Performance Analysis of Effective Symbolic Methods for Solving Band Matrix SLAEs

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Objectives

This paper presents an experimental performance study of implementations of three symbolic algorithms for solving band matrix systems of linear algebraic equations (SLAEs) with heptadiagonal (HD), pentadiagonal (PD) and tridiagonal (TD) coefficient matrices, the latter two of which were introduced and investigated in Veneva and Ayriyan (EPJ-WoC (2018), arXiv:1710.00428v2 and arXiv:1804.09666v1). The only assumption on the coefficient matrix is nonsingularity. These algorithms are implemented using the `GiNaC` library of `C++` and `SymPy` library of `Python`. Performance analysis of the implementations is done using the high-performance computing (HPC) platforms "HybriLIT" and "Avitohol". To that purpose, the experimental setup and the results from the conducted computations on the individual computer systems are presented and discussed.

Investigated Methods

- SHDM: based on LU decomposition;
- SPDM: based on LU decomposition [1];
- STDM: symbolic Thomas method [2];
- implementations:
  - `C++` and `GiNaC` library;
  - `Python` and `SymPy` library;
- 5 different classes for data storing;
- assumptions on the coefficient matrix: nonsingularity.

Experimental Setup

- Computer systems:
  - heterogeneous computational platform "HybriLIT" at LIT, JINR in Dubna, Russia (`http://hlit.jinr.ru/en/`);
  - cluster computer system "Avitohol" at ACDC, IICT, BAS in Sofia, Bulgaria (`http://www.hpc.acad.bg/`).

Computer system	Processor	Rpeak [TFlop/s]	FREQ [GHz]	Cache [MB]
"HybriLIT"	Intel Xeon E5-2695v2	500	2.40	30
"Avitohol"	Intel Xeon E5-2650v2	412.32	2.60	20

Software

Software		"HybriLIT"	"Avitohol"
OS		Scientific Linux 7.4	Red Hat Linux
C++	Compilers	GCC (4.9.3)	GCC (6.2.0)
C++	Libraries	GiNaC (1.7.2), CLN (1.3.4)	
C++	Optimization	-O0	
Python	Version	Anaconda (5.0.1): Py2.7	
Python	Library	SymPy (1.1.1)	

Results – "HybriLIT"

	GiNaC ("lst" – var-size) := Impl. 1		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	0.476306	0.109828	0.082774
10^4	66.135339	17.518928	7.957098
10^5	14820.286663	5991.496290	2857.548384

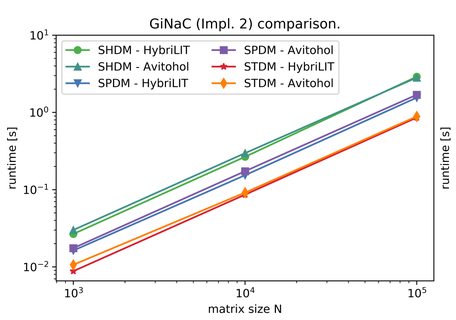
	GiNaC ("matrix" – fixed-size) := Impl. 2		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	0.026697	0.016281	0.008808
10^4	0.266264	0.153711	0.086114
10^5	2.901681	1.553468	0.858090

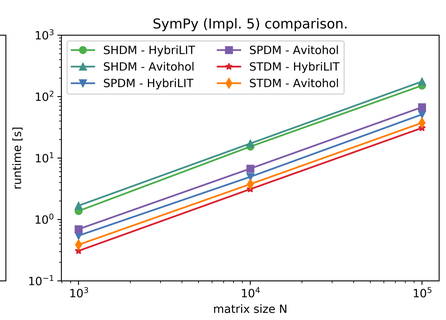
	SymPy ("Matrix" – var-size) := Impl. 3		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^2	0.809609	0.541518	0.288752
10^3	62.801114	47.205663	23.203961
10^4	6159.227881	4587.017868	2294.136577

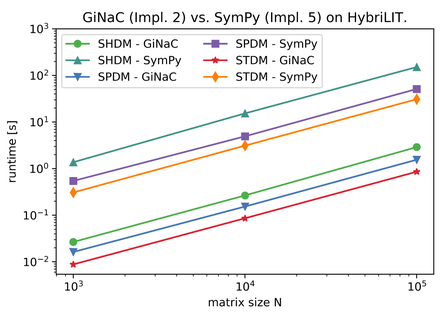
	SymPy ("Matrix" – fixed-size) := Impl. 4		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	1.514612	0.726237	0.417078
10^4	15.592185	5.780376	2.909977
10^5	167.711683	60.360366	29.446712

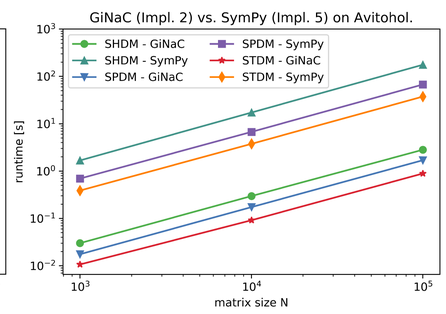
	SymPy ("Ar" – fixed-size) := Impl. 5		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	1.374093	0.545021	0.308278
10^4	15.362392	4.941960	3.115306
10^5	151.636368	51.304943	30.917188

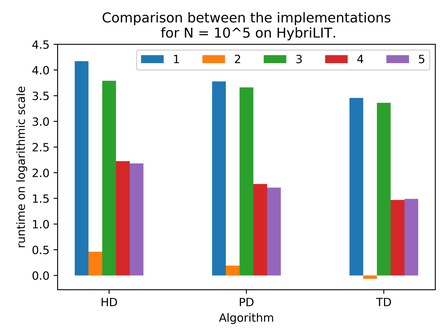
Graphic Results

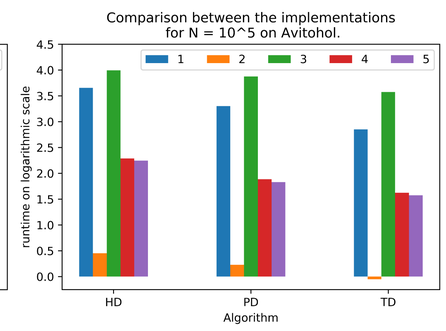


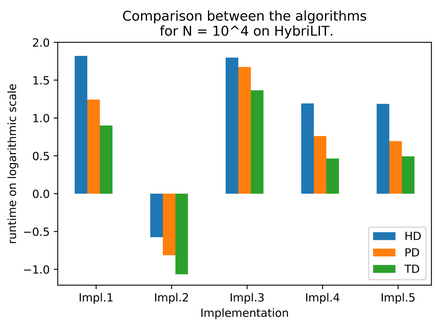


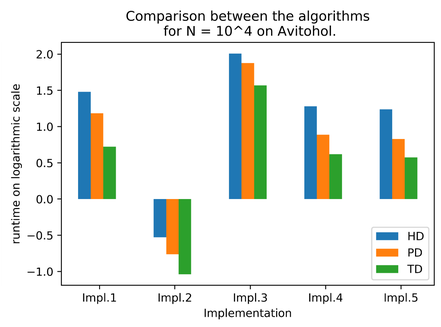






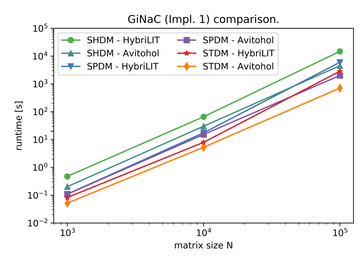






Order of growth	Implementation
$O(N)$	2, 4, 5
$O(N^2)$	1^1, 3

[1] a bit growing on "HybriLIT"



Results – "Avitohol"

	GiNaC ("lst" – var-size) := Impl. 1		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	0.203257	0.108980	0.051806
10^4	30.189948	15.238341	5.280648
10^5	4514.675670	2009.600485	711.940280

	GiNaC ("matrix" – fixed-size) := Impl. 2		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	0.030020	0.017507	0.010651
10^4	0.296938	0.173316	0.091560
10^5	2.822281	1.694611	0.888428

	SymPy ("Matrix" – var-size) := Impl. 3		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^2	1.211271	0.854763	0.450166
10^3	102.026049	75.408729	36.983316
10^4	9927.730700	7517.447892	3751.046286

	SymPy ("Matrix" – fixed-size) := Impl. 4		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	1.845628	0.788081	0.429694
10^4	19.005279	7.738780	4.163307
10^5	194.507700	77.026990	41.930210

	SymPy ("Ar" – fixed-size) := Impl. 5		
	Wall-clock time [s]		
N	SHDM	SPDM	STDM
10^3	1.677360	0.694794	0.387745
10^4	17.256714	6.726764	3.759721
10^5	176.235569	67.404472	37.466753

[1] S. S. Askar, A. A. Karawia. *On Solving Pentadiagonal Linear Systems via Transformations. Mathematical Problems in Engineering.*, 2015, 9 (2015).

[2] M. El-Mikkawy. *A Generalized Symbolic Thomas Algorithm. App. Math.*, 3, 4. (2012).

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