# String corrected Near-Horizon Geometries

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# Black holes in higher dimensions

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  **Black ring**, BH with horizon topology $S^1 \times S^2$, discovered in

  - Einstein gravity [Emparan, Reall],
  - $\mathcal{N} = 2$ minimal supergravity [Elvang, Emparan, Mateos, Reall].

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  **BMPV**, class of supersymmetric BHs [Breckenridge, Myers, Peet, Vafa]
- String/M-theory suggests us to look at gravitational systems in ten and eleven dimensions. Exotic black hole solutions are expected.
- Finding full BH solutions is difficult (based on ansatz)

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  - Assume supersymmetry
- Supersymmetric near-horizon geometries often experience a doubling of preserved supersymmetries (*supersymmetry enhancement*).
- Enhanced supersymmetry $\Longrightarrow$ Symmetry enhancement symmetry of the full solution, generally at least $\mathfrak{sl}(2,\mathbb{R})$.

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- e.g. $\mathcal{N} = 2, D = 5$ sugra + higher-derivative corrections:

  $\exists$ new class of near-horizon solutions, which do not enjoy the susy enhancement. [Gutowski, Klemm, Sabra, Sloane]

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Aim of this work:

- Investigate properties of $D = 10$ near-horizon geometries with higher-derivative corrections.
- Theory: heterotic supergravity

# Outline

- Near-Horizon Geometries
- Heterotic Near-Horizon Geometries
- Supersymmetry enhancement?
- Lichnerowicz Theorem

# Near-Horizon Geometries

**Assumption**

- spacetime contains an (extremal) Killing horizon, i.e. a null-hypersurface $\mathcal{H}$ associated with the Killing vector $V$.

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One can introduce a Gaussian Null Co-ordinate system $\{u, r, y^I\}$, such that $V = \frac{\partial}{\partial u}$, the horizon $\mathcal{H}$ is located at $r = 0$, and the metric is

$$ds^2 = 2drdu + 2rh_I dudy^I - r^2 \Delta dudu + \gamma_{IJ} dy^I dy^J$$

[Isenberg, Moncrief]

where $\Delta$, $h_I$ and $\gamma_{IJ}$ are analytic in $r$, $u$-independent scalar, 1-form and metric of the 8-dim horizon spatial cross section $\mathcal{S}$, which we shall assume **smooth** and **compact without boundary**.

Then we perform the **near-horizon limit**

$$r \to \varepsilon r \qquad u \to \frac{u}{\varepsilon} \qquad y^I \to y^I \qquad \varepsilon \to 0$$

the metric remains invariant in form, and the near-horizon data $\{\Delta, h_I, \gamma_{IJ}\} = \{\Delta(y), h_I(y), \gamma_{IJ}(y)\}$.

In light-cone basis:

$$\mathbf{e}^+ = du \qquad \mathbf{e}^- = dr + rh - \frac{1}{2}r^2\Delta du \qquad \mathbf{e}^i = e^i{}_J dy^J$$

$$ds^2 = 2\mathbf{e}^+\mathbf{e}^- + \delta_{ij}\mathbf{e}^i\mathbf{e}^j$$

The near-horizon limit only exists for *extremal* black holes.

# Heterotic Near-Horizon Geometries

The bosonic fields of heterotic supergravity are the metric $g$, a real scalar dilaton field $\Phi$, a real 3-form $H$, and a non-abelian 2-form field $F$.

They must be well-defined and regular in the near-horizon limit $\varepsilon \to 0$.

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| | |
|---|---|
| dilaton | $\Phi = \Phi(y)$ |
| 3-form | $H = \mathbf{e}^+ \wedge \mathbf{e}^- \wedge N + r\mathbf{e}^+ \wedge Y + W$ |
| 2-form | $A = r\mathcal{P}\mathbf{e}^+ + \mathcal{B}\ , \qquad F = dA + A \wedge A$ |

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Assume all fields, including spinors, admit a Taylor series expansion in $\alpha'$

$$\Delta = \Delta^{[0]} + \alpha' \Delta^{[1]} + \mathcal{O}(\alpha'^2)$$

# Supersymmetry

We further assume that the solution is *supersymmetric*, i.e. there exists a Majorana-Weyl Killing spinor $\epsilon$, well defined on $\mathcal{H}$, satisfying the KSE:

$$\nabla^{(+)}_M \epsilon \equiv \big(\nabla_M - \frac{1}{8} H_{MN_1N_2}\Gamma^{N_1N_2}\big)\epsilon = \mathcal{O}(\alpha'^2) \qquad \text{gravitino}$$

$$\big(\Gamma^M \nabla_M \Phi - \frac{1}{12} H_{N_1N_2N_3}\Gamma^{N_1N_2N_3}\big)\epsilon = \mathcal{O}(\alpha'^2) \qquad \text{dilatino}$$

$$F_{MN}\Gamma^{MN}\epsilon = \mathcal{O}(\alpha') \qquad \text{gaugino}$$

[Bergshoeff, de Roo]

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$\nabla$ is the Levi-Civita connection.

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We shall integrate the gravitino KSE along the $\mathbf{e}^+$ and $\mathbf{e}^-$ directions ($u, r$ dependence of all bosonic fields is known).

Split the Killing spinors into positive and negative light-cone chiralities

$$\epsilon = \epsilon_+ + \epsilon_- \ , \qquad \Gamma_\pm \epsilon_\pm = 0$$

Integrating the gravitino KSE along $\mathbf{e}^+$ and $\mathbf{e}^-$

$$\begin{aligned} \epsilon_+ &= \eta_+ + \frac{1}{4} u (h+N)_i \Gamma^i \Gamma_+ \eta_- + \mathcal{O}(\alpha'^2) \\ \epsilon_- &= \eta_- + \frac{1}{4} r (h-N)_i \Gamma^i \Gamma_- \eta_+ + \frac{1}{8} r u (h-N)_i (h+N)_j \Gamma^i \Gamma^j \eta_- + \mathcal{O}(\alpha'^2) \end{aligned}$$

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Global analysis (maximum principle) on $\tilde{\nabla}^2 \parallel \eta_\pm \parallel^2$ implies:

$$\Delta = \mathcal{O}(\alpha'^2) \qquad N = h \qquad Y = dh$$

Theorem (Completion of $AdS$ classification):
No $AdS_2$ solutions in heterotic supergravity, up to $\mathcal{O}(\alpha'^2)$, for which $\mathcal{S}$ is smooth and compact without boundary, and all fields are smooth.

# Supersymmetry enhancement

We simplified the reduced KSE to the following minimal set of KSE:

$$\begin{aligned} \tilde{\nabla}_i^{(+)}\eta_\pm &\equiv \left(\tilde{\nabla}_i - \frac{1}{8}W_{ijk}\Gamma^{jk}\right)\eta_\pm = \mathcal{O}(\alpha'^2) \\ \mathcal{A}\eta_\pm &\equiv \left(\Gamma^i\tilde{\nabla}_i\Phi \pm \frac{1}{2}h_i\Gamma^i - \frac{1}{12}W_{ijk}\Gamma^{ijk}\right)\eta_\pm = \mathcal{O}(\alpha'^2) \end{aligned}$$

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- Zeroth order in $\alpha'$:

$\eta_+$ satisfies " $+$ " $\Longrightarrow$ $\eta_- = \Gamma_-\Gamma^i h_i \eta_+$ satisfies " $-$ "

and conversely

$\eta_-$ satisfies " $-$ " $\Longrightarrow$ $\eta_+ = \Gamma_+\Gamma^i h_i \eta_-$ satisfies " $+$ "

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Key ingredient: $\tilde{\nabla}^{(+)} h = \mathcal{O}(\alpha')$
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- First order in $\alpha'$:

Susy enhancement if $\exists$ at least one $\eta_-^{[0]} \neq 0$.
Reason: if $\exists\ \eta_-^{[0]} \neq 0$, extra eqn. + local analysis $\Longrightarrow \tilde{\nabla}^{(+)} h = \mathcal{O}(\alpha'^2)$.

# Lichnerowicz Theorem

**Can we make the statement:**

$$\textbf{Killing Spinors } \eta_\pm \quad \stackrel{1:1}{\Longleftrightarrow} \quad \textbf{solutions of a Dirac equation} \quad \textbf{?}$$

(It works in $D = 11$ sugra, type IIA, IIB and for $AdS$ geometries )

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*Proof*: Define the modified connection with torsion:

$$\hat{\nabla}_i \equiv \tilde{\nabla}_i^{(+)} + \kappa \Gamma_i \mathcal{A}$$

and the modified near-horizon Dirac operator:

$$\mathcal{D} \equiv \tilde{\nabla\!\!\!/}^{(+)} + q\mathcal{A}$$

$\kappa, q \in \mathbb{R}$, and $\quad \tilde{\nabla}^{(+)}\eta_\pm = \mathcal{A}\eta_\pm = \mathcal{O}(\alpha'^2)$ are the KSE.

Consider the functional

$$\mathcal{I} \equiv \int_{\mathcal{S}} e^{c\Phi} \Big( \parallel \hat{\nabla}_i \eta_\pm \parallel^2 - \parallel \mathcal{D}\eta_\pm \parallel^2 \Big) \quad , \qquad c \in \mathbb{R}$$

Manipulate $\mathcal{I}$ and finally assume $\mathcal{D}\eta_\pm = \mathcal{O}(\alpha'^2)$.

- Zeroth order in $\alpha'$:

$$\int_{\mathcal{S}} e^{-2\Phi} \parallel \hat{\nabla}\eta_\pm \parallel^2 + \left(\frac{1}{6}\kappa - 8\kappa^2\right) \int_{\mathcal{S}} e^{-2\Phi} \parallel \mathcal{A}\,\eta_\pm \parallel^2 \quad = \quad \mathcal{O}(\alpha')$$

$(q = \frac{1}{12}, c = -2, 0 < \kappa < \frac{1}{48})$.

$$\Longrightarrow \quad \tilde{\nabla}^{(+)}\eta_\pm = \mathcal{O}(\alpha')\ , \qquad\qquad \mathcal{A}\eta_\pm = \mathcal{O}(\alpha')$$

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$$\int_{\mathcal{S}} e^{-2\Phi}\left[\parallel \hat{\nabla}\eta_\pm \parallel^2 + (\frac{1}{6}\kappa - 8\kappa^2) \parallel \mathcal{A}\,\eta_\pm \parallel^2 + \frac{\alpha'}{64}\left(2 \parallel d\!\!\!/h\,\eta_\pm \parallel^2 + \parallel \tilde{F\!\!\!/}\eta_\pm \parallel^2\right)\right] = \mathcal{O}(\alpha'^2)$$

$$\Longrightarrow \quad \tilde{\nabla}^{(+)}\eta_\pm = \mathcal{O}(\alpha')\ , \qquad \mathcal{A}\eta_\pm = \mathcal{O}(\alpha') \qquad \text{(again!)}$$

and the extra conditions

$$\tilde{F}_{ij}\Gamma^{ij}\eta_\pm = \mathcal{O}(\alpha')\ , \qquad dh_{ij}\Gamma^{ij}\eta_\pm = \mathcal{O}(\alpha')$$

Lichnerowicz Theorem is not enough to establish susy enh. at $\mathcal{O}(\alpha'^2)$.

# Conclusions

## Summary

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- First order in $\alpha'$: global analysis ($\tilde{\nabla}^2 h^2$ and Lichnerowicz) are insufficient to imply susy enhancement.
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- First order in $\alpha'$: global analysis ($\tilde{\nabla}^2 h^2$ and Lichnerowicz) are insufficient to imply susy enhancement. The reason is our no-control on $\mathcal{O}(\alpha'^2)$ corrections.
- Found a sufficient condition for susy enhancement. There must exists at least one non-vanishing $\eta_-^{[0]}$.

## Open questions for heterotic horizons

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- Susy enhancement when all $\eta_-^{[0]} \equiv 0$ ?
- Extension into the bulk:

$$\text{near-horizon geometry} \quad \longrightarrow \quad \text{BH solutions ?}$$

  - Taylor expand the horizon fields at first order in $r$ (moduli).
  - Show that the moduli must satisfy an elliptic system of PDEs.

$\Longrightarrow$ The moduli space is finite dimensional (see Carmen Li's talk)

What happens to the moduli space when string corrections are considered?