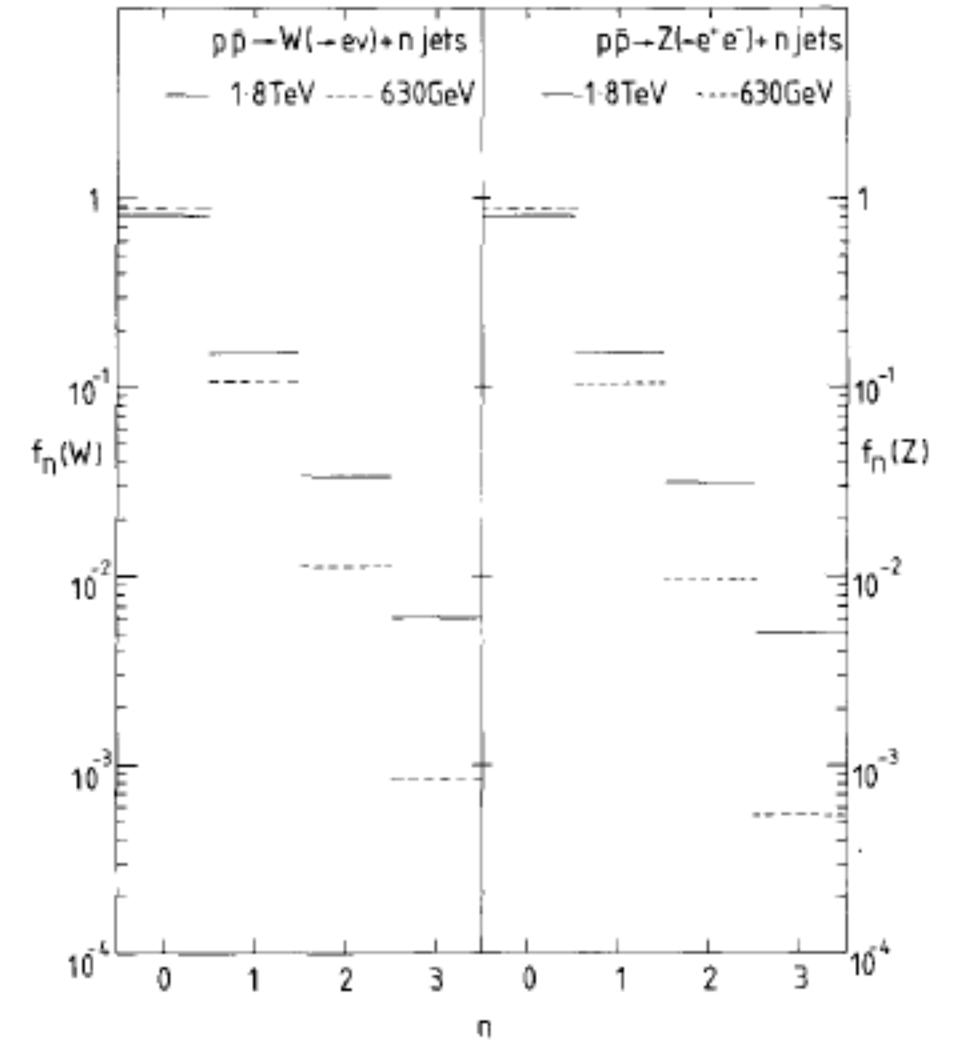
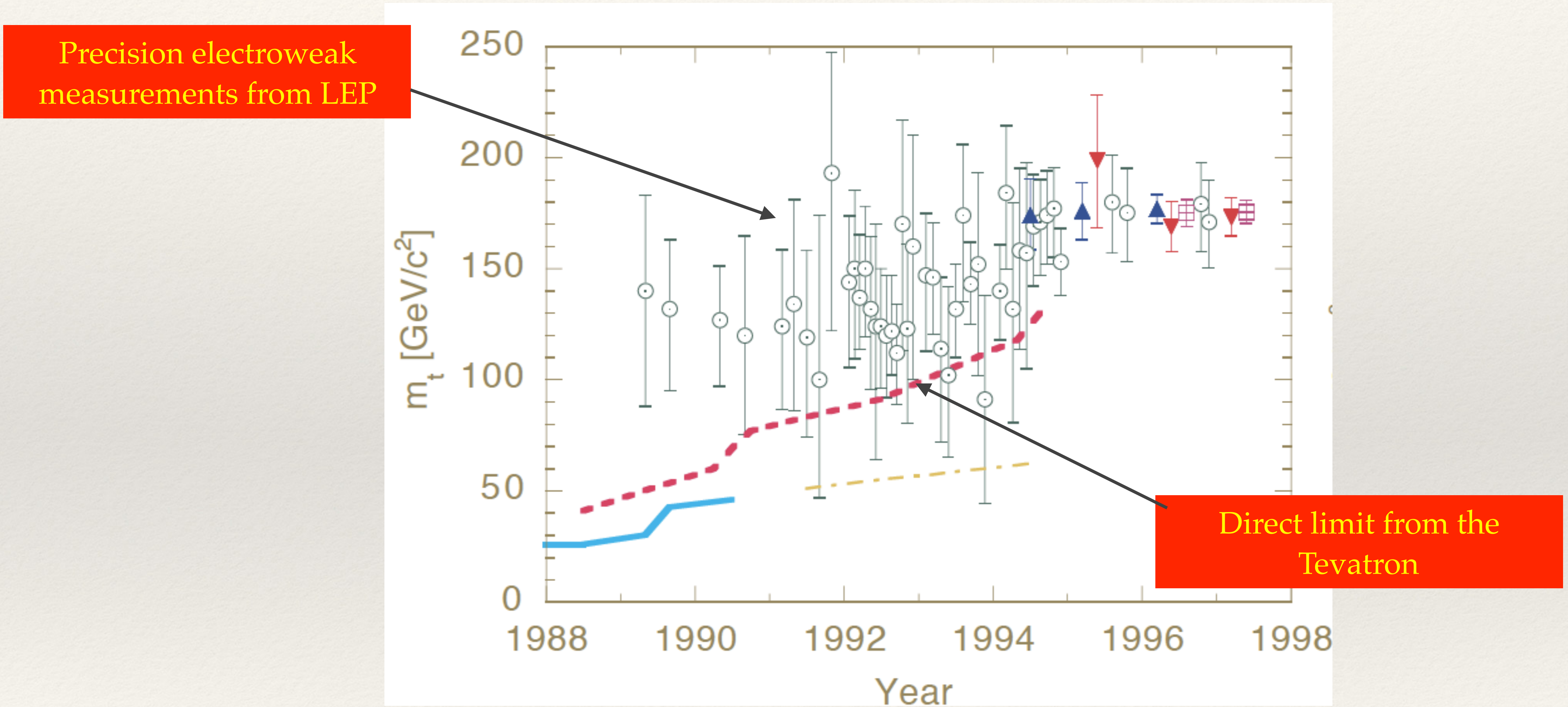


Fig. 2. Jet multiplicities at $\sqrt{s} = 630$ GeV (dashed lines) and 1.8 TeV (solid lines) for (a) $W \to e\nu$ and (b) $Z \to e^+e^-$, with the full set of lepton and jet cuts defined in eqs. (3.1), (3.2).

- Estimated of b-tagged W+jets background relies on W+jet Monte Carlo, incorporating, in some measure, Berends-Giele scaling.

Constraints on top quark mass

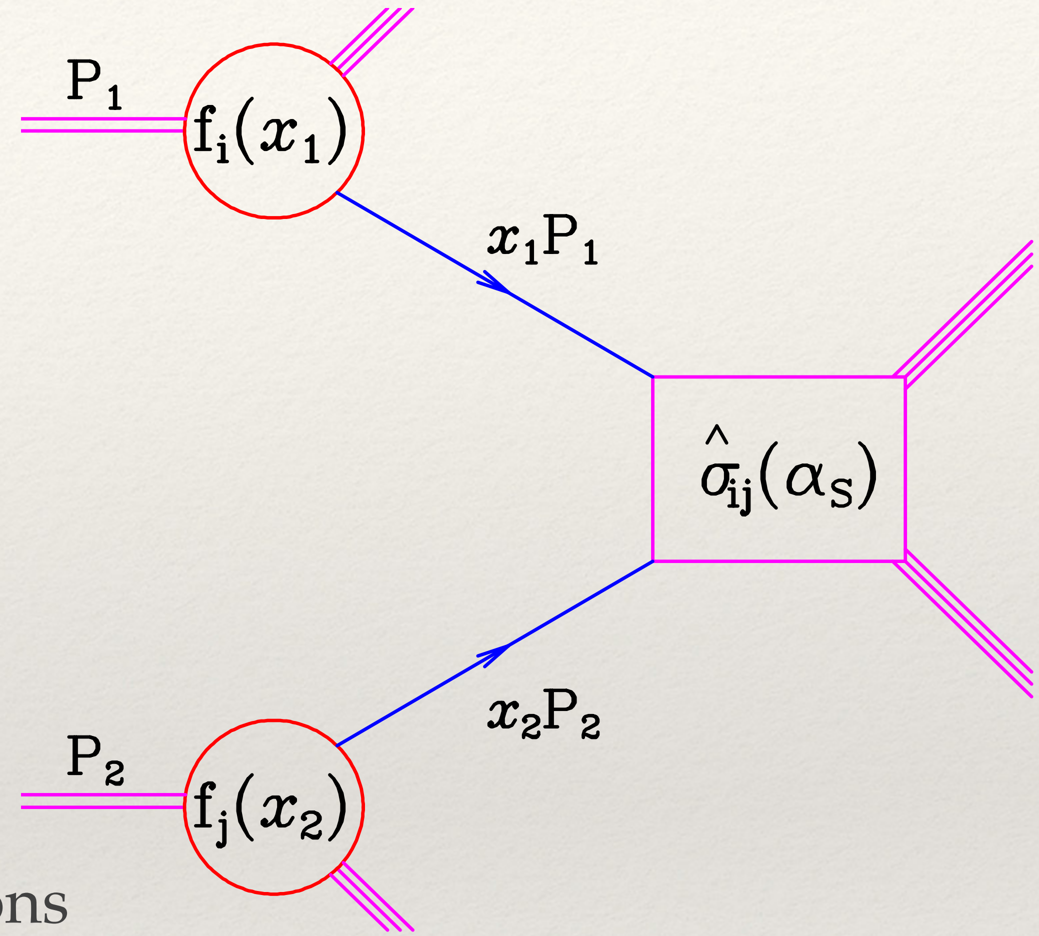
Quigg, arXiv:hep-ph/9704332



- Indirect precision limits track direct bounds, or is it vice versa?

How precise is precise?

- Hard cross section is represented as a convolution of a parton scattering cross section and non-perturbative parton distribution functions.
- Power corrections of order Λ/Q, for Q=100GeV, set a bound on the achievable precision of the factorisation formula of about 1%.
- The luminosity measurement at the LHC is in the range 2-5%, this also sets a scale for the precision to be aimed for.
- e.g. $\delta L/L$=1.9%, ATLAS measurement of luminosity for 22.7 inverse fb of data taken at $\sqrt{s}$=8TeV in 2012, [1608.03953]



Parton distributions

$$d\sigma(P_1,P_2) = \sum_{i,k} \int dx_1 dx_2\; f_i(x_1,\mu^2) f_k(x_2,\mu^2)\; d\hat{\sigma}_{ik}(p_1,p_2,p_J,\alpha_s(\mu^2),Q^2/\mu^2)F_J(p_J) + O(\Lambda/Q).$$

Physical cross section

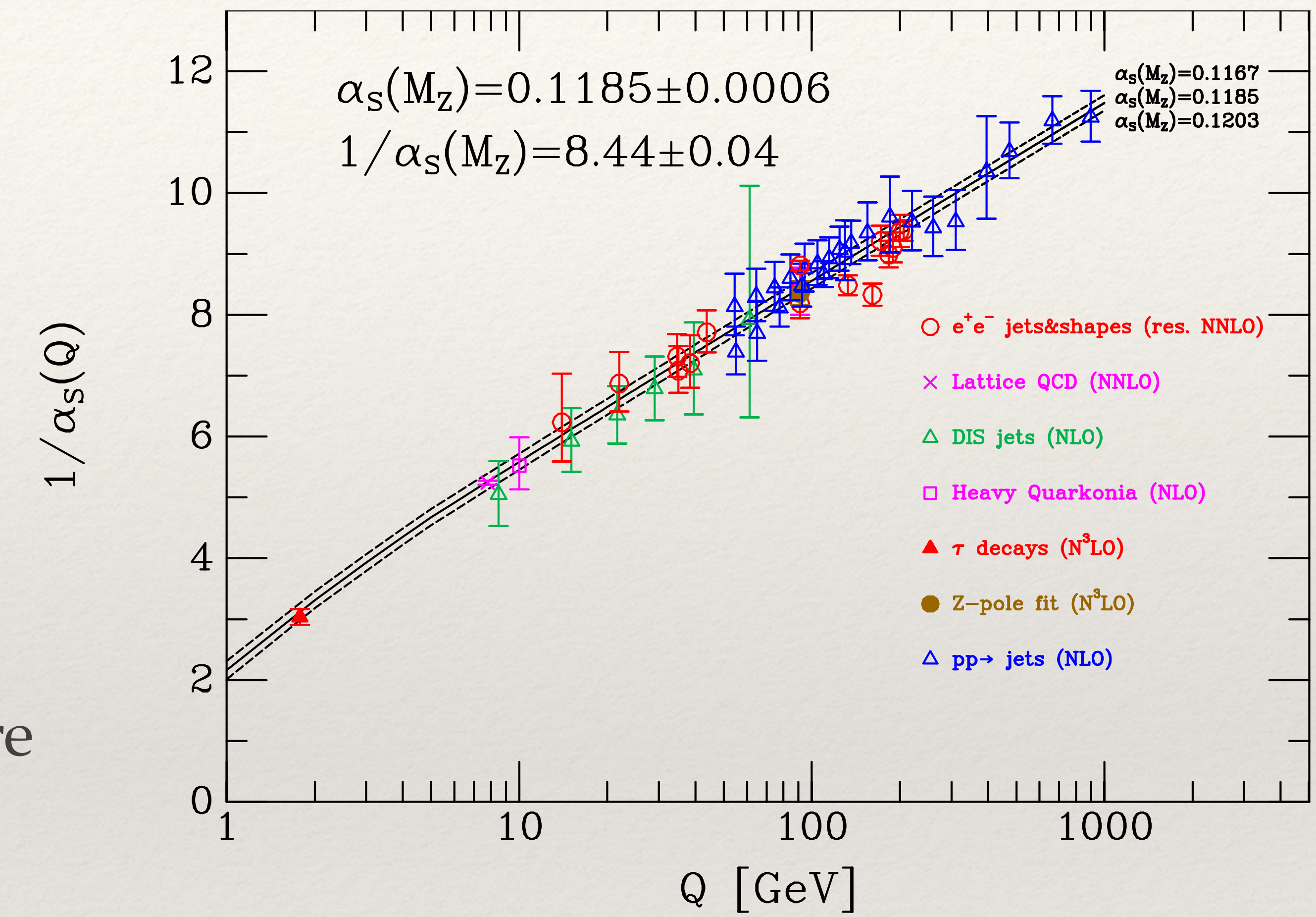
Factorization scale

Renormalization scale

Power corrections

Measurements of α_s

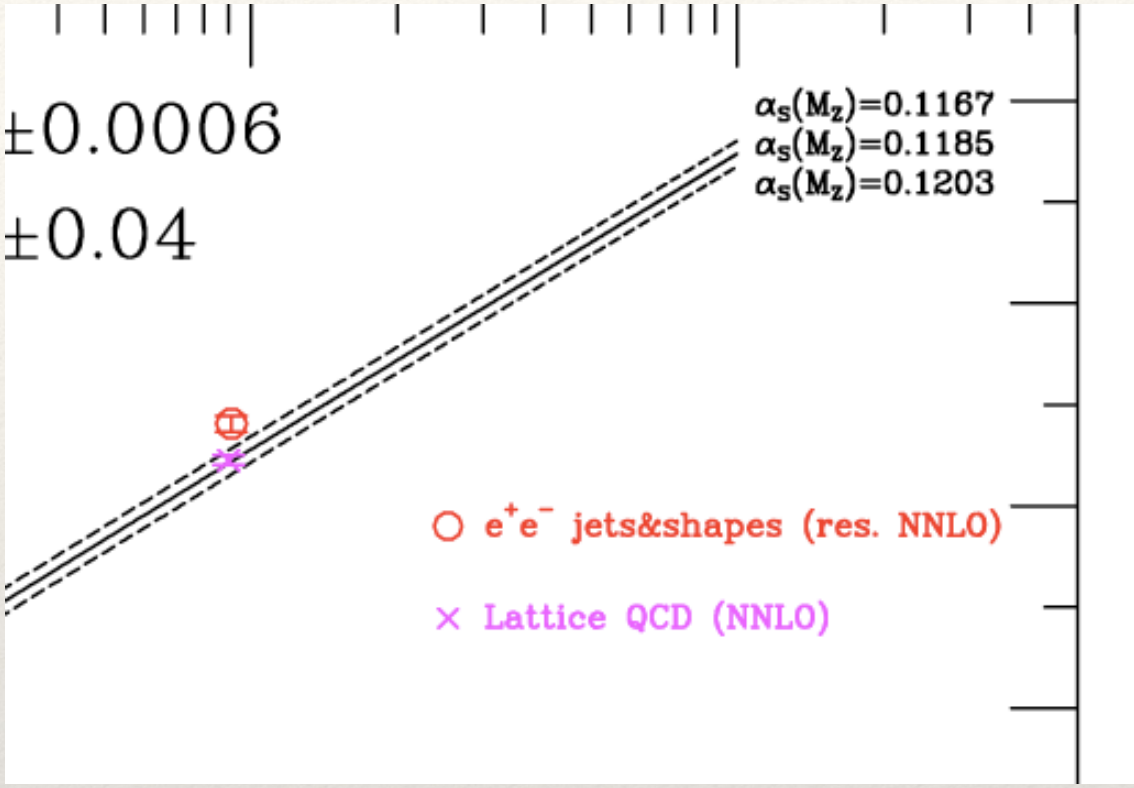
- Incontrovertible fact that **α**s is smallish at energies accessible with current machines.
- $1/\alpha_s$ as grows as ~log (Q).
- $1/\alpha_s(Mz)=8.44\pm0.04$
- c.f QED: $1/\alpha=128....137$
- Radiative corrections ~15 times more important in QCD than QED.
- The standard view is that $\alpha_s(Mz)$ is known to ~1%.

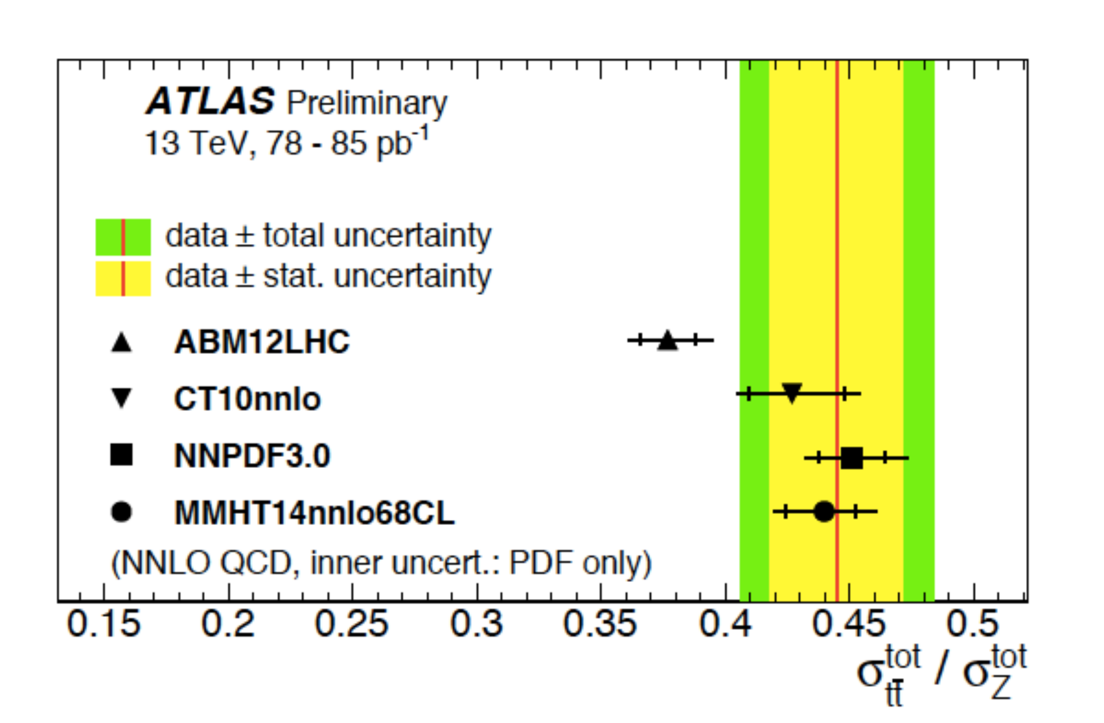


Data from PDG September, 2013

Measurements of α_s

- Disagreement between the two most precise measurements
- $\alpha_s(M_Z)$=0.1183±0.0007, Lattice QCD 1004.4285,1408.4169
- $\alpha_s(M_Z)$=0.1135±0.0010 Thrust in e+e- 1006.3080,1501.04111,1501.04753





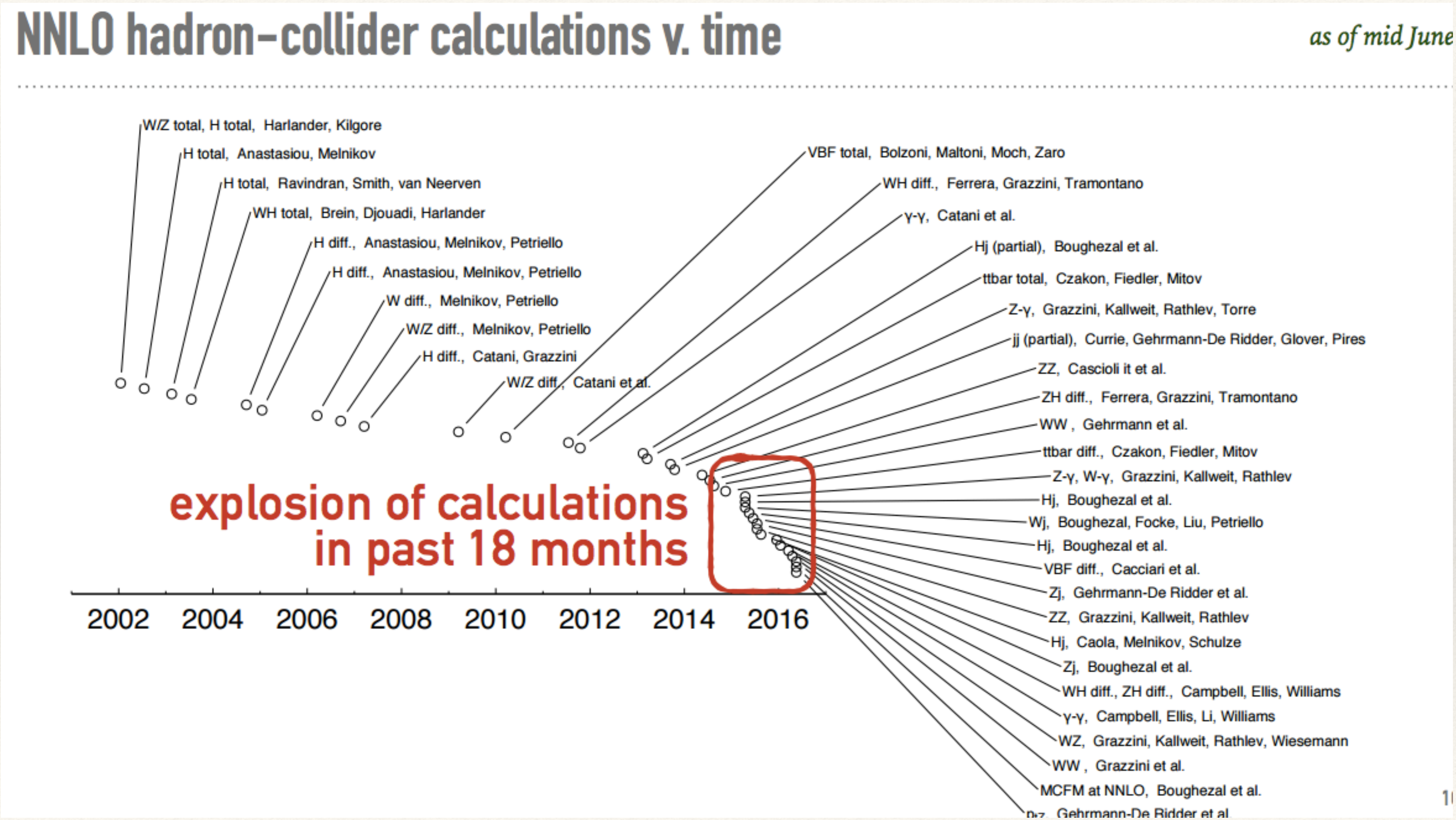
- Ratio of top/Z cross section offers some discrimination,
- ABM12LHC:$\alpha_s(M_Z)$=0.1132
- MMHT:$\alpha_s(M_Z)$=0.1172

Expectations for parton distributions from LHC

- **Gluon**
  - Inclusive jets, dijets,trijets → medium/large x
  - Isolated photon and photon+jets → medium/large x
  - ttbar spectrum → large x
  - Z pt spectrum → small/medium x
- **Quarks**
  - W and Z rapidity spectrum → medium x
  - High pt W+jets → medium/large x
  - Low mass and high mass Drell Yan → small/large x
  - W + c rapidity spectrum → strange at medium x
  - Single top differential → medium/large x

Credit:V. Radescu

Burgeoning NNLO activity



Credit: G Salam

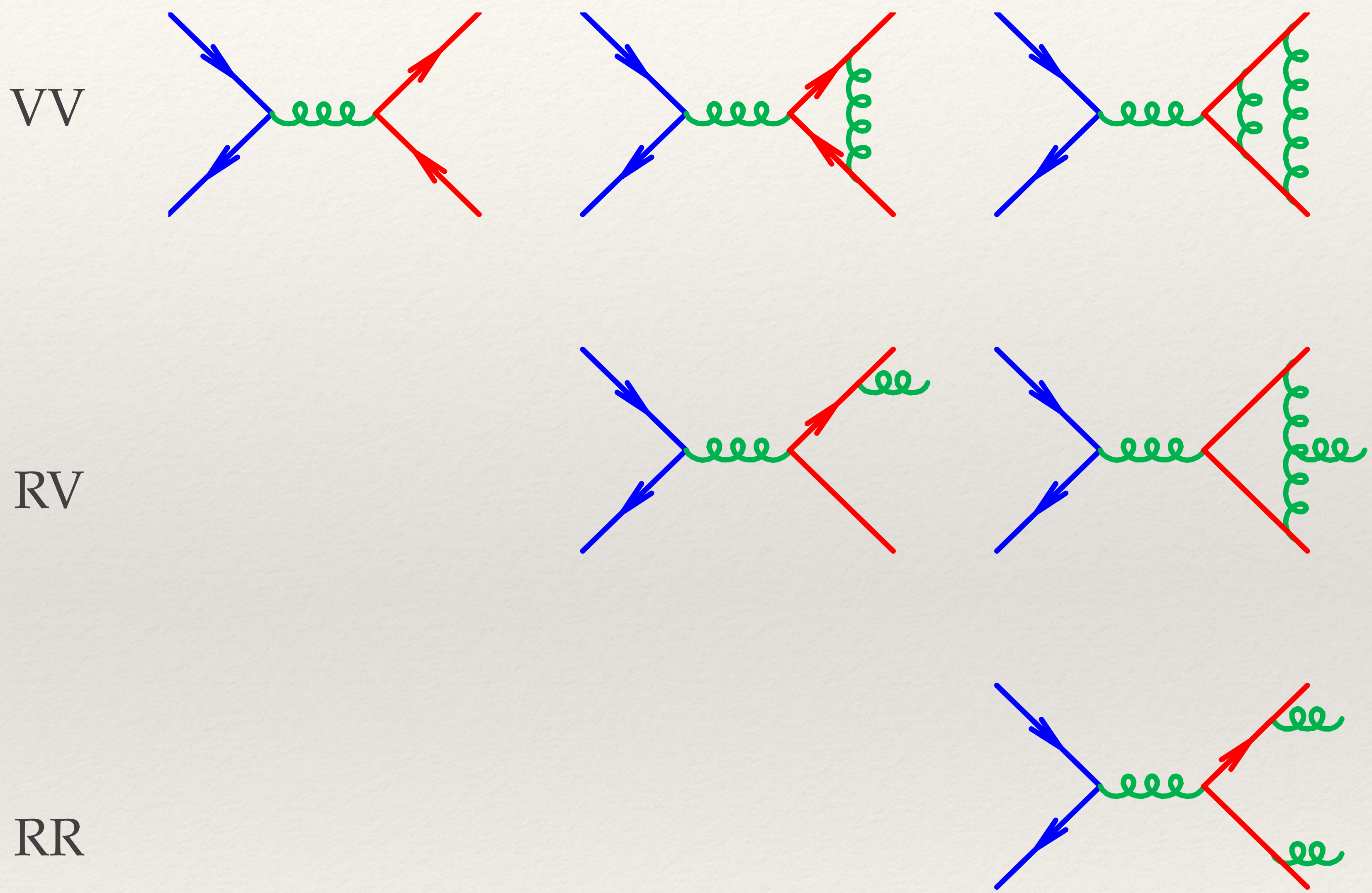
Recent NNLO calculations

dijets	gluon-gluon	PDFs,strong couplings,BSM	1407.5558	
H+0jet	fully inclusive N^3LO	Higgs couplings	1503.06056,	
H+1jet	fully exclusive	Higgs couplings, GGH vertex	1408.5325,1504.07922, 1505.03893,1607.08817	
tt pair	fully exclusive, stable tops	mass pt, FB asymmetry,PDFs	1601.05375	
single top	fully exclusive, stable tops, t-	Vtb,width, PDfs	1404.7116	
WBF	exclusive VBF cuts	Higgs couplings	1506.02660	
W+j	fully exclusive, decays	PDFs	1504.02131	
Z+j	decay, off-shell effects	PDFs	1601.04569,1507.20850, 1507.02850, 1605.04295	
ZH	decays to bb at NLO	Higgs couplings	1407.4747,1601.00658	
WH	fully exclusive	Higgs couplings	1312.1669, 1601.00658	
ZZ	fully exclusive, off-shell	trilinear gauge couplings,BSM	1405.2219, 1507.06257	
WW	fully inclusive	trilinear gauge couplings,BSM	1408.5243	
$W\gamma$,$Z\gamma$	fully exclusive	trilinear gauge couplings,BSM	1601.06751	
$\gamma\gamma$	fully differential	Background studies	1110.2375,1603.02663	
top decay	exclusive	Top couplings	1301.7133	
H-bb	exclusive, massless	Higgs couplings boosted	1110.2368	
Single inclusive jet	Inclusive	PDFs	1611.04295	New
Direct photon	Inclusive	PDFs, Monojets	1612.04333	New

Adapted from K. Melnikov

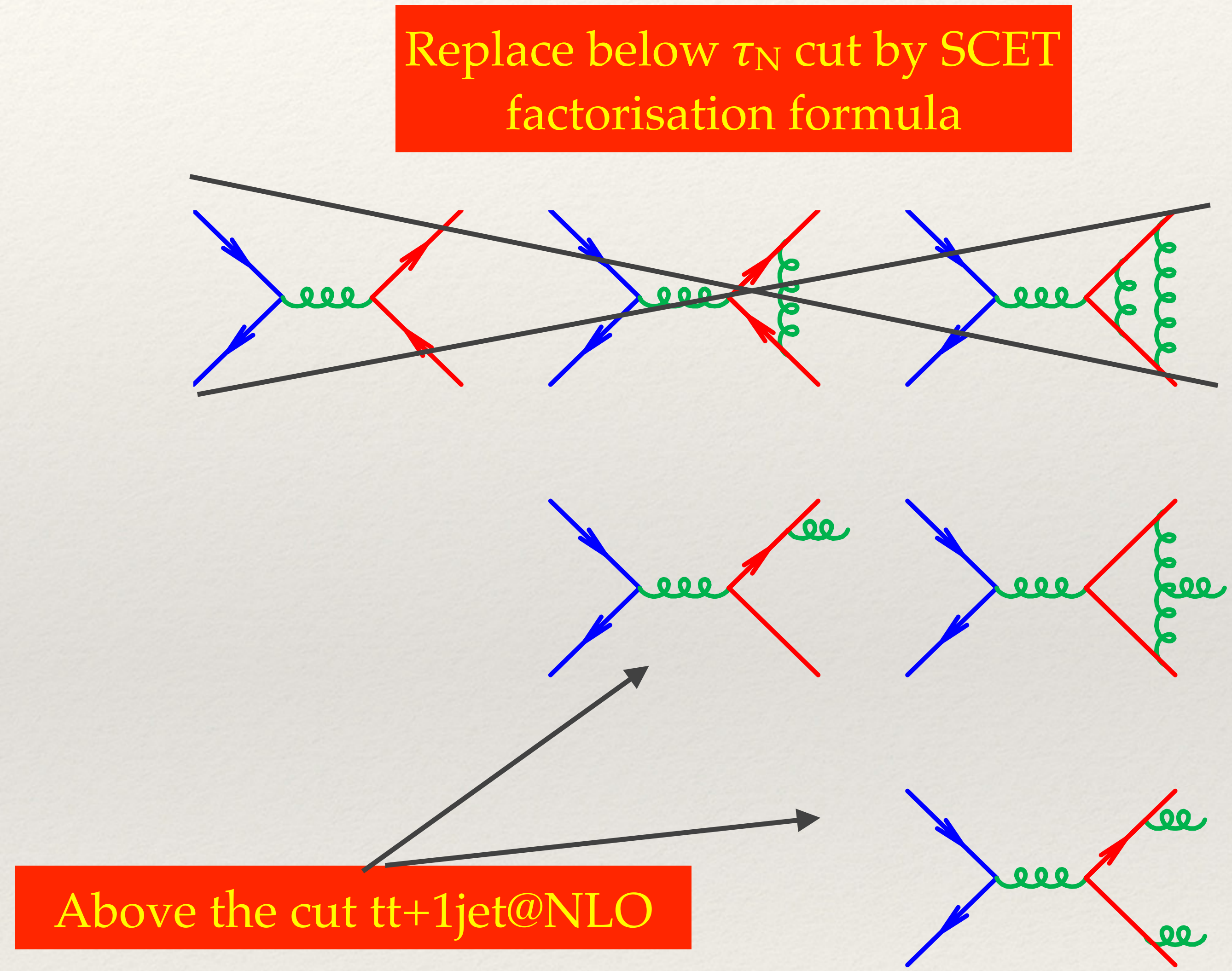
Challenge of NNLO (and higher)

- Tension between:-
  - Cancellation of singularities due to soft and collinear emission in more inclusive quantities.
  - Desire for an exclusive description of the event, to allow comparison data coming from detectors with limited acceptance.



Non-local subtraction/slicing

- Use a physical jet-resolution variable to control the IR behaviour.
- Originally applied using the transverse momentum of colourless final state as a resolution variable, (Catani&Grazzini, hep-ph/0703012)
- Current incarnation using τ_N-jettiness (Stewart et al, 1004.2489) as resolution parameter
- At NLO (NNLO) isolates the single (doubly) unresolved region
- Subtraction/Slicing terms correspond to the singular limits of a physical cross section.
    - Allows recycling of existing NLO calculations (e.g. MCFM)
    - Conceptually straightforward to extend to even higher orders.
    - With increased computer power, non-locality may be less of a problem.



N-jettiness, τ_N

- N-jettiness $\tau_N(p_j)$ of a parton j, defined wrt to N-jets with momentum q_i.

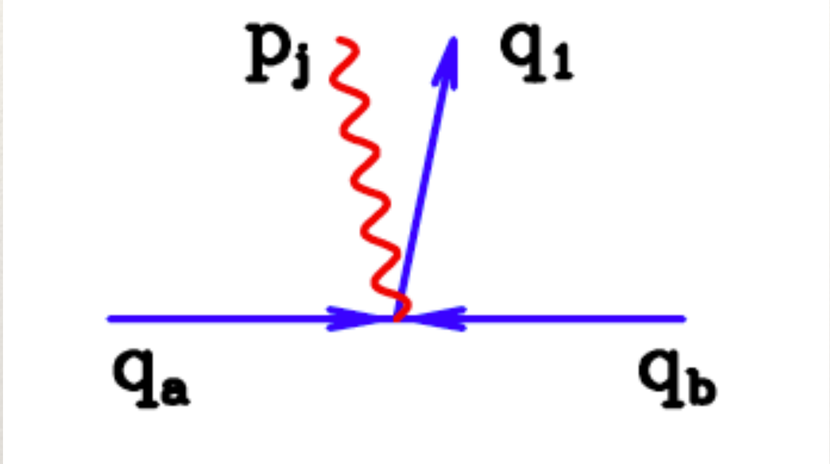
$$\mathcal{T}_N(p_j) = \min_{i=a,b,1,\ldots,N} \left\{ \frac{2\, q_i \cdot p_j}{Q_i} \right\}$$

- Event jettiness defined as the sum over all jettiness values for all final-state partons

$$\mathcal{T}_N = \sum_{k=1}^{M} \mathcal{T}_N(p_k) = \sum_{k=1}^{M} \min_{i=a,b,1,\ldots,N} \left\{ \frac{2\, q_i \cdot p_k}{Q_i} \right\}$$

- Q_i set equal to $2E_i$

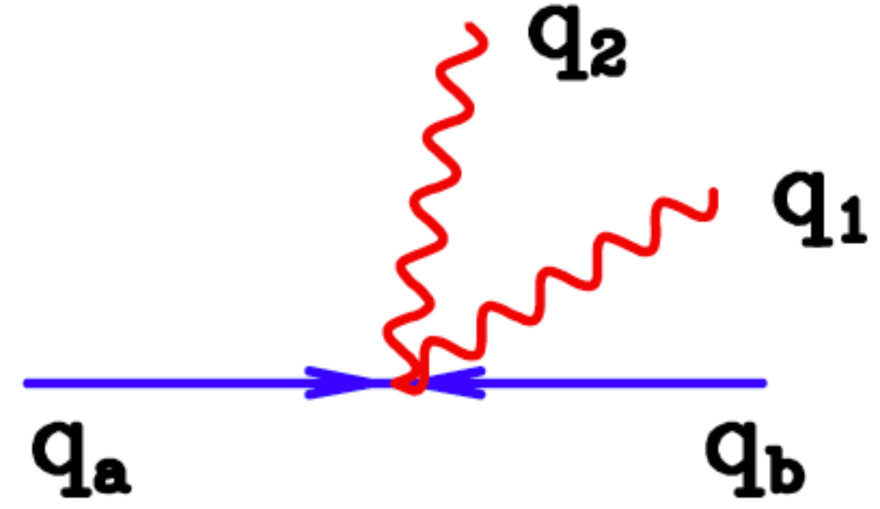
Illustration of τ_1 jettiness



Colour singlet final states

- For color singlet final states*, we can limit ourselves to τ_0.
- At NLO (NNLO) the $\tau_0 > \tau_0^{cut}$ contribution is given by the LO (NLO) results with one additional parton.
- By demanding $\tau_0 < \tau_0^{cut}$ at NNLO (NLO) one isolates the doubly (singly) unresolved regions of phase space.

Illustration of τ_0-jettiness



* with no coloured final state parton at Born level

SCET

- Analytic results for τ_0-jettiness in the soft collinear region are available using soft/collinear effective theory.

Hard function

$$\frac{d\sigma}{d\mathcal{T}_0} = \sum_{ab} \int dx_a dx_b \int d\Phi_B(p_a, p_b; p_{\rm singlet})\, \Theta(p_{\rm singlet})\, H_{ab}(\Phi_B, \mu)\, \frac{d\Delta_{ab}}{d\mathcal{T}_0} + \dots$$

Power corrections, negligible for $\tau_0 << Q$

$$\frac{d\Delta_{ab}}{d\mathcal{T}_0} = B_a \otimes B_b \otimes S_{ab}$$
$$\equiv \int dt_{B_a} dt_{B_b} dt_S\, \delta(\mathcal{T}_0 - t_{B_a} - t_{B_b} - t_S)\, B_a(t_{B_a}, x_a, \mu)\, B_b(t_{B_b}, x_b, \mu)\, S_{ab}(t_S, \mu)\ .$$

Beam functions

Soft function

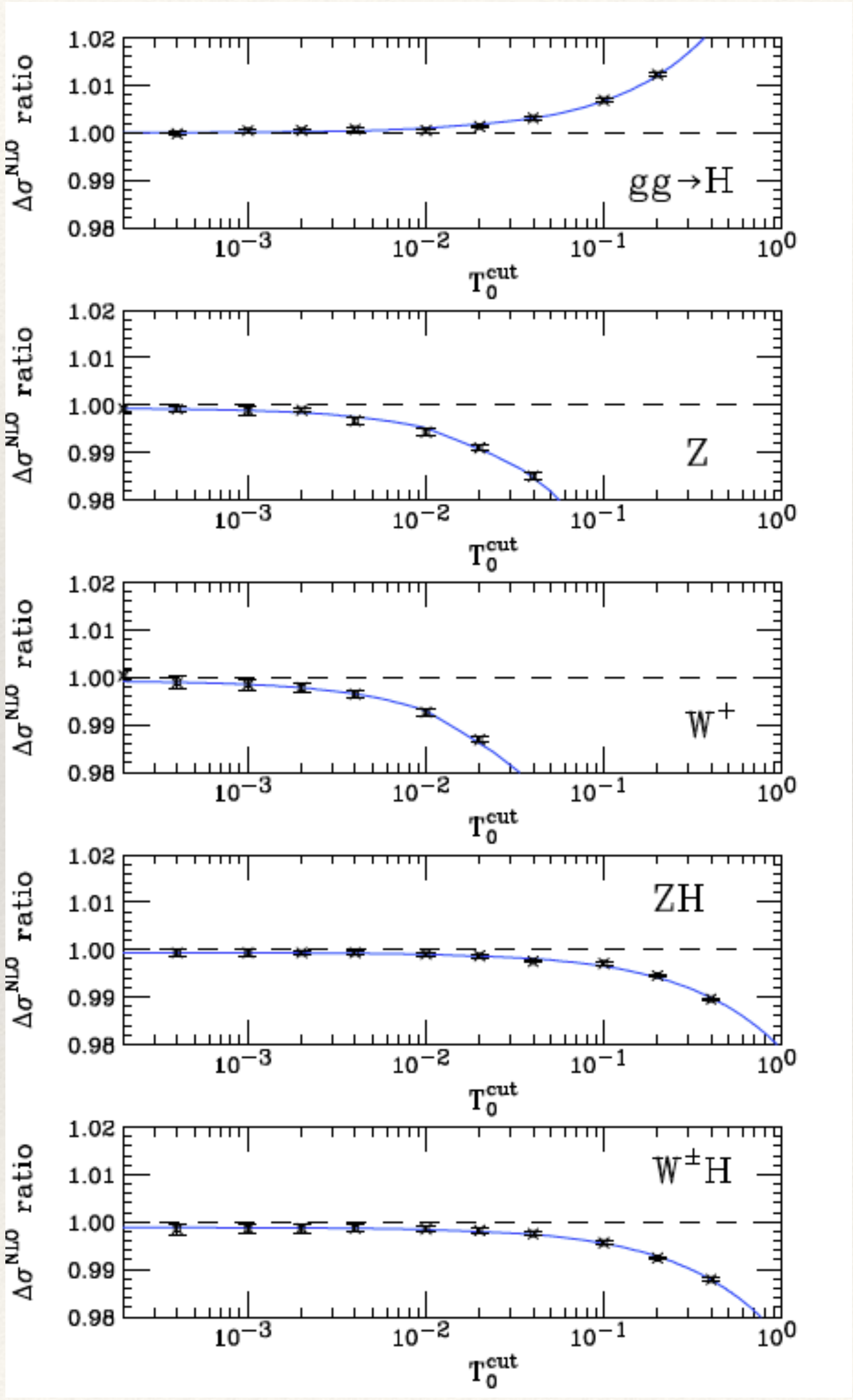
- Hard-function: effect of hard-virtual corrections
- Beam-function: effects of initial state collinear radiation
- Soft function: jettiness contributions of soft radiation.

Stewart, Tackmann, Waalewijn,………

Verifying the method : NLO

- Processes considered H production in EFT , Z,W, ZH,WH.
- At NLO we can calculate the total cross sections using three different methods, (useful to fix input parameters)
    - Standard MCFM (subtraction)
    - MCFM with τ_0 slicing
    - Purpose-built codes from other authors for total cross sections.

$$\Delta\sigma^{NLO}_{\rm jettiness}(\mathcal{T}_0^{cut}) = \Delta\sigma^{NLO} + c \times \left(\frac{\mathcal{T}_0^{cut}}{Q}\right) \times \log\left(\frac{\mathcal{T}_0^{cut}}{Q}\right) \,,$$



The ratio N-jettiness, vs MCFM subtraction result

Verifying the method

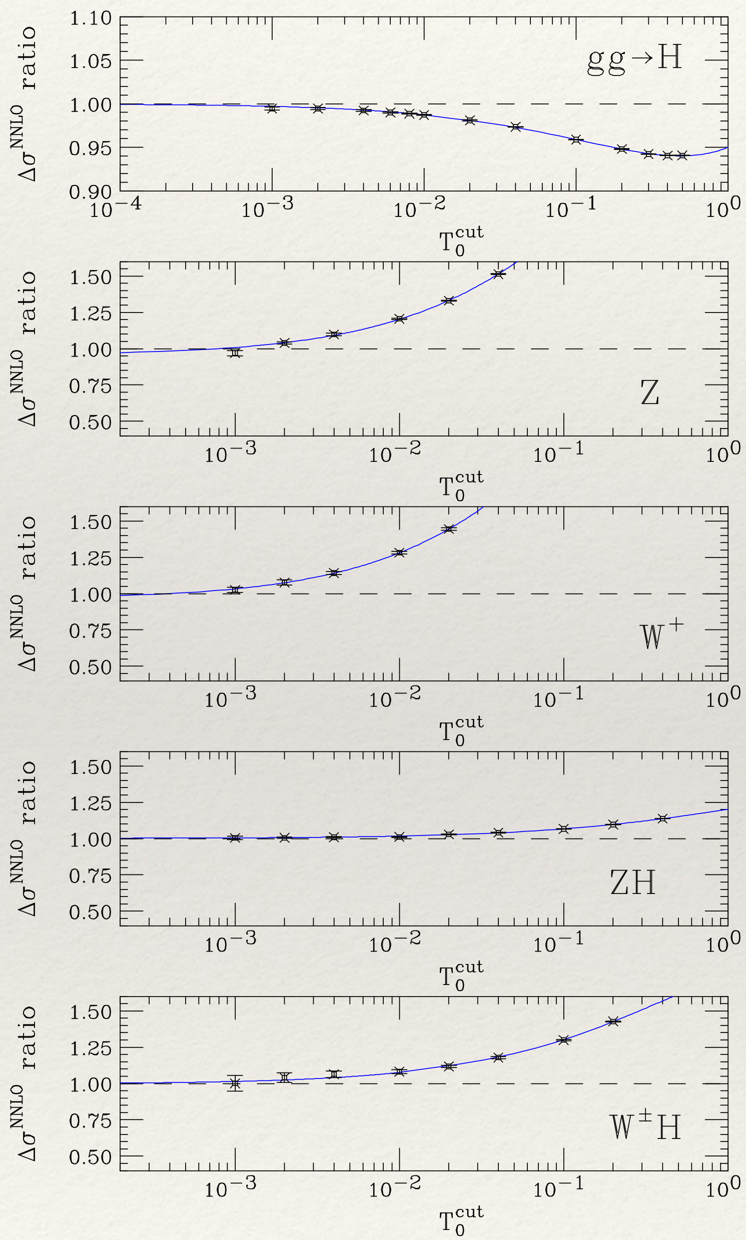
- We compare wth the results of inclusive codes at LO and NLO to ensure that parameters are set up correctly

Process	μ_R	μ_F	Cross-section to NNLO	Reference
$gg \to H$	M_H	M_H	$12.937 \times (1 + 1.28 + 0.77)$ pb	`ggh@nnlo` [76]
Z	$2M_Z$	$M_Z/2$	$44.303 \times (1 + 0.22 + 0.05)$ nb	`ZWMS` [77]
W^+	$2M_W$	$M_W/2$	$81.561 \times (1 + 0.23 + 0.06)$ nb	`ZWMS` [77]
ZH	$\sqrt{q^2}$	$\sqrt{q^2}$	$0.68255 \times (1 + 0.16 + 0.10)$ pb	`vh@nnlo` [78, 79]
$W^+H + W^-H$	$\sqrt{q^2}$	$\sqrt{q^2}$	$1.2593 \times (1 + 0.16 + 0.02)$ pb	`vh@nnlo` [78, 79]

- Comparison with inclusive results of Harlander et al, and Van Neerven et al.

Color singlet production:NNLO

Verification at NNLO



- We compare wth the results of inclusive codes at LO and NLO to ensure that parameters are set up correctly

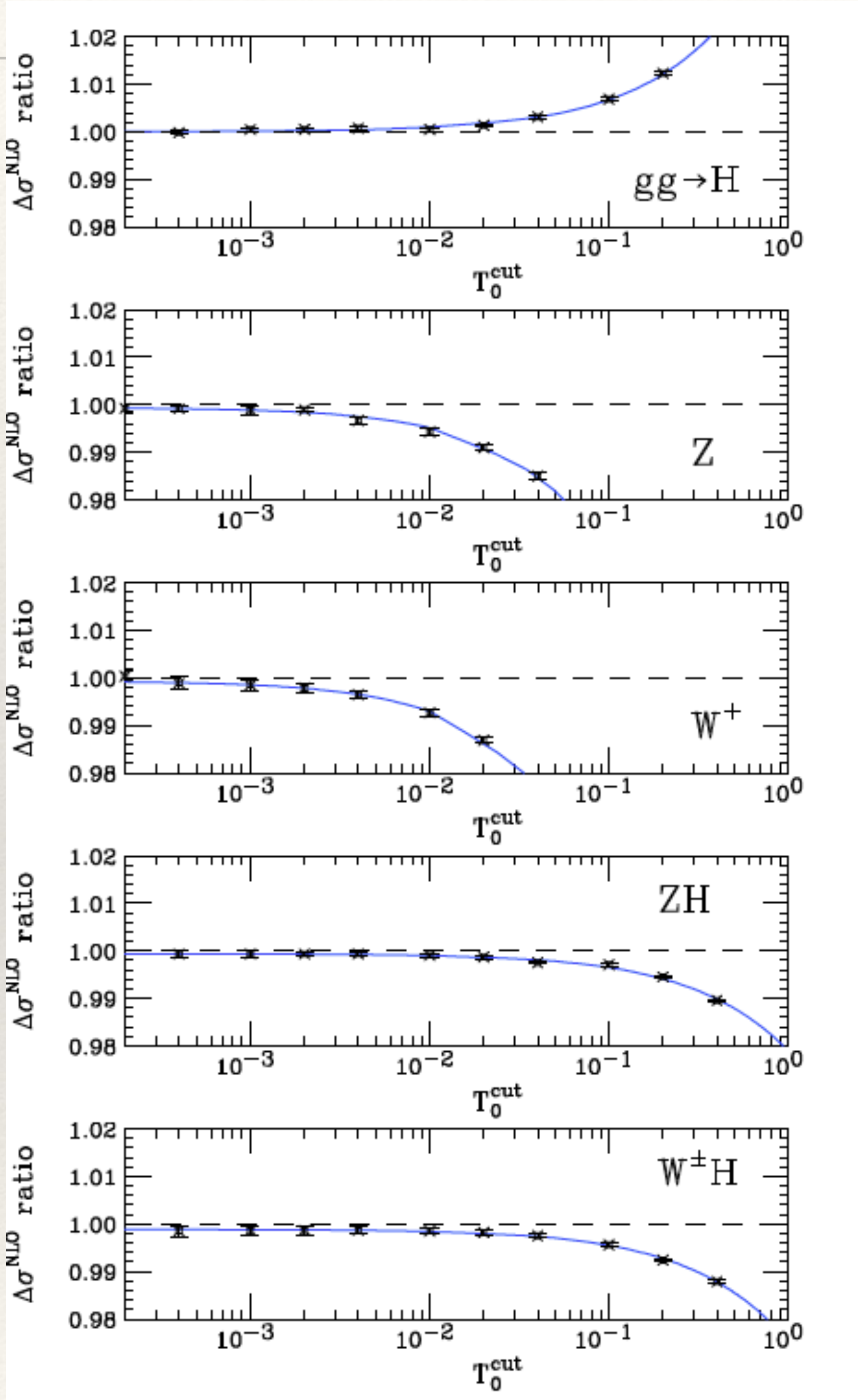
Process	μ_R	μ_F	Cross-section to NNLO	Reference
$gg \to H$	M_H	M_H	$12.937 \times (1 + 1.28 + 0.77)$ pb	ggh@nnlo [76]
Z	$2M_Z$	$M_Z/2$	$44.303 \times (1 + 0.22 + 0.05)$ nb	ZWMS [77]
W^+	$2M_W$	$M_W/2$	$81.561 \times (1 + 0.23 + 0.06)$ nb	ZWMS [77]
ZH	$\sqrt{q^2}$	$\sqrt{q^2}$	$0.68255 \times (1 + 0.16 + 0.10)$ pb	vh@nnlo [78, 79]
$W^+H + W^-H$	$\sqrt{q^2}$	$\sqrt{q^2}$	$1.2593 \times (1 + 0.16 + 0.02)$ pb	vh@nnlo [78, 79]

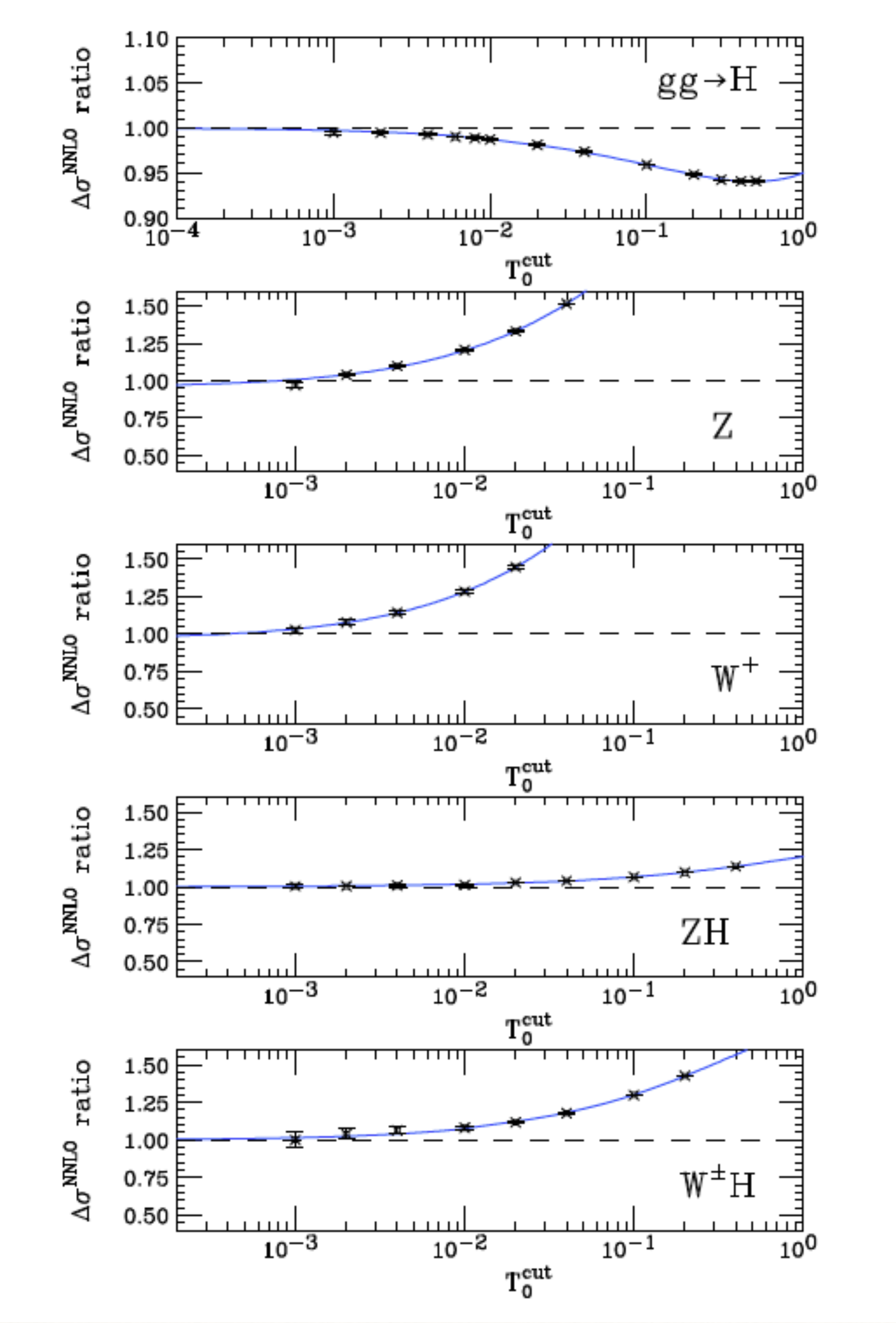
- Note that the plots show the NNLO contribution, not the total cross section.

$$\Delta\sigma^{NNLO}_{\text{jettiness}}(\mathcal{T}_0^{cut}) = \Delta\sigma^{NNLO} + c_3 \times \left(\frac{\mathcal{T}_0^{cut}}{Q}\right) \times \log^3\left(\frac{\mathcal{T}_0^{cut}}{Q}\right) + c_2 \times \left(\frac{\mathcal{T}_0^{cut}}{Q}\right) \times \log^2\left(\frac{\mathcal{T}_0^{cut}}{Q}\right)$$

Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello and Williams, 1605.08011

NLO vs NNLO





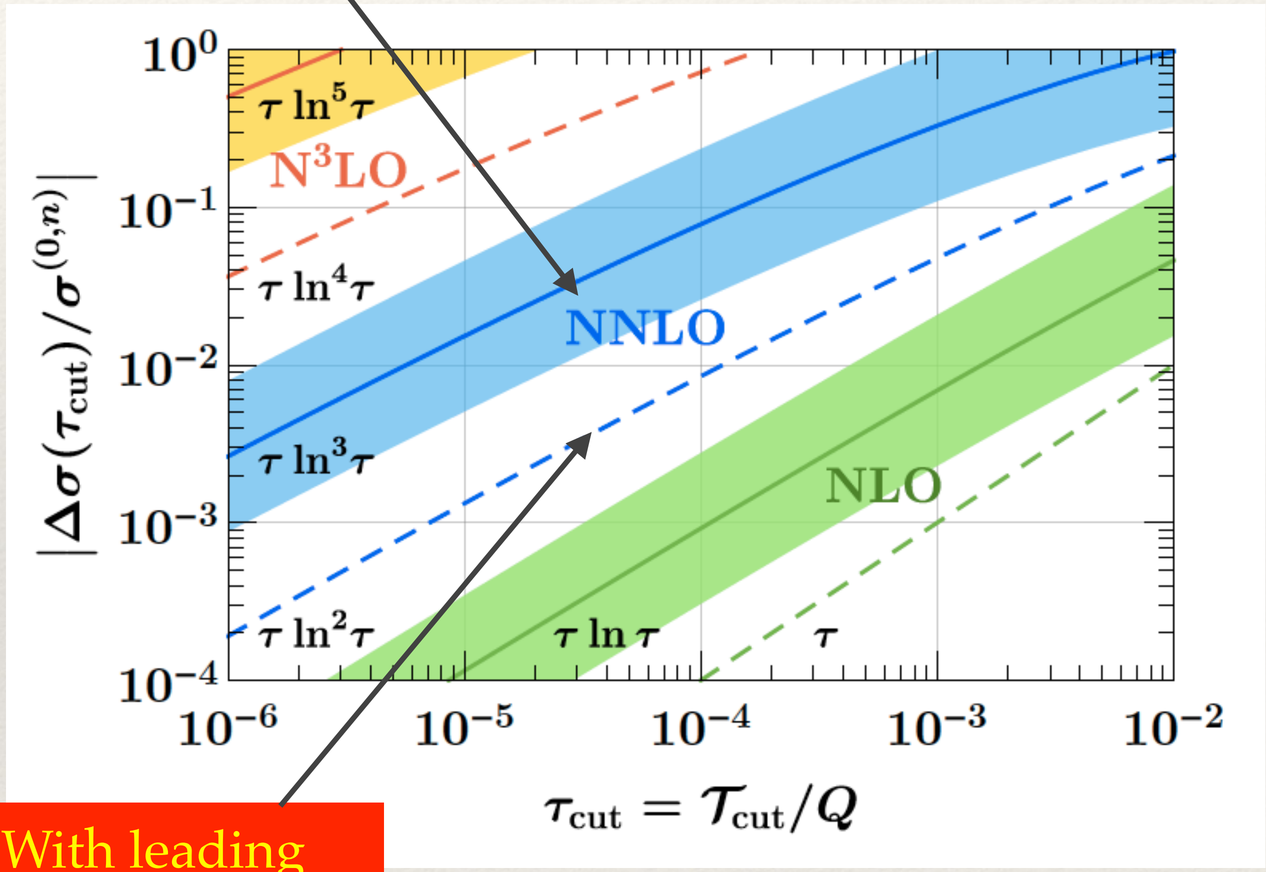
Plots look similar (modulo sign of approach), **but note that NNLO scale is bigger by a factor between 5 and 30**

Power corrections

- Leading power corrections
  - NLO: $\tau \ln \tau$
  - N^2LO: $\tau \ln^3 \tau$
  - N^3LO: $\tau \ln^5 \tau$
- Size of missing terms grows rapidly with loop order
- Power corrections essential from N^3LO

Without leading power correction

Estimate of size of power correction
Moult et al, 1612.00450



With leading power correction

- Expectation is that inclusion of power corrections will reduce error at fixed tau, or allow extraction with the same error at larger tau.
- Improvement possible in MCFM results

Moult et al, 1612.00450, Boughezal et al, 1612.02911

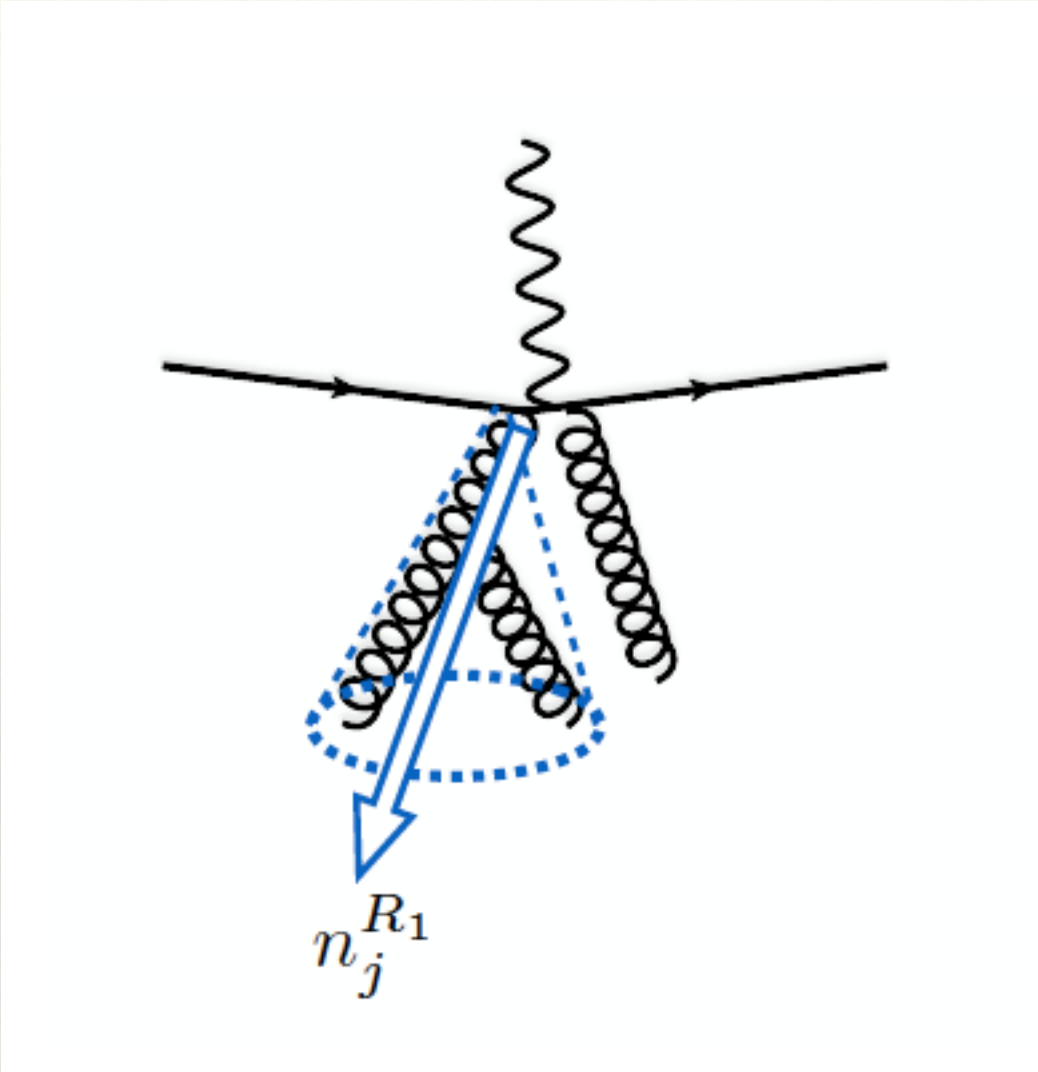
Direct photon production

Direct photon production at NNLO

- Direct photon and related gamma+jet processes are the highest rate EW processes at the LHC.
- Similarity to Z+jet process can be exploited to constrain mono-jet production process, $Z(\rightarrow \nu\nu)$+jet. (Useful to constrain backgrounds, for dark matter, supersymmetry, especially at the highest p_T where $Z(\rightarrow e^+e^-)$+jet is statistically hampered.
- Presence of a coloured final-state parton at the Born-level introduces a new feature.

Direct photon

- No final-state jet is required, but the non-zero pt of the photon mandates some coloured radiation in the final state.
- Final state coloured parton introduces singularities, that are not regulated by a cut on τ_0 jettiness.
- So we have to define a jet axis, define τ_1 jettiness and perform a cut on τ_1^{cut}



τ_1-jettiness calculation

- As before the double-unresolved IR poles occur for τ_1=0
- Below the cut, recycling of beam, soft and jet functions from Z+jet, W+jet calculations.
- Hard function adapted from Anastasiou et al, (hep/ph0201274)
- Above the cut implementation requires a NLO calculation of γ+2jet, which is a new process for MCFM. (previously studied by Bern et al, 1106.1423)

Dependence on jet parameters

- In order to be well defined (i.e. smallish power corrections), the contribution from the jet direction must correspond to a sufficiently hard jet.
    - Born phase space: $p_T > p_T^{\gamma}$
    - Real-virtual phase space: $p_T > p_T^{\gamma}/2$
    - Double real phase space: $p_T > p_T^{\gamma}/3$
- Jettiness axis depends on the value chosen for jet cone radius R

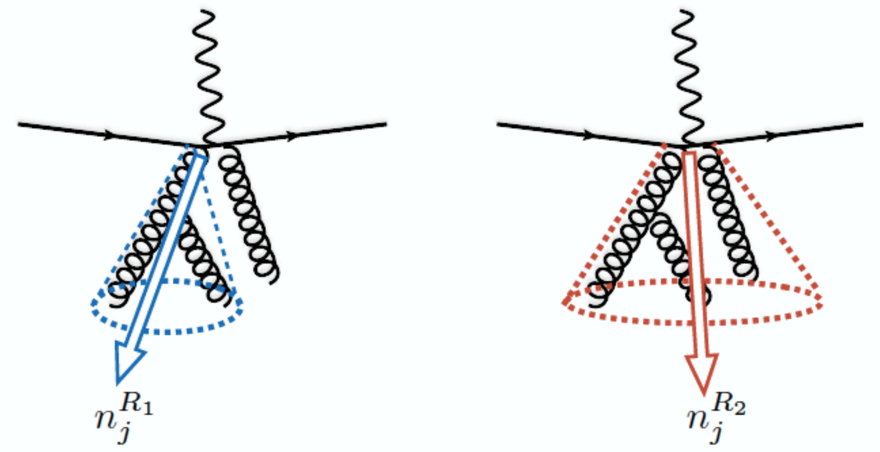
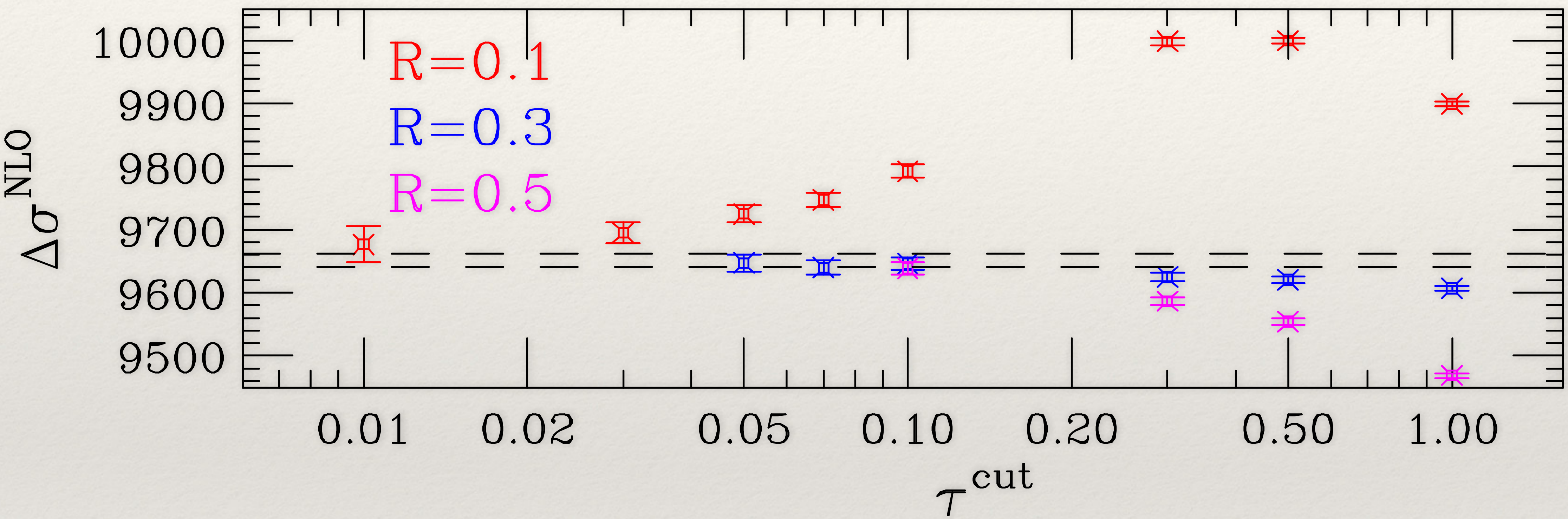


Figure 1: An illustration of the dependence on R in the N-jettiness algorithm. The same event, clustered with two different R values, results in differing orientations of the jettiness axis. This results in different power corrections for $\tau_1 > 0$. In the limit $\tau_1 \to 0$ the dependence on R vanishes.

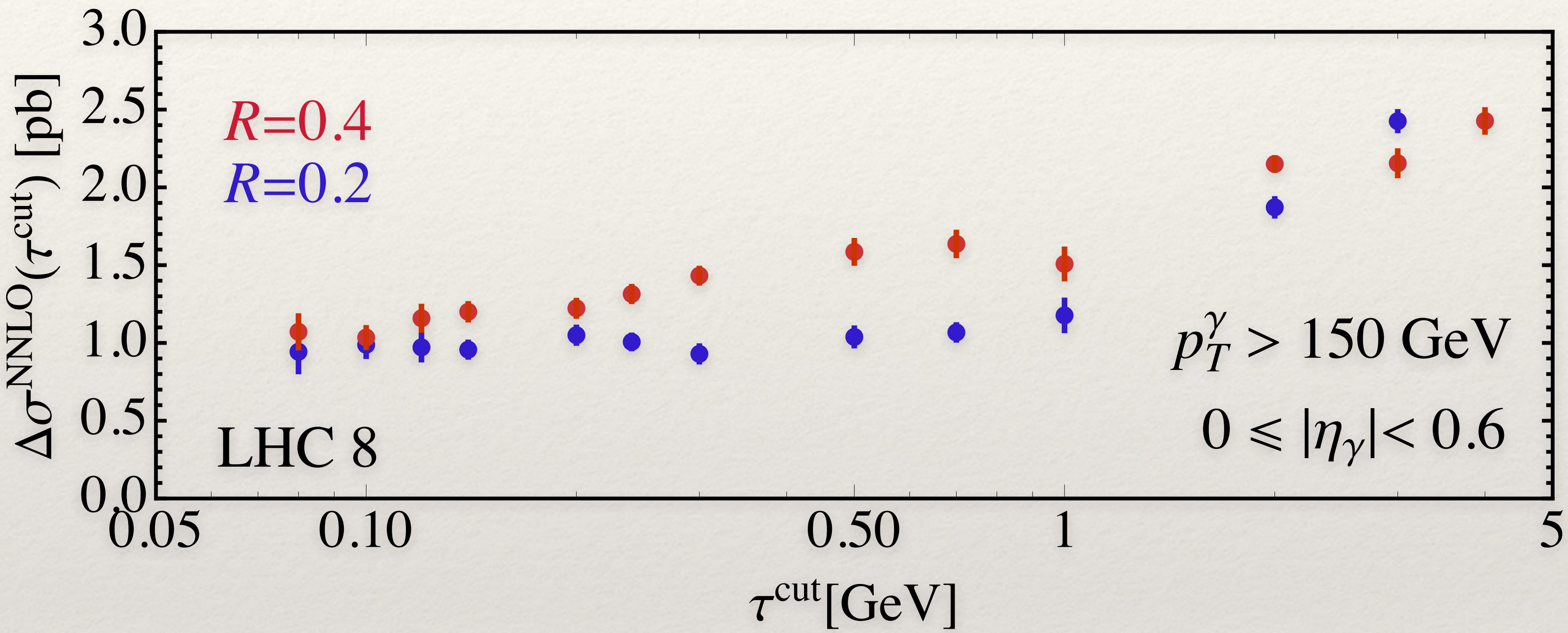
Dependence on cone size:NLO



- Investigate at NLO
- Dependence on cone size vanishes as $\tau_1{}^{\mathrm{cut}} \rightarrow 0$

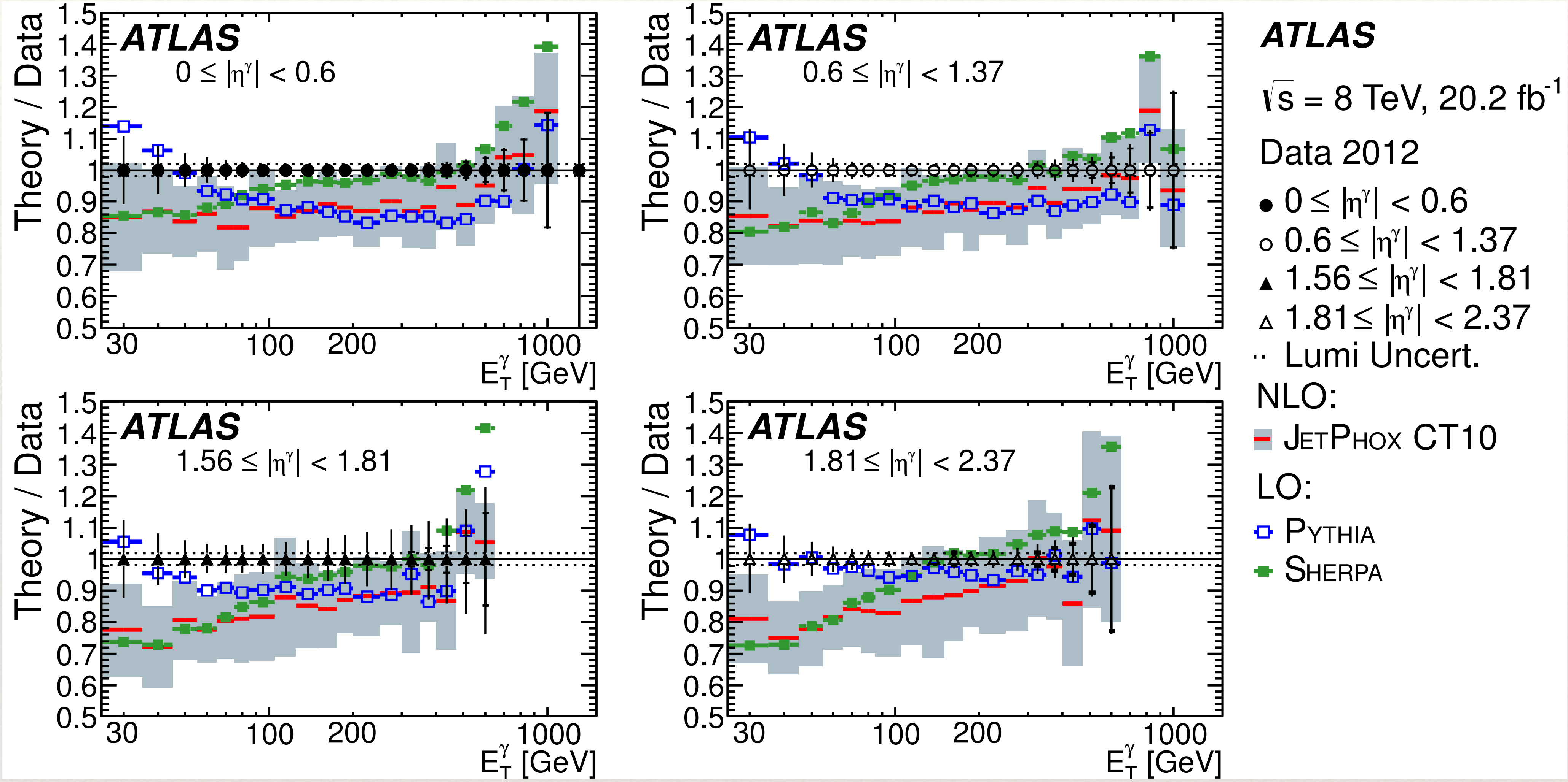
Dependence on the cone-size:NNLO

- At finite cone size the two jettiness axis differ, but as $\tau_1^{cut} \to 0$ the difference vanishes.
- We are free to choose the cone size for our own convenience.



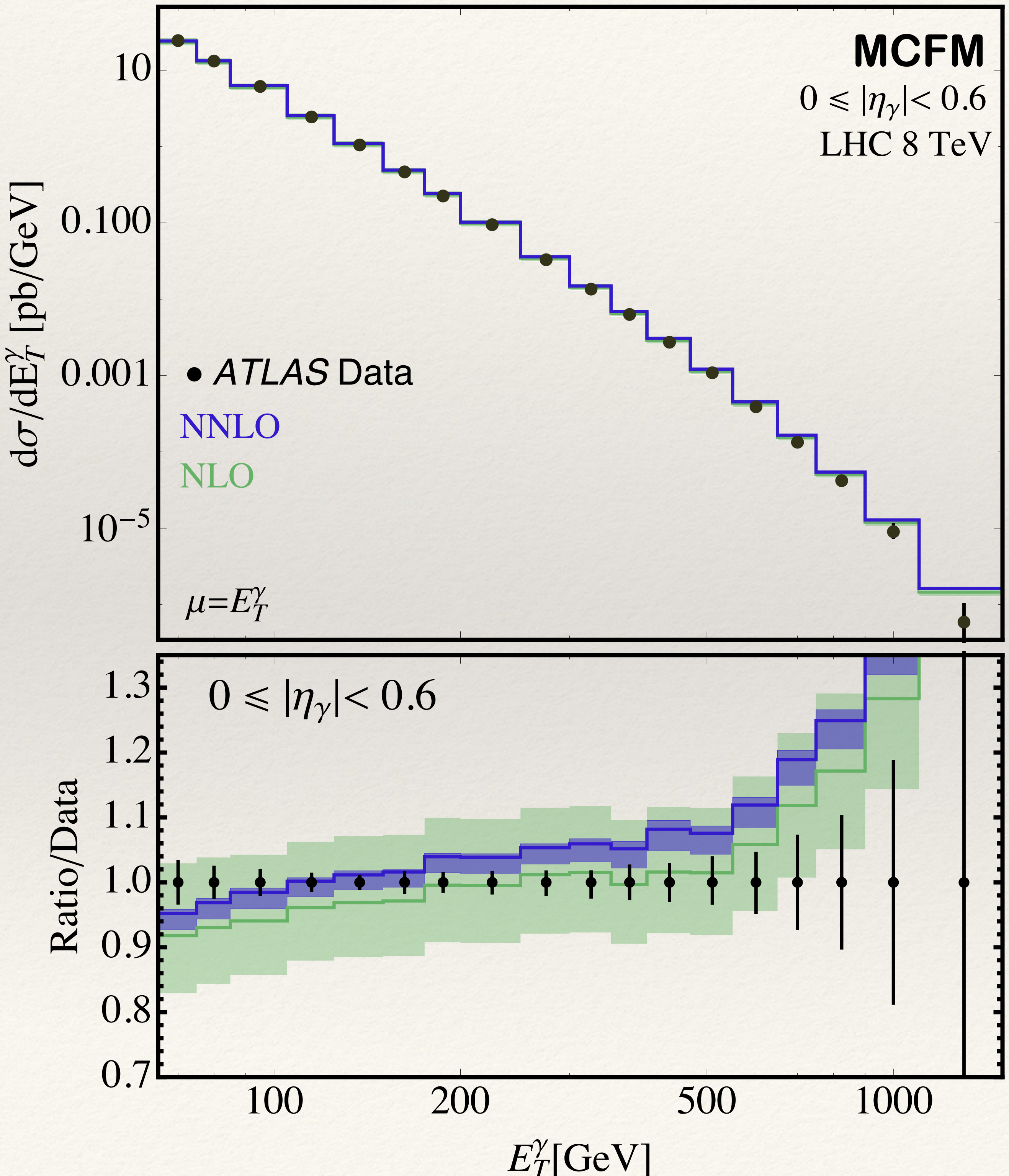
- The dependence of the NNLO coefficient on the parameter, τ_1^{cut} and the clustering cone size R.

Direct photon data



- ATLAS 8 TeV data 1605.04935
- Comparisons have so far been made with JetPhox, Pythia and Sherpa

Direct photon results



- Comparison of MCFM predictions for the transverse momentum of the photon to ATLAS 8 TeV data 1605.04935
- Six-point renormalisation and factorisation scale variation

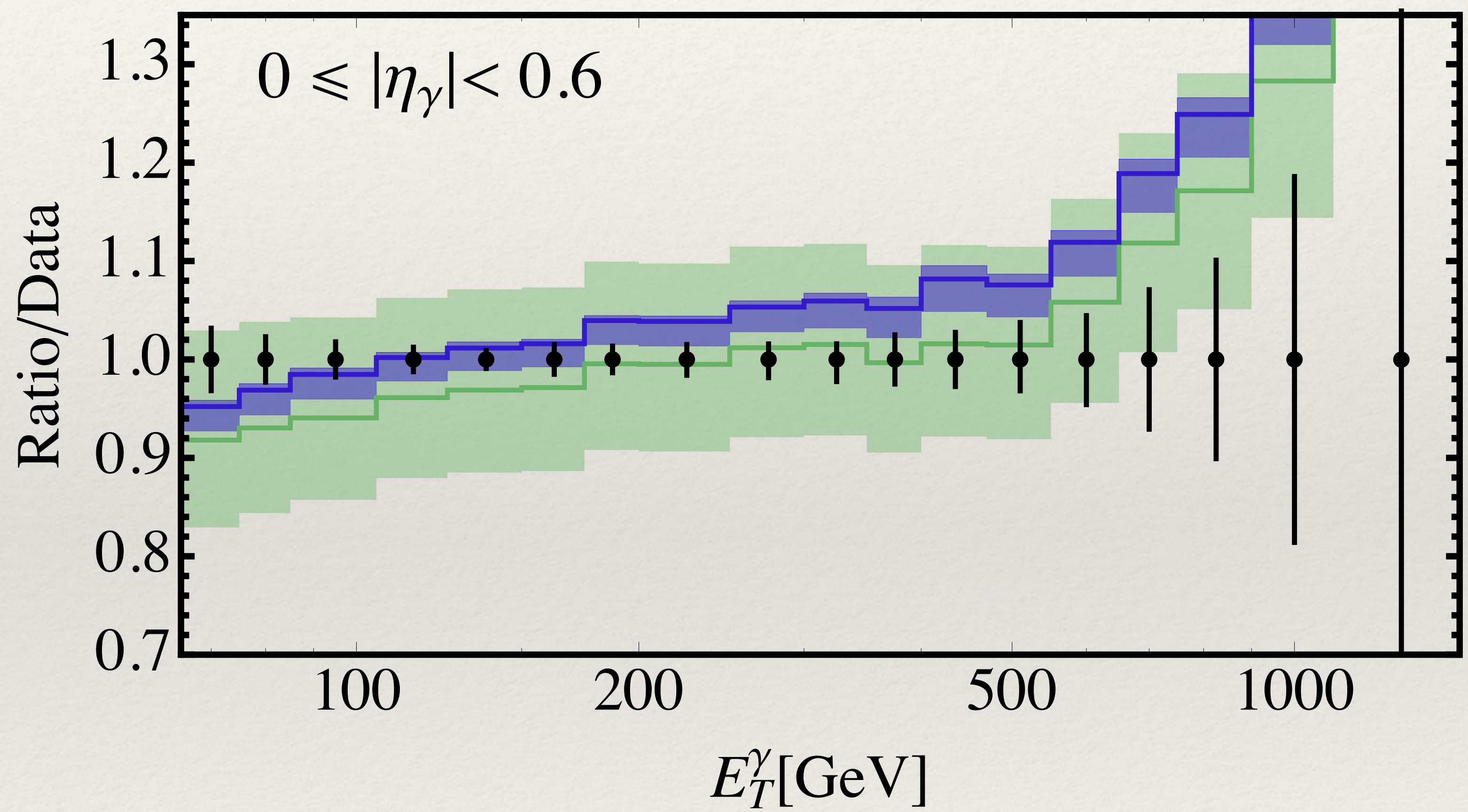
$\{\mu_R, \mu_F\} = \{\lambda_1 p_T^\gamma, \lambda_2 p_T^\gamma\}$ with $\lambda_i \in \{2, 1, 1/2\}$ and $\lambda_1 \neq \lambda_2^{-1}$.

- $\alpha(M_Z)$=1/127.9, larger than default value for DiPhox. (Until EW corrections are added one is free to choose either 137 or 127.9)

Direct photon results

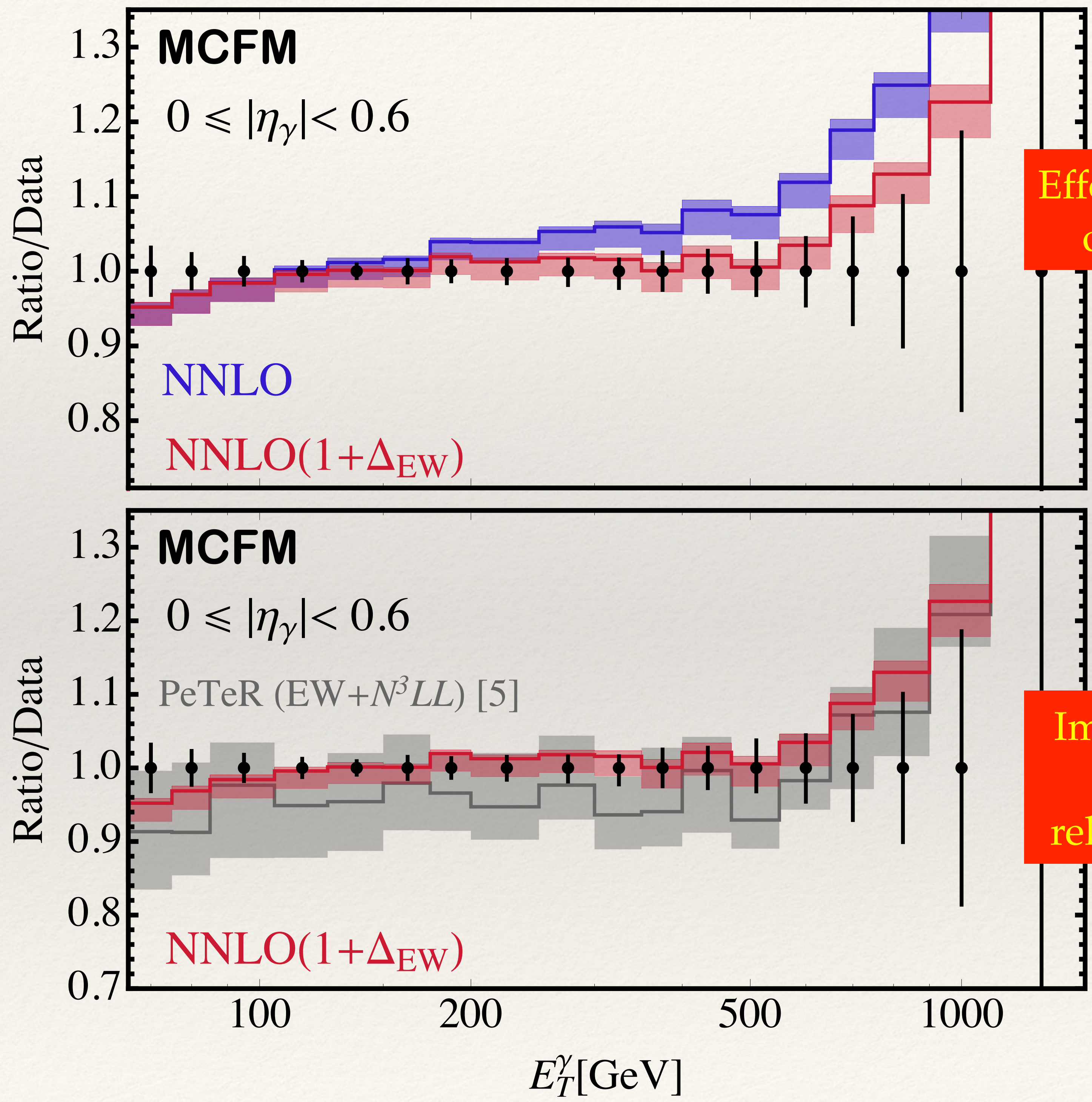
- The NNLO curve has much improved scale dependence, but fails to describe the shape of the data
- This especially the case at high momentum, where the theory would be expected to work best.

ATLAS data,1606.03495



Adding EW; comparison with N^{3}LL

- NNLO and EW together do a good job of describing the data
- Interesting that the data requires the inclusion of EW effects.
- Improvement of scale error relative to PeTeR, 1509.01961, 1606,1423 (pink vs grey).



Effect of inclusion of EW effects

Improvement of scale error relative to PeTeR

Conclusion

- Slicing based results for many colour singlet final states — proof of the method.
- Code for W,Z,H,WH available in MCFM-8.0 at mcfm.fnal.gov
- Doubly unresolved region isolated by a cut on τ_N (jettiness)
- Behaviour below the cut mandated by SCET.
- First NNLO results for direct photon.
- Extension to photon + jet imminent.
- New developments for power corrections, offer the possibility of a publicly distributed code for more complicated processes.