Hydra: A framework for data analysis in massively parallel platforms

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HYDRA
Multithreaded Data
Analysis Framework

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Hydra

Hydra is a header-only, templated C++11 framework designed to perform common tasks found in HEP data analyses on massively parallel platforms.

- It is implemented on top of the C++11 Standard Library and a variadic version of the Thrust library.
- Hydra is designed to run on Linux systems and to deploy parallelism using
  - OpenMP. Directive-based implementation of multithreading.
  - TBB (Threading Building Blocks). C++ template library developed by Intel for parallel programming on multi-core processors.
  - CUDA. Parallel computing platform and application programming interface (API) model created by Nvidia for compatible GPUs.

Design

- Static polymorphic structure.
- Optimized containers to store polymorphic and multidimensional data-sets using SoA layout.
- Enforced type and thread-safeness and strong separation between algorithms and data.
- All supported back-ends can run concurrently in the same program using the suitable policies:
  - `hydra::omp::sys`
  - `hydra::cuda::sys`
  - `hydra::tbb::sys`
  - `hydra::cpp::sys`
  - `hydra::host::sys`
  - `hydra::device::sys`

The source files written using Hydra and standard C++ compile for GPU and CPU just exchanging the extension from .cu to .cpp and one or two compiler flags. There is no need to re-factory or double code.

Features

- Interface to `ROOT::Minuit2` minimization package, to perform binned and unbinned multidimensional fits.
- Parallel calculation of S-Plots.
- Phase-space generator and integrator.
- Multidimensional p.d.f. sampling.
- Parallel function evaluation over multidimensional data-sets.
- Numerical integration: plain and VEGAS Monte Carlo, Gauss-Kronrod and Genz-Malik quadratures.
- Dense and sparse multidimensional histogramming.
- Support to C++11 lambdas, filters, smart-ranges,... etc.

Functors and C++11 lambdas

- Hydra calls user's code using functors.
- The framework adds features and type information to generic functors using the CRTP idiom.
- All functors derive from `hydra::BaseFunctor<Func,ReturnType,NPars>` and needs to implement the `Evaluate(...)` method.
- C++11 lambdas are supported via `hydra::wrap_lambda()`.
- Some of the built-in functors:

  - `ArgusShape`
  - `BifurcatedGaussian`
  - `BreitWignerLineShape`
  - `Chebychev`
  - `ChiSquare`
  - `CosHelicityAngle`
  - `CrystalBallShape`
  - `Exponential`
  - `Gaussian`
  - `M12PhaseSpaceLineShape`
  - `PlanesDeltaAngle`
  - `Polynomial`
  - `WignerDFunctions`
  - `ZemachFunctions`

PDFs representation

- PDFs are represented by the `hydra::Pdf<Functor, Integrator>` class template and can be conveniently built using the function `hydra::make_pdf( functor, integrator)`.
- The PDF evaluation and normalization can executed in different back-ends.
- PDF objects cache the normalization integrals results. The user can inspect the cached values and corresponding errors.
- It is also possible to represent models composed by the sum of two or more PDFs.
- Hydra implements classes and interfaces to allow the definition of FCNs suitable to perform maximum likelihood fits on unbinned and binned data-sets.
- The different types of log-likelihood FCNs are covered specializing the class template `hydra::LogLikelihoodFCN<PDF, Iterator, Extensions...>`, using the function template `hydra::make_likelihood_fcn(...)`.

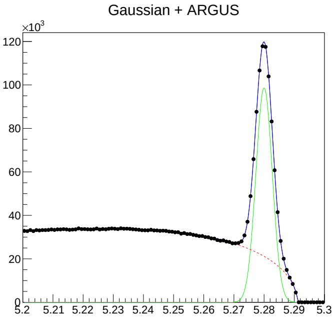
Example 1: Gaussian + Argus

```cpp
//Analysis range
double min =  5.20, max = 5.30;

//Gaussian: parameters definition using ``named parameter idiom''
auto  mean  = Parameter::Create("Mean").Value( 5.28).Error(0.0001).Limits(5.27,5.29);
auto  sigma = Parameter::Create("Sigma").Value(0.0027).Error(0.0001).Limits(0.0025,0.0029);

//Gaussian: PDF definition using analytical integration
auto Signal_PDF = make_pdf( Gaussian<>(mean, sigma), GaussianAnalyticalIntegral(min, max));

//Argus: parameters definition
auto  m0     = Parameter::Create("M0").Value(5.291).Error(0.0001).Limits(5.28, 5.3);
auto  slope  = Parameter::Create("Slope").Value(-20.0).Error(0.0001).Limits(-50.0, -1.0);
auto  power  = Parameter::Create("Power").Value(0.5).Fixed();

//Argus: PDF definition using analytical integration
auto Background_PDF = make_pdf( ArgusShape<>(m0, slope, power), ArgusShapeAnalyticalIntegral(min, max));

//Signal and Background yields
Parameter N_Signal("N_Signal"        ,500, 100, 100 , nentries) ;
Parameter N_Background("N_Background",2000, 100, 100 , nentries) ;

//Make model
auto Model = hydra::add_pdfs( {N_Signal, N_Background}, Signal_PDF, Background_PDF);
```

Example 1: Gaussian + Argus

```
...
//1D device buffer
device::vector<double>  data(nentries);

//generator
Random<> Generator();

//Generate data
auto data_range = Generator.Sample(data, min, max, model.GetFunctor());

//Make model and fcn
auto fcn = make_loglikehood_fcn( model, data_range );

//Execute the fit using ROOT::Minuit2...

//minimization strategy
MnStrategy strategy(2);

//create Migrad minimizer
MnMigrad migrad_d(fcn, fcn.GetParameters().GetMnState() ,  strategy);

//minimization
FunctionMinimum minimum_d =  FunctionMinimum(migrad_d(500, 5));

...
```

Example 1: Gaussian + Argus



Unbinned fit with 2 million events.

- FCN calls: 789
- Intel® Core^{TM} i7-4790 CPU @ 3.60 GHz (1 thread):146,531 s
- Intel® Core^{TM} i7-4790 CPU @ 3.60 GHz (8 threads):26,875 s
- NVidia TitanZ GPU: 3,75 s

Example 2: $D^+ \to K^-\pi^+\pi^+$

PHYSICAL REVIEW D 78, 052001 (2008)

Mode	Parameter	E791	CLEO-c
NR	a	$1.03 \pm 0.30 \pm 0.16$	$7.4 \pm 0.1 \pm 0.6$
	$\phi(°)$	$-11 \pm 14 \pm 8$	$-18.4 \pm 0.5 \pm 8.0$
	FF (%)	$13.0 \pm 5.8 \pm 4.4$	$8.9 \pm 0.3 \pm 1.4$
$\bar{K}^*(892)\pi^+$	a	1 (fixed)	1 (fixed)
	$\phi(°)$	0 (fixed)	0 (fixed)
	FF (%)	$12.3 \pm 1.0 \pm 0.9$	$11.2 \pm 0.2 \pm 2.0$
$\bar{K}_0^*(1430)\pi^+$	a	$1.01 \pm 0.10 \pm 0.08$	$3.00 \pm 0.06 \pm 0.14$
	$\phi(°)$	$48 \pm 7 \pm 10$	$49.7 \pm 0.5 \pm 2.9$
	FF (%)	$12.5 \pm 1.4 \pm 0.5$	$10.4 \pm 0.6 \pm 0.5$
	m (MeV/c^2)	$1459 \pm 7 \pm 12$	$1463.0 \pm 0.7 \pm 2.4$
	Γ (MeV/c^2)	$175 \pm 12 \pm 12$	$163.8 \pm 2.7 \pm 3.1$
$\bar{K}_2^*(1430)\pi^+$	a	$0.20 \pm 0.05 \pm 0.04$	$0.962 \pm 0.026 \pm 0.050$
	$\phi(°)$	$-54 \pm 8 \pm 7$	$-29.9 \pm 2.5 \pm 2.8$
	FF (%)	$0.5 \pm 0.1 \pm 0.2$	$0.38 \pm 0.02 \pm 0.03$
$\bar{K}^*(1680)\pi^+$	a	$0.45 \pm 0.16 \pm 0.02$	$6.5 \pm 0.1 \pm 1.5$
	$\phi(°)$	$28 \pm 13 \pm 15$	$29.0 \pm 0.7 \pm 4.6$
	FF (%)	$2.5 \pm 0.7 \pm 0.3$	$1.28 \pm 0.04 \pm 0.28$
$\kappa\pi^+$	a	$1.97 \pm 0.35 \pm 0.11$	$5.01 \pm 0.04 \pm 0.27$
	$\phi(°)$	$-173 \pm 8 \pm 18$	$-163.7 \pm 0.4 \pm 5.8$
	FF (%)	$47.8 \pm 12.1 \pm 5.3$	$33.2 \pm 0.4 \pm 2.4$
	m (MeV/c^2)	$797 \pm 19 \pm 43$	$809 \pm 1 \pm 13$
	Γ (MeV/c^2)	$410 \pm 43 \pm 87$	$470 \pm 9 \pm 15$

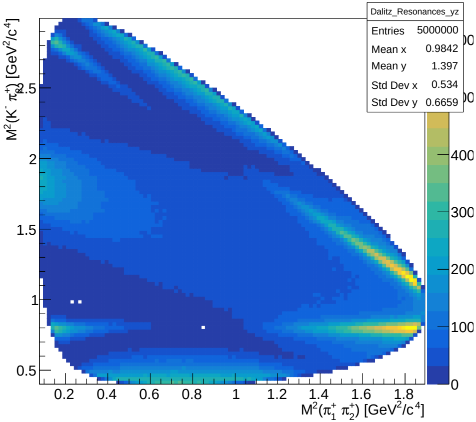
- Masses and widths from PDG-2017.
- Phases and magnitudes from paper above(see page 12, table 7).
- Mimics the corresponding EvtGen's DDalitz model.

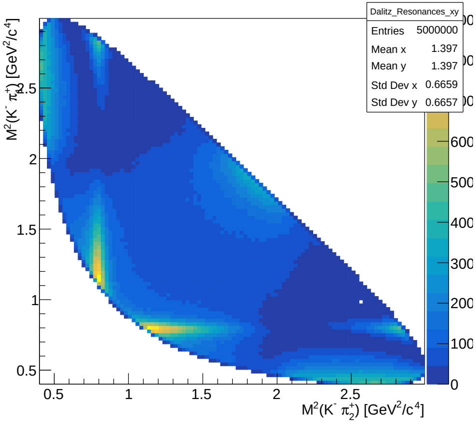
$D^+ \to K^-\pi^+\pi^+$: contributions and toy data

- Contributions for each $K\pi$ channel: N.R., κ, $K^*(892)^0$, $K_0^*(1425)$, $K_2^*(1430)$ and $K_1(1780)$. The total number of parameters is 22: complex coefficients, masses and widths.
- Resonances are represented by the template `class Resonance<Channel, L>`, where *Channel* $= 1, 2, 3$ and L is a `hydra::Wave` object.
- Non-resonant contribution represented by `class NonResonant`.
- Each entry of the dataset (toy data) contains the four-vectors of the three final states.
- Dataset generation is run in parallel using the class `hydra::PhaseSpace<3>`
- Hydra provides:
  - `hydra::BreitWignerLineShape<hydra::Wave L>`
  - `hydra::ZemachFunction<hydra::Wave L>`
  - `hydra::CosTheta`
  - `hydra::complex` ... etc.

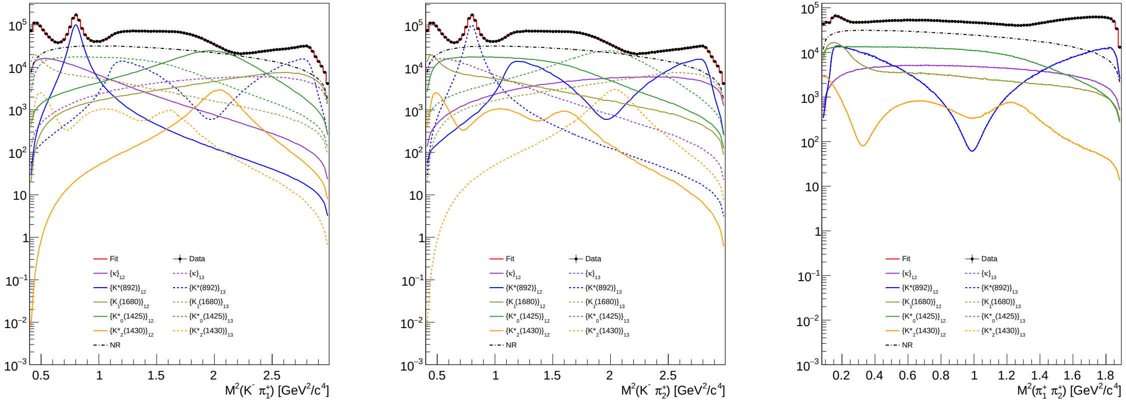
$D^+ \to K^-\pi^+\pi^+$: Dataset

Toy data (5,000,000 events)





$D^+ \to K^-\pi^+\pi^+$: Fit result



- Resonances identified by color.
- Solid lines for $K\pi_1$-channel.
- Dashed lines for $K\pi_2$-channel.
- Lines are superposed in $\pi_1\pi_2$-channel.

Performance: CPU with OpenMP

The table below summarizes the time spent to perform a fit with 2.5 Million events.

Parallel system	Threads	Time (sec/min)	FCN Calls	Time/Call (sec)
i7-4790 CPU @ 3.60GHz	1	5060,578 (1.4 hours)	1030	4.91
	8	750.245 (12.50)	"	0.73
Xeon(R) CPU E5-2680 v3 @ 2.50GHz	1	5128.480 (1,42 hours)	"	4.98
	8	784.252 (13.1)	"	0.76
	12	612.278 (10.2)	"	0.59
	24	371.838 (6.2)	"	0.36
	48	247.787 (4.1)	"	0.24

Performance: CPU with TBB

The table below summarizes the time spent to perform a fit with 2.5 Million events.

Parallel system	Threads	Time (s/min)	FCN Calls	Time/Call (s)
i7-4790 CPU @ 3.60GHz	8	746.684 (12.4)	1030	0.72
Xeon(R) CPU E5-2680 v3 @ 2.50GHz	48	184.779 (3.01)	"	0.18

Performance: GPU with CUDA

The table below summarizes the time spent to perform a fit with 2.5 Million events.

Parallel system	Time (s/min)	FCN Calls	Time/Call (s)
GeForce GTX Tesla P100	221.114 (3.68)	"	0.21
GeForce GTX Titan Z (GPU 1)	336.672 (5.61)	"	0.33
GeForce GTX 1050 Ti	729.165 (12,15)	"	0.71
GeForce GTX 970M (video)	744.247 (12,40)	"	0.72

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Integrating Hydra with ROOT's using ACLIC and CLING

- From ROOT 6.13/03 and Hydra 2.1.0 it is possible to use Hydra interactively through ROOT, in both prompt and batch modes.
- Configuration: `export ROOT_INCLUDE_PATH=/path-to-hydra/`
- Example: `root -l -b my_macro_with_hydra.C++`
- The code will parallelize using TBB instance controlled by ROOT.
- Limitations: ROOT can't deploy GPUs yet.

Comments

- Same code compiled and executed on hardware with different architectures, providing numerically identical results and showing consistent scale over the available resources.
- Observed speed-ups by a factor O(10-100) on data fits. All other operations take maximum two or three dozens of milliseconds.
- It is not really a necessary to be a C++ expert to code your model on Hydra: no previous experience or specific knowledge on CUDA, OpenMP or TBB is required.
- Code is absolutely portable: you can run it on CERN's lxplus machines, on your desktop, laptop, in summary, one can share its code or migrate calculations between different platforms without major concerns.

Summary

- The project is hosted on GitHub: https://github.com/MultithreadCorner/Hydra
- The manual is available online: https://hydra-documentation.readthedocs.io
- The package includes a suite of examples covering: ROOT integration, fit, phase-space Monte Carlo, parallel and polymorphic containers, numerical integration, PDF sampling and random number generation etc.
- It is being used on the Measurement of the Kaon mass at LHCb.

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Backup

Functors

- Hydra calls user's code using functors.
- The framework adds features and type information to generic functors using the CRTP idiom.
- All functors derive from `hydra::BaseFunctor<Func,ReturnType,NPars>` and needs to implement the `Evaluate(...)` method.

A generic functor with N parameters is represented like this:

```
struct MyFunctor: public hydra::BaseFunctor<MyFunctor,double,N>
{
// constructors and assignment operator omitted
...
// implement the Evaluate() method for arrays
template<typename T> __hydra_dual__
inline double Evaluate(T* x) { /*actual calculation*/ }

// implement the Evaluate() method for tuples
template<typename T> __hydra_dual__
inline double Evaluate(T x) { /*actual calculation*/ }
};
```

Arithmetic operations and composition with functors

If `A`, `B` and `C` are Hydra functors, the code below is completely legal.

```
...
//basic arithmetic operations
auto A_plus_B  = A + B;
auto A_minus_B = A - B;
auto A_times_B = A * B;
auto A_per_B   = A/B;
//any composition of basic operations
auto any_functor = (A - B)*(A + B)*(A/C);
// C(A,B) is represented by:
auto compose_functor = hydra::compose(C, A, B)
...
```

These operations are lazy and there is no intrinsic limit on the number of functors participating on arithmetic or composition mathematical expressions.

Support for C++11 lambdas I

Lambda functions are fully supported in Hydra.

- The user can define a C++11 lambda function and convert it into a Hydra functor using `hydra::wrap_lambda()`:

```
...
double two = 2.0;

//define a lambda capturing 'two' and convert it to a Hydra functor
auto my_lamba_wrapped  = hydra::wrap_lambda(
  [=] __hydra_dual__ (unsigned n, double* x){

      return two*sin(x[0]);
  } );

...
```

Support for C++11 lambdas II

It is also possible to add named parameters to C++11 lambdas. In Hydra's jargon: "parametric lambdas"

```
...
//named parameter
auto multiplier = hydra::Parameter::Create().Name("multiplier").Value(2.0);

//
auto my_lamba_wrapped  = hydra::wrap_lambda(
  [] __hydra_dual__ (unsigned nparams, hydra::Parameter* param, unsigned n, double* x){

        return param[0]*sin(x[0]);

    }, multiplier);

//set the multiplier to a different value
my_lamba_wrapped.SetParameter("multiplier", 3.0);
...
```

This feature is very usefull for quickly prototyping new functors or to combine the existing ones.

Parameters representation

- Parameters are represented by the `hydra::Parameter` class and can hold name, limits and error.
- `hydra::Parameter` objects are thread safe and automatically tracked and managed by the `hydra::BaseFunctor<Func,ReturnType,NPars>` interface.
- Can be instantiated using the *named parameter idiom*:

```
auto P1 = hydra::Parameter::Create().Name("P1").Value(5.291).Error(0.0001).Limits(5.28, 5.3);
auto P2 = hydra::Parameter::Create("P3").Value(5.291).Limits(5.28, 5.3).Error(0.0001);
```

- Can be instantiated using the *parameter list idiom*

```
//name, value, error, minimum, maximum
hydra::Parameter P3("P3" ,5.291 ,0.0001 , 5.28, 5.3) ;
```

Not all members in a functor are required to be represented by `hydra::Parameter` objects.

PDFs representation

- PDFs are represented by the hydra::Pdf<Functor, Integrator> class template and can be conveniently built using the function hydra::make_pdf(functor, integrator).
- The PDF evaluation and normalization can executed in different back-ends.
- PDF objects cache the normalization integrals results. The user can monitor the cached values and corresponding errors.
- It is also possible to represent models composed by the sum of two or more PDFs. Such models are represented by the class templates
  - hydra::PDFSumExtendable<Pdf1, Pdf2,...>
  - hydra::PDFSumNonExtendable<Pdf1, Pdf2,...>

  and can be built using the function hydra::add_pdfs({yield1, yield2,...}, pdf1, pdf2,...);

FCNs representation

The FCN is defined binding a PDF to the data the PDF is supposed to describe.

- Hydra implements classes and interfaces to allow the definition of FCNs suitable to perform maximum likelihood fits on unbinned and binned data-sets.
- The different typed of log-likelihood FCNs are covered specializing the class template `hydra::LogLikelihoodFCN<PDF, Iterator, Extensions...>` .
- Objects representing likelihood-based FCNs are conveniently instantiated using the function templates:
  - `hydra::make_likelihood_fcn(data.begin(), data.end() , pdf)`
  - `hydra::make_likelihood_fcn(data.begin(), data.end() , weights.begin(), pdf)`

  where `data.begin()`, `data.end()` and `weights.begin()` are iterators pointing to the data-set range, its weights or bin-contents.

$D^+ \to K^-\pi^+\pi^+$: contributions

Defining a contribution:

```
//K*(892)
//parameters
auto mass = hydra::Parameter::Create().Name("MASS_KST_892" ).Value(KST_892_MASS )
                                    .Error(0.0001).Limits(KST_892_MASS*0.95,  KST_892_MASS*1.05 );

auto width = hydra::Parameter::Create().Name("WIDTH_KST_892").Value(KST_892_WIDTH)
                                    .Error(0.0001).Limits(KST_892_WIDTH*0.95, KST_892_WIDTH*1.05);

auto coef_re = hydra::Parameter::Create().Name("A_RE_KST_892" ).Value(KST_892_CRe)
                                    .Error(0.001).Limits(KST_892_CRe*0.95,KST_892_CRe*1.05).Fixed();

auto coef_im = hydra::Parameter::Create().Name("A_IM_KST_892" ).Value(KST_892_CIm)
                                    .Error(0.001).Limits(KST_892_CIm*0.95,KST_892_CIm*1.05).Fixed();
//contributions per channel
Resonance<1, hydra::PWave> KST_892_Resonance_12(coef_re, coef_im, mass, width, D_MASS, K_MASS, PI_MASS, PI_MASS , 5.0);

Resonance<3, hydra::PWave> KST_892_Resonance_13(coef_re, coef_im, mass, width, D_MASS, K_MASS, PI_MASS, PI_MASS , 5.0);

//total contribution
auto KST_892_Resonance = (KST_892_Resonance_12 - KST_892_Resonance_13);
```

The other resonances are defined in a similar way.

$D^+ \to K^-\pi^+\pi^+$: model

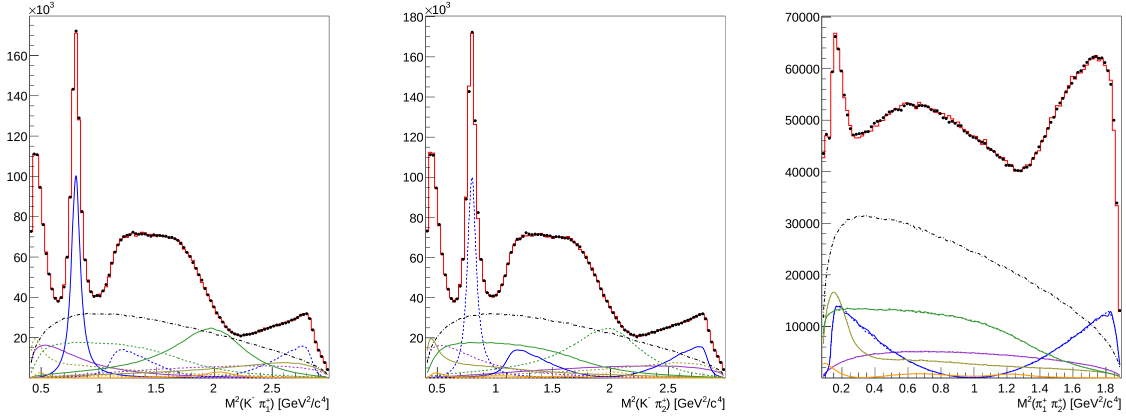
```
//NR
coef_re = hydra::Parameter::Create().Name("A_RE_NR" ).Value(NR_CRe).Error(0.001).Limits(NR_CRe*0.95,NR_CRe*1.05);
coef_im = hydra::Parameter::Create().Name("A_IM_NR" ).Value(NR_CIm).Error(0.001).Limits(NR_CIm*0.95,NR_CIm*1.05);

auto NR = NonResonant(coef_re, coef_im);

//Total model |N.R + \sum{ Resonaces }|^2
auto Norm = hydra::wrap_lambda(
    []__host__  __device__ (unsigned int n, hydra::complex<double>* x) {
          hydra::complex<double> r(0,0);
          for(unsigned int i=0; i< n;i++) r += x[i];
          return hydra::norm(r);}
    );

//Functor
auto Model = hydra::compose(Norm, K800_Resonance, KST_892_Resonance,
              KST0_1430_Resonance, KST2_1430_Resonance, KST_1680_Resonance, NR);

//PDF
auto Model_PDF = hydra::make_pdf( Model,
                hydra::PhaseSpaceIntegrator<3, hydra::device::sys_t>(D_MASS, {K_MASS, PI_MASS, PI_MASS}, 500000));
```

$D^+ \to K^-\pi^+\pi^+$: data generation, management and fit

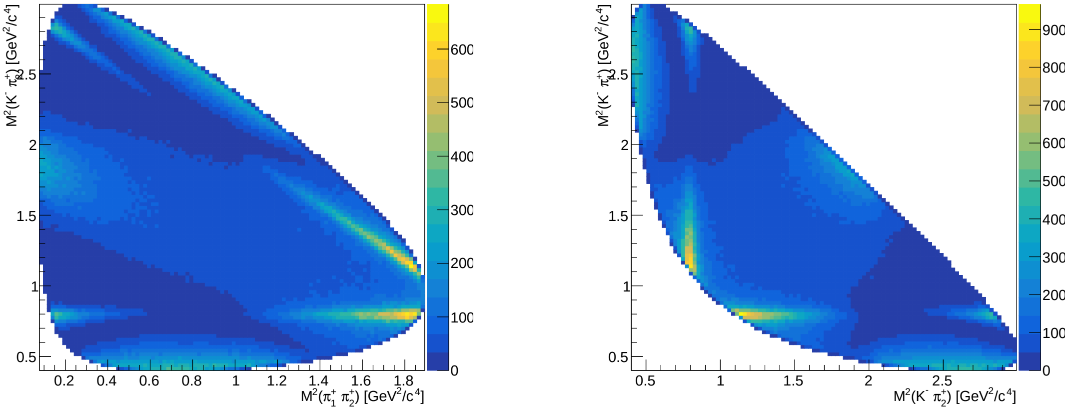
- Each entry of the dataset contains the four-vectors of the three final states.
- Dataset generation is managed by the template `class hydra::PhaseSpace<N>`
- The data is generated sampling the model on the device, in bunches of hundred of thousands events, which are then stored in a `hydra::Decays<N, Backend >` container allocated on the host memory space.
- When necessary, the data-set is transferred to the suitable device to perform the fit, histograming etc.

```
...
//get the fcn
auto fcn = hydra::make_loglikehood_fcn(Model_PDF, particles.begin(), particles.end());
//minimization strategy
MnStrategy strategy(2);
//create Migrad minimizer
MnMigrad migrad_d(fcn, fcn.GetParameters().GetMnState() ,  strategy);
//fit...
FunctionMinimum minimum_d =  FunctionMinimum( migrad_d(5000, 5) );
```

$D^+ \to K^-\pi^+\pi^+$: Projections



$D^+ \to K^-\pi^+\pi^+$: Fit result



$D^+ \to K^-\pi^+\pi^+$:Fit fractions

```
KST800_12_FF :0.0782446
KST800_13_FF :0.0784398
KST892_12_FF :0.101073
KST892_13_FF :0.100459
KST1425_12_FF :0.17922
KST1425_13_FF :0.178935
KST1430_12_FF :0.00996452
KST1430_13_FF :0.00994939
KST1680_12_FF :0.0732225
KST1680_13_FF :0.0730777
NR_FF :0.44089
Sum :1.32348
```

$D^+ \to K^-\pi^+\pi^+$: data generation

```
//Mother particle
hydra::Vector4R D(D_MASS, 0.0, 0.0, 0.0);

// create PhaseSpace object for D-> K pi pi
hydra::PhaseSpace<3> phsp{K_MASS, PI_MASS, PI_MASS};

//allocate memory to hold the final states particles
hydra::Decays<3, hydra::device::sys_t > Events( nentries );

//generate the final state particles
phsp.Generate(D, Events.begin(), Events.end());

//container hold the unweighted dataset on the host
hydra::Decays<3, hydra::host::sys_t > toy_data;

//unweighted on device
auto last = Events.Unweight(Model, 1.0);

//allocate memory to hold the unweighted dataset
toy_data.resize(last);

//copy
hydra::copy(Events.begin(), Events.begin()+last, toy_data.begin());
```

Previous presentation

The package has been presented in several computing conferences and workshops:

- **Hydra: Accelerating Data Analysis in Massively Parallel Platforms**- University of Washington, 21-25 August 2017, Seattle
- **Hydra: A Framework for Data Analysis in Massively Parallel Platforms** - NVIDIA's GPU Technology Conference, May 8-11, 2017 - Silicon Valley, US
- **Hydra** - HSF-HEP analysis ecosystem workshop, 22-24 May 2017 Amsterdam
- **MCBooster and Hydra: two libraries for high performance computing and data analysis in massively parallel platforms**- Perspectives of GPU computing in Science September 2016, Rome
- **Efficient Python routines for analysis on massively multi-threaded platforms-Python bindings for the Hydra C++ library** -Google Summer of Code project 2017

Functor example: Gaussian

```
template<unsigned int ArgIndex=0>
class Gaussian: public BaseFunctor<Gaussian<ArgIndex>, double, 2>
{
public:
        //copy constructor and assignment operator omitted
        Gaussian(Parameter const& mean, Parameter const& sigma ):
          BaseFunctor<Gaussian<ArgIndex>, double, 2>({mean, sigma})
          {}

        template<typename T>
        __hydra_host__ __hydra_device__ inline
        double Evaluate(unsigned int, T*x) const {
                double m2 = (x[ArgIndex] - _par[0])*(x[ArgIndex] - _par[0] );
                double s2 = _par[1]*_par[1];
                return  exp(-0.5*m2/s2);
        }

        template<typename T>
        __hydra_host__ __hydra_device__ inline
        double Evaluate(T x)  const {
                double m2 = ( get<ArgIndex>(x) - _par[0])*(get<ArgIndex>(x) - _par[0] );
                double s2 = _par[1]*_par[1];
                return exp(-0.5*m2/s2);
        }
};
```

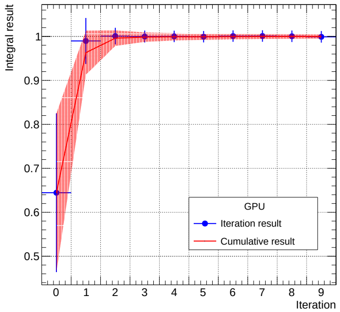
NVidia GPUs

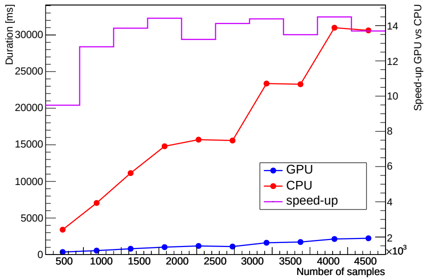
GPU Architecture: Kepler
CUDA Cores 5760
Base Clock (MHz) 705
Single-Precision Performance 4.3 - 5.0 TeraFLOPS
Double-Precision Performance 1.4 - 1.7 TeraFLOPS
Memory Interface 12GB GDDR5

GPU Architecture: Pascal
CUDA Cores 3584
Base Clock (GHz) 1.126
Double-Precision Performance 4.7 TeraFLOPS
Single-Precision Performance 9.3 TeraFLOPS
Memory Interface 16GB CoWoS HBM2 at 732 GB/s

Vegas-like multidimensional numerical integration

Integrating a normalized Gaussian distribution in 10 dimensions.

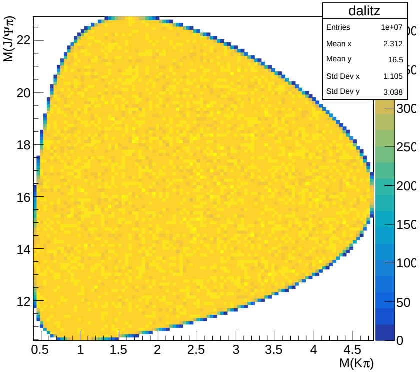


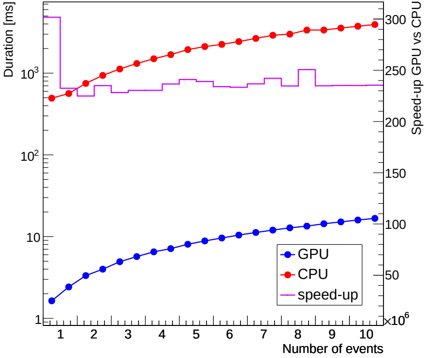


System configuration:

- GPU model: Tesla K40c
- CPU: Intel® Xeon(R) CPU E5-2680 v3 @ 2.50GHz (one thread)

Phase-Space Monte Carlo





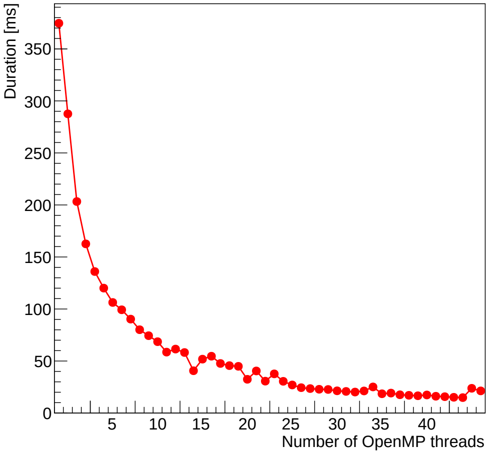
System configuration:

- GPU model: Tesla K40c
- CPU: Intel® Xeon(R) CPU E5-2680 v3 @ 2.50GHz (one thread)

Phase-Space Monte Carlo

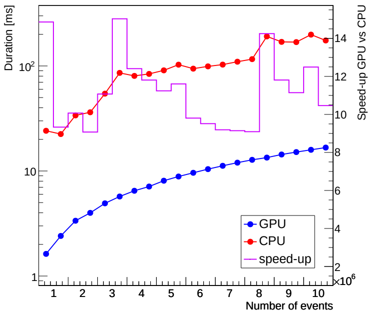
System configuration:

- CPU: Intel® Xeon(R) CPU E5-2680 v3 @ 2.50GHz x 48

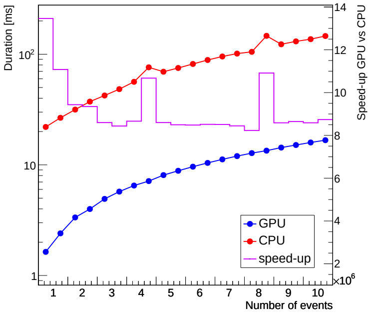


Phase-Space Monte Carl0

GPU vs OpenMP



GPU vs TBB



System configuration:

- GPU model: Tesla K40c
- CPU: Intel® Xeon(R) CPU E5-2680 v3 @ 2.50GHz x 48

Vegas-like multidimensional numerical integration

System configuration:

- CPU: Intel® Xeon(R) CPU E5-2680 v3 @ 2.50GHz x 48