# QCD Showers beyond leading order

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# Disclaimer

*I have never written a QCD shower and should therefore be considered an outsider to the field. I use my knowledge of QCD and resummation, as well as experience in interfacing with parton showers to give you my personal views on the subject.*

*I encourage the experts in the audience to add anything I might miss.*

**Theoretical calculations can be performed in three different limits of field theory**

| | |
|---|---|
| Fixed perturbation theory | $\alpha_s \to 0$ |
| Logarithmic resummation | $\alpha_s \to 0$, $\alpha_s L^2$ fixed |
| Kinematic expansion (parton shower) | $\theta_{ij} \to 0$ |

Each expansion important in different regions

# Theoretical calculations can be performed in three different limits of field theory

## Fixed order perturbation theory

Best precision for inclusive observables
(only one relevant scale in problem)

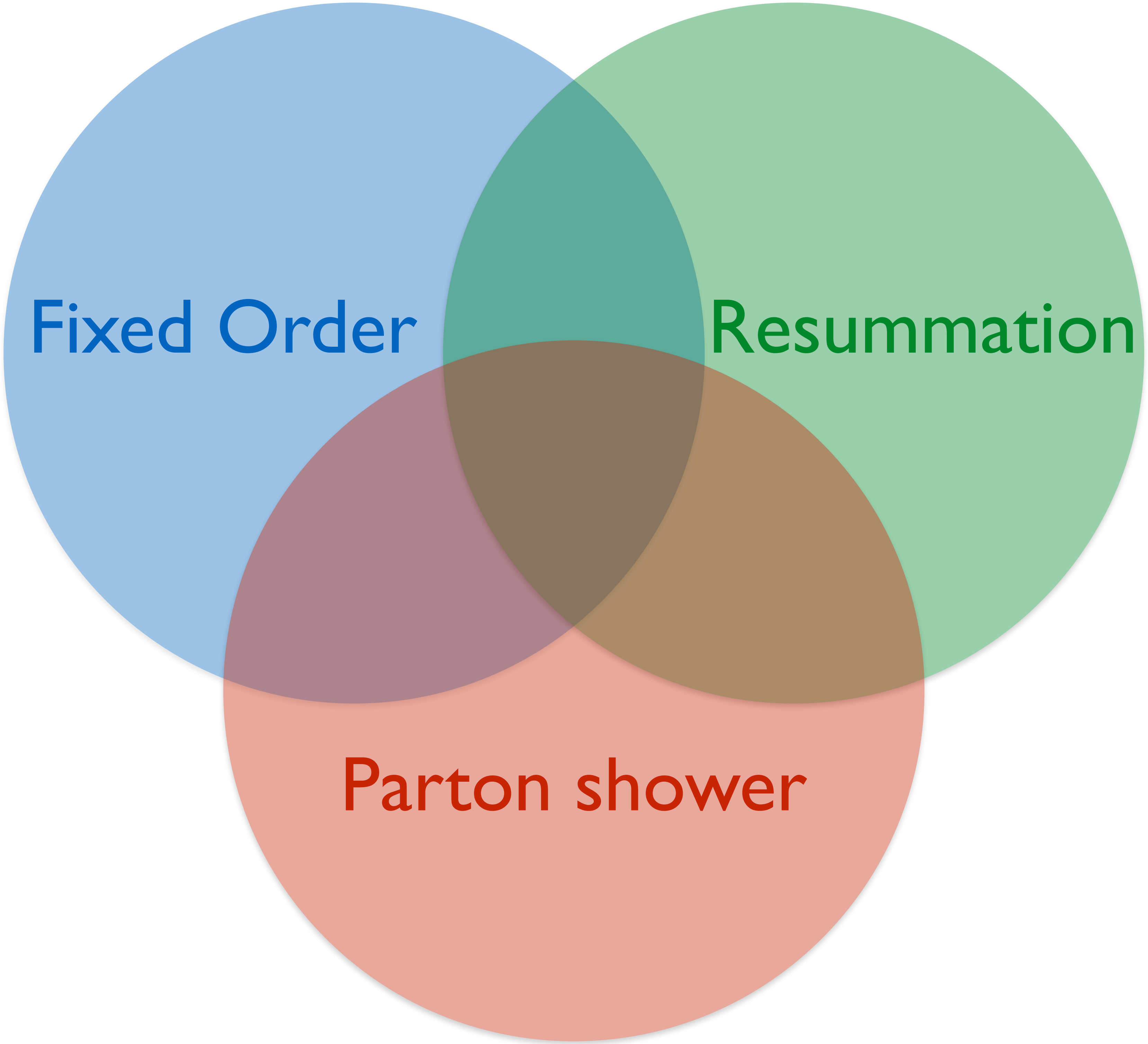
## Logarithmic resummation

Best precision for semi-inclusive observables (large ratio(s)
of scale in problem)

## Parton shower

Only tool for events with arbitrary multiplicity
(event simulation)

## Lots of efforts to combine the various limits



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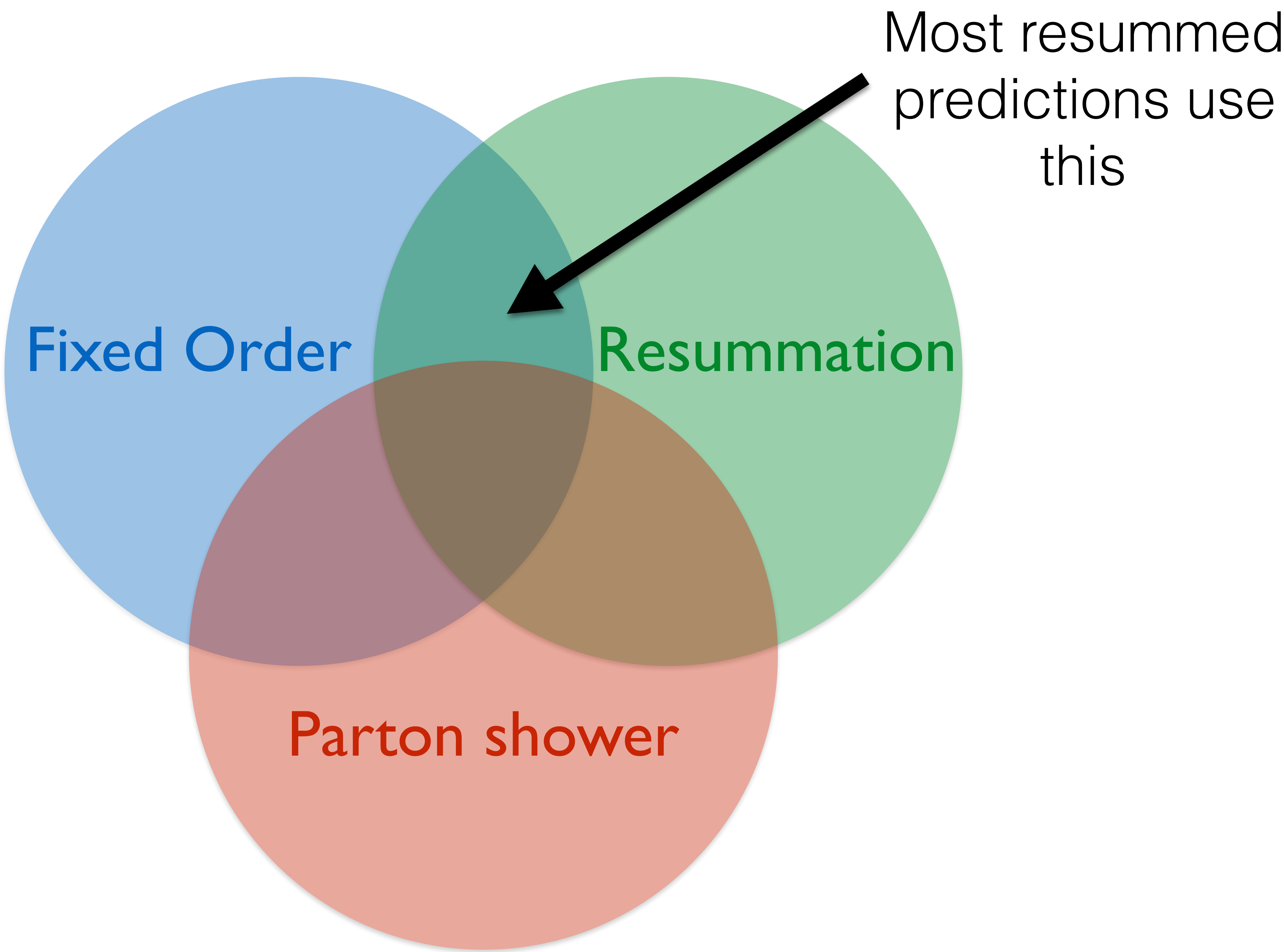

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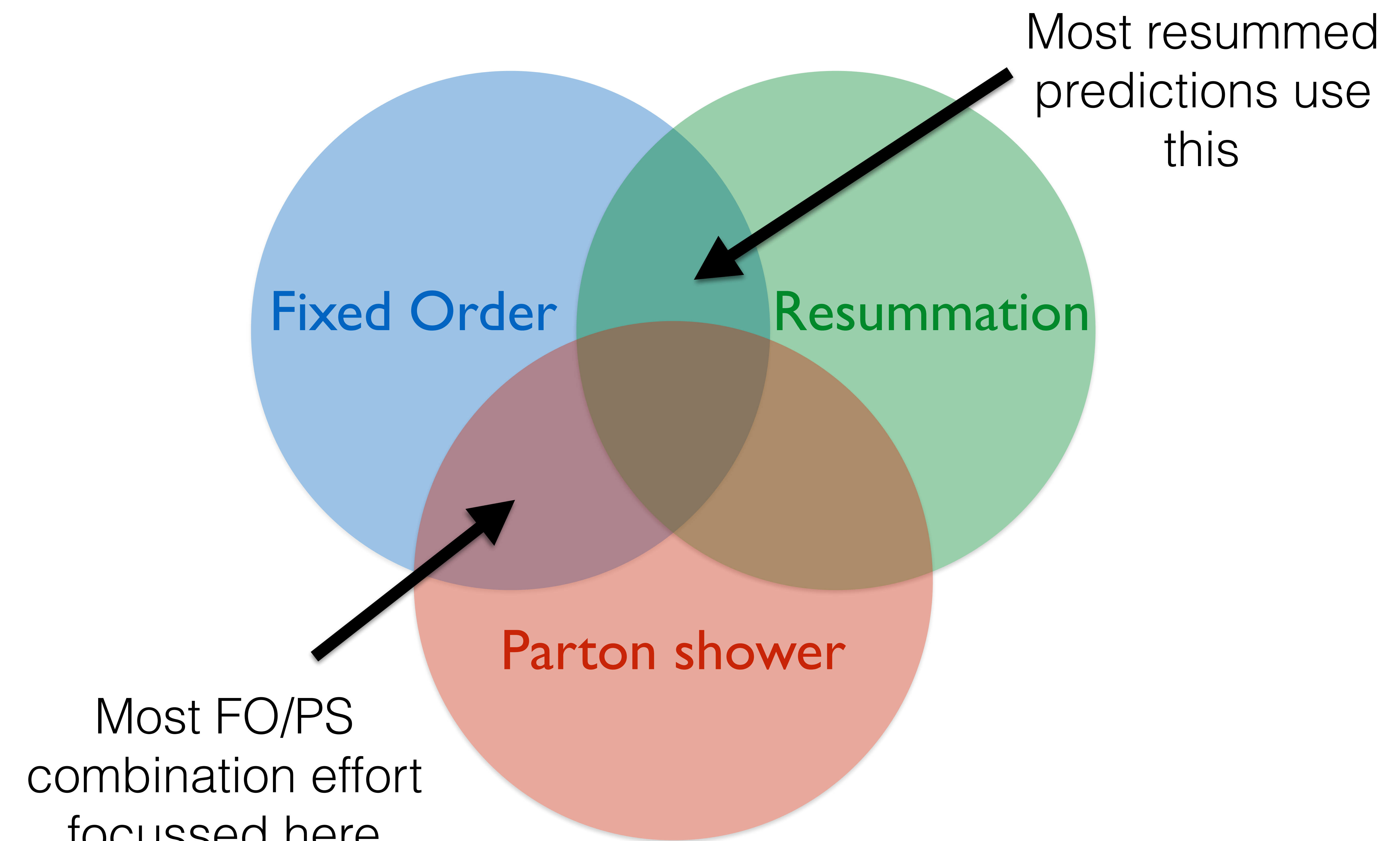
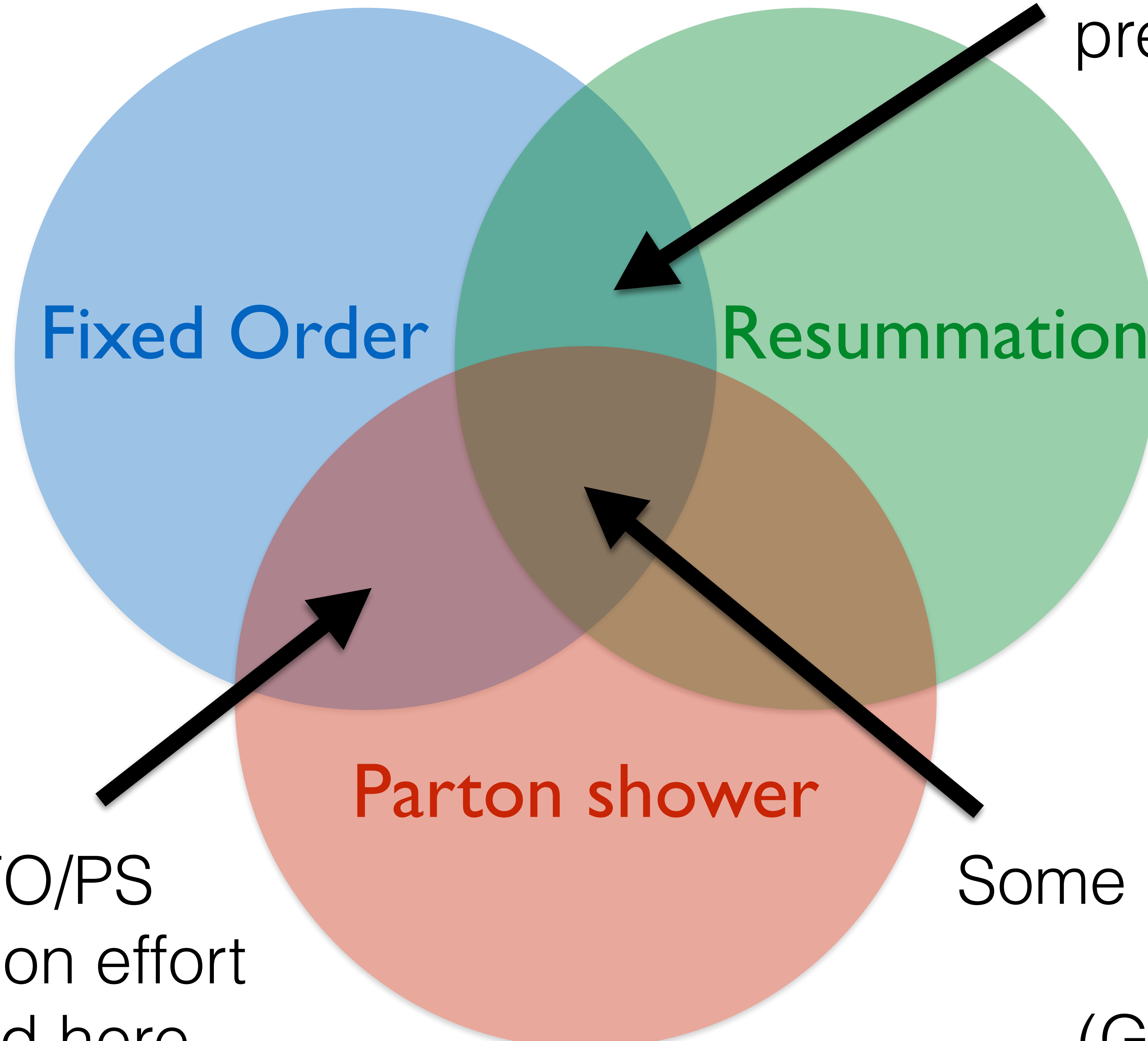


**Lots of efforts to combine the various limits**

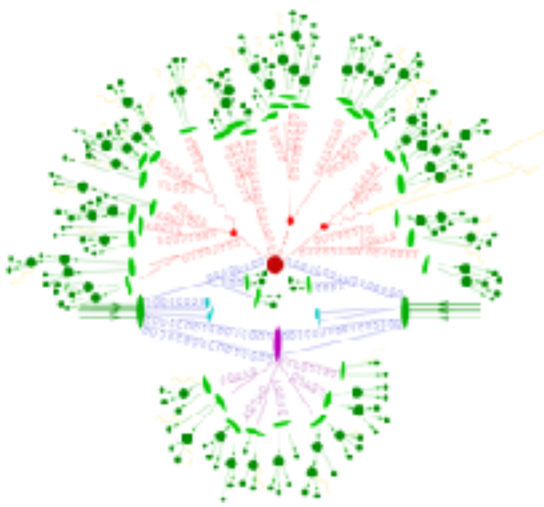
Most resummed predictions use this



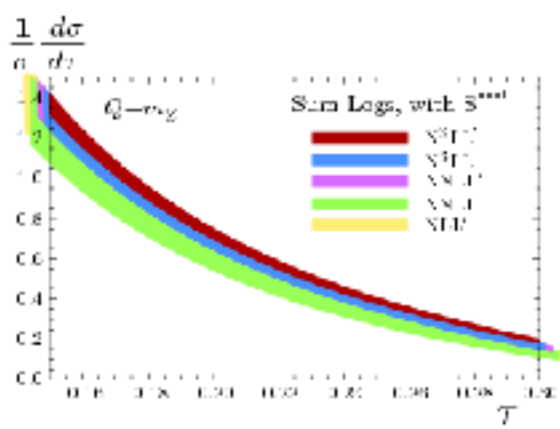

Most FO/PS combination effort focussed here

Some recent effort here (GENEVA)

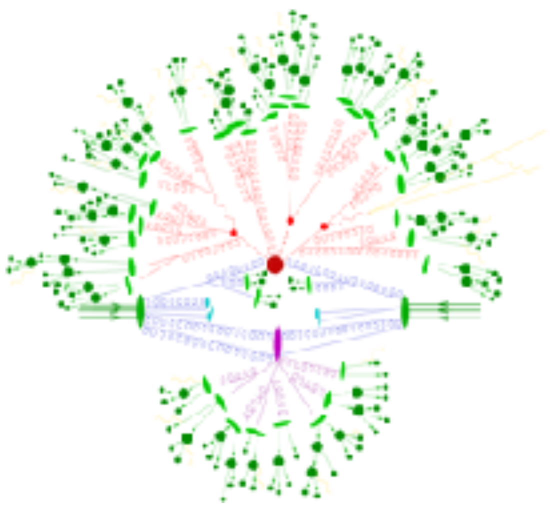
# The talk today will only focus on the parton shower



Theory behind perturbative
parton showers

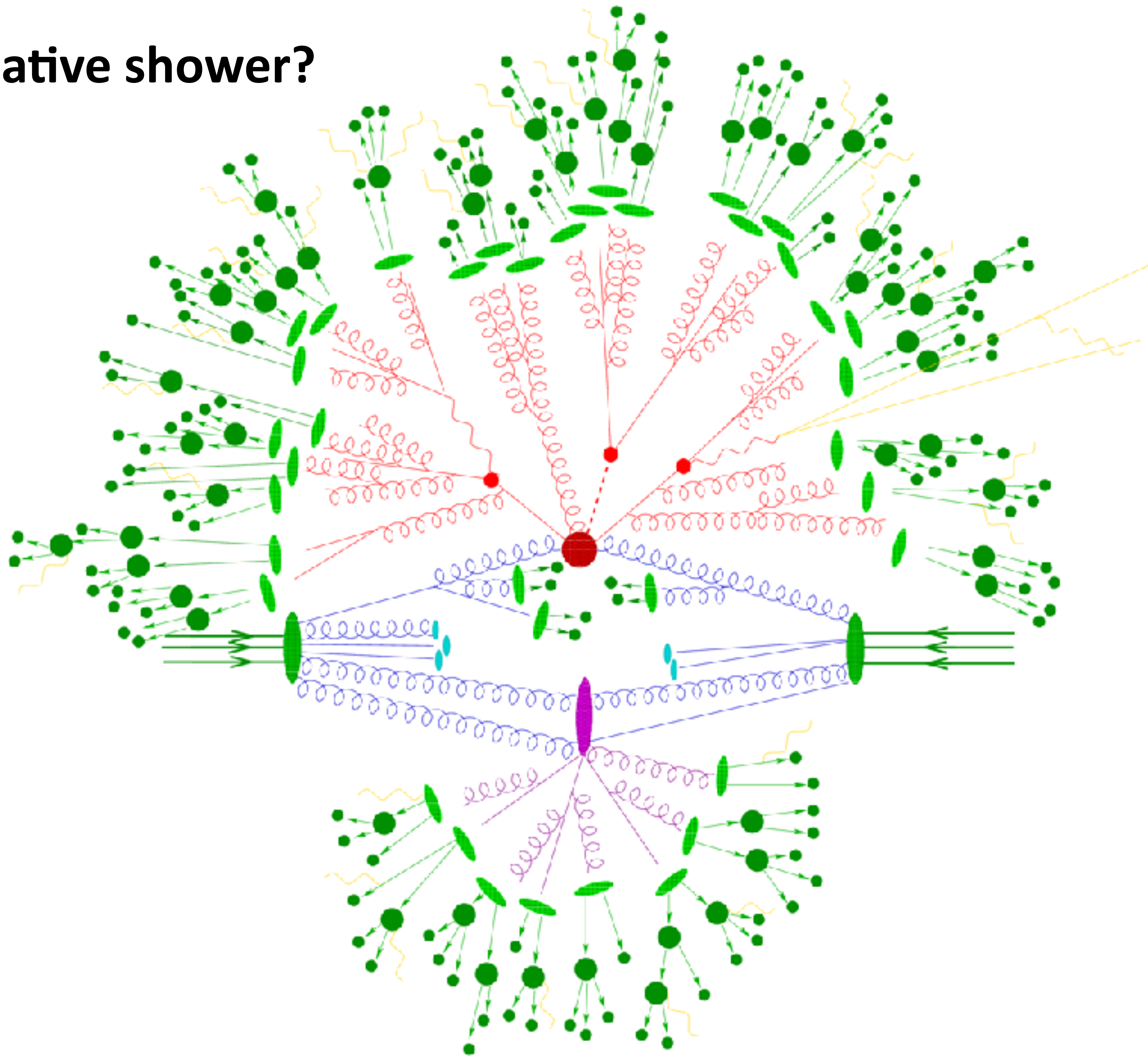


Theory behind resummation
(Why it is hard to go beyond LL
in showers)

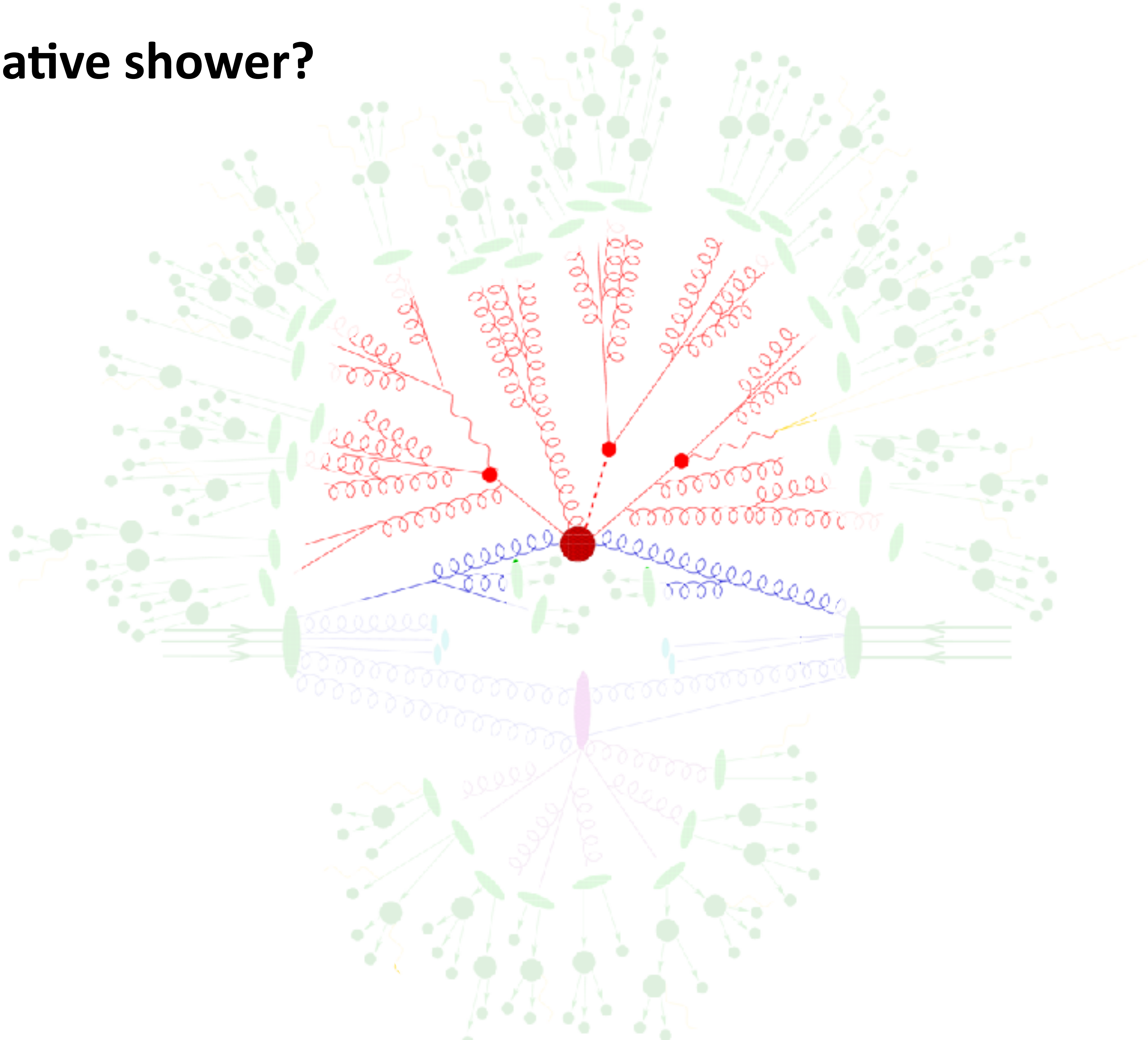


# Theory behind perturbative parton showers

# Perturbative shower?



# Perturbative shower?



## The basic idea of a parton shower

The basic equation underlying a parton shower is

$$\langle O \rangle = G_N(Q_N, O)$$

<O>: expectation value of observable

$G_N$: Shower generating functional

N: Multiplicity of hard interaction

Generating functional can symbolically be written as

$$G_N(t,O) = \Pi_N(t,t_c)\langle O \rangle_N + \int_{t_c}^{t} \mathrm{d}t' \, \Pi_N(t,t') \, \mathrm{SP}(t_1) \, G_{N+1}(t',O)$$

This gives recursive definition (with $t_c$) being shower cutoff

## The basic idea of a parton shower

$$G_N(t,O) = \Pi_N(t,t_c)\langle O\rangle_N + \int_{t_c}^{t} \mathrm{d}t'\,\Pi_N(t,t')\,\mathrm{SP}(t_1)\,G_{N+1}(t',O)$$

Expand recursive definitions to a few orders (with N=2)

$$\begin{aligned}\langle O\rangle = {} & \Pi_2(Q,t_c)\langle O\rangle_2 + \int_{t_c}^{Q}\mathrm{d}t_1\,\Pi_2(Q,t_1)\,\mathrm{SP}(t_1)\,\Pi_3(t_1,t_c)\langle O\rangle_3 \\ & + \int_{t_c}^{Q}\mathrm{d}t_2\,\int_{t_c}^{t_1}\mathrm{d}t_2\,\Pi_2(Q,t_1)\,\mathrm{SP}(t_1)\,\Pi_3(t_1,t_2)\,\mathrm{SP}(t_2)\Pi_4(t_2,t_c)\langle O\rangle_4 \\ & + \ldots\end{aligned}$$

A parton shower is probabilistic description that relies on

$\Pi_N(t,t_c)$: probability that N-body system does not change between $t_1$ and

$\mathrm{SP}(t)$: probability of one emission at scale t

## Probabilistic evolution requires unitarity

The two main building blocks of a parton shower are

$\Pi_N(t, t_c)$: probability that N-body system does not change between $t_1$ and

$\mathrm{SP}(t)$ : probability of one emission at scale t

Probability conservation (unitarity) requires

$$P_{\text{no branch}} = 1 - P_{\text{branch}}$$

This gives a relation between the splitting function and no-branching probability

$$\Pi_N(t, t_c) = \exp\left\{ - \int_{t_c}^{t} \mathrm{d}t' \sum_{i=1}^{N} \mathrm{SP}_i(t') \right\}$$

## Probabilistic evolution requires unitarity

$$\Pi_N(t,t_c) = \exp\left\{ -\int_{t_c}^{t} \mathrm{d}t' \sum_{i=1}^{N} \mathrm{SP}_i(t') \right\}$$

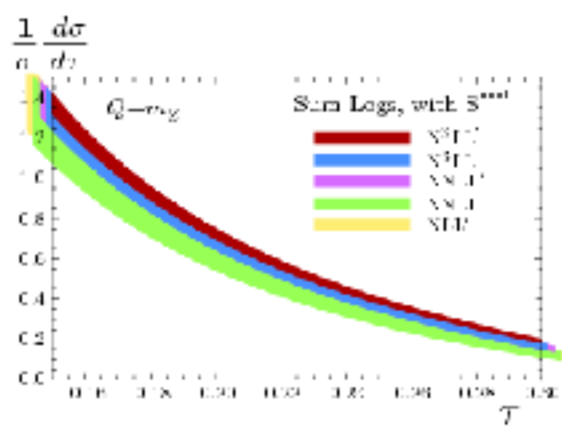
This immediately implies that the no-branching probability is the product of independent Sudakov factors

$$\Pi_N(t,t_c) = \prod_{i=1}^{N} \Pi^{(i)}(t,t_c)$$

In particular, it is has to be independent of

- The kinematics of event
- The color structure of event

**These conditions have to be violated for NLL resummation**



# Theory behind resummation (Why it is hard to go beyond LL in showers)

## The basic idea of resumation

The basic equation underlying a resummed calculation

$$\langle O \rangle = H_N \otimes S_N \otimes \prod_{i \in N} J_i \, \delta \left[ O - O(\Phi) \right]$$

<O>: expectation value of observable

$H_N$: Hard function

$S_N$: Soft function

$J_i$: Jet function

$O(\Phi)$: measurement function

Each function has to be evaluated at its now characteristic scale, and evolution to common scale resums logarithms

**To understand any relation to the parton shower, useful to only keep collinear physics for a moment**

$$\langle O\rangle = H_N \otimes S_N \otimes \prod_{i\in N} J_i \, \delta\left[O - O(\Phi)\right]$$

- Jet function depends only on the type of "splitting", and there is one function for each final state particle
- Perturbative expansion of jet function is

$$J_i(t_i) = U_i(Q_N, t_i) + U_i(Q_N, t_i)\, \mathrm{SP}(t_i) + \ldots$$

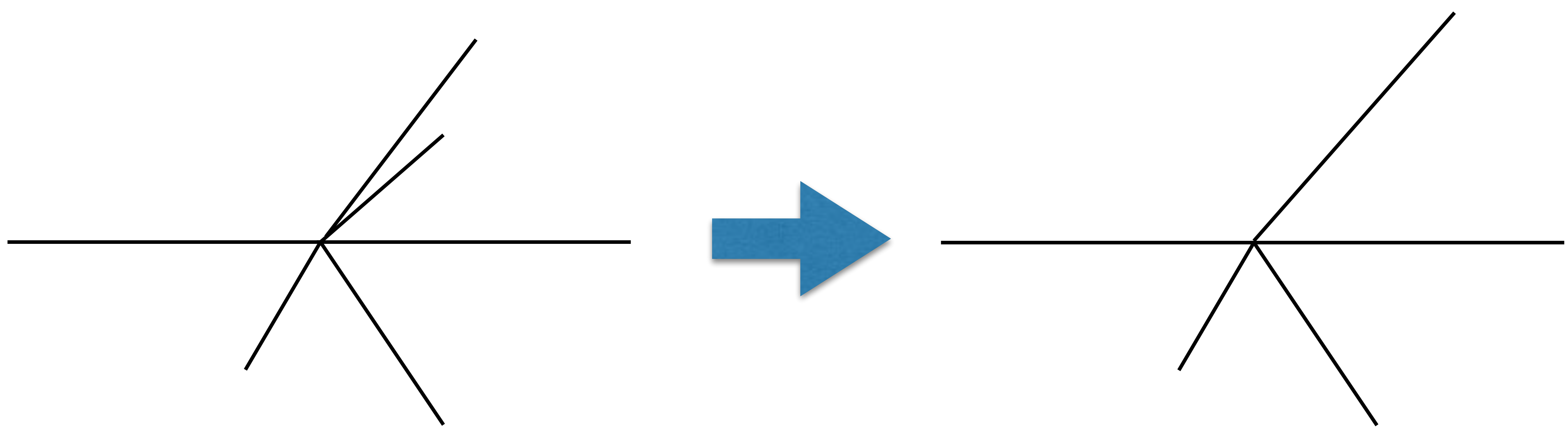
So a single emission starts to look a lot like what a partons shower would give

How about multiple emissions?

**To understand any relation to the parton shower, useful to only keep collinear physics for a moment**

$$\langle O \rangle = H_N \otimes S_N \otimes \prod_{i \in N} J_i \, \delta \left[ O - O(\Phi) \right]$$

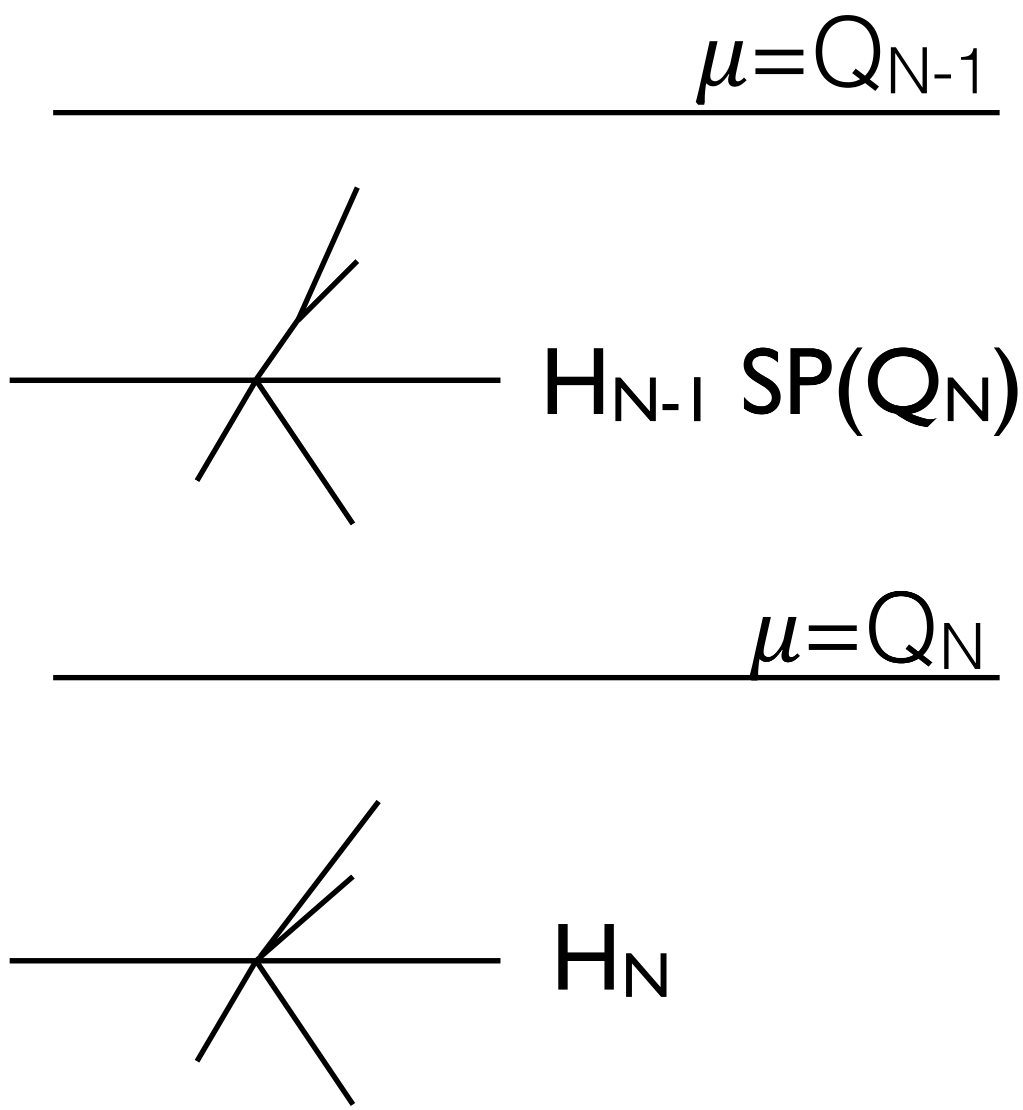
The multiplicity of the hard function always depends on the resolution available



Can be understood quantitively using SCET

**To understand any relation to the parton shower, useful to only keep collinear physics for a moment**

$$\langle O \rangle = H_N \otimes S_N \otimes \prod_{i \in N} J_i \, \delta\left[O - O(\Phi)\right]$$




- One-to-one correspondence between resummation and parton shower
- To LL accuracy, resummation reproduces expressions in the parton shower
- Can perform all SCET calculations to higher order and beyond LL

but. . .

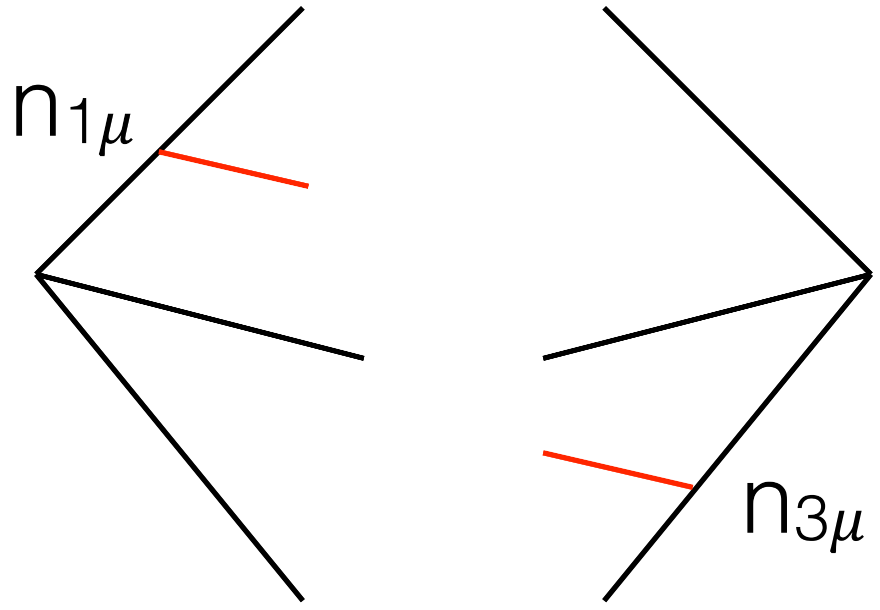
# Soft physics complicates the issue considerably

$$\langle O \rangle = H_N \otimes S_N \otimes \prod_{i \in N} J_i \, \delta\left[O - O(\Phi)\right]$$

1. Soft interactions come entirely from interference effects
2. Both hard and soft functions are matrices in color space
3. In general, soft anomalous dimension depends on kinematics of the full N-body final state
4. Matching calculations in SCET significantly complicated by presence of soft physics

**All these complications start at NLL**

**Treatment of soft interactions and quantum interference breaks beyond LL**




Eikonal coupling of soft particles implies that only interference terms survive in squared amplitudes ($n_i^2=0$)

- Main effect is destructive interference at large angles
- Using angular ordering, interference properly treated
- By using angular veto, most of this effect included

Thus, soft physics can be included in the purely collinear evolution of a parton shower

**Beyond LL accuracy, this no longer true**

**Trivial color structure in the evolution breaks beyond LL**

Both soft and hard functions are complicated matrices in color space

$$H_N \otimes S_N \equiv H_N^{\alpha_1 \alpha_2 \ldots \alpha_N} \otimes S_{N,\alpha_1 \alpha_2 \ldots \alpha_N}$$

- To LL accuracy, matrices are diagonal, and evolution is identical for each element in matrix
- At NLL and beyond, evolution is itself evolution in color space, requiring exponentiation of color matrix

**Exponential in NLL resummation needs to depend on color structure of entire event**

**Anomalous dimension of the soft physics depends on the kinematics of the full N-body kinematics beyond LL**

Important requirements for probabilistic shower was independence of no-splitting probability from event kinematics

$\Pi_N(t, t_c)$: On N (types of particles in event) and scales t, $t_c$

- At LL, the soft divergences are completely given by the collinear divergences
- Beyond LL, evolution depends on angles between all particles in the event (due to the $n_i \cdot n_j$)
- While this dependence is known in principle, not clear if / how combined with unitarity

**Exponential in NLL resummation needs to depend on kinematics of entire event**

**There are many subleading effects that parton showers include (any many groups are actively working to including many more)**

- All parton showers include running couplings
- Simplest sub-leading terms in splitting functions included for most showers
- Most parton showers do some sort of matching to tree-level
- Using dipole showers or changing evolution variables will change subheading behavior
- Very recent work on using full NLO splitting functions

Höche, Krauss, Prestel ('17)

**While this has improved accuracy of parton showers, systematic improvement over LL still elusive**

**There are also efforts in the direction of changing the basic idea of the parton shower to maybe go towards NLL in the future**

- In a series of papers, Nagy and Soper have outlined a theoretical framework
- Some results using several approximations are available

Nagy, Soper ('07-now)
Plätzer

- Simon Plätzer is also working on an amplitude based shower
- There might be other efforts I am forgetting or am not aware of

**So there hope that at some point we could break through the LL barrier.**
**But I think we need to rethink basic setup**