Size, angular momentum and mass for objects*

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FaMAF-UNC and IFEG-CONICET

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Slowly rotating neutron star

In the Newtonian limit, the total energy of a rotating star could be written as:

$$E \approx E_0 + \frac{J^2}{2I}$$

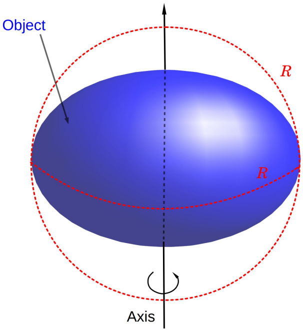
For an ordinary object one expects

$$m_\Omega \leq \frac{\mathcal{R}}{2}.$$

Let $\mathcal{R}_C = \max_\Omega(\rho)$, then $I \approx m_\Omega \mathcal{R}_C^2$

Then we obtain a lower bound to the total energy:

$$E \gtrsim E_0 + \frac{J^2}{\mathcal{R}\mathcal{R}_C^2}$$



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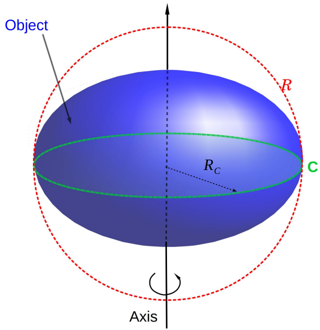

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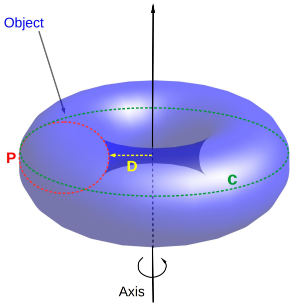
Geometrical Inequalities for Objects

- Reiris (2014) - On the shape of bodies in General Relativistic regimes.

$$8\pi|J| \leq \left(1 + \frac{P}{\pi D}\right) C^2$$



- Dain (2013) - Inequality between size and angular momentum for bodies.

$$|J| \leq \mathcal{R}_{Dain} \qquad \mathcal{R}_{Dain} = \frac{2}{\pi}\frac{\left(\int_\Omega \sqrt{\eta} dV\right)}{\mathcal{R}_{SY}}$$

- Khuri (2015) - Existence of Black Holes Due to Concentration of Angular Momentum.

Setting and Tools

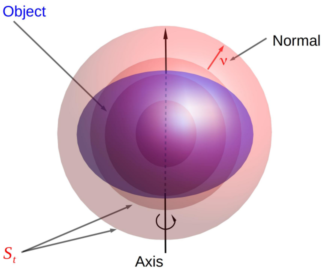
Initial Data $(M, \bar{g}, K; \mu, j)$

- Asymptotically flat
- Axially symmetric, $J(S) = \frac{1}{8\pi}\int_S K_{ij}\eta^i\nu^j ds$
- Dominant energy condition, (DEC), $\mu \geq |j|$.
- Maximal, $Tr(K) = 0$
- Has no minimal surfaces
- j has compact support in some axially symmetric, compact, and connected region Ω.

Setting and Tools

We use the Inverse Mean Curvature flow (IMCF) of surfaces S_t, and the Geroch Energy

$$E_G(S_t) := \frac{A_t^{1/2}}{(16\pi)^{3/2}}\left(16\pi - \int_{S_t} H^2 ds\right)$$



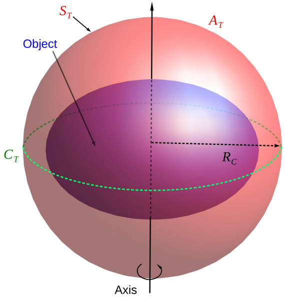
The Geroch Energy

- Is positive $E_G(S_t) \geq 0$
- When the flow goes to infinity, $\lim_{t\to\infty} E_G(S_t) = m_{ADM}$.
- Is non-decreasing along the flow $\frac{d}{dt}E_G \geq \frac{A_t^{1/2}}{(16\pi)^{3/2}} \int_{S_t} \bar{R}$

Theorem

Let $(M, \bar{g}, K; \mu, j)$ that satisfies the previous hypotheses. Assume there exists a smooth IMCF of surfaces S_t on M starting from some point on axis inside the object and going to spheres at infinity, and **let T be such that S_t is convex for $t \geq T$ and S_T encloses the object**. Then

$$m_{ADM} \geq m_T + \frac{1}{5} \frac{J^2}{\mathcal{R}_A \mathcal{R}_C^2}$$



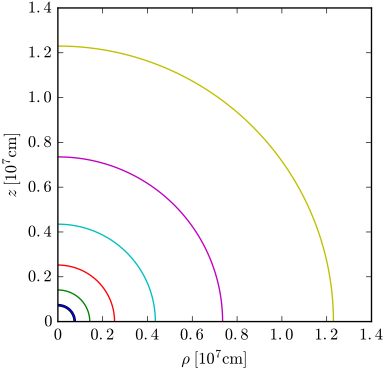
$$\mathcal{R}_A := \sqrt{\frac{A_T}{4\pi}}, \quad \mathcal{R}_C := \frac{\mathcal{C}_T}{2\pi} = \max_{S_T}(\eta)$$

$$m_T := \frac{1}{4\pi} \int_0^{\mathcal{R}_A} d\xi \int_{S_\xi} \bar{R} ds$$

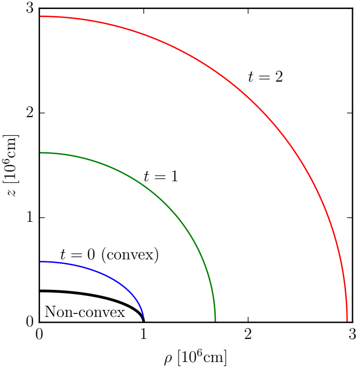
Numerical Tests

IMCF in a conformally flat initial data

Neutron Star



Very Oblate Neutron Star

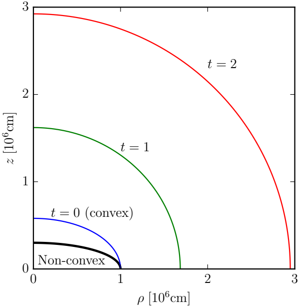


Numerical Tests

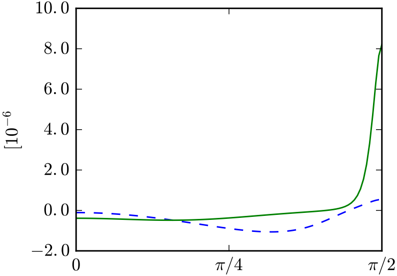
IMCF in a conformally flat initial data

S_t is convex if $\lambda_1, \lambda_2 > 0$, where λ_1, λ_2 are the principal curvatures of the surface.

IMCF for a Very Oblate Neutron Star

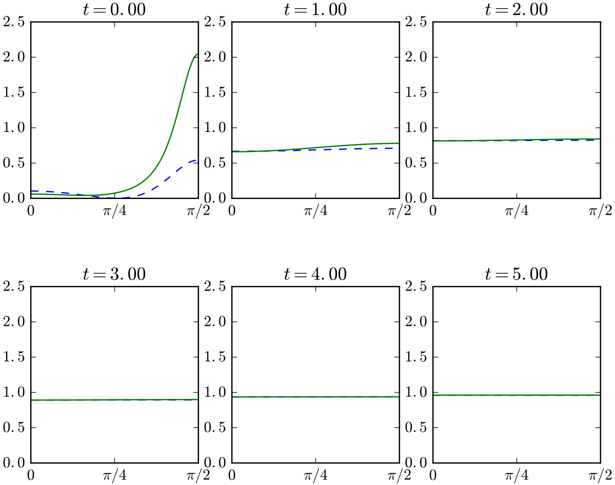


Principal curvatures of the surface of the object



Numerical Tests

Grafics of $\lambda_1 r, \lambda_2 r$ for a Very Oblate Neutron Star



Remarks

$$m_{ADM} \geq m_T + \frac{1}{5}\frac{J^2}{\mathcal{R}_A \mathcal{R}_C^2}$$

- Is global in nature as it involves the ADM mass
- No equation of state is assumed, we only assume the DEC
- Disregarding m_T we have:

$$\mathcal{R}_C^2 \geq \frac{1}{5}\frac{J^2}{m_{ADM}\mathcal{R}_A}$$

Remaks

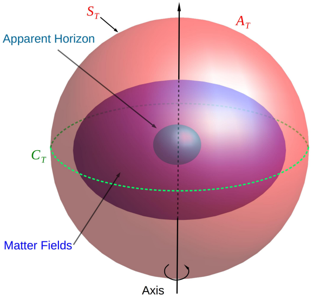
For an initial data $(M, \partial M, \bar{g}, K; \mu, j^i)$ that has an apparent horizon ∂M wich is a compact and connected minimal surface:

$$m_{ADM} \geq m_T + \frac{\mathcal{R}_A}{2} + \frac{1}{5}\frac{J^2}{\mathcal{R}_A(T)\mathcal{R}_C^2(T)}$$

where $\mathcal{R}_A$ is the area of the minimal surface

$$J(S_T) = J(\partial M) - \int_{V(S_T)} j_i \eta^i dv,$$

$$m_T := \frac{1}{16\pi}\int_{\mathcal{R}_A}^{\mathcal{R}_A(T)} d\xi \int_{S_\xi} \bar{R} dS$$



Extra comments

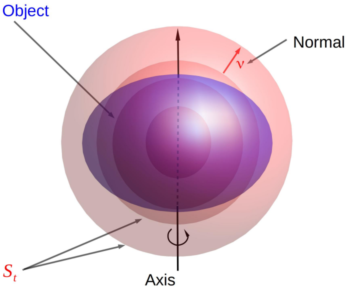
Convexity condition in the proof

Without using the convexity condition we have:

$$m_{ADM} \geq \lim_{t\to\infty} E_G(S_t) \geq m_T + \sqrt{\pi}J^2 \int_T^\infty \frac{A_t^{1/2}}{\int_{S_t} \eta} dt.$$

The Inverse Mean Curvature flow (IMCF)

$$\frac{\partial X}{\partial t} = \frac{\nu}{H}$$



Properties

- $A_t = A_0 e^t$
- $\frac{\partial}{\partial t}\eta = 2\eta \frac{\lambda_2}{H}$.