What else can we say that has not already been said

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“The visionary lies to himself, the liar only to others”, Friedrich Nietzsche

I don’t have visions, I have nightmares

The lesson of experiments 1973 - today:

extremely difficult to find a flaw in the SM

Maybe the SM includes elements of a truly fundamental theory.

But then how can one hope to make progress?

- With experimental guidance?
- Paying close attention to what we don’t understand precisely about the SM?
- Trying to better understand links between SM and mathematics?

From LEP to LHC: a paradigm shift

- LEP

  Hypothesis SM, a weakly coupled, strictly renormalizable[1] theory with one unknown, M_{H}

  Strategy test hypothesis vs. M_{H}. EWPD must be understood within this framework

[1] A theory with n Lagrangian parameters, requiring n data points, requiring n calculations; the $(n+1)$th calculation is a prediction

- LHC after the discovery, with a lack of direct evidence for new physics phenomena but having all the ingredients required to asses (in)consistency of the SM

$\mathbf{Theory_0}$ = SM, set $n = 1$

1. **$\mathbf{Model}_n$** = **$\mathbf{Theory}_{n-1}$** augmented with the inclusion of coupling modifiers, fails above a certain scale, Λ_n.
2. identify **$\mathbf{Theory}_n$** (strictly renormalizable and respecting perturbative unitarity) explaining the deviations found with **$\mathbf{Model}_n$**. Does it explain everything?
3. IF Yes THEN STOP
4. ELSE (but it should be able to explain something more than **$\mathbf{Theory}_{n-1}$**), set $n = n + 1$, GO TO **1**

Methodological antireductionism

It is possible that at some very large energy scale, all nonrenormalizable interactions disappear. This seems unlikely, given the difficulty with gravity. It is possible that the rules change drastically, it may even be possible that there is no end, simply more and more scales (Georgi).

This prompts the important question whether there is a last fundamental theory in this tower of EFTs which supersede each other with rising energies. Some people conjecture that this deeper theory could be a string theory, i.e. a theory which is not a field theory any more.

Epistemological antifoundationalism

Or should one ultimately expect from physics theories that they are only valid as approximations and in a limited domain? (Hartmann, Castellani)

What to do?

Strategy: define a "theory" of SM deviations

A "theory" deals with the well founded theoretical results obtained from first principles, while phenomenology deals with not so well founded effective "models" with a smaller domain of application[2]. Therefore,

- Experiments occur at finite energy and "measure" an effective action $\mathbf{S^{eff}(\Lambda)}$
- whatever QFT should give low energy $\mathbf{S^{eff}(\Lambda)}, \forall \mathbf{\Lambda} < \infty$
- One also assumes that there is no fundamental scale above which $\mathbf{S^{eff}(\Lambda)}$ is not defined [Costello2011] and
- $\mathbf{S^{eff}(\Lambda)}$ loses its predictive power in a process where the scale E approaches $\mathbf{\Lambda}$; indeed, $E = \mathbf{\Lambda}$ requires ∞ renormalized parameters [Preskill:1990]. Calculationally this is similar to the way renormalizable theories satisfy unitarity in perturbation theory; a perturbative expansion in $E/\mathbf{\Lambda}$ always becomes difficult at high enough energy

[2] For a definition see Hartmann (Studies in History and Philosophy of Modern Physics)

Why do we need a "theory" of SM deviations and a "model" is not enough?

The prototype of a model is the original kappa-framework

$$\mathscr{L}_{\mathrm{SM}}(\{m\},\{g\}) \quad \rightarrow \quad \mathscr{L}(\{m\},\{\kappa_g\, g\})$$

where $\{m\}, \{g\}$ denote the SM masses and couplings and κ_g are the scaling parameters. This is the framework used during Run 1.

It is a procedure used at LO, partially accommodating factorizable QCD corrections (but there are contributions which induce sizeable corrections unrelated to the SM ones [Gauld:2016kuu]) but not EW corrections.

Provisionally or permanently:

a theory of SM deviations

A possible solution is the SMEFT [3]. Priority goes to QCD NLO SMEFT, including those corrections that are unrelated to the SM. They provide a necessary ingredient for performing a global EFT fit to the LHC data at NLO accuracy. Examples:

① the production cross section of a Higgs boson in association with a $\overline{\mathbf{t}}\mathbf{t}$ pair at the LHC,

② production of Higgs bosons in loop-induced processes at the LHC, such as inclusive Higgs, $\mathbf{Hj}$ and $\mathbf{HH}$ production,

∋ all relevant $\mathrm{dim} = 6$ operators are included and not only a subset, [arXiv:1607.05330], (accurately avoiding field reparameterization). Subsets are, in general, not closed under renormalization. Wilson coefficients mix, e.g. there is a mixing between $\mathbf{a_{uBW}}(\mathbf{a_{dBW}})$ and $\mathbf{a_{ug}}(\mathbf{a_{dg}})$ from $\mathbf{Z} \to \overline{\mathbf{u}}\mathbf{u}(\overline{\mathbf{d}}\mathbf{d})$.

But there is more ...

[3] For your convenience ▸ Warsaw

How to include the corresponding (EW) THU? Conservatively

$$\mathrm{SM}\,(1+\delta_{EW})\,\kappa \;\pm\; \mathrm{SM}\,\delta_{EW}\,\kappa$$
$$\delta_{EW} = \text{ SM EW corrections}$$

The point, however, is that the EW corrections are not defined in the kappa framework if the kappa's are meant as coupling modifiers. Solution? Again SMEFT, where it's

(EW SM) + (EFT couplings in EW loops)

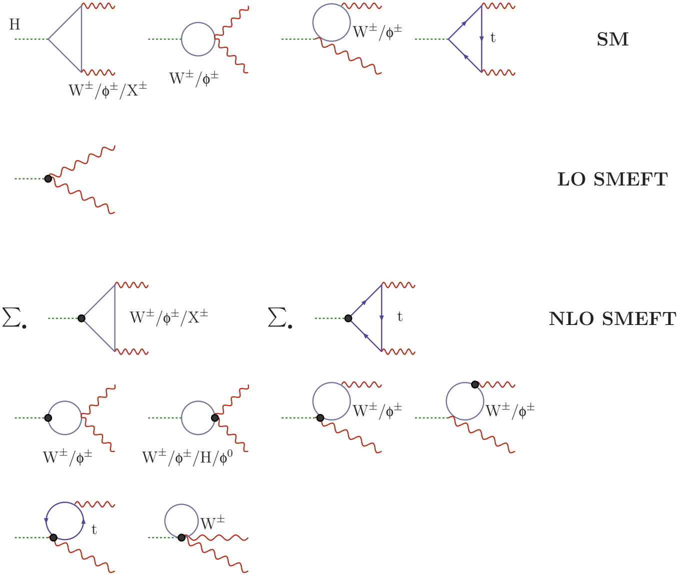
Effects should be computed and not covered by some unclear prescription. Obviously, kappas were introduced to "find" deviations not for "precision physics"

 Why not using a phenomenological approach, e.g. an extension of the SM Lagrangian with a limited number of interactions? Why the full SMEFT?

Phenomenological approaches are a reasonable starting point to describe limits on SM deviations. However, this outcome is much less desirable than dealing with a consistent SMEFT.

Furthermore, SMEFT is the generalized κ-framework, i.e. generalized kappas are linear combinations of Wilson coefficients, [arXiv:1505.03706]. Moreover, NLO SMEFT allows for generalized, resolved, scaling factors.

A brief introduction to SMEFT



Compendium Records

$\mathrm{CT}_{4,6} + \mathrm{Mix}$

$$\mathrm{A} = g^{N}\, \mathrm{A}^{(4)}_{\mathrm{LO}}(\{p\}) + g^{N}\, g_6\, \mathrm{A}^{(6)}_{\mathrm{LO}}(\{p\}\,,\,\{a\}) + \frac{1}{16\,\pi^2}\, g^{N+2}\, \mathrm{A}^{(4)}_{\mathrm{NLO}}(\{p\}) + \frac{1}{16\,\pi^2}\, g^{N+2}\, g_6\, \mathrm{A}^{(6)}_{\mathrm{NLO}}(\{p\}\,,\,\{a\})$$

CT_4

$\{p\} = \{g\,,\, g_s\,,\, \sin\theta_{\mathrm{W}}\,,\, M\,,\, M_{\mathrm{H}}\,,\, M_{\mathrm{f}}\} \in \mathrm{SM}$

$\{a\} =$ Wilson coeff. $\in$ Warsaw basis

$$\{p\}\,,\,\{a\} \longrightarrow \{p_{\mathrm{ren}}\}\,,\,\{a_{\mathrm{ren}}\} \longrightarrow \mathrm{IPS}\,,\,\{a_{\mathrm{ren}}(\mu_R)\}$$

$$\nearrow \underbrace{\mathrm{G}_F\,,\,\alpha_s\,,\,M_{\mathrm{W}}\,,\,M_{\mathrm{Z}}\,,\,M_{\mathrm{H}}}$$

CT = counterterm

$\mathscr{G}$

$$\begin{aligned}
\mathrm{p} &= \mathrm{p}_{\mathsf{ren}} \Big[1 + \frac{g^2}{16\,\pi^2} \left(\delta^{(4)}\,\mathrm{p} + g_6\,\delta^{(6)}\,\mathrm{p} \right) \Delta_{\mathrm{UV}} \\
&+ \frac{g_{\mathrm{S}}^2}{16\,\pi^2} \left(\delta_{\mathrm{S}}^{(4)}\,\mathrm{p} + g_6\,\delta_{\mathrm{S}}^{(6)}\,\mathrm{p} \right) \Delta_{\mathrm{UV}} \Big]
\end{aligned}$$

$$\begin{aligned}
\mathrm{Z}_{\psi_{\mathrm{L,R}}} &= 1 - \frac{g^2}{16\,\pi^2} \left(\delta^{(4)}\,\mathrm{Z}_{\psi_{\mathrm{L,R}}} + g_6\,\delta^{(6)}\,\mathrm{Z}_{\psi_{\mathrm{L,R}}} \right) \Delta_{\mathrm{UV}} \\
&- \frac{g_{\mathrm{S}}^2}{16\,\pi^2} \left(\delta_{\mathrm{S}}^{(4)}\,\mathrm{Z}_{\psi_{\mathrm{L,R}}} + g_6\,\delta_{\mathrm{S}}^{(6)}\,\mathrm{Z}_{\psi_{\mathrm{L,R}}} \right) \Delta_{\mathrm{UV}}
\end{aligned}$$

$$\Delta_{\mathrm{UV}} = \frac{2}{4-n} - \gamma_{\mathrm{E}} - \ln\pi - \ln\frac{\mu_{\mathrm{R}}^2}{\mu^2}$$

p	$\delta_{\mathrm{S}}^{(6)}\,\mathrm{p}$	ψ	$\delta_{\mathrm{S}}^{(6)}\,\mathrm{Z}_{\psi_{\mathrm{L,R}}}$
m_{u}	$+3\,\frac{M_{\mathrm{u}}}{M_{\mathrm{W}}}\,\mathrm{a_{d\,g}}$	u	$+\frac{M_{\mathrm{u}}}{M_{\mathrm{W}}}\,\mathrm{a_{u\,g}}$
m_{d}	$-3\,\frac{M_{\mathrm{d}}}{M_{\mathrm{W}}}\,\mathrm{a_{d\,g}}$	d	$-\frac{M_{\mathrm{u}}}{M_{\mathrm{W}}}\,\mathrm{a_{d\,g}}$

$$a_i = \mathrm{Z}_{ij}\, a_j^{\mathrm{R}} \qquad i,j \in \text{ Warsaw basis}$$

$$\mathrm{Z}_{ij} = \delta_{ij} + \frac{g^2}{16\,\pi^2}\,\delta\mathrm{Z}_{ij}\,\Delta_{\mathrm{UV}} + \frac{g_{\mathrm{S}}^2}{16\,\pi^2}\,\delta\mathrm{Z}_{ij}^{\mathrm{s}}\,\Delta_{\mathrm{UV}}$$

QCD induced mixings

i/j	$\mathrm{a}_{\mathrm{d\,g}}$	$\mathrm{a}_{\mathrm{A\,Z}}$	$\mathrm{a}_{\mathrm{u\,g}}$	$\mathrm{a}_{\mathrm{u\,BW}}$	$\mathrm{a}_{\mathrm{d\,BW}}$
$\mathrm{a}_{\mathrm{d}\,\phi}$	✓				
$\mathrm{a}_{\phi\,\mathrm{u\,V}}$		✓	✓		
$\mathrm{a}_{\phi\,\mathrm{d\,V}}$	✓	✓			
$\mathrm{a}_{\phi\,\mathrm{u\,A}}$			✓		
$\mathrm{a}_{\phi\,\mathrm{d\,A}}$	✓				
$\mathrm{a}_{\mathrm{u\,BW}}$			✓	✓	
$\mathrm{a}_{\mathrm{d\,BW}}$	✓				✓
		etc.			

What Could Possibly Go Wrong?

Fitting and interpreting data

Assume that nature has selected for us the singlet extension of the SM and that the heavy Higgs is (evidently) heavy so that we can integrate it out and use the corresponding EFT.

- Actually, the correct mathematical statement is: $\mathbf{v_S} >> \mathbf{v_{SM}}$, where $\mathbf{v_S}$ is the singlet VEV.
- The scale of NP ($\boldsymbol{\Lambda}$) is dictated by $\mathbf{v_S}$. In this limit also the mixing angle, defining $\mathbf{h}$ (light) and $\mathbf{H}$ (heavy), is small and should be expanded.
- Λ is not necessarily the mass of the next resonance, e.g.

$$M_{\mathrm{H}}^2 = 4\,\frac{\lambda_1}{g^2}\,\Lambda^2\left[1 + \frac{\lambda_{12}^2}{\lambda_1^2}\,\frac{M^2}{\Lambda^2} + \mathscr{O}\left(\Lambda^{-4}\right)\right]$$

Q1) Do we predict a change in $\mathbf{h} \to \gamma\gamma$ in the top-down approach? Yes, of course, due to mixing

Q2) Do we have a faithful simulation of the singlet extension of the SM in the bottom-up approach of EFT?

❶ When we integrate out $\mathbf{H}$ there is no "contact" $\mathbf{h}-\gamma-\gamma$ vertex ($\mathbf{h}/\mathbf{H}$ do not couple to photons). Therefore, in fitting (within the bottom-up approach) we should find that the "contact" term is zero

❷ The shift in $\mathbf{h} \to \gamma\gamma$ is due to the fact that in loops $\mathbf{h}$ interacts with fermions with a coupling proportional to

$$\cos\alpha \sim 1[\mathrm{SM}] + \mathcal{O}(\Lambda^{-2})[\dim = 6]$$

Therefore, the fit should tell us that the preferred solution is "loops".

Q3 However, we have analyzed the data with **LO SMEFT** and have interpreted the shift in terms of some "contact" $\mathbf{dim = 6}$ operator (loops remain strictly SM). Is it clever?

A better way? Pseudo-observable parametrization

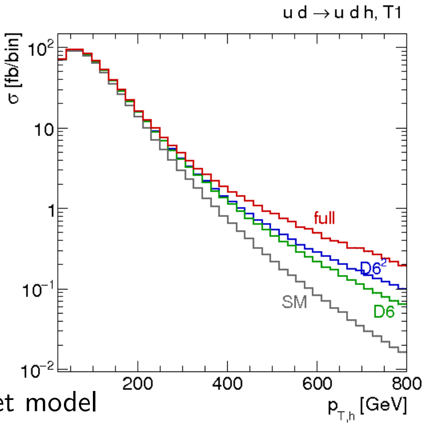
- The data will tell us there is a non-SM PO for $\mathbf{h \to \gamma\gamma}$. There are many possible interpretations, we don't know what is the correct one.
- One possible interpretation is that there is mixing between a singlet and the Higgs. Anyone who wants to test that specific model can use the constraints on the PO factor to infer a range of possible couplings/mass.
- Anyone wanting to avoid UV assumptions will use SMEFT. In doing so, they should ideally check how much their interpretation would be affected by accounting for NLO SMEFT contributions.


Questions for SMEFT

Is SMEFT good?

D6 vs. D6^2 in MHOU

open for discussion



Full = vector triplet model

Quoting T. Plehn "Forcing the EFT approach into a spectacular breakdown was the original aim of this paper, but to our surprise this did not happen"

Does EFT break down?

Obviously the scale should be such that $E \ll \Lambda$.

❶ There is complementarity between (H) "pole" vs. "tail" measurements: derivative operators influence tail observables and pole observables in a different way.

❷ Tails are interesting, the accessible Λ are

$$\Lambda_{\mathrm{pole}} \approx \frac{M_{\mathrm{H}}}{\sqrt{\delta_{\mathrm{exp}}(\mathrm{pole})}} \qquad \Lambda_{\mathrm{tail}} \approx \frac{Q}{\sqrt{\delta_{\mathrm{exp}}(\mathrm{tail})}}$$

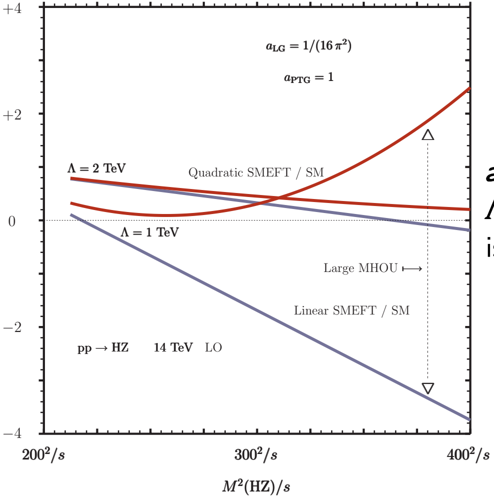
Unfortunately the SM will break in "tails" (or new physics will be seen before the breaking); projecting data into the SMEFT will have a large intrinsic uncertainty, i.e. we do not know what exactly is going on because the SMEFT interpretation becomes a series where the expansion parameter is close to 1 and/or the perturbative unitarity bound is saturated.

Understanding positivity and unitarity: LO helicity amplitudes for $\overline{\mathbf{q}}\mathbf{q} \to \mathbf{HZ}$, large M_{HZ} behavior

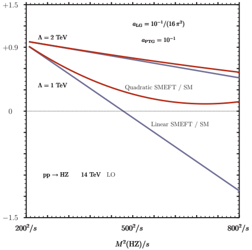
$$\mathrm{A}_{\lambda_{\overline{\mathrm{q}}}\,,\lambda_{\mathrm{q}}\,,\lambda_{\mathrm{Z}}}\,(M_{\mathrm{HZ}}\,,\cos\theta\ \text{fixed})$$

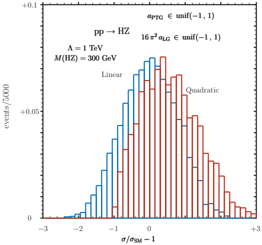
helicity	SM	one insertion	two insertions
$-+-$	$\frac{G_{\mathrm{F}}\,M_{\mathrm{Z}}^3}{M_{\mathrm{HZ}}}$	$\frac{M_{\mathrm{Z}}\,M_{\mathrm{HZ}}}{\Lambda^2}$	$\frac{M_{\mathrm{Z}}\,M_{\mathrm{HZ}}}{G_{\mathrm{F}}\Lambda^4}$
Wilson	$-$	$\mathrm{a}_{\mathrm{ZZ}}\,,\mathrm{a}^{(3)}_{\phi\mathrm{q}}\,,\mathrm{a}^{(1)}_{\phi\mathrm{q}}$	$\mathrm{a}_{\mathrm{AA}}\,,\mathrm{a}_{\mathrm{AZ}}\,,\mathrm{a}_{\mathrm{ZZ}}\,,\mathrm{a}_{\phi\mathrm{D}}\,,\mathrm{a}_{\phi\Box}\,,\mathrm{a}^{(3)}_{\phi\mathrm{q}}\,,\mathrm{a}^{(1)}_{\phi\mathrm{q}}$
$-+0$	$G_{\mathrm{F}}\,M_{\mathrm{Z}}^2$	$\frac{M^2_{\mathrm{HZ}}}{\Lambda^2}$	$\frac{M^2_{\mathrm{HZ}}}{G_{\mathrm{F}}\Lambda^4}$
Wilson	$-$	$\mathrm{a}^{(3)}_{\phi\mathrm{q}}\,,\mathrm{a}^{(1)}_{\phi\mathrm{q}}$	$\mathrm{a}_{\mathrm{AA}}\,,\mathrm{a}_{\mathrm{AZ}}\,,\mathrm{a}_{\mathrm{ZZ}}\,,\mathrm{a}_{\phi\mathrm{D}}\,,\mathrm{a}_{\phi\Box}\,,\mathrm{a}^{(3)}_{\phi\mathrm{q}}\,,\mathrm{a}^{(1)}_{\phi\mathrm{q}}$
$-++$	$\frac{G_{\mathrm{F}}\,M_{\mathrm{Z}}^3}{M_{\mathrm{HZ}}}$	$\frac{M_{\mathrm{Z}}\,M_{\mathrm{HZ}}}{\Lambda^2}$	$\frac{M_{\mathrm{Z}}\,M_{\mathrm{HZ}}}{G_{\mathrm{F}}\Lambda^4}$
Wilson	$-$	$\mathrm{a}_{\mathrm{ZZ}}\,,\mathrm{a}^{(3)}_{\phi\mathrm{q}}\,,\mathrm{a}^{(1)}_{\phi\mathrm{q}}$	$\mathrm{a}_{\mathrm{AA}}\,,\mathrm{a}_{\mathrm{AZ}}\,,\mathrm{a}_{\mathrm{ZZ}}\,,\mathrm{a}_{\phi\mathrm{D}}\,,\mathrm{a}_{\phi\Box}\,,\mathrm{a}^{(3)}_{\phi\mathrm{q}}\,,\mathrm{a}^{(1)}_{\phi\mathrm{q}}$

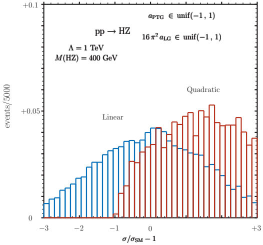
Note a/Λ^2 degeneracy

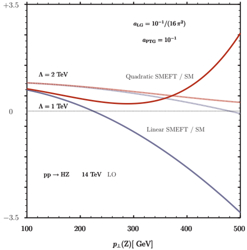


a fixed
Λ small
is large MHOU


+1.5
+0.9
0
−1.5
$a_{LG} = 10^{-1}/(16\,\pi^2)$
$a_{PTG} = 10^{-1}$
Λ = 2 TeV
Λ = 1 TeV
Quadratic SMEFT / SM
Linear SMEFT / SM
pp → HZ 14 TeV LO
$200^2/s$
$500^2/s$
$800^2/s$
$M^2(\mathrm{HZ})/s$


+0.1
+0.05
0
events/5000
$a_{PTG} \in \mathrm{unif}(-1\,,\,1)$
pp → HZ
$16\,\pi^2\,a_{LG} \in \mathrm{unif}(-1\,,\,1)$
Λ = 1 TeV
M(HZ) = 300 GeV
Linear
Quadratic
−3
−2
−1
0
+3
$\sigma/\sigma_{SM} - 1$


+0.1
+0.05
0
events/5000
$a_{PTG} \in \mathrm{unif}(-1\,,\,1)$
pp → HZ
$16\,\pi^2\,a_{LG} \in \mathrm{unif}(-1\,,\,1)$
Λ = 1 TeV
M(HZ) = 400 GeV
Quadratic
Linear
−3
−2
−1
0
+3
$\sigma/\sigma_{SM} - 1$


+3.5
0
−3.5
$a_{LG} = 10^{-1}/(16\,\pi^2)$
$a_{PTG} = 10^{-1}$
Λ = 2 TeV
Quadratic SMEFT / SM
Λ = 1 TeV
Linear SMEFT / SM
pp → HZ 14 TeV LO
100
200
300
400
500
$p_\perp(\mathrm{Z})[\,\mathrm{GeV}]$

FAQ and misunderstandings

- The "to square or not to square problem" has been analyzed (for specific model) only at LO
  - At NLO we have the "not to square",

$$| g^{\mathrm{N}} \mathscr{A}^{(4)}_{\mathrm{N}} + g^{\mathrm{K}} g_6 \mathscr{A}^{(6)}_{\mathrm{K},1,1} |^2 \rightsquigarrow | g^{\mathrm{N}} \mathscr{A}^{(4)}_{\mathrm{N}} |^2 + 2 g^{\mathrm{N}+\mathrm{K}} g_6 \, \mathrm{Re} \left[\mathscr{A}^{(4)}_{\mathrm{N}} \right]^{\dagger} \mathscr{A}^{(6)}_{\mathrm{K},1,1}$$

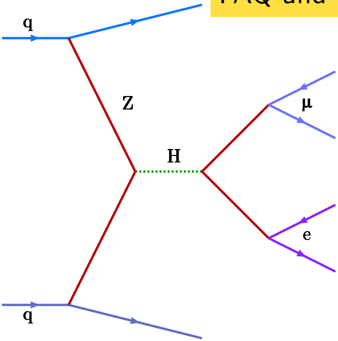
and the "square", i.e. the addition of

$$| g^{\mathrm{K}} g_6 \mathscr{A}^{(6)}_{\mathrm{K},1,1} |^2$$

  - In both cases we are missing something: e.g. at NLO the interference of $\mathscr{A}^{(4)}_{\mathrm{N}}$ with $g_6^2 \mathscr{A}^{(6)}_{\mathrm{K},2,1}$, i.e. double insertion of $\dim = 6$ operators (not to mention $\dim = 8$ operators)

$$\mathscr{A} = \sum_{n=\mathrm{N}}^{\infty} g^n \mathscr{A}^{(4)}_n + \sum_{n=\mathrm{N}_6}^{\infty} \sum_{l=1}^{n} \sum_{k=1}^{\infty} g^n g^l_{4+2k} \mathscr{A}^{(4+2k)}_{n\,l\,k}$$

FAQ and misunderstandings



$$\mathcal{A}_{\rm LO} = \alpha^3\, \mathcal{A}^{(\rm dim=4)} + \alpha^3 \sum_{n=1,6} g_6^n\, \mathcal{A}_n^{(\rm dim=6)}$$

where n denotes the number of $\mathbf{dim} = 6$ insertions

"not to square" $+\, \alpha^6\, g_6 \left[\mathcal{A}^{(\rm dim=4)}\right]^\dagger \mathcal{A}_1^{(\rm dim=6)}$

"to square" $+\, \alpha^6\, g_6^2\, \mid \mathcal{A}_1^{(\rm dim=6)} \mid^2$

"not to forget" $+\, \alpha^6\, g_6^2 \left[\mathcal{A}^{(\rm dim=4)}\right]^\dagger \mathcal{A}_2^{(\rm dim=6)}$

FAQ and misunderstandings

- The "to square" argument: "There are phase space region with a suppressed $\mathrm{dim} = 4$ prediction where the SMEFT expansion holds"
  - "expansion holds" is a questionable statement and, most likely, means "positive"
  - if $(\mathrm{dim} = 4) \times (\mathrm{dim} = 6)$ (LO) is suppressed what about $(\mathrm{dim} = 4) \times (\mathrm{dim} = 6)^2$ (LO)?
  - if $(\mathrm{dim} = 4) \times (\mathrm{dim} = 6)$ (LO) is suppressed what about $(\mathrm{dim} = 4) \times (\mathrm{dim} = 6)$ (NLO)?
- "to square" vs. "not to square" is certainly part of MHOU, the problem (process dependent) is on the central value

It is process and distribution dependent. The validity range of the perturbative expansion in SMEFT is poorly known and unitarity conditions can be used to get additional (rough) informations. Having said that ... EFTs could enjoy some range of non-perurbative validity before new physics is manifest (classicalization, asymptotic safety ...)

$\mathbf{V_L V_L \to V_L V_L}$ scattering with $M^2_{\mathrm{W}}, M^2_{\mathrm{Z}}, M^2_{\mathrm{H}} \ll s$. The SM result is well-known,

$$\frac{d}{dt}\,\sigma_{\mathrm{V_L V_L \to V_L V_L}} = \frac{\Big| T(s,t) \Big|^2}{16\,\pi\,s^2}, \qquad \mathrm{T}^0_{\mathrm{LO}} = \frac{1}{16\,\pi\,s}\int_{-s}^{0} dt\; T_{\mathrm{LO}}$$

$$\mathrm{T}^0_{\mathrm{LO}}\left(\mathrm{W}^+_{\mathrm{L}}\mathrm{W}^-_{\mathrm{L}} \to \mathrm{W}^+_{\mathrm{L}}\mathrm{W}^-_{\mathrm{L}}\right) \sim -\frac{G_{\mathrm{F}}\,M^2_{\mathrm{H}}}{4\,\sqrt{2}\,\pi} \qquad s \to \infty$$

with a critical mass

$$\Big|\mathbf{T^0_{LO}}\left(M_{\mathbf{H}} = M_c\right)\Big| = 1 \qquad M^2_c = \frac{4}{3}\sqrt{2}\,\pi\,G_{\mathbf{F}}^{-1}.$$

Perturbative unitarity is violated in SMEFT. However, one has to be careful in formulating the problem: the region of interest is $M^2_{\mathrm{W}}, M^2_{\mathrm{Z}}, M^2_{\mathrm{H}} \ll s \ll \Lambda^2$.

When s approaches Λ^2 the effective theory must be replaced by a complete renormalizable, unitary Lagrangian [4] and it makes no sense to study the limit $s \to \infty$ in the effective theory[5]. Consider partial-wave expansion, e.g.

$$\begin{aligned} \mathrm{T}^0_{\mathbf{SMEFT}} &= \frac{s}{16\pi\lambda\left(s, M^2_{\mathrm{W}}, M^2_{\mathrm{W}}\right)} \int_{-s+4M^2_{\mathrm{W}}}^{-t_0 s} dt \\ &\times \; T_{\mathbf{SMEFT}}\left(\mathrm{W}^+_{\mathrm{L}}\mathrm{W}^-_{\mathrm{L}} \to \mathrm{W}^+_{\mathrm{L}}\mathrm{W}^-_{\mathrm{L}}\right) \end{aligned}$$

with a cut $t_0 >> M^2_{\mathrm{W}}/s$ to avoid the Coulomb pole.

[4] i.e. there will be new physics *somewhere* in the neighborhood of the energy where SMEFT becomes so strongly coupled as to be difficult to treat in perturbation theory

[5] However, it is well known that heavy degrees of freedom may induce effects of *delayed* unitatity cancellation in the intermediate region and these effects could easily be detectable [Ahn:1988fx]

In the limit $M_{\mathrm{W}}^2, M_{\mathrm{Z}}^2, M_{\mathrm{H}}^2 \ll s \ll \Lambda^2$ we obtain the following result

$$\mathrm{T}^0_{\mathrm{SMEFT}} = -\frac{1}{32\pi} g_6 \, G_{\mathrm{F}} \, s \, (\mathrm{a}_{\phi \mathrm{D}} + 2\mathrm{a}_{\phi \Box}) + \mathscr{O}(1)$$

As expected the SM part contributes to the constant part while the part proportional to g_6 has a positive power of s (single diagrams grow as s^2). The leading behavior is controlled by $\mathrm{a}_{\phi \mathrm{D}} + 2\mathrm{a}_{\phi \Box}$, i.e.

$$\mid \mathrm{T}^0_{\mathrm{SMEFT}} \mid \sim \frac{1}{32\pi} (\mathrm{a}_{\phi \mathrm{D}} + 2\mathrm{a}_{\phi \Box}) \frac{s}{\Lambda^2} = 1$$

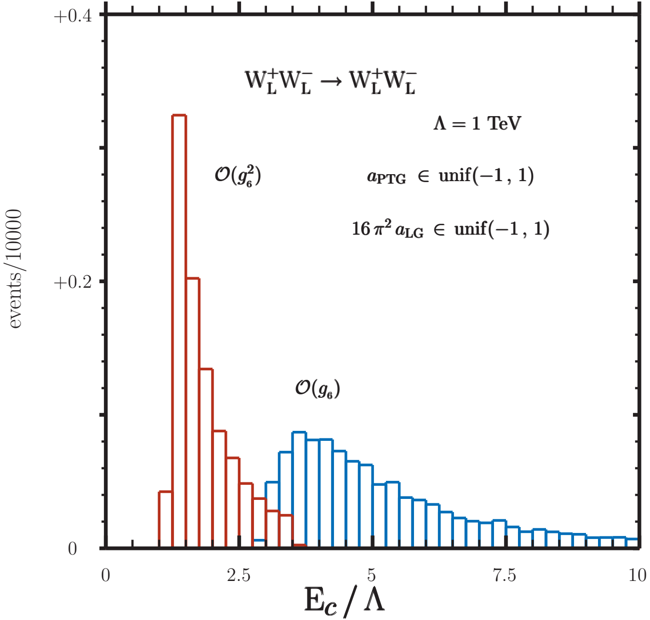
$$s \lesssim \frac{32\pi}{\mathrm{a}_{\phi \mathrm{D}} + 2\mathrm{a}_{\phi \Box}} \Lambda^2 = s_c$$

To be completed with subleadings and NLO; asymptotic LO requires $\mathrm{a}_{\phi \mathrm{D}} + 2\mathrm{a}_{\phi \Box} > 32\pi$ to have $s_c < \Lambda^2$

$j = 0$ coupled channels

	$W_L^+ W_L^-$	$\frac{1}{\sqrt{2}} Z_L Z_L$	$\frac{1}{\sqrt{2}} HH$
$W_L^+ W_L^-$	$-\frac{1}{32\pi}(a_{\phi D} + 2a_{\phi\Box})\frac{s}{\Lambda^2}$	$\frac{1}{32\sqrt{2}\pi}(a_{\phi D} - 4a_{\phi\Box})\frac{s}{\Lambda^2}$	$-\frac{1}{32\sqrt{2}\pi}(a_{\phi D} - 4a_{\phi\Box})\frac{s}{\Lambda^2}$
$\frac{1}{\sqrt{2}} Z_L Z_L$	$\frac{1}{32\sqrt{2}\pi}(a_{\phi D} - 4a_{\phi\Box})\frac{s}{\Lambda^2}$	const	$\frac{1}{32\pi}(a_{\phi D} + 2a_{\phi\Box})\frac{s}{\Lambda^2}$
$\frac{1}{\sqrt{2}} HH$	$-\frac{1}{32\sqrt{2}\pi}(a_{\phi D} - 4a_{\phi\Box})\frac{s}{\Lambda^2}$	$\frac{1}{32\pi}(a_{\phi D} + 2a_{\phi\Box})\frac{s}{\Lambda^2}$	const

N.B. with a restricted set, e.g. $\{a_{\phi WB}, a_{\phi W}, a_{\phi B}\}$ the leading behavior is constant it is "perturbative" unitarity, so what happens when we include double insertions?



▸ VH

Keep in mind, these are not statements that unitarity is violated in SMEFT. Unitarity *would be violated*, if we could trust the perturbative expansion, which we can't. There are perturbative unitarity bounds, but the bounds also imply that loops *must* be important

$$\mathrm{A}_j = \mathrm{A}_j^{\mathrm{LO}} + \mathrm{A}_j^{\mathrm{NLO}} + \dots$$

$$\mathrm{A}_j^{\mathrm{LO}} = \mathscr{A}_j\, s \quad \mathrm{A}_j^{\mathrm{NLO}} = \left[\mathscr{B}_j(\mu_{\mathrm{R}}) + \mathscr{C}_j \ln\frac{s}{\mu_{\mathrm{R}}^2} + \mathscr{D}_j \ln\frac{-s}{\mu_{\mathrm{R}}^2}\right] s^2$$

I one believes in Froissart bound (single dispersion relation in the momentum transfer is required) the total cross-section for two hadron interaction cannot grow faster than $\ln^2 E$.

SMEFT and the UV

How to use SMEFT?

- The pattern of suppressions for Wilson coefficients is not a SMEFT prediction but must be determined experimentally. Of course, it depends on the underlying UV completion but can be determined experimentally solely by using "low-energy" measurements that

  can be computed by using SMEFT

  - Of course this does not contradict the statement that a low energy effective theory is unable to accurately predict the scale of the next new physics

Low-energy theories, next resonances and all that

- $M_{\rm W}$ from Fermi theory, see section III.C of [arxiv:1601.07551]
- SM before LEP: how to use low-energy ($Q^2 \ll M^2_{\rm W}$) data points

$$g^2 = 4\,\pi\alpha(0) \quad M^2_{\rm W} = \frac{g^2}{4\,\sqrt{2}\,G_{\rm F}} \quad s^2_{\rm W} \text{ from } \frac{\sigma(\bar{\nu}_e\,e)}{\sigma(\nu_e\,e)}$$

plus radiative corrections. Back in the Eighties it was

$$\boldsymbol{M_{\rm W}} = \frac{37.281\ \ GeV}{s_{\rm W}}(1 + \text{radiative corr.})$$

$$s^2_{\rm W}\,|_{\rm exp} = 0.224 \pm 0.014 \quad \text{SLAC e-deuteron DIS}$$

not bad for a low-energy theory (SM expanded in Q^2) ......

- SM after LEP and before LHC: $M_{\rm H}$? Look at the blue-band! ($M_{\rm H} \to \infty$ breaks unitarity)

Once again, there is presently a lack of direct evidence for new physics phenomena at the accelerator energy frontier. From this state of affairs arises the need for a consistent theoretical framework in which deviations from the SM predictions can be calculated.

Such a framework should be applicable to comprehensively describe measurements in all sectors of particle physics: LHC Higgs measurements, past electroweak precision data, etc.

How to deal with EWPD?

Conventions at Lep: examples

❶ The quantity $\sigma_h = \sum \sigma_0^{\bar{f}f}$ is the de-convoluted hadronic peak cross-section, which by definition includes only the Z exchange

❷ For the de-convoluted forward-backward asymmetry, typically only the Z exchange is included and

- initial and final state QED corrections plus
- the eventual final state QCD corrections

☛ are assumed to be subtracted from the experimental data

Include $\dim = 6$ operators: QED deconvolution is not the same, LEP fits to be redone according to current hypothesis

$$\Gamma^{Z\to\bar{l}l}_{QED} = \frac{3}{4}\,\Gamma^{l}_{0}\,\frac{\alpha}{\pi}\left(1 + g_6\,\Delta^{(6)}_{QED}\right) \qquad \Gamma^{l}_{0} = \frac{G_F\,M^3_Z}{24\,\sqrt{2}\,\pi}\left(v_l^2 + 1\right)$$

$$\Delta^{(6)}_{QED} = 2\left(2 - s^2_\theta\right) a_{AA} + 2\,s^2_\theta\,a_{ZZ} + 2\left(\frac{c^3_\theta}{s_\theta} + \frac{512}{26}\,\frac{v_l}{v_l^2 + 1}\right) a_{AZ}$$

$$- \frac{1}{2}\,\frac{c^2_\theta}{s^2_\theta}\,a_{\phi D} + \frac{1}{v_l^2 + 1}\,\delta^{(6)}_{nfc}$$

$$\delta^{(6)}_{nfc} = \left(1 - 6\,v_l - v_l^2\right)\frac{1}{c^2_\theta}\left(s_\theta\,a_{AA} - \frac{1}{4}\,a_{\phi D}\right)$$

$$+ \left(1 + 2\,v_l - v_l^2\right)\left(a_{ZZ} + \frac{s_\theta}{c_\theta}\,a_{AZ}\right)$$

$$+ \frac{2}{c^2_\theta}\left(a_{\phi l A} + v_l\,a_{\phi l V}\right)$$

From [arXiv:1611.09879]

1. At one loop the number of SMEFT parameters contributing to the precise LEP1 POs exceeds the number of measurements
2. As a result the SMEFT parameters contributing to LEP data are formally unbounded when the size of loop corrections are reached until other data is considered in a global analysis
3. The size of these loop effects is generically a correction of order $\sim\%$ to leading effects in the SMEFT, with multiple large numerical coefficients

$$
\begin{aligned}
\mathscr{A}^{\mu}\,(\mathrm{Z}\to\bar{\mathrm{l}}\mathrm{l}) &= \mathscr{A}^{\mu}_{\mathrm{SM}} + \mathscr{A}^{\mu}_{\mathrm{LO\,SM}}\left[1+\frac{g_6}{4\sqrt{2}}\,\delta\mathscr{A}_{\mathrm{fc}}\right] + \frac{g\,g_6}{4\sqrt{2}}\,\delta\mathscr{A}^{\mu}_{\mathrm{nfc}} + \mathscr{O}(g^3\,g_6) \\
\delta\mathscr{A}_{\mathrm{fc}} &= 4\,s^2_\theta\,\mathrm{a_{A\,A}} - \mathrm{a_{\phi\,D}} + 8\,\mathrm{a_{\phi 1\,A}} \\
\delta\mathscr{A}^{\mu}_{\mathrm{nfc}} &= \delta\mathrm{V}\,\gamma^\mu + \delta\mathrm{A}\,\gamma^\mu\,\gamma^5, \quad \left(\delta\mathrm{T}\,\sigma^{\mu\nu}\,P_\nu \propto M_l\right) \\
\delta\mathrm{V} &= 8\,c_\theta\,s^2_\theta\,\mathrm{a_{A\,A}} - (5-4\,c^2_\theta)\,c_\theta\,\mathrm{a_{Z\,Z}} - (1-4\,c^2_\theta)\,s_\theta\,\mathrm{a_{A\,Z}} - 2\,c_\theta\,\mathrm{a_{\phi\,D}} + 2\,\frac{1-4\,s^2_\theta}{c_\theta}\,\mathrm{a_{\phi 1\,A}} - 2\,\frac{1}{c_\theta}\,\mathrm{a_{\phi 1\,V}} \\
\delta\mathrm{A} &= s_\theta\,\mathrm{a_{A\,Z}} - s_\theta\,\mathrm{a_{Z\,Z}}
\end{aligned}
$$

Finally: is SMEFT consistent?

- All sets of gauge invariant, dimension d operators, none of which is redundant, form a basis and all bases are equivalent. For a formal definition of redundancy see [Einhorn:2013kja]. Avoid field reparameterization[6]

① What about closure w.r.t. renormalization?

② What about IR/collinear singularities?

[6] For different opinions 

① Technically speaking this would require proving removal of UV poles for ALL off-shell Green's functions, too much in the present moment. Closure is proven for all on-shell Green's functions relevant for Higgs physics and EWPD, [arXiv:1607.01236]

② Yes, they cancel

Anomalies?

Inclusion of triple/quadrupole gauge couplings: this brings us to gauge anomalies and anomaly cancellation; perhaps, a deeper understanding of SMEFT, a low-energy limit of an underlying anomaly-free theory?

Proposition

SMEFT anomalies are UV finite (it is good for renormalizability), restoring gauge invariance order-by-order by adding finite counterterms, i.e. it is possible to quantize an anomalous theory in a manner that respects WSTI [Preskill:1990fr] and local. The latter is good for unitarity, another tiny step forward.

UV: beware of mixing

Finding “one” large scale that controls the low energy behavior is not trivial

SM decoupling limit requires additional conditions. Furthermore:

More than one scalar? Hierarchy of VEVs? Again a serious fine-tuning. Are the small mixings accidental or systematic, i.e. symmetry?

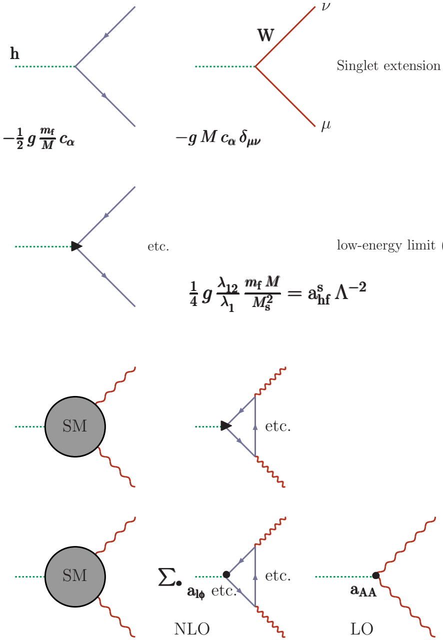
$$\begin{aligned}\mathscr{L}_{\rm S} &= -\left(\mathrm{D}_\mu\,\Phi\right)^\dagger \mathrm{D}_\mu\,\Phi - \partial_\mu\,\chi\,\partial_\mu\,\chi - \mu_2^2\,\Phi^\dagger\,\Phi - \mu_1^2\,\chi^2 - \frac{1}{2}\,\lambda_2\,\left(\Phi^\dagger\,\Phi\right)^2 \\ &- \frac{1}{2}\,\lambda_1\,\chi^4 - \lambda_{12}\,\chi^2\,\Phi^\dagger\,\Phi\end{aligned}$$

- The SM decoupling limit cannot be obtained by making only assumptions about one parameter (the "large" scale). In the mass eigenbasis the **large scale is $M_{\rm s}$**:

$$\chi = \frac{1}{\sqrt{2}}\,(\mathrm{h}_1 + \mathrm{v}_{\rm s}) \qquad M_{\rm s}^2 = \frac{1}{4}\,g^2\,\mathrm{v}_{\rm s}^2$$

- The behavior of the mixing angle is not selected a priori but follows from the hierarchy of VEVs. Additional suppression of the heavy mode can be imposed by requiring $\lambda_{12} \propto g^2\,M/M_{\rm s}$, i.e. this additional suppression is an independent condition. In any case, the SM decoupling limit cannot be obtained by making only assumptions about one parameter

Singlet extension of the SM



low-energy limit ($M_s \to \infty$)

$$\frac{1}{4}\, g\, \frac{\lambda_{12}}{\lambda_1}\, \frac{m_f\, M}{M_s^2} = a^s_{hf}\, \Lambda^{-2}$$

$$\mathscr{L}_{\mathrm{THDM}} = -\sum_{i=1,2}\left(\mathrm{D}_\mu\,\Phi_i\right)^\dagger \mathrm{D}_\mu\,\Phi_i + \sum_{i=1,2}\mu_i^2\,\Phi_i^\dagger\,\Phi_i + \mu_3^2\left(\Phi_1^\dagger\,\Phi_2 + \Phi_2^\dagger\,\Phi_1\right)$$
$$+ \frac{1}{2}\sum_{i=1,2}\lambda_i\left(\Phi_i^\dagger\,\Phi_i\right)^2 + \lambda_3\,\Phi_1^\dagger\,\Phi_1\,\Phi_2^\dagger\,\Phi_2 + \lambda_4\,\Phi_1^\dagger\,\Phi_2\,\Phi_2^\dagger\,\Phi_1 + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\,\Phi_2\right)^2 + \left(\Phi_2^\dagger\,\Phi_1\right)^2\right]$$

$$\Phi_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{h}_i + \sqrt{2}\,\mathrm{v}_i + i\,\phi^0 \\ \sqrt{2}\,i\,\phi^- \end{pmatrix}$$

$$\mathrm{h}_1 = -\mathrm{v}_1 + \cos\beta\left(\mathrm{h}_1' + \mathrm{v}\right) - \sin\beta\,\mathrm{h}_2' \quad \phi_1^0 = \cos\beta\,\phi^0 - \sin\beta\,\mathrm{A}^0 \quad \phi_1^\pm = \cos\beta\,\phi^\pm - \sin\beta\,\mathrm{H}^\pm$$
$$\mathrm{h}_2 = -\mathrm{v}_2 + \sin\beta\left(\mathrm{h}_1' + \mathrm{v}\right) - \cos\beta\,\mathrm{h}_2' \quad \phi_2^0 = \sin\beta\,\phi^0 + \sin\beta\,\mathrm{A}^0 \quad \phi_2^\pm = \sin\beta\,\phi^\pm + \cos\beta\,\mathrm{H}^\pm$$

with $\mathrm{v}^2 = \mathrm{v}_1^2 + \mathrm{v}_2^2$.

$$\mathrm{h}_1' = \cos(\alpha-\beta)\,\mathrm{H} - \sin(\alpha-\beta)\,\mathrm{h} \qquad \mathrm{h}_2' = \sin(\alpha-\beta)\,\mathrm{H} + \cos(\alpha-\beta)\,\mathrm{h}$$

$$\mu_3^2 = \sin\beta\,\cos\beta\,\overline{M}^2 \quad \mathrm{v}^2\,\lambda_5 = M_{\mathrm{A}\,0}^2 - \overline{M}^2 \quad \mathrm{v}^2\,\lambda_4 = 2\,\beta_1 + 2\,\frac{\cos^2\beta - \sin^2\beta}{\sin\beta\cos\beta}\,\beta_2 + 2\,M_{\mathrm{H}\,\pm}^2 - M_{\mathrm{A}\,0}^2 - \overline{M}^2$$

$$\mathrm{v}^2\,\lambda_2 = \mathrm{v}^2\left(2\,\overline{\lambda} - \lambda_1\right) + \overline{M}^2 - M_{22}^2\sin^2\beta\cos^2\beta$$
$$\mathrm{v}^2\,\overline{\lambda} = \frac{1}{2}\left(\mathrm{v}^2\,\lambda_1\,\frac{\sin^4\beta - \cos^4\beta}{\sin^2\beta} + \frac{M_{22}^2 - \overline{M}^2}{\cos^2\beta} - \frac{M_{11}^2}{\sin^2\beta}\right)$$
$$\mathrm{v}^2\,\lambda_1 = 2\tan\beta\,M_{12}^2 + \tan^2\beta\left(\overline{M}^2 - M_{22}^2\right) - M_{11}^2$$

The low energy scenario for the THDM is defined by $\Lambda = \overline{M} >> \mathrm{v}$ and **$\beta = 1/2(\pi - \delta_\beta)$, $\alpha = 1/2\,\delta_\alpha$**

- The task is not to see what no else has seen but to think what no else has thought about that which everyone else has seen
- The problem is not how to imagine wild scenarios, the problem is how to arrive to the correct scenario by making only small steps, without having to make unreasonable assumptions[7].
- We have the Standard Model of particle physics with coupling strengths that we do not know how to derive, but which can be measured accurately.

[7] Resisting the temptation to admit that the regulative ideal of an ultimate theory remains a powerful aesthetic ingredient

Thank you for your attention


Backup Slides

Appendix C. Dimension-Six Basis Operators for the SM[22].

Einhorn, Wudka

Grzadkowski, Iskrzynski, Misiak, Rosiek

X^3 (LG)		φ^6 and $\varphi^4 D^2$ (PTG)		$\psi^2\varphi^3$ (PTG)	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$ (LG)		**$\psi^2 X\varphi$ (LG)**		**$\psi^2\varphi^2 D$ (PTG)**	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\,\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\,\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

Table C.1: Dimension-six operators other than the four-fermion ones.

[22] These tables are taken from [5], by permission of the authors.

More in the next slide

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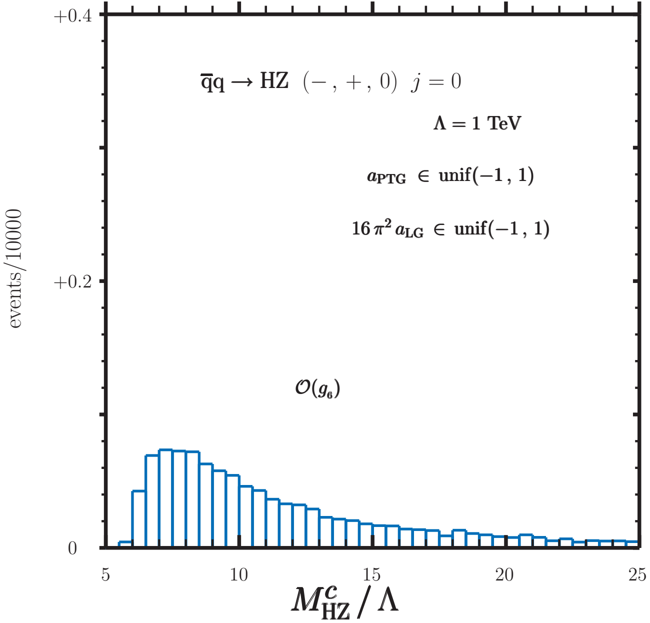
Table: Definition of linear combinations of Wilson coefficients. Here $s_\theta(c_\theta)$ denotes the sine(cosine) of the renormalized weak-mixing angle

$$
\begin{aligned}
\mathrm{a}_{\mathrm{AA}} &= c_\theta^2\, \mathrm{a}_{\phi\mathrm{B}} + s_\theta^2\, \mathrm{a}_{\phi\mathrm{W}} + s_\theta\, c_\theta\, \mathrm{a}_{\phi\mathrm{WB}} \\
\mathrm{a}_{\mathrm{ZZ}} &= s_\theta^2\, \mathrm{a}_{\phi\mathrm{B}} + c_\theta^2\, \mathrm{a}_{\phi\mathrm{W}} - s_\theta\, c_\theta\, \mathrm{a}_{\phi\mathrm{WB}} \\
\mathrm{a}_{\mathrm{AZ}} &= 2\, c_\theta\, s_\theta \left(\mathrm{a}_{\phi\mathrm{W}} - \mathrm{a}_{\phi\mathrm{B}}\right) + \left(2\, c_\theta^2 - 1\right) \mathrm{a}_{\phi\mathrm{WB}}
\end{aligned}
$$

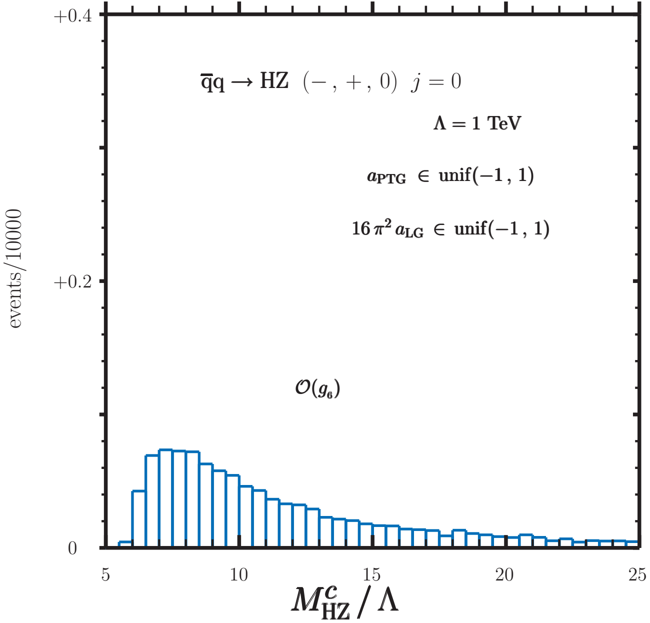
In Tab. 2 $l, u/d$ (e.g. in $\mathrm{a}_{\mathrm{l\,W}}$) denote arbitrary leptons, up/down quarks. The whole renormalization procedure does not assume flavor universality.

Table: Definition of linear combinations of Wilson coefficients. Here $s_\theta(c_\theta)$ denotes the sine(cosine) of the renormalized weak-mixing angle

$\mathrm{a}_{\mathrm{l\,W}} = s_\theta\, \mathrm{a}_{\mathrm{l\,WB}} + c_\theta\, \mathrm{a}_{\mathrm{l\,BW}}$	$\mathrm{a}_{\mathrm{l\,B}} = s_\theta\, \mathrm{a}_{\mathrm{l\,BW}} - c_\theta\, \mathrm{a}_{\mathrm{l\,WB}}$
$\mathrm{a}_{\mathrm{d\,W}} = s_\theta\, \mathrm{a}_{\mathrm{d\,WB}} + c_\theta\, \mathrm{a}_{\mathrm{d\,BW}}$	$\mathrm{a}_{\mathrm{d\,B}} = s_\theta\, \mathrm{a}_{\mathrm{d\,BW}} - c_\theta\, \mathrm{a}_{\mathrm{d\,WB}}$
$\mathrm{a}_{\mathrm{u\,W}} = s_\theta\, \mathrm{a}_{\mathrm{u\,WB}} + c_\theta\, \mathrm{a}_{\mathrm{u\,BW}}$	$\mathrm{a}_{\mathrm{u\,B}} = c_\theta\, \mathrm{a}_{\mathrm{u\,WB}} - s_\theta\, \mathrm{a}_{\mathrm{u\,BW}}$
$\mathrm{a}^{(3)}_{\phi\mathrm{l}} - \mathrm{a}^{(1)}_{\phi\mathrm{l}} = \frac{1}{2}\left(\mathrm{a}_{\phi\mathrm{l\,V}} + \mathrm{a}_{\phi\mathrm{l\,A}}\right)$	$\mathrm{a}_{\phi\mathrm{l}} = \frac{1}{2}\left(\mathrm{a}_{\phi\mathrm{l\,A}} - \mathrm{a}_{\phi\mathrm{l\,V}}\right)$
$\mathrm{a}_{\phi\mathrm{u\,V}} = \mathrm{a}^{(3)}_{\phi\mathrm{q}} + \mathrm{a}_{\phi\mathrm{u}} + \mathrm{a}^{(1)}_{\phi\mathrm{q}}$	$\mathrm{a}_{\phi\mathrm{u\,A}} = \mathrm{a}^{(3)}_{\phi\mathrm{q}} - \mathrm{a}_{\phi\mathrm{u}} + \mathrm{a}^{(1)}_{\phi\mathrm{q}}$
$\mathrm{a}_{\phi\mathrm{d\,V}} = \mathrm{a}^{(3)}_{\phi\mathrm{q}} - \mathrm{a}_{\phi\mathrm{d}} - \mathrm{a}^{(1)}_{\phi\mathrm{q}}$	$\mathrm{a}_{\phi\mathrm{d\,A}} = \mathrm{a}^{(3)}_{\phi\mathrm{q}} + \mathrm{a}_{\phi\mathrm{d}} - \mathrm{a}^{(1)}_{\phi\mathrm{q}}$



$\bar{q}q \to HZ \ (-,+,0) \ j=0$
$\Lambda = 1$ TeV
$a_{PTG} \in$ unif$(-1, 1)$
$16\pi^2 a_{LG} \in$ unif$(-1, 1)$
$\mathcal{O}(g_6)$
events/10000
+0.4
+0.2
0
5
10
15
20
25
M^c_{HZ} / Λ



$\bar{q}q \to HZ \;(-,+,0)\; j=0$
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$\mathcal{O}(g_6)$
events/10000
+0.4
+0.2
0
5
10
15
20
25
M^c_{HZ} / Λ

Summary of summaries

positivity violation

$$| \mathrm{T}_j |^2 \approx | \mathrm{T}_j^{(4)} |^2 + 2\,\mathrm{T}_j^{(4)}\,\mathrm{T}_j^{(6)} = 0$$

perturbative unitarity violation

$$| \mathrm{T}_j |^2 \approx | \mathrm{T}_j^{(4)} + \mathrm{T}_j^{(6)} |^2 \leq | \mathrm{T}_j^{(4)} + \mathrm{T}_j^{(6)} | = 1$$

"to square" has to stop somewhere; unfortunately $| T_j^{\mathrm{LO}} | = 1$ is too conservative, giving scales that are too far away from where positivity breaks down. LO is not enough and loops are needed, e.g.

$$| \mathrm{T}_j^{\mathrm{LO}} |^2 + 2\,\mathrm{T}_j^{\mathrm{LO}}\,\mathrm{Re}\,\mathrm{T}_j^{\mathrm{NLO}} \leq \mathrm{Im}\,\mathrm{T}_j^{\mathrm{NLO}} + \mathrm{Im}\,\mathrm{T}_j^{\mathrm{NNLO}}$$

SMEFT a longer story

How to connect intermediate POs with Wilson coefficients?

Example:

❶ The amplitude for the process $\mathrm{H}(P) \to \gamma_\mu(p_1)\gamma_\nu(p_2)$ can be written as

$$\mathrm{A}^{\mu\nu}_{\mathrm{HAA}} = i\,\mathscr{T}_{\mathrm{HAA}}\,\mathrm{T}^{\mu\nu} = -i\,\frac{2}{\mathrm{v_F}\,M^2_{\mathrm{H}}}\,\varepsilon_{\gamma\gamma}\,\mathrm{T}^{\mu\nu}$$

$$M^2_{\mathrm{H}}\,\mathrm{T}^{\mu\nu} = p_2^\mu\,p_1^\nu - p_1\cdot p_2\,g^{\mu\nu}$$

❷ A convenient way for writing the amplitude is the following: after renormalization we neglect all fermion masses but $m_{\mathrm{t}}, m_{\mathrm{b}}$ and write

$$\mathscr{T}_{\mathrm{HAA}} = \frac{g^3_{\mathrm{F}}\,\mathrm{s}^2_{\mathrm{W}}}{8\,\pi^2}\sum_{\mathrm{I=W,t,b}}\rho^{\mathrm{HAA}}_{\mathrm{I}}\,\mathscr{T}^{\mathrm{I}}_{\mathrm{HAA;LO}} + g_{\mathrm{F}}\,g_6\,\frac{M^2_{\mathrm{H}}}{M_{\mathrm{W}}}\,\mathrm{a_{AA}} + \frac{g^3_{\mathrm{F}}\,g_6}{\pi^2}\,\mathscr{T}^{\mathrm{nf}}_{\mathrm{HAA}}$$

$\mathcal{G}$

❸ Introduce $g_F^2 = 4\sqrt{2}\, G_F\, M_W^2$ and $c_w = M_W / M_Z$ (note that, at this point we have selected the $\{G_F\,,\,M_Z\,,\,M_W\}$ IPS, alternatively one could use the $\{\alpha\,,\,G_F\,,\,M_Z\}$) and derive

$$
\begin{aligned}
\kappa_I^{HAA} &= \frac{g_F^3\, s_W^2}{8\,\pi^2}\, \rho_I^{HAA} \\
\kappa_c^{HAA} &= g_F\, \frac{M_H^2}{M_W}\, a_{AA}
\end{aligned}
$$

$$
\begin{aligned}
a_{ZZ} &= s_W^2\, a_{\phi B} + c_W^2\, a_{\phi W} - s_W\, c_W\, a_{\phi WB} \\
a_{AA} &= c_W^2\, a_{\phi B} + s_W^2\, a_{\phi W} + s_W\, c_W\, a_{\phi WB} \\
a_{AZ} &= 2\, c_W\, s_W\, (a_{\phi W} - a_{\phi B}) + \left(2\, c_W^2 - 1\right) a_{\phi WB}
\end{aligned}
$$

𝒢

❹ The (process dependent) **ρ**-factors are given by

$$\rho^{\text{proc}}_{\mathrm{I}} = 1 + g_6\, \Delta\rho^{\text{proc}}_{\mathrm{I}}$$

and there are additional, non-factorizable, contributions. For $\mathrm{H} \to \gamma\gamma$ the $\Delta\rho$ factors are as follows:

$$\begin{aligned}
\Delta\rho^{\mathrm{HAA}}_{\mathrm{t}} &= \frac{3}{16}\,\frac{M^2_{\mathrm{H}}}{\mathrm{s_W}\, M^2_{\mathrm{W}}}\,\mathrm{a_{t\,WB}} + (2 - \mathrm{s^2_W})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\,\mathrm{a_{AZ}} + (6 - \mathrm{s^2_W})\,\mathrm{a_{AA}} \\
&- \frac{1}{2}\left[\mathrm{a_{\phi\,D}} + 2\,\mathrm{s^2_W}\,(\mathrm{c^2_W}\,\mathrm{a_{ZZ}} - \mathrm{a_{t\,\phi}} - 2\,\mathrm{a_{\phi\,\Box}})\right]\frac{1}{\mathrm{s^2_W}} \\
\Delta\rho^{\mathrm{HAA}}_{\mathrm{b}} &= -\frac{3}{8}\,\frac{M^2_{\mathrm{H}}}{\mathrm{s_W}\, M^2_{\mathrm{W}}}\,\mathrm{a_{b\,WB}} + (2 - \mathrm{s^2_W})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\,\mathrm{a_{AZ}} + (6 - \mathrm{s^2_W})\,\mathrm{a_{AA}} \\
&- \frac{1}{2}\left[\mathrm{a_{\phi\,D}} + 2\,\mathrm{s^2_W}\,(\mathrm{c^2_W}\,\mathrm{a_{ZZ}} + \mathrm{a_{b\,\phi}} - 2\,\mathrm{a_{\phi\,\Box}})\right]\frac{1}{\mathrm{s^2_W}} \\
\Delta\rho^{\mathrm{HAA}}_{\mathrm{W}} &= (2 + \mathrm{s^2_W})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\,\mathrm{a_{AZ}} + (6 + \mathrm{s^2_W})\,\mathrm{a_{AA}} - \frac{1}{2}\left[\mathrm{a_{\phi\,D}} - 2\,\mathrm{s^2_W}\,(2\,\mathrm{a_{\phi\,\Box}} + \mathrm{c^2_W}\,\mathrm{a_{ZZ}})\right]\frac{1}{\mathrm{s^2_W}}
\end{aligned}$$

$\mathscr{G}$

❺ In the PTG [8] scenario we only keep $\mathbf{a_{t\phi}, a_{b\phi}, a_{\phi D}}$ and $\mathbf{a_{\phi\Box}}$. These results tell us that κ-factors can be introduced also at the loop level; they are combinations of Wilson coefficients but we have to extend the scheme with the inclusion of process dependent, non-factorizable, contributions.

We also derive the following result for the non-factorizable part of the amplitude (in the PTG scenario all non-factorizable amplitudes for $\mathrm{H} \to \gamma\gamma$ vanish):

$$\mathscr{T}^{\mathrm{nf}}_{\mathrm{HAA}} = M_{\mathrm{W}} \sum_{a \in \{A\}} \mathscr{T}^{\mathrm{nf}}_{\mathrm{HAA}}(a)\, a$$

$$\{A\} = \{\mathrm{a_{t\,WB}}, \mathrm{a_{b\,WB}}, \mathrm{a_{AA}}, \mathrm{a_{AZ}}, \mathrm{a_{ZZ}}\}$$

[8] Potentially Tree Generated