A new method of constructing binary black hole initial data

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Outline:

1. Motivations
2. The parabolic-hyperbolic form of the constrains
3. Kerr-Schild black holes and the superposed ones
4. Solving the constraints as an initial-boundary value problem
5. Input parameters and ADM charges
6. Summary

Motivations:

GW observations:

- inspiral and merger of binary black holes is of distinguished importance for the emerging field of gravitational wave astronomy
- non-linearities necessitate the use of accurate numerical approaches in determining the emitted waveforms
- precision of these simulations—in particular, their initializations—is of critical importance in enhancing the detection of gravitational wave signals

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- I. Rácz: *Constraints as evolutionary systems*, Class. Quantum Grav. **33** 015014 (2016)
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- I. Rácz: *On the ADM charges of multiple black holes* , arXiv:1608.02283

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Lichnerowicz A (1944) and York J W (1972):

- replace

$$h_{ij} = \phi^4\, \widetilde{h}_{ij} \quad \text{and} \quad K_{ij} - \tfrac{1}{3}\, h_{ij}\, K^l{}_l = \phi^{-2}\, \widetilde{K}_{ij}$$

- using these variables the constraints are put into the **semilinear elliptic system**

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- either "constancy" of $K^l{}_l$ or "smallness" of the TT part of $\widetilde{K}_{ij}$ is required
- it is highly implicit due to its elliptic character and the replacements $h_{ij} = \phi^4\,\widetilde{h}_{ij}$ and $K_{ij} = \frac{1}{3}\,\phi^4\,\widetilde{h}_{ij}\,K^l{}_l + \phi^{-2}\,\widetilde{K}_{ij}$ $\Longrightarrow$
  - no direct control on the physical parameters of the initial data specifications
- **boundary conditions:**
  - are known to influence solutions everywhere in their domains
  - the inner boundary conditions—they are applied with **excision** in the black hole interior—cannot simply be supported by intuition (**trumpet data** ...)
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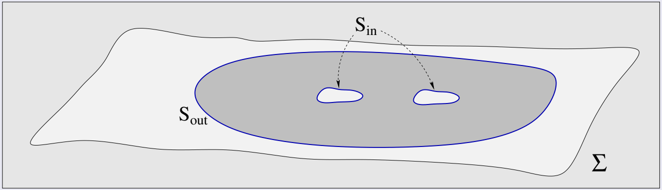

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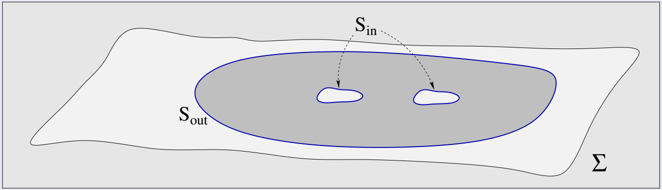

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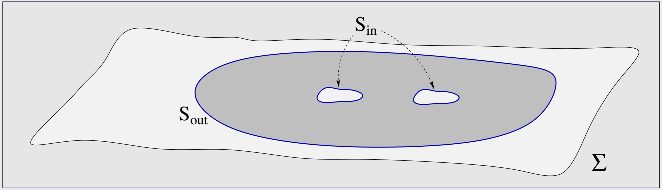

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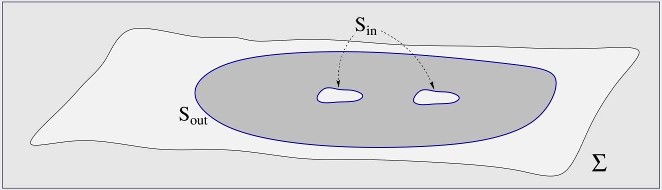

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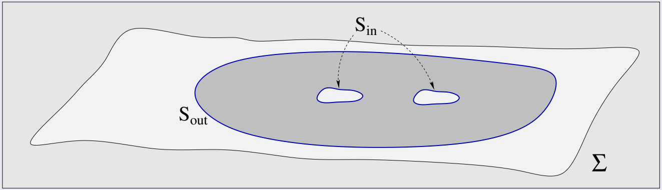

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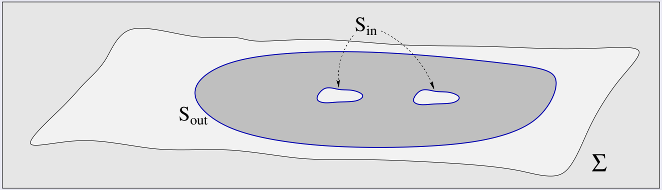

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- the metric h_{ij} can then be given as

$$h_{ij} = \widehat{\gamma}_{ij} + \widehat{n}_i\widehat{n}_j \quad \Longleftrightarrow \quad \{\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}\}$$

$2+1$ decompositions:

Splitting of the symmetric tensor field K_{ij}:

- $$K_{ij} = \boldsymbol{\kappa}\,\widehat{n}_i\widehat{n}_j + [\widehat{n}_i\,\mathbf{k}_j + \widehat{n}_j\,\mathbf{k}_i] + \mathbf{K}_{ij}$$

  where

  $$\boldsymbol{\kappa} = \widehat{n}^k\widehat{n}^l\,K_{kl}\,, \quad \mathbf{k}_i = \widehat{\gamma}^k{}_i\widehat{n}^l\,K_{kl} \quad \text{and} \quad \mathbf{K}_{ij} = \widehat{\gamma}^k{}_i\widehat{\gamma}^l{}_j\,K_{kl}$$

- the **trace** and **trace free** parts of $\mathbf{K}_{ij}$

  $$\mathbf{K}^l{}_l = \widehat{\gamma}^{kl}\,\mathbf{K}_{kl} \quad \text{and} \quad \mathring{\mathbf{K}}_{ij} = \mathbf{K}_{ij} - \tfrac{1}{2}\,\widehat{\gamma}_{ij}\mathbf{K}^l{}_l$$

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- these variables retain the physically distinguished nature of h_{ij} and K_{ij}

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- the **trace** and **trace free** parts of $\mathbf{K}_{ij}$

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The new variables:

- $$(h_{ij}, K_{ij}) \quad \Longleftrightarrow \quad (\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$$

- these variables retain the physically distinguished nature of h_{ij} and K_{ij}

$2+1$ decompositions:

Splitting of the symmetric tensor field K_{ij}:

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The parabolic-hyperbolic form of the constraints:

An evolutionary system for the constrained fields $\widehat{N}, \mathbf{k}_i$ and $\mathbf{K}^l{}_l$:

$$\overset{\star}{K}\,[\,(\partial_\rho \widehat{N}) - \widehat{N}^l(\widehat{D}_l\widehat{N})\,] - \widehat{N}^2(\widehat{D}^l\widehat{D}_l\widehat{N}) - \mathcal{A}\,\widehat{N} + \mathcal{B}\,\widehat{N}^3 = 0$$

$$\mathscr{L}_{\widehat{n}}\mathbf{k}_i - \tfrac{1}{2}\,\widehat{D}_i(\mathbf{K}^l{}_l) - \widehat{D}_i\boldsymbol{\kappa} + \widehat{D}^l\overset{\circ}{\mathbf{K}}_{li} + \widehat{N}\overset{\star}{K}\,\mathbf{k}_i + [\,\boldsymbol{\kappa} - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,]\,\dot{\widehat{n}}_i - \dot{\widehat{n}}^l\,\overset{\circ}{\mathbf{K}}_{li} = 0$$

$$\mathscr{L}_{\widehat{n}}(\mathbf{K}^l{}_l) - \widehat{D}^l\mathbf{k}_l - \widehat{N}\overset{\star}{K}\,[\,\boldsymbol{\kappa} - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,] + \widehat{N}\,\overset{\circ}{\mathbf{K}}_{kl}\overset{\star}{K}{}^{kl} + 2\,\dot{\widehat{n}}^l\,\mathbf{k}_l = 0\,,$$

where $\widehat{D}_i$ denotes the covariant derivative operator associated with $\widehat{\gamma}_{ij}$

$$\overset{\star}{K} = \tfrac{1}{2}\,\widehat{\gamma}^{ij}\mathscr{L}_\rho\widehat{\gamma}_{ij} - \widehat{D}_j\widehat{N}^j$$

$$\overset{\star}{K}_{ij} = \tfrac{1}{2}\mathscr{L}_\rho\widehat{\gamma}_{ij} - \widehat{D}_{(i}\widehat{N}_{j)}\,, \qquad \dot{\widehat{n}}_k = \widehat{n}^l D_l\widehat{n}_k = -\widehat{D}_k(\ln\widehat{N})$$

$$\mathcal{A} = (\partial_\rho\overset{\star}{K}) - \widehat{N}^l(\widehat{D}_l\overset{\star}{K}) + \tfrac{1}{2}[\,\overset{\star}{K}{}^2 + \overset{\star}{K}_{kl}\overset{\star}{K}{}^{kl}\,]$$

$$\mathcal{B} = \tfrac{1}{2}\,[\,\widehat{R} + 2\,\boldsymbol{\kappa}\,(\mathbf{K}^l{}_l) + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \overset{\circ}{\mathbf{K}}_{kl}\,\overset{\circ}{\mathbf{K}}{}^{kl}\,]$$

The parabolic-hyperbolic system:

The parabolic-hyperbolic system:

- no restriction applies to $\widehat{N}^i, \widehat{\gamma}_{ij}, \boldsymbol{\kappa}$ and $\mathring{\mathbf{K}}_{ij}$ $\Longrightarrow$ they are freely specifiable throughout Σ
- the parabolic equation is uniformly parabolic in those subregions of Σ, where $\overset{\star}{K}$ is either positive or negative
- $\overset{\star}{K}$ depends exclusively on the freely specifiable fields $\widehat{\gamma}_{ij}$ and $\widehat{N}^i$ $\Longrightarrow$ its sign can be tailored according to the desire of the investigated problem
- the combined evolutionary system is (locally) well-posed
    - if suitable initial values for the constrained fields $\widehat{N}, \mathbf{k}_i$ and $\mathbf{K}^l{}_l$ are given, on some level surface $\mathscr{S}_0$ in Σ, then, in the domain of dependence of $\mathscr{S}_0$, unique solution exists to the evolutionary system
    - the fields h_{ij} and K_{ij} that can be reconstructed from the freely specifiable and constrained variables do satisfy the Hamiltonian and momentum constraints

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The Kerr black hole:

In Kerr-Schild form:

- $$g_{\alpha\beta} = \eta_{\alpha\beta} + 2H\ell_\alpha\ell_\beta$$
- inertial coordinates (t, x, y, z) adapted to the Minkowski background $\eta_{\alpha\beta}$

$$H = \frac{r^3 M}{r^4 + a^2 z^2}$$

$$\ell_\alpha = \left(1, \frac{r\,x + a\,y}{r^2 + a^2}, \frac{r\,y - a\,x}{r^2 + a^2}, \frac{z}{r}\right)$$

- the Boyer-Lindquist radial coordinate r is related to the spatial part of the inertial coordinates as

$$r^4 - (x^2 + y^2 + z^2 - a^2)\,r^2 - a^2\,z^2 = 0$$

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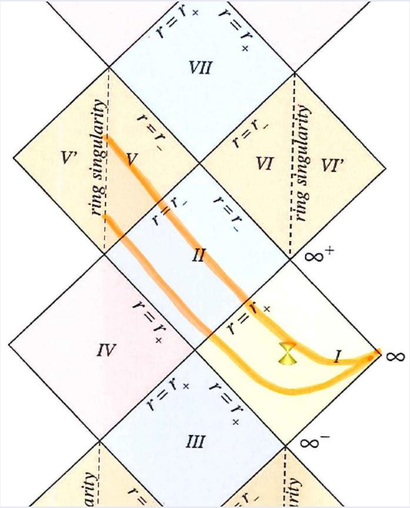
$t = const$ slices in Kerr spacetime:



the $\theta = \frac{\pi}{2}$ section

$$\Sigma \approx \mathbb{R}^3 \setminus \{\text{“ring singularity”}\}$$

Generic Kerr-Schild black holes:

- the Kerr-Schild metrics are form-invariant under Lorentz transformations
  - if a Lorentz transformation $x'^{\alpha} = \Lambda^{\alpha}{}_{\beta}\, x^{\beta}$ is performed
  - the metric retains its distinguished Kerr-Schild form

$$g'_{\alpha\beta} = \eta_{\alpha\beta} + 2H'\ell'_{\alpha}\ell'_{\beta}$$

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$$H' = H\left([\Lambda^{\alpha}{}_{\beta}]^{-1} x'^{\beta}\right), \quad \ell'_{\beta} = \Lambda^{\alpha}{}_{\beta}\, \ell_{\alpha}\left([\Lambda^{\varepsilon}{}_{\varphi}]^{-1} x'^{\varphi}\right)$$

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Superposed Kerr-Schild black holes:

A binary system will be approximated by:

- $$g_{\alpha\beta} = \eta_{\alpha\beta} + 2H^{[1]}\ell_\alpha{}^{[1]}\ell_\beta{}^{[1]} + 2H^{[2]}\ell_\alpha{}^{[2]}\ell_\beta{}^{[2]} \qquad (*)$$
  - $H^{[n]}$ and $\ell_\alpha{}^{[n]}$ correspond to the Kerr-Schild data for individual black holes
- good approximation close to the individual black holes
- Einstein tensor falls off at the rate $\mathcal{O}(|\vec{x}|^{-4})$, where $|\vec{x}| = \sqrt{x^2+y^2+z^2}$

Choice for the free data:

- $\widehat{N}^i, \widehat{\gamma}_{ij}, \boldsymbol{\kappa}$ and $\mathring{\mathbf{K}}_{ij}$ as if (*) solved the Einstein equations
- $\widehat{N}, \mathbf{K}^l{}_l$ and $\mathbf{k}_i$ on some level surface $\mathscr{S}_0$ in Σ deduced from (*) [only on $\mathscr{S}_0$!]

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The initial-boundary value problem:

the $\mathscr{S}_\rho$ surfaces have tacitly been assumed to be compact without boundary:

- in numerical approaches Σ is chosen to be a large but bounded subset of $\mathbb{R}^3$
- the product structure $\Sigma \approx \mathbb{R} \times \mathscr{S}$ can be guaranteed by choosing the $\mathscr{S}_\rho$ leaves to be diffeomorphic to a closed disk in $\mathbb{R}^2$

choose Σ to be a cube centered at the origin in $\mathbb{R}^3$:

- for large enough value of A ...
- boundary of Σ:

  six squares each with edges of size $2A$
- the black holes are assumed to be located on the $z = 0$ plane
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- foliation by $z = const$ level surfaces
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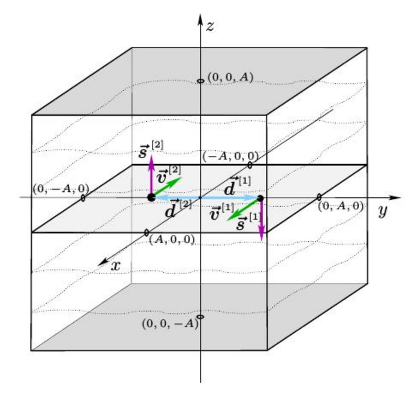

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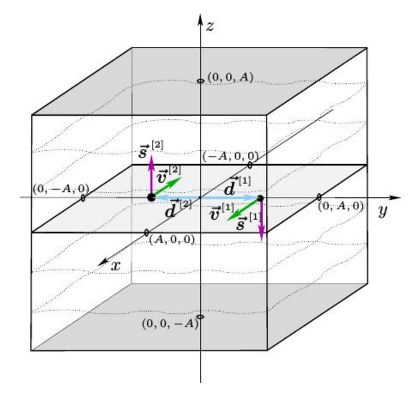

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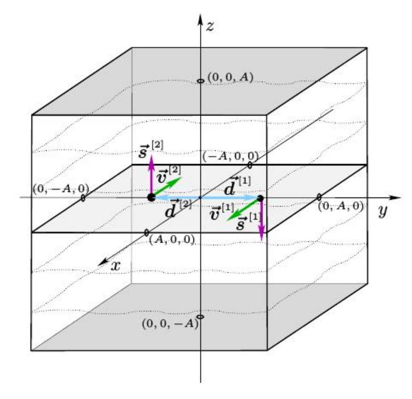
choose Σ to be a cube centered at the origin in $\mathbb{R}^3$:



- for large enough value of A ...
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  six squares each with edges of size $2A$
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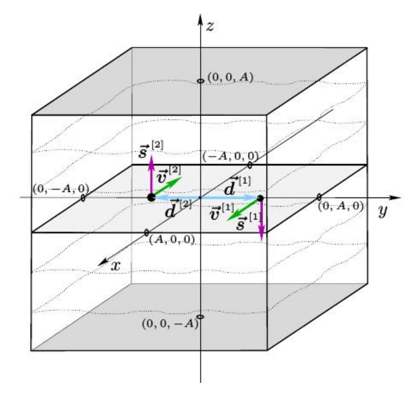

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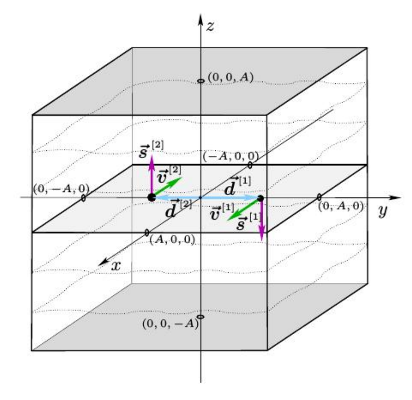

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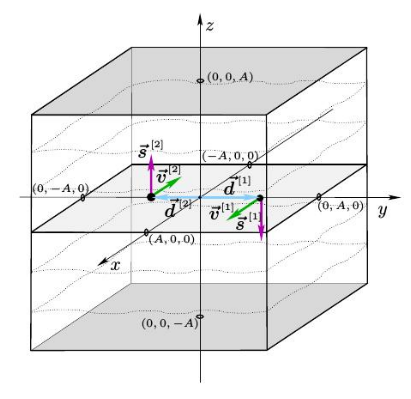

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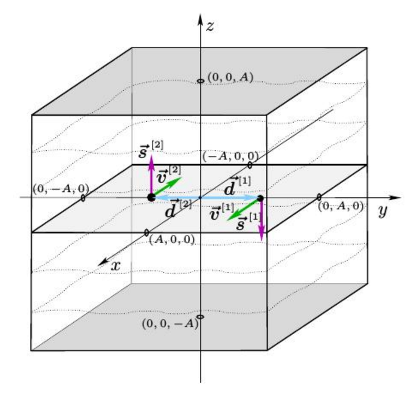

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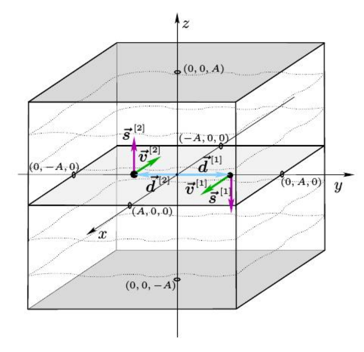

- the sign of $\overset{\star}{K}$ decides whether the parabolic-hyperbolic system evolves in the positive or negative ρ-direction

$$\overset{\star}{K}\,[\,(\partial_\rho \widehat{N}) - \widehat{N}^l(\widehat{D}_l\widehat{N})\,] = \widehat{N}^2(\widehat{D}^l\widehat{D}_l\widehat{N}) + \mathcal{A}\,\widehat{N} + \mathcal{B}\,\widehat{N}^3$$

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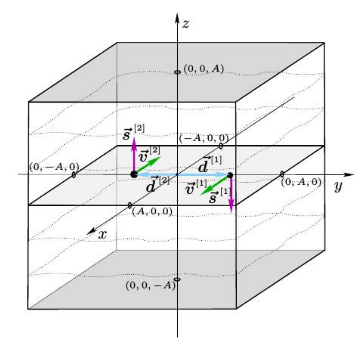

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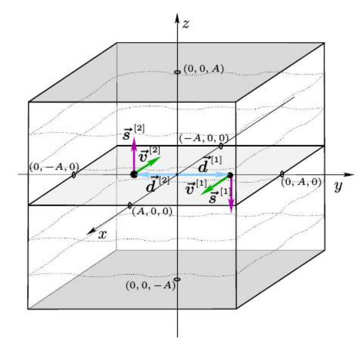

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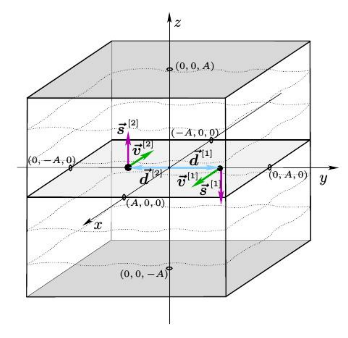

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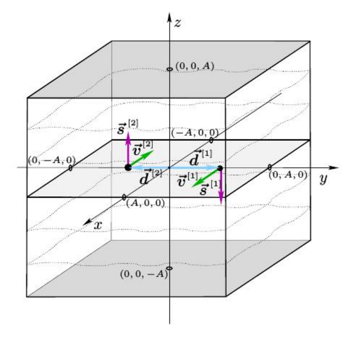

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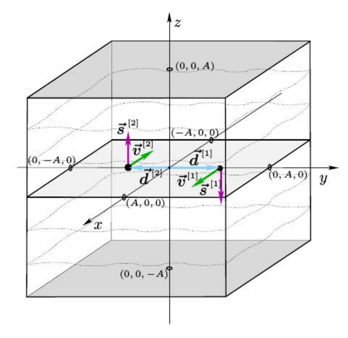

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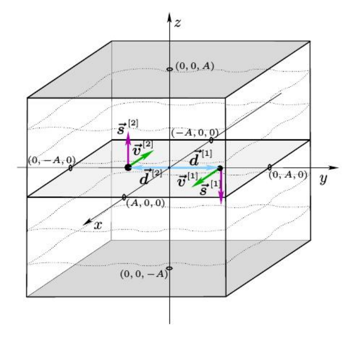

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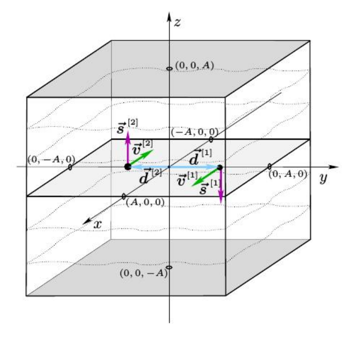

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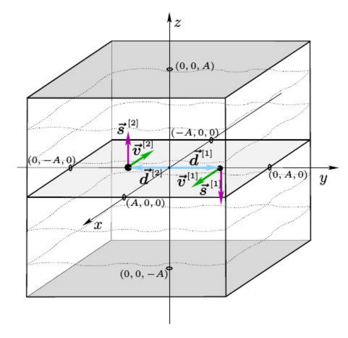

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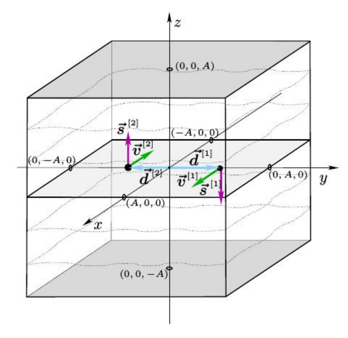

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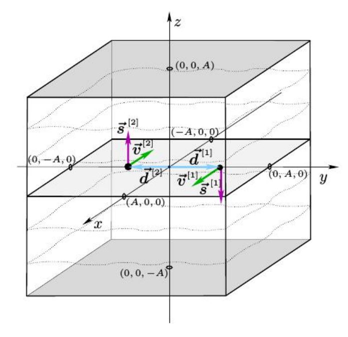

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Input parameters and global ADM charges:

- **Input parameters:** the rest masses $M^{[n]}$, displacements $\vec{d}^{\,[n]}$, speeds $\vec{v}^{\,[n]}$ and spins $a^{[n]}\vec{s}_\circ^{[n]}$ of the involved black holes
  - essentially the same as used in post-Newtonian description of binaries **!!!**
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   - a parabolic-hyperbolic formulation of constraint equations
   - superposing Kerr-Schild black holes
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4. !!! Anna Nakonieczna's talk in the afternoon: construct initial data by integrating numerically the parabolic-hyperbolic form of the constraints
5. the input parameters—the rest masses, speeds, spins and displacements—are essentially the same as used in PN **!!!**
6. each of the ADM charges can be given in terms of the input parameters
7. no use of boundary conditions in the strong field regime (tidal deformations)

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The ADM quantities as flux integrals:

- in the applied admissible coordinates the ADM mass, center of mass, linear and angular momenta are determined by the flux integrals

$$M^{ADM} = \frac{1}{16\pi}\oint_\infty [\,\partial_i h_{ij} - \partial_j h_{ii}\,]\, n^j \mathrm{d}S$$

$$M^{ADM} d_i = \frac{1}{16\pi}\oint_\infty \Big\{ x_i\,[\,\partial_k h_{kj} - \partial_j h_{kk}\,] - \Big[\, h_{kj}\,\delta^k{}_i - h_{kk}\,\delta_{ij} \Big] \Big\}\, n^j \mathrm{d}S$$

$$P_i^{ADM} = \frac{1}{8\pi}\oint_\infty \Big[K_{ij} - h_{kj}\, K^l{}_l \Big]\, n^j \mathrm{d}S$$

$$J_i^{ADM} = \frac{1}{8\pi}\oint_\infty \Big[K_{kj} - h_{kj}\, K^l{}_l \Big]\, Y_i^k n^j \mathrm{d}S$$

- the symbol $\oint_\infty$ is meant to denote limits of integrals over spheres while their radii tend to infinity
- n^i and $\mathrm{d}S$ denote the outward normal and the volume element of the individual spheres in the sequences
- the symbol $Y_i^k = \epsilon_i{}^{jk} x_j$ denote the components of the three rotational Killing vector fields, defined with respect to the applied admissible asymptotically Euclidean coordinates