Linear perturbations of spherically symmetric black holes in Lovelock theories

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Lovelock theory

The Einstein-Gauss-Bonnet (second order in curvature) theory:

$$\mathcal{L} = -2\Lambda + R + \frac{\alpha}{2}(R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 4\,R_{\mu\nu}R^{\mu\nu} + R^2),$$

where α is a coupling constant.
Ghost-free generalization:

$$\mathcal{L} = -2\Lambda + \sum_{m=1}^{\infty}\frac{\alpha_m}{m}\frac{1}{2^m}\delta^{\mu_1\nu_1\ldots\mu_m\nu_m}_{\lambda_1\sigma_1\ldots\lambda_m\sigma_m}\,R_{\mu_1\nu_1}{}^{\lambda_1\sigma_1}\ldots R_{\mu_m\nu_m}{}^{\lambda_m\sigma_m},$$

$$\alpha_1 = \frac{1}{16\pi G} = 1,\; \delta^{\mu_1\nu_1\ldots\mu_m\nu_m}_{\lambda_1\sigma_1\ldots\lambda_m\sigma_m} = \det\begin{pmatrix} \delta^{\mu_1}_{\lambda_1} & \delta^{\mu_1}_{\sigma_1} & \cdots & \delta^{\mu_1}_{\sigma_m} \\ \delta^{\nu_1}_{\lambda_1} & \delta^{\nu_1}_{\sigma_1} & \cdots & \delta^{\nu_1}_{\sigma_m} \\ \vdots & \vdots & \ddots & \vdots \\ \delta^{\nu_m}_{\lambda_1} & \delta^{\nu_m}_{\sigma_1} & \cdots & \delta^{\nu_m}_{\sigma_m} \end{pmatrix}.$$

Motivation: AdS/CFT

- Quark-gluon plasma is formed due to high-energy collisions of heavy ions (such as lead or gold nuclei) in RHIC or LHC when quarks deconfine.
- Quantum chromodynamics does not describe quark-gluon plasma, because it is not perturbative in the regime of strong coupling g.
- AdS/CFT correspondence: large g in quantum field theory corresponds to the weak-field regime of gravity in AdS.

AdS/CFT dictionary

Quantum system	**Gravity**
D dimensions	$(D+1)$ dimensions
strong coupling ($\lambda = g^2 N$)	weak coupling ($\lambda = \frac{R^4}{l_s^4}$)
equilibration and thermalization	formation of a stable black hole event horizon
temperature of plasma	Hawking temperature of the black hole
poles of the retarded Green function	quasinormal modes
time-scale for perturbation relaxation	the least damped (dominant) mode

AdS/CFT

- Kovtun, Son, and Starinets, PRL 94 (2005) 111601 showed that the AdS/CFT correspondence predicts that

$$\frac{\eta}{s} \approx \frac{\hbar}{4\pi k_B},$$

where η is the shear viscosity, s is volume density of entropy. In this approach, instead of $SU(3)$ group, $SU(N)$ Yang-Mills theory is used in the limit $N \to \infty$.

- In 2008 experiments on RHIC this universal ration was confirmed with good accuracy!
- CFT is not a real quantum chromodynamics (although works at **large coupling**).
  At **weak coupling** we have perturbative theory.
  What happens at **intermediate coupling**?

AdS/CFT and Lovelock gravity

- M. Brigante, R. Myers et.al, PRD **77** (2008) 126006 showed that the Gauss-Bonnet correction violates the ratio and suggested the $(D = 5)$ Einstein-Gauss-Bonnet black brane background as a candidate for the model for quantum fluids

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{GB}) \qquad \left(\lambda_{GB} = \frac{\alpha_2}{L^2} = -\frac{\alpha_2\Lambda}{6}\right).$$

- Higher curvature corrections to the Einstein action, such as Gauss-Bonnet or Lovelock may represent corrections to the results when 't Hooft coupling is large:
S. Waeber, et. al, JHEP 1511 (2015) 087;
S. Grozdanov, et. al, JHEP 1607 (2016) 151;
T. Andrade, et. al, JHEP 1702 (2017) 016.

Spherically symmetric black holes

$$ds^2 = -(1 - r^2\,\psi(r))dt^2 + \frac{dr^2}{1 - r^2\,\psi(r)} + r^2\,d\Omega_n^2$$

where $d\Omega_n^2$ is a $(n = D - 2)$-dimensional sphere, and

$$\frac{\mu}{r^{n+1}} = -\frac{\Lambda}{(n+1)} + \frac{n}{2}\left(\psi + \alpha_2\frac{(n-1)(n-2)}{2}\psi^2 + \alpha_3\frac{(n-1)(n-2)(n-3)(n-4)}{3}\psi^3\ldots\right) \equiv W(\psi),$$

where μ is a constant, proportional to mass.

Problems

1. Nonphysical branches.
   Example for the Gauss-Bonnet theory
   ($\widetilde{\alpha} = \alpha_2(n-1)(n-2)/2$):

$$\psi(r) = \frac{4\left(\frac{\mu}{r^{n+1}} + \frac{\Lambda}{n+1}\right)}{n \pm \sqrt{n^2 + 8\widetilde{\alpha}n\left(\frac{\mu}{r^{n+1}} + \frac{\Lambda}{n+1}\right)}}.$$

2. Parametric space.
   - $\mu > \dfrac{n\left(-2\widetilde{\alpha}\right)^{(n-1)/2}}{4}\left(1 + \dfrac{8\widetilde{\alpha}\Lambda}{n(n+1)}\right) \qquad (\widetilde{\alpha} < 0),$
   - In the AdS space $\widetilde{\alpha}$ has an upper bound, e.g. $\lambda_{GB} \leq 1/4$.
3. Stability.

Solutions in Lovelock gravity

For the physically relevant configurations we observe that ψ is a **monotonic function** of r outside the black hole, spanning

$$\psi_H \geq \psi(r) \geq \psi_A,$$

where $\psi_H = r_H^{-2}$ (r_H is the event horizon radius),
$\psi_H = 0$ for black branes;
$\psi_A = r_C^{-2}$ for de Sitter (r_C is the cosmological horizon),
$\psi_A = 0$ for the asymptotically flat space,
$\psi_A < 0$ in AdS.

With the allowed values for ψ, through analysis of polynomials of ψ, we obtain:

1. $\psi(r)$ and all the derivatives with arbitrary precision;
2. test if the set of parameters is allowed;
3. test for the eikonal instability.

Example of parametric space bounds

$$\widetilde{\alpha} \equiv \alpha_2 \frac{(n-1)(n-2)}{2}, \quad \widetilde{\beta} \equiv \alpha_3 \frac{(n-1)(n-2)(n-3)(n-4)}{3}.$$

$$r_H > \begin{cases} 0, & \widetilde{\alpha} \geq 0, \quad \widetilde{\beta} \geq 0; \\ \sqrt{\sqrt{\widetilde{\alpha}^2 - 3\widetilde{\beta}} - \widetilde{\alpha}}, & \widetilde{\alpha} \geq 0, \quad \widetilde{\beta} < 0; \\ 0, & \widetilde{\alpha} < 0, \quad \widetilde{\beta} > \dfrac{\widetilde{\alpha}^2}{3}; \\ \sqrt{-\widetilde{\alpha}\left(1 + \sqrt{1 - \dfrac{3\widetilde{\beta}}{\widetilde{\alpha}^2}}\right)}, & \widetilde{\alpha} < 0, \quad \widetilde{\beta} \leq \dfrac{\widetilde{\alpha}^2}{3}. \end{cases}$$

and in AdS (for $\widetilde{\beta} \leq \widetilde{\alpha}^2/3$) $\widetilde{\alpha} + \sqrt{\widetilde{\alpha}^2 - 3\widetilde{\beta}} \leq R^2$, where R is the AdS radius.

Eikonal instability of Lovelock black holes

Eikonal instability: the perturbations for large multipole number ℓ are more unstable. Hence summation over multipoles is divergent.

- In the parametric region of the eikonal instability the perturbation equations become nonhyperbolic
  H. Reall, N. Tanahashi, B. Way, CQG **31** (2014) 205005.
- For the eikonal instability it is sufficient that the dominant in ℓ term of the corresponding effective potential has a negative gap
  T. Takahashi, J. Soda, Prog. Theor. Phys. **124** (2010) 711.
- These terms are proportional to a polynomials in ϕ.

Effective potentials

Classification according to the irreducible representations of the rotation group on $(D-2)$-sphere (hydrodynamic analogy):

- Tensor type (scalar channel):

$$V_t(r) = \frac{\ell^2 f(r) T''(r)}{(n-2) r T'(r)} + \mathcal{O}(\ell),$$

- Vector type (shear channel):

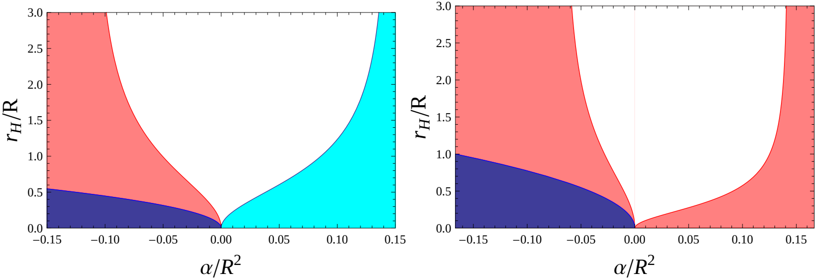
$$V_v(r) = \frac{\ell^2 f(r) T'(r)}{(n-1) r T(r)} + \mathcal{O}(\ell),$$

- Scalar type (sound channel):

$$V_s(r) = \frac{\ell^2 f(r)(2T'(r)^2 - T(r)T''(r))}{n r T'(r) T(r)} + \mathcal{O}(\ell),$$

where $T(r) = r^{n-1} dW/d\psi = n r^{n-1}(1/2 + \widetilde{\alpha}\psi(r))$.

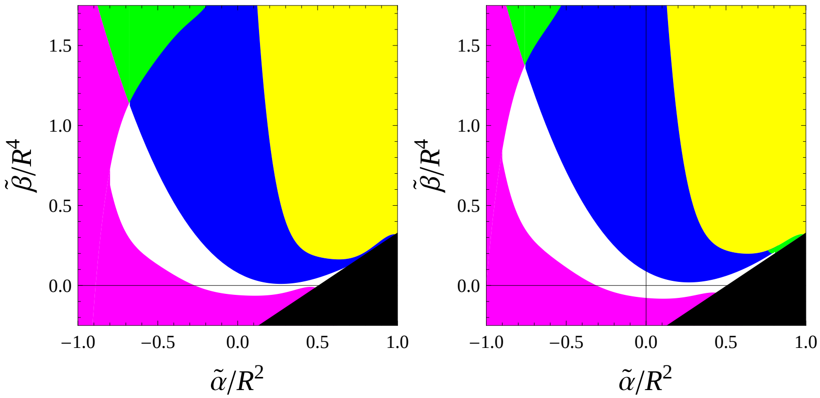
Regions of eikonal instability for AdS black holes



Parametric regions of the eikonal instability for tensor-type perturbations (red) and scalar-type perturbations (cyan) of Einstein-Gauss-Bonnet-AdS black holes for
$D = 5$ (left) and $D = 6$ (right).
For $D = 5$ AdS black holes are stable for

$$-\frac{R^2 r_H^2}{2}\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}R^2+\sqrt{2}r_H^2} \leq \alpha \leq \frac{R^2 r_H^2}{2}\frac{\sqrt{2}-1}{R^2+\sqrt{2}r_H^2}.$$

Eikonal instability of black branes ($r_H \to \infty$)



$D = 7$ (left) and $D = 8$ (right): The excluded parametric region is black, vector-type (ghost) instability – yellow, scalar-type instability – blue, tensor-type instability – magenta, scalar-type and tensor-type instability – green.

Results

1. We obtained a comprehensive method for accurate finding of the parametric space and region of eikonal instability for spherically symmetric black holes in the Lovelock theory of gravity.
2. For the valid parameters we can find numerically the metric, its derivatives, and all the effective potentials with arbitrary precision.
3. Black holes with well-posed initial value problem exist/can be formed in AdS only when the parameters $\alpha_2, \alpha_3 \ldots$ are sufficiently small.

Quasinormal spectrum for the Gauss-Bonnet case

4. The obtained quasinormal spectrum consists from the two essentially different types of modes: perturbative and non-perturbative in the Gauss-Bonnet coupling α.
5. The sound and hydrodynamic modes of the perturbative branch can be expressed as linear corrections in α to the damping rates of their Schwarzschild-AdS limits: $\omega \approx \mathrm{Re}(\omega_{SAdS}) + \mathrm{Im}(\omega_{SAdS})(1 - \alpha \cdot ((D+1)(D-4)/2R^2))i$.
6. The non-perturbative branch of modes consists of purely imaginary modes, whose damping rates unboundedly increase when α goes to zero. The instability is "driven" by these purely imaginary modes.
7. We find only eikonal instability for AdS black holes in the Gauss-Bonnet theory while for dS (and flat) there is also a "normal" instability.

Open questions

1. Stability (non-eikonal) and quasinormal ringing of spherically symmetric black holes in the Lovelock theory.
2. Causality violation in the Lovelock theory.
   In the $D = 5$ Gauss-Bonnet theory causality is violated for the AdS black branes, which have no eikonal instability: M. Brigante, et. al PRL **100** (2008) 191601.
3. Black hole formation in the Lovelock theory.
   In the $D = 5$ Gauss-Bonnet theory in AdS there is a low bound for mass, for which black hole formation is possible. Nils Deppe, et. al JHEP 1610 (2016) 087.