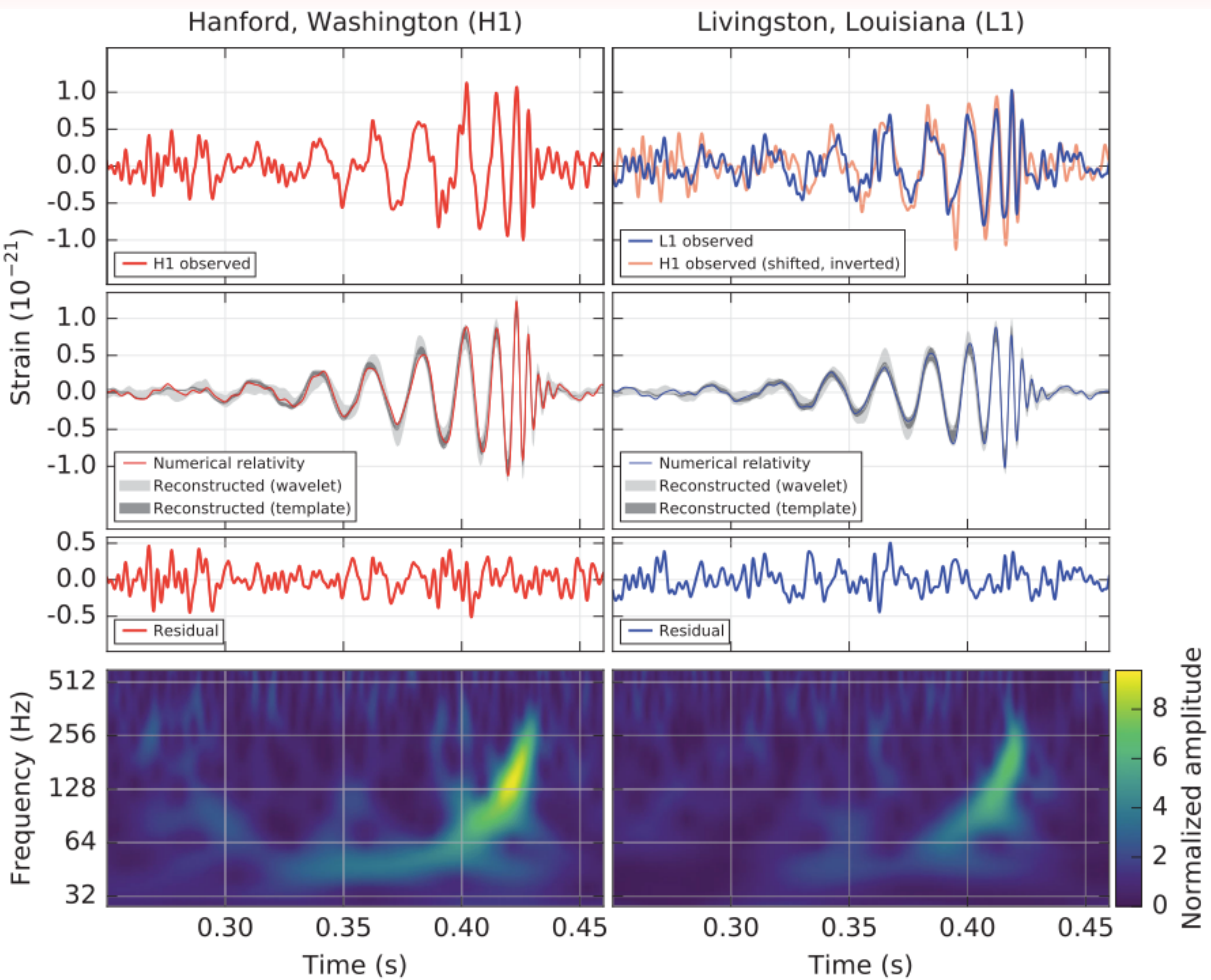
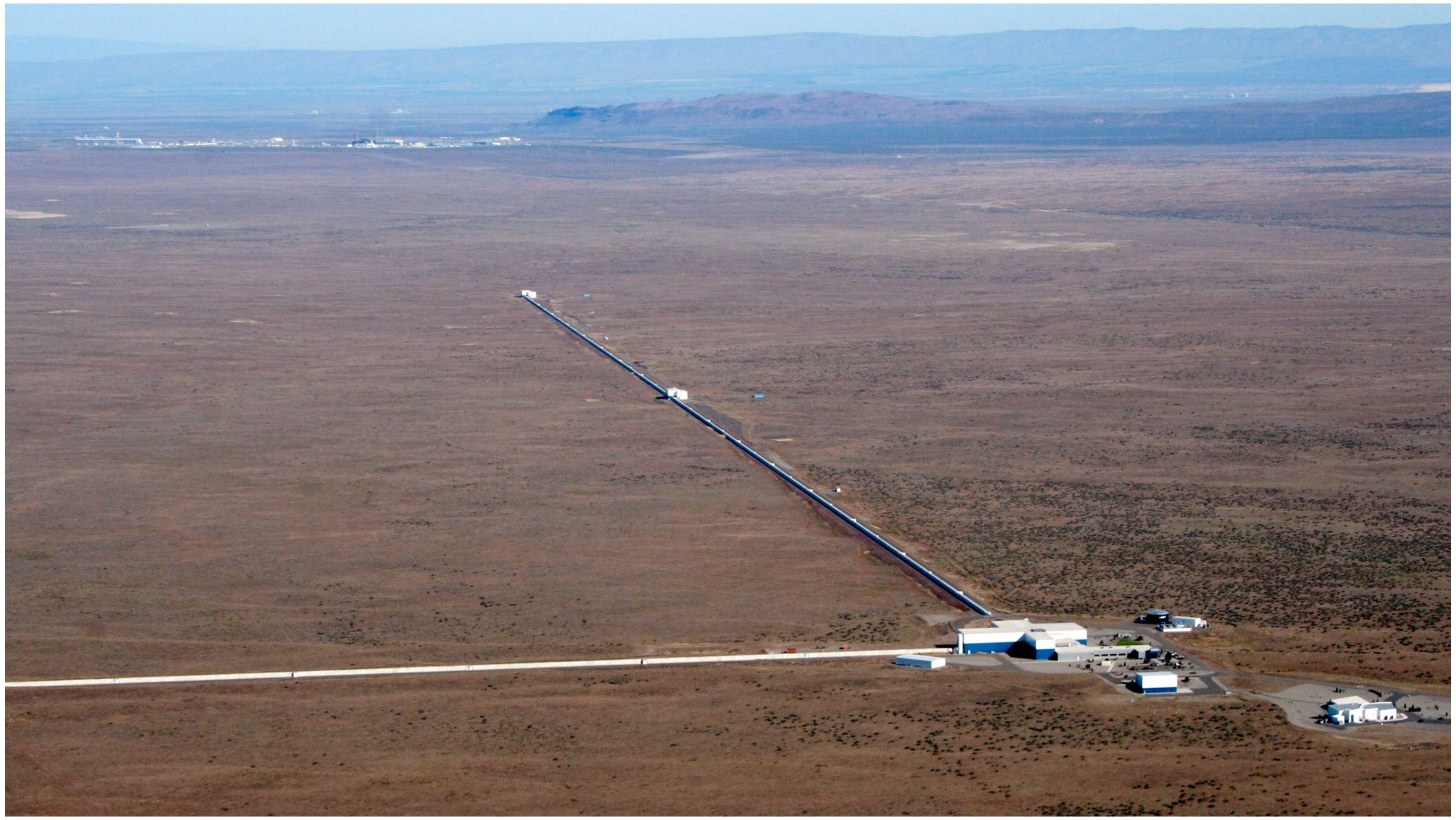
Raffaele Tito D’Agnolo
EuCARD-2 XBEAM Strategy Workshop 13/12/2017

STORAGE RINGS AS GRAVITATIONAL WAVE ANTENNAS

A WONDERFUL OBSERVATION



LIGO



- BIG
  (few km)
- VERY PRECISE
  (Position of mirrors, …)

LHC

- BIG
  (few km)
- VERY PRECISE
  (Position of protons)

LIGO vs LHC

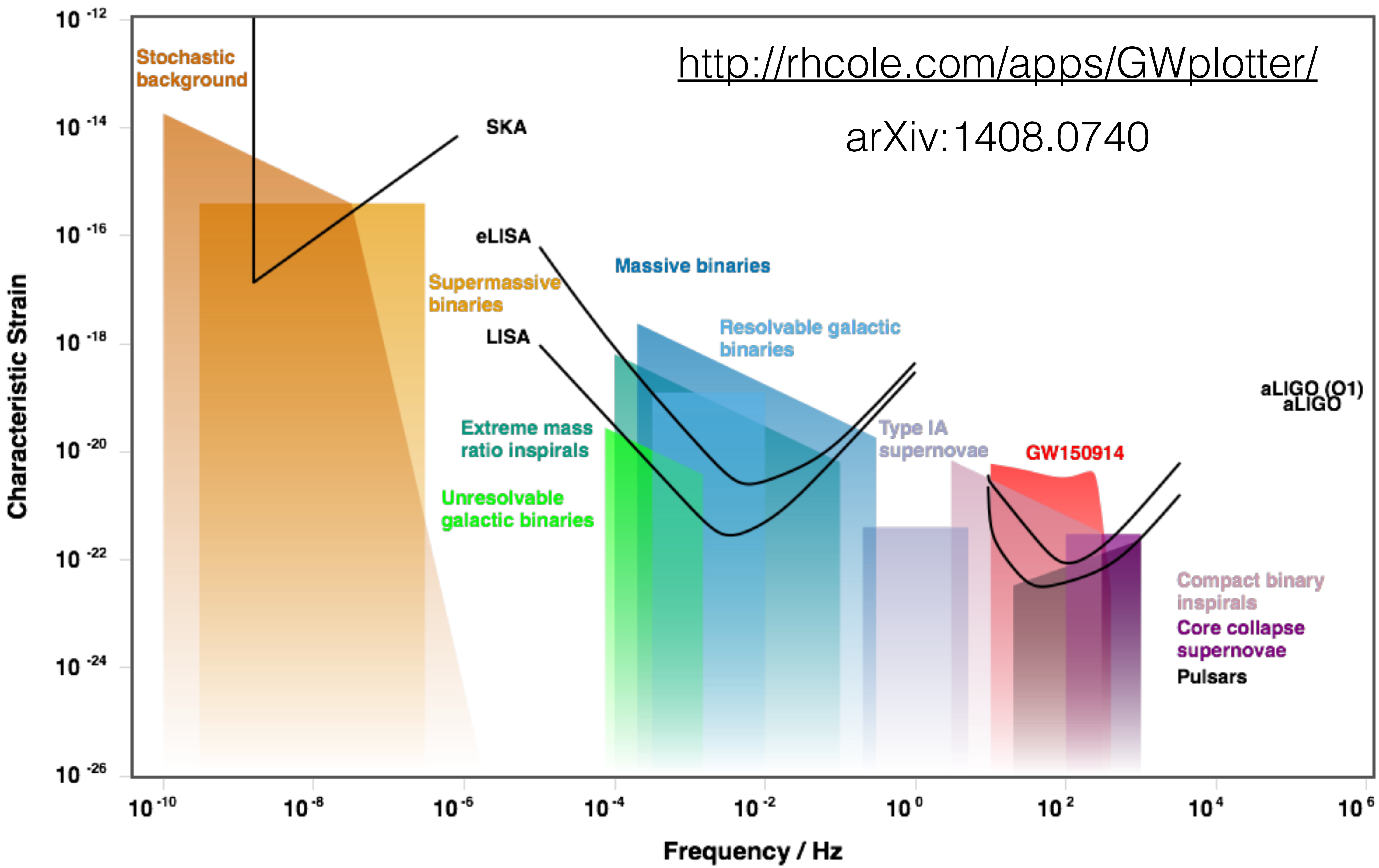
LASERS ARE COHERENT

$$\Delta\phi = \frac{\Delta l}{\lambda} \approx 10^6 \Delta l(m)$$

BUT TeV PROTON BUNCHES ARE NOT

WE WILL EXPLOIT RESONANT EFFECTS

SOURCES AND DETECTORS



THE LAY OF THE LAND

WITHOUT RESONANCES THERE IS NO HOPE

LHC circumference

$$\Delta L \sim h\overline{L} \approx 10^{-7}\ \mathrm{nm}$$

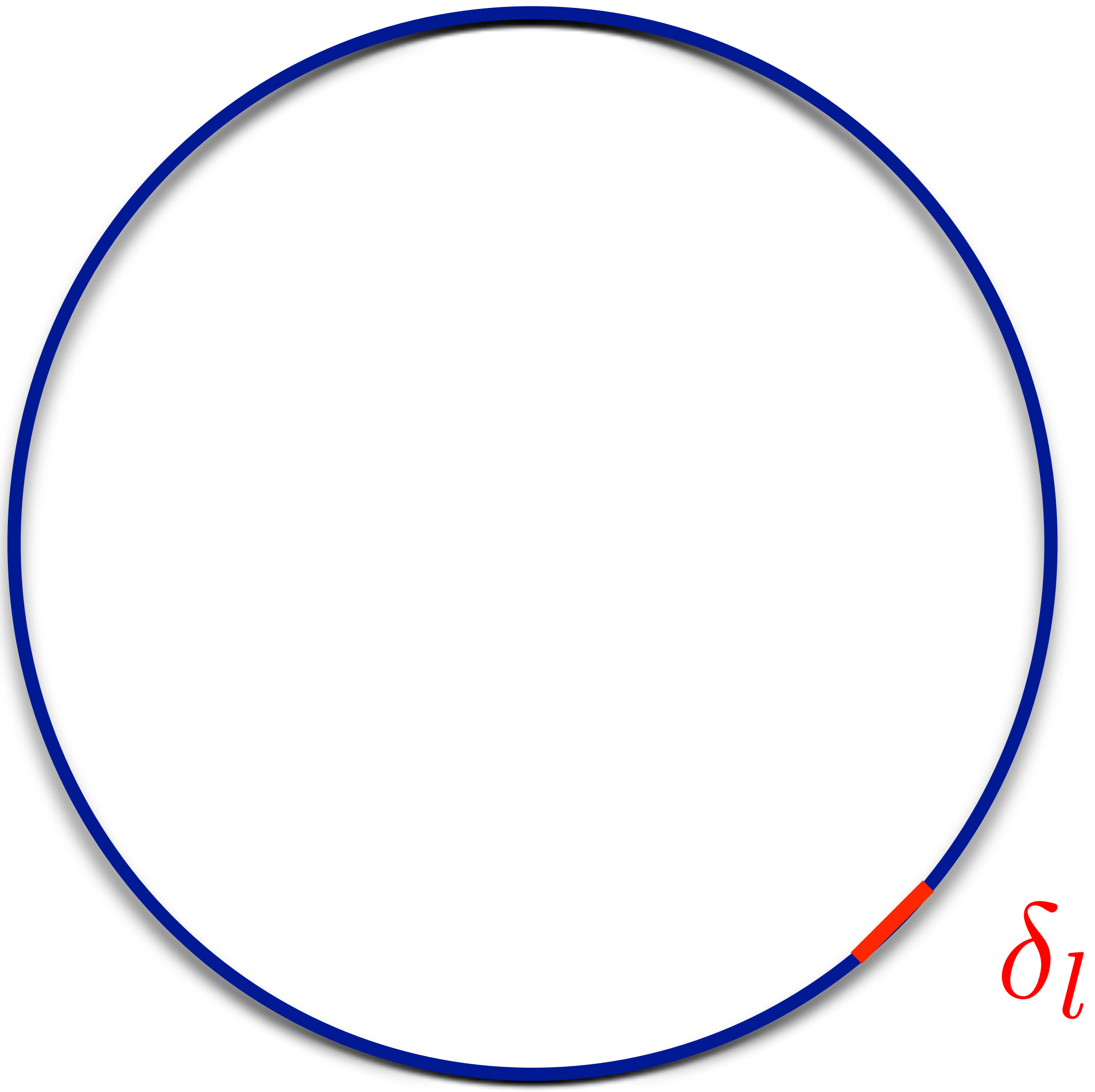
$$\Delta T \sim h\underline{T} \approx 10^{-12}\ \mathrm{ps}$$

Revolution Period of the protons

THE LAY OF THE LAND

DISCLAIMER: TODAY ONLY VERY RUDIMENTARY TOY MODELS

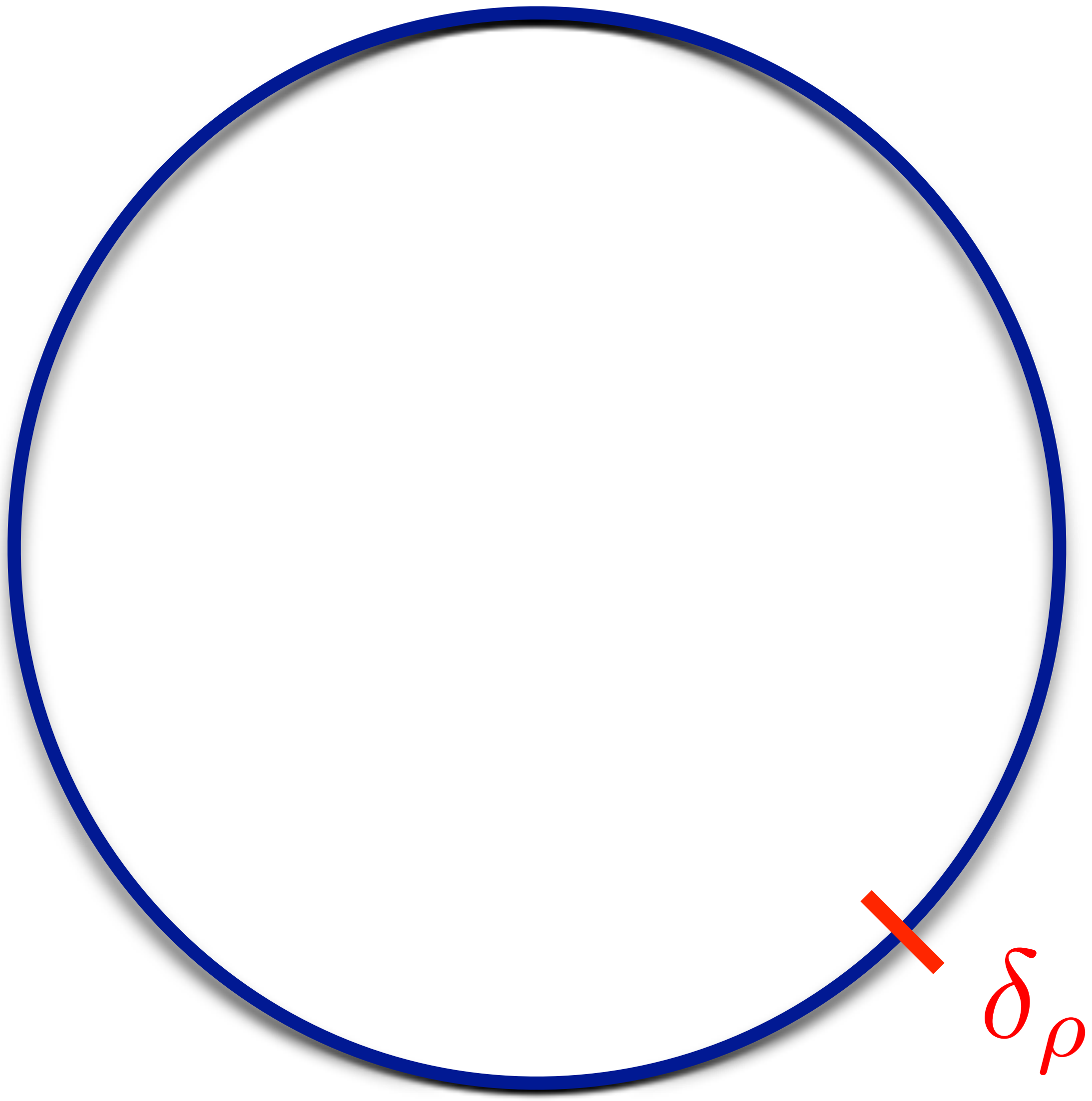
THE HARMONIC OSCILLATOR(S)



$$\ddot{\delta}_l + \omega_l^2 \delta_l = 0$$

$$\omega_l \approx \omega_0 \sqrt{\frac{\tilde{h} \alpha_C q V_{\mathrm{RF}}}{E}} \approx 10 \text{ Hz}$$

THE HARMONIC OSCILLATOR(S)

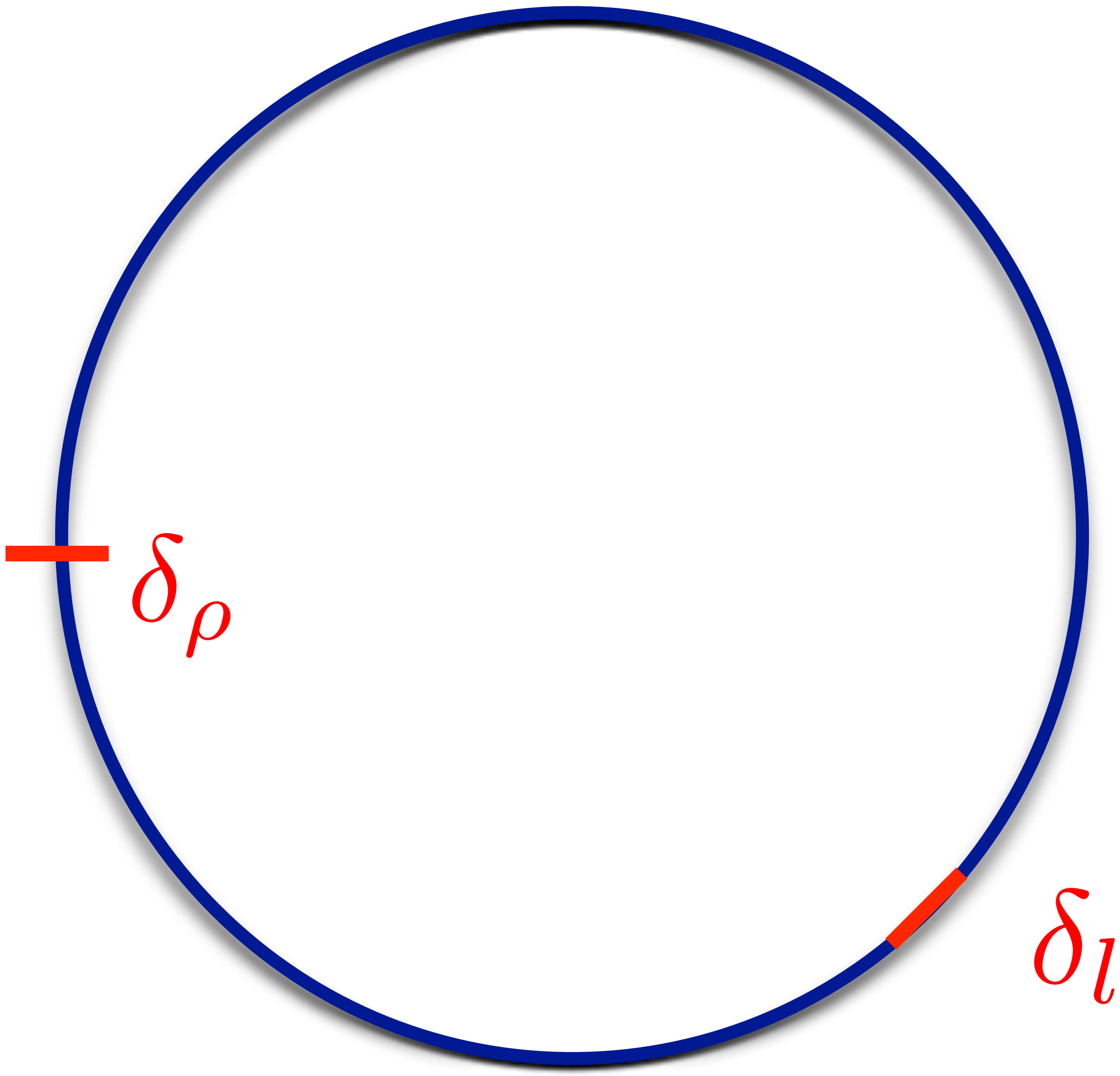


$$\ddot{\delta}_\rho + \omega_\rho^2 \delta_\rho = 0$$

$$\omega_\rho \approx \omega_0 \approx 10 \text{ kHz}$$

Only dipoles for simplicity

THE WAVE



$$\ddot{\delta}_l + \omega_l^2 \delta_l = \omega_g^2 L f(\omega_g t)$$

$$\ddot{\delta}_\rho + \omega_\rho^2 \delta_\rho = \omega_g^2 \rho h(\omega_g, \omega_0, t)$$

GRAVITATIONAL WAVE BASICS

$$ds^2 = -dt^2 + (\delta_{ij} + h_{ij}^{TT})dx^i dx^j$$

$$h_{ij}^{TT} = hH_{ij}\cos(\omega_g t - \omega_g \vec{n}\cdot\vec{x})$$

Tiny dimensionless number

Two polarizations
(i.e. sparse matrix with
O(1) nonzero elements)

I HAVE CHOSEN A GAUGE, BUT THIS IS STILL A VERY REDUNDANT DESCRIPTION

GRAVITATIONAL WAVE BASICS

$$ds^2 = -dt^2 + (\delta_{ij} + h_{ij}^{TT})dx^i dx^j$$

$$V \sim \frac{1}{r}$$ As EM field

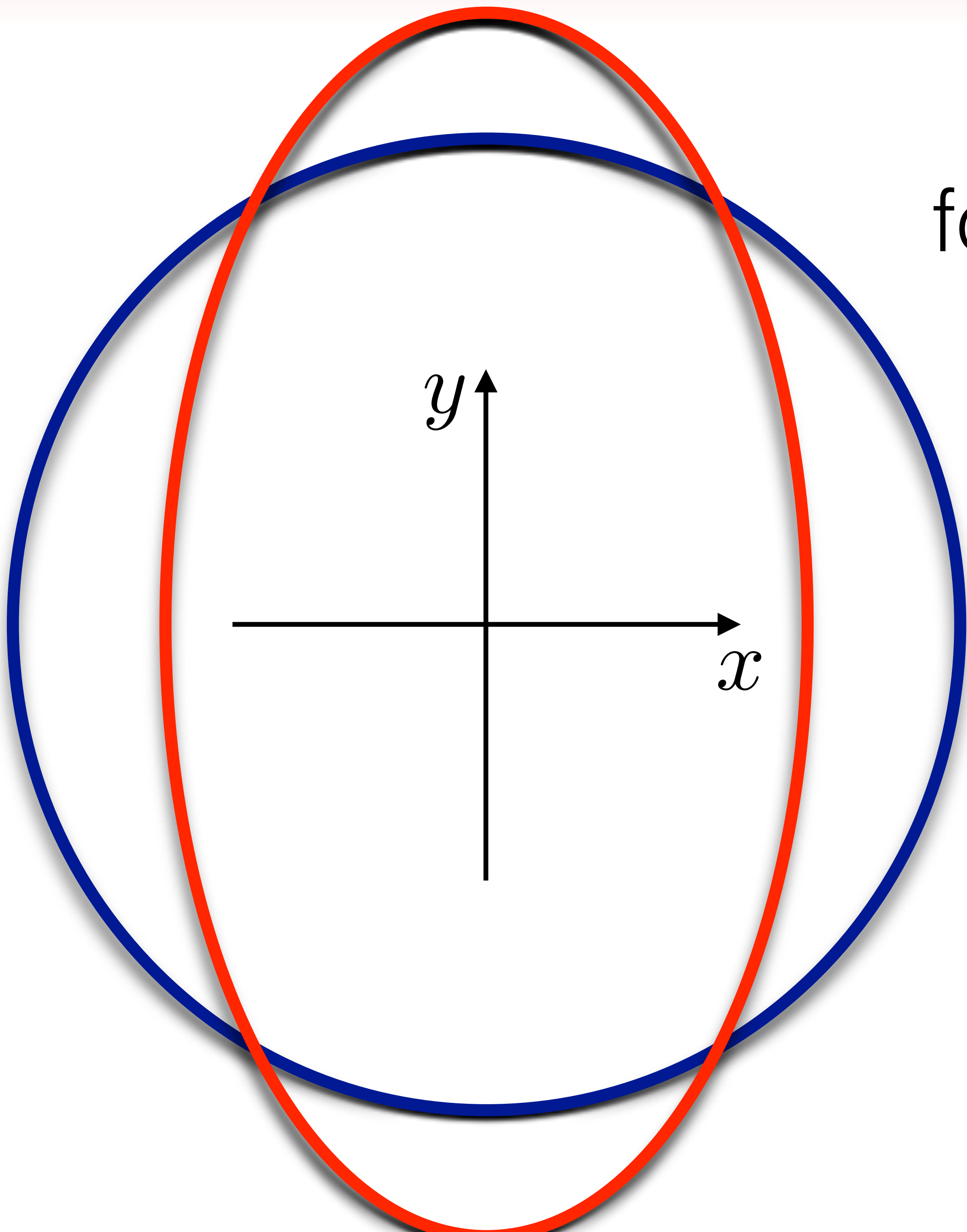
$$n^i h_{ij}^{TT} = 0$$

Transverse
No effect in the direction of propagation

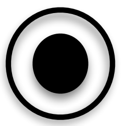
$$h_{ii}^{TT} = 0$$

Traceless
Two dofs

DIRECTION OF PROPAGATION



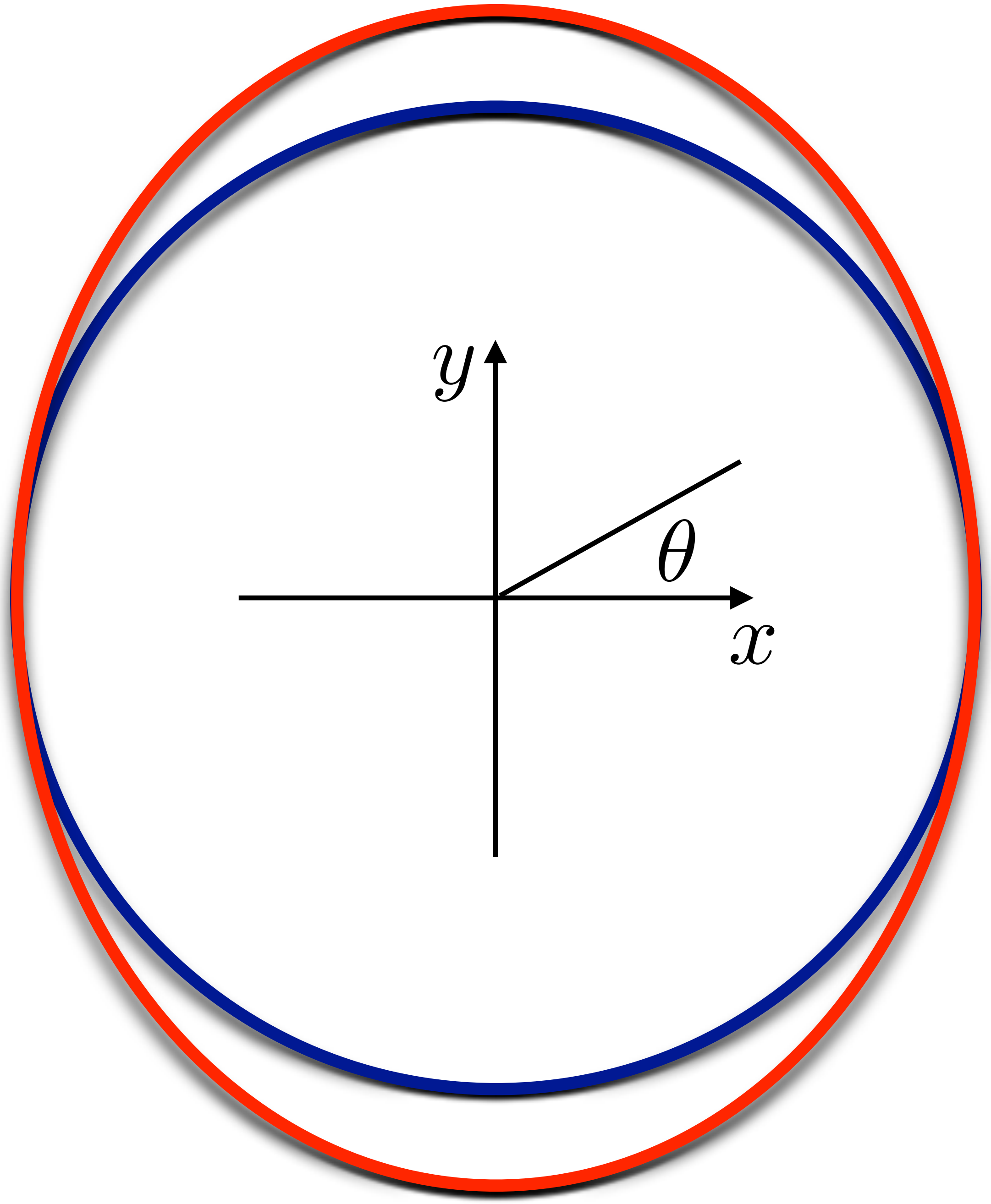
Familiar from
focusing-defocusing quadrupoles



$$\Delta L = \mathcal{O}(h^2)$$

$$\Delta R \sim hR$$

DIRECTION OF PROPAGATION

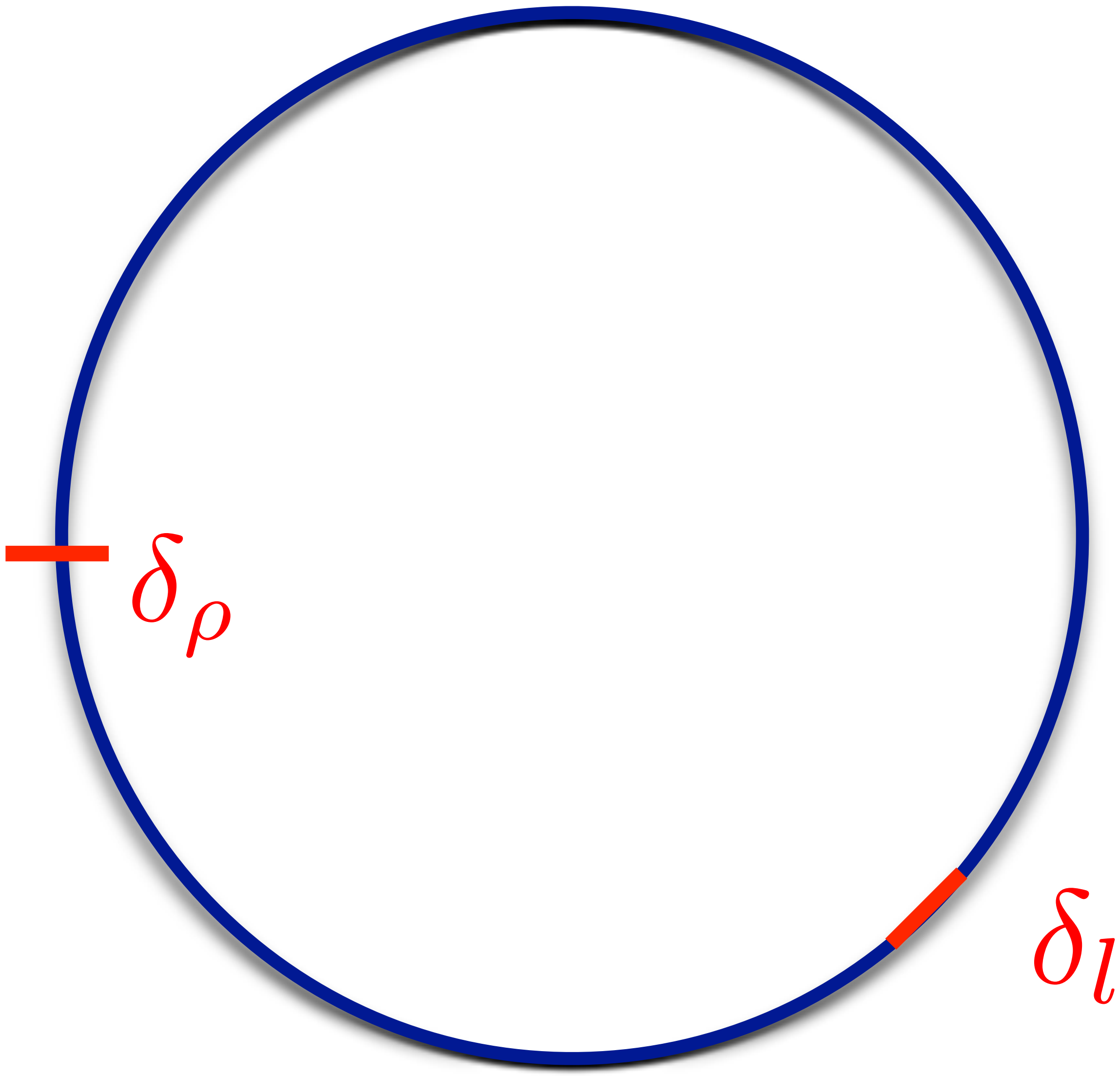


$\longrightarrow \hat{x}$

$$\Delta L \sim hL$$

$$\Delta R \sim hR \sin\theta$$

LONGITUDINAL MOTION



$$\ddot{\delta}_l + \omega_l^2 \delta_l = \boxed{\omega_g^2 L f(\omega_g t)}$$

$$\ddot{\delta}_\rho + \omega_\rho^2 \delta_\rho = \omega_g^2 \rho h(\omega_g, \omega_0, t)$$

THE QUANTITY THAT WE MEASURE

$$\ddot{\delta}_t + \omega_l^2 \delta_t = \boxed{\omega_g^2 T f(\omega_g t)}$$

$$\longrightarrow \hat{x}$$

NOBODY LIKES TEDIOUS GR COMPUTATIONS

$$\ddot{\delta}_t + \omega_l^2 \delta_t = \boxed{\omega_g^2 T f(\omega_g t)} \longrightarrow \hat{x}$$

1) $f(\omega_g t) \sim h$

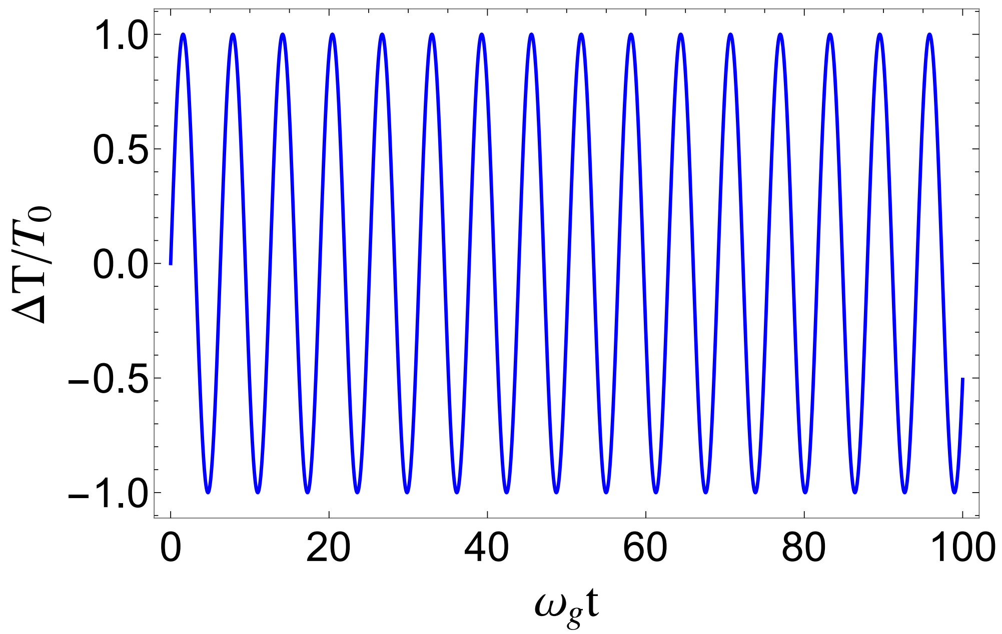
2) $f(\omega_g t) = f(\omega_g (t + T)) + \mathcal{O}\left(\frac{\omega_g}{\omega_0} \approx 10^{-3}\right) \qquad T = \frac{2\pi}{\omega_g}$

SO $\boxed{f(\omega_g t) = ch\cos(\omega_g t + \phi) \quad c = \mathcal{O}(1)}$

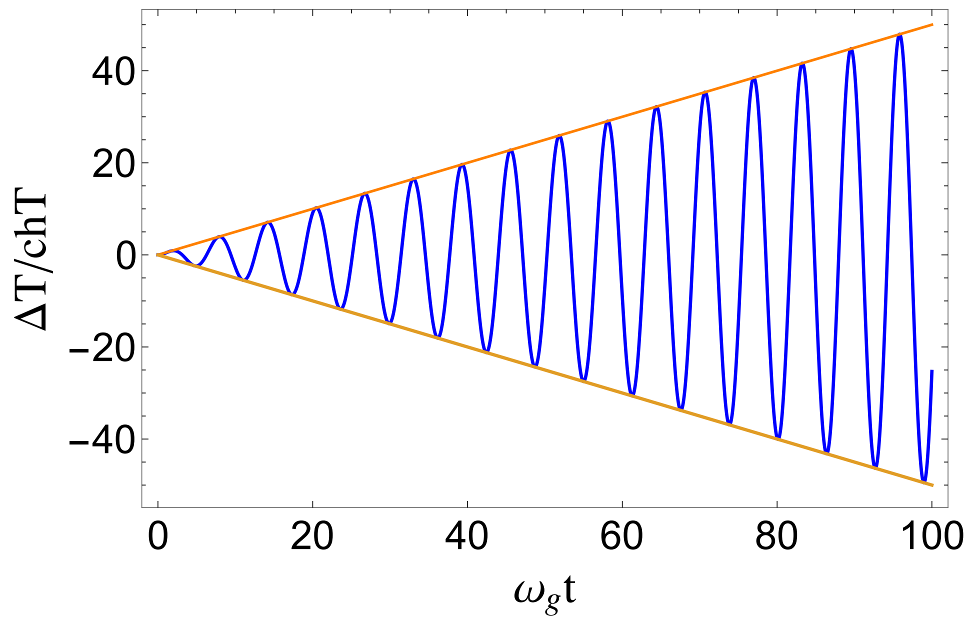
FORCED HARMONIC OSCILLATOR

$$\ddot{\delta}_t + \omega_l^2 \delta_t = \omega_g^2 chT \cos\left(\omega_g t + \phi\right)$$

$$\omega_g = \omega_l$$



$$+$$



$$|\Delta T|^h_{\max} \sim (hT)(\omega_g \tau)$$

FORCED HARMONIC OSCILLATOR

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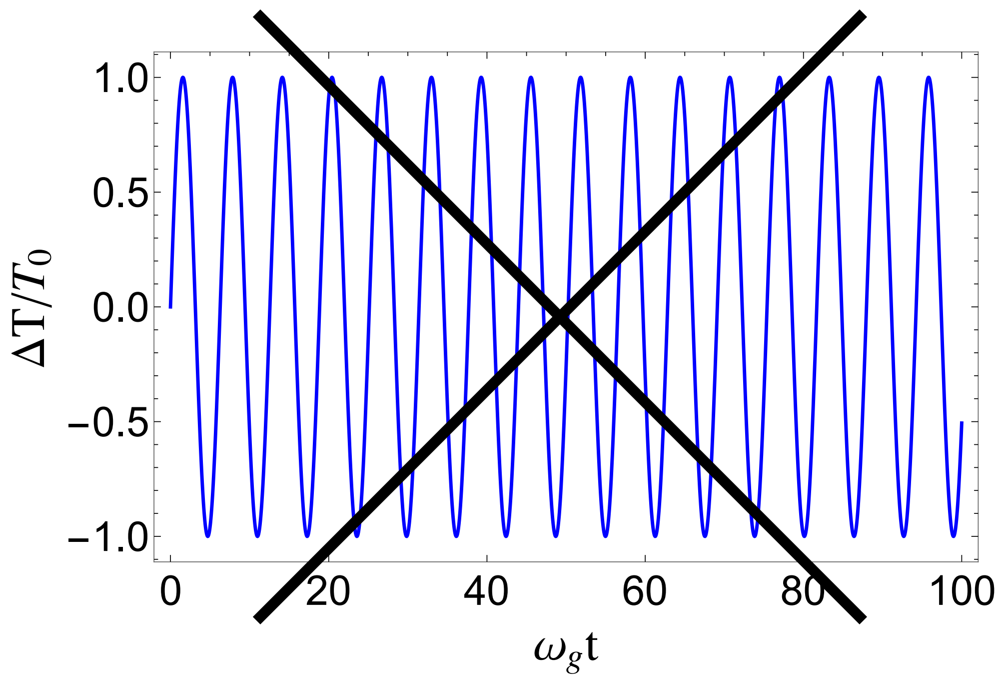

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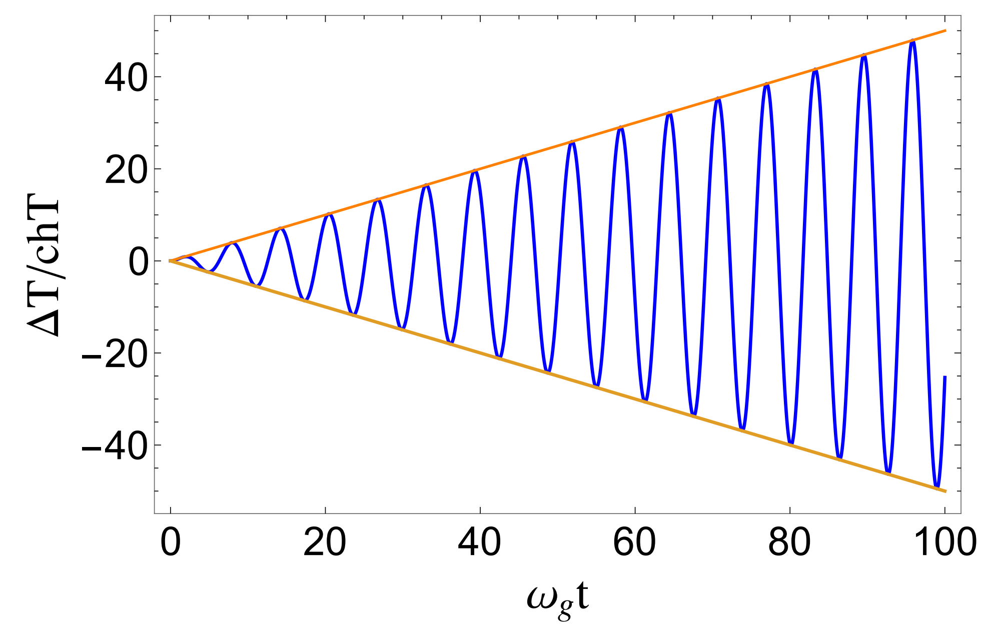


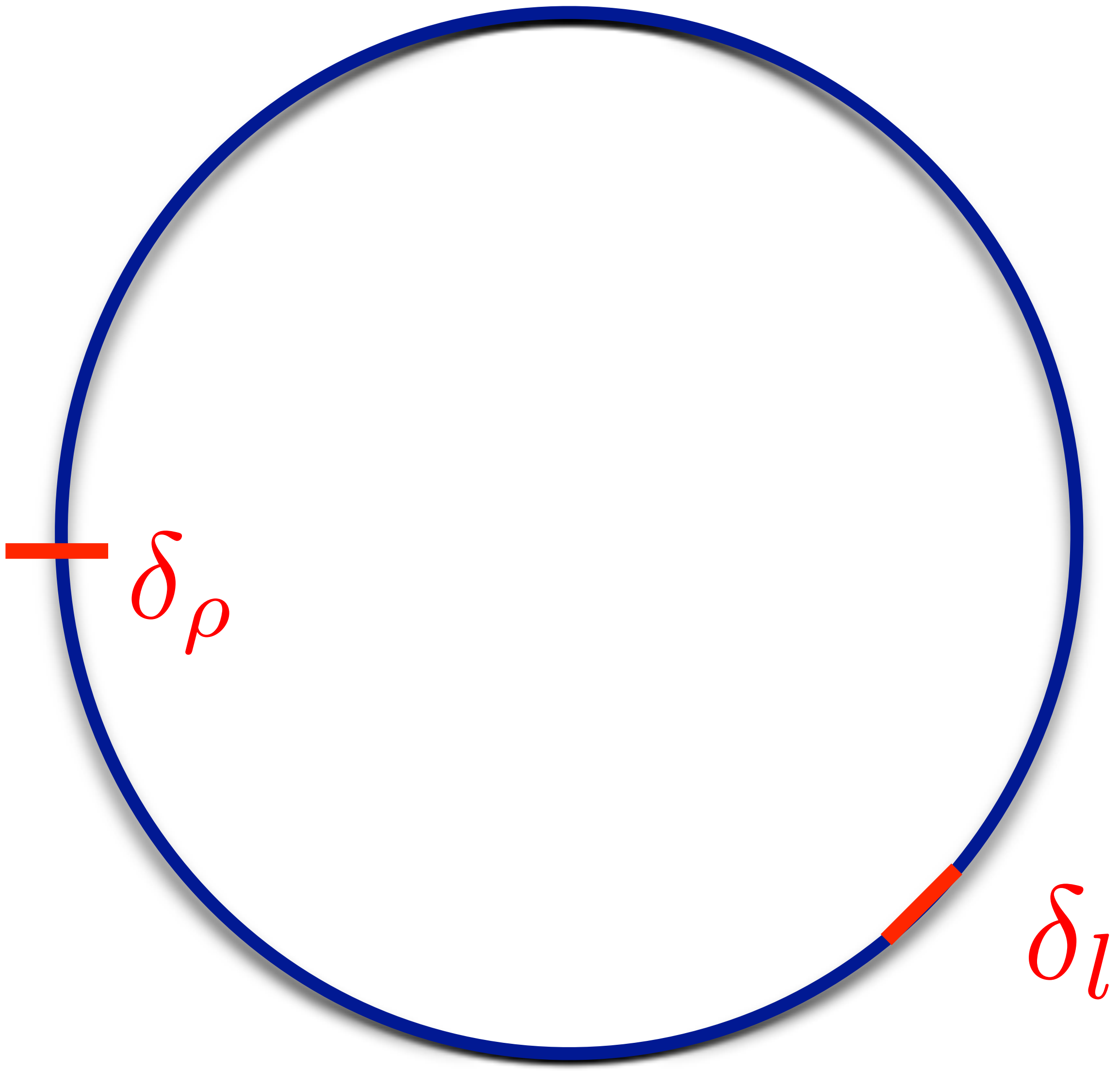
ASSUMPTION: WE CAN SUBTRACT THE CONSTANT COMPONENT PAYING AN O(1) PRICE IN SENSITIVITY $(\Delta T/T)_{\rm exp} \approx 10^{-7}$

SENSITIVITY

$$h \gtrsim 10^{-11} \left(\frac{1\ \mathrm{h}}{\tau}\right) \left(\frac{10\ \mathrm{Hz}}{\omega_g}\right)$$

- AT LEAST 10 ORDERS OF MAGNITUDE ABOVE KNOWN SOURCES
- BUT THERE IS A CLEAR WAY FORWARD
  - (MUCH) SLOWER PROTONS $|\Delta T|^h_{\mathrm{max}} \sim (hT)(\omega_g \tau)$
  - IMPROVE TIME RESOLUTION
  - IMPROVE BEAM STABILITY
    - THE BEAM INTENSITY CAN BE CONSIDERABLY REDUCED
    - THE LOWER THE ENERGY THE BETTER
    - A SMALLER RING WITH SLOWER PROTONS CAN HAVE THE SAME SENSITIVITY

RADIAL MOTION



$$\ddot{\delta}_l + \omega_l^2 \delta_l = \omega_g^2 L f(\omega_g t)$$

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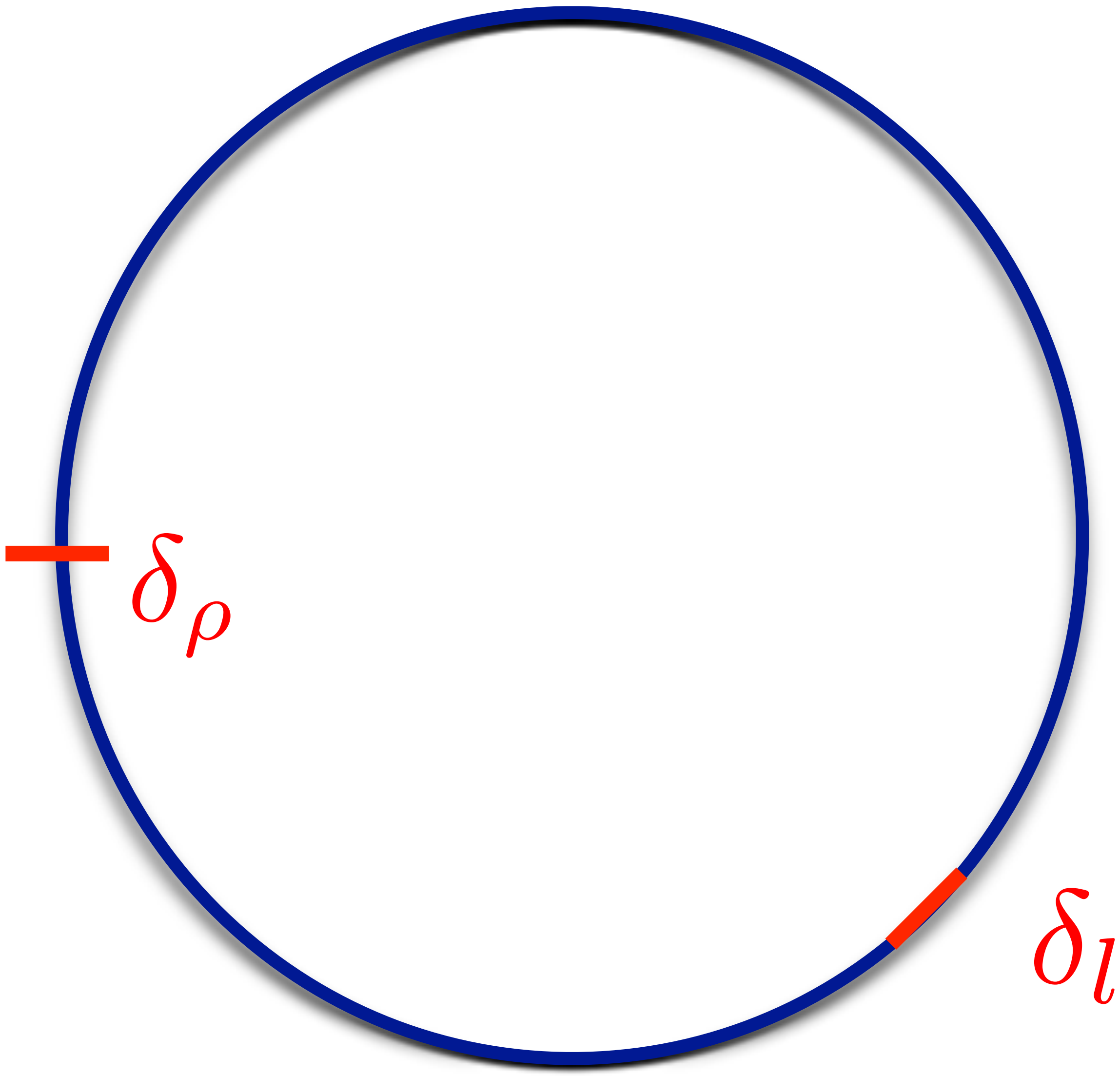
RADIAL MOTION



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No shortcut

$$\omega_0 \sim \omega_\rho \sim \omega_g$$

RADIAL MOTION

$$\ddot{\delta}_\rho + \omega_\rho^2 \delta_\rho = -\frac{\omega_g^2}{2} h\rho \left[e_+ \cos(2\omega_0 t) + e_X \sin(2\omega_0 t)\right] \cos(\omega_g t + \phi)$$

Separation between protons and magnets $\ll R$

RADIAL MOTION

$$\ddot{\delta}_\rho + \omega_\rho^2 \delta_\rho = -\frac{\omega_g^2}{2} h\rho \left[e_+ \cos(2\omega_0 t) + e_X \sin(2\omega_0 t)\right] \cos(\omega_g t + \phi)$$

Revolution Frequency

For dipoles $\omega_\rho = \omega_0$

(no resonance)

Neglected z displacement

RADIAL MOTION

$$\ddot{\delta}_\rho + \omega_\rho^2 \delta_\rho = -\frac{\omega_g^2}{2} h\rho \left[e_+ \cos(2\omega_0 t) + e_X \sin(2\omega_0 t)\right] \cos(\omega_g t + \phi)$$

Rough approximation: $\omega_\rho =$ weighted average between dipoles and quadrupoles, in all the machine.

We sit on the resonance $\omega_g = |\omega_\rho \pm 2\omega_0|$ and assume that the homogenous solution can be subtracted.

RADIAL MOTION

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$$|\delta_\rho|^h_{\max} \approx \mu\text{m} \left(\frac{h}{10^{-12}}\right) \left(\frac{\rho}{2\ \text{cm}}\right) \left(\frac{\tau}{s}\right)$$

RADIAL MOTION

$$|\delta_\rho|^h_{\max} \approx \mu\mathrm{m} \left(\frac{h}{10^{-12}}\right) \left(\frac{\rho}{2\ \mathrm{cm}}\right) \left(\frac{\tau}{s}\right)$$

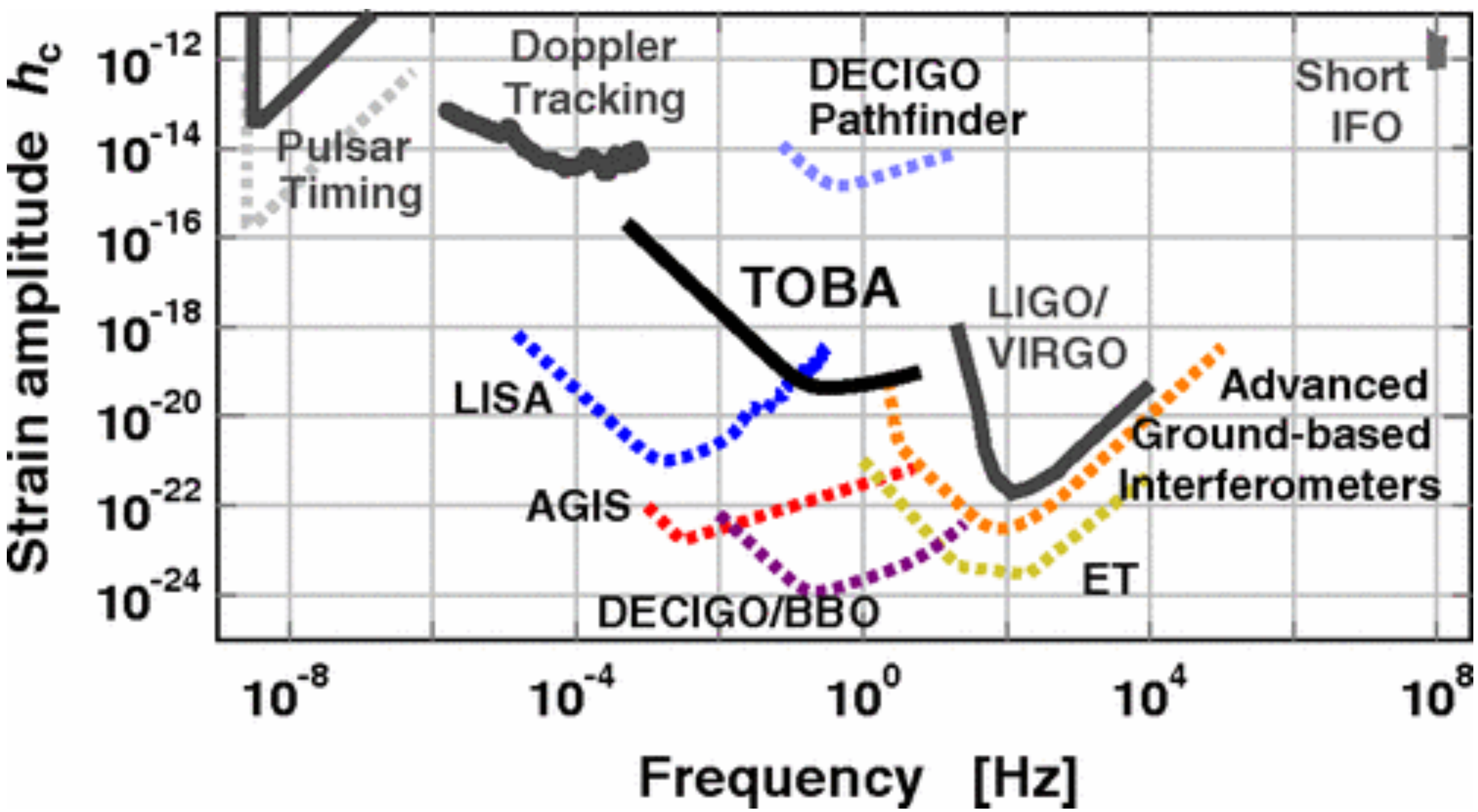
- THE APPROXIMATIONS ON THE BEHAVIOR OF THE MACHINE ARE NOT OBVIOUSLY JUSTIFIED
- THERE IS NOT A CLEAR WAY FORWARD THIS TIME
  - IMPROVE BY MORE THAN 10 ORDERS OF MAGNITUDE THE BEAM STABILITY?
  - NO KNOWN SOURCES IN THIS FREQUENCY RANGE
  - INCREASING THE RESONANCE FREQUENCY IS NOT REALLY AN OPTION

CONCLUSION

- MEASURING GRAVITATIONAL WAVES USING STORAGE RINGS MIGHT BE HOPELESS
- HOWEVER MEASURING GRAVITATIONAL WAVES WAS CONSIDERED ALTOGETHER HOPELESS FOR DECADES AND WE WERE WRONG
- THE OUT-OF-THE-BOX SENSITIVITY OF THE LHC TO SPATIAL DEFORMATION OF A PART IN TEN BILLIONS IS QUITE REMARKABLE
- THE PRESENCE OF A POTENTIAL PATH TOWARDS GW-LEVEL SENSITIVITY FOR LONGITUDINAL DEFORMATIONS IS EVEN MORE REMARKABLE AND MIGHT DESERVE FURTHER STUDY

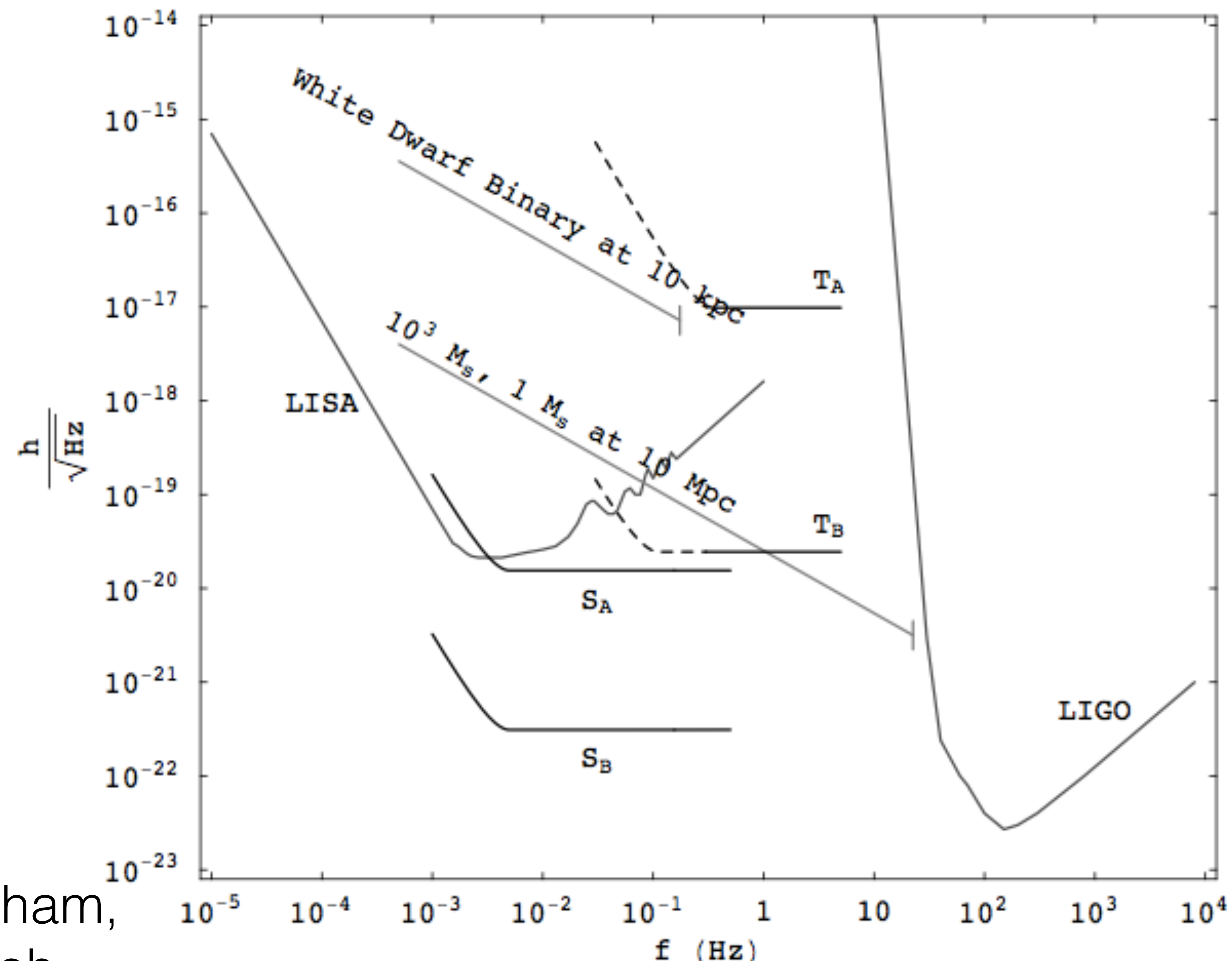
BACKUP

TORSION-BAR ANTENNA



Masaki Ando, Koji Ishidoshiro, Kazuhiro Yamamoto, Kent Yagi, Wataru Kokuyama, Kimio Tsubono, and Akiteru Takamori, Phys. Rev. Lett. 105, 161101

ATOM INTERFEROMETRY



Dimopoulos, Graham, Hogan, Kasevich, Rajendran
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