Vector boson production in joint resummation

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Introduction

- Electro-weak boson distributions reached percent level uncertainty *[ATLAS, CMS, LHCb, 15]*
- Differential → Additional scales → Different types of logarithms
- Threshold resummation improved precision and small q_T logarithms

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Joint threshold and Q_T resummation

- Original formalism for joint q_T and threshold *[Laenen, Sterman, Vogelsang, '00]*
- Applied at NLL to prompt photon *[Laenen, Sterman, Vogelsang, '00]*, Higgs and DY *[Kulesza, Sterman, Vogelsang, '02,'03]*, top *[Banfi, Laenen, '05]*, EW SUSY *[Fuks et al., '13]*
- Recent resurgence *[Li, Neill, Zhu, '16]* *[Lustermans, Waalewijn, Zuene, '16][Forte, Muselli, Ridolfi, '17]*
- Parton shower with threshold resummation *[Nagy, Soper, '16]*
- This talk → extension to NNLL *[Marzani, VT, '16]*

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Definition of Threshold

Threshold variable $\hat{\tau} = \frac{Q^2}{\hat{s}}$

Q^2: the invariant mass final state particles

$$1 - \hat{\tau} = 1 - \frac{Q^2}{\hat{s}}$$

$$\sim \frac{\text{energy of the emitted gluons}}{\text{total available energy}}$$

The IR divergences lead to logarithms:

$$(1-\hat{\tau})^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta(1-\hat{\tau}) + \left(\frac{1}{1-\hat{\tau}}\right)_+ - 2\epsilon\left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}}\right)_+$$

$$\boxed{\alpha_s^n \left(\frac{\log^m(1-\hat{\tau}_Q)}{1-\hat{\tau}_Q}\right)_+}$$

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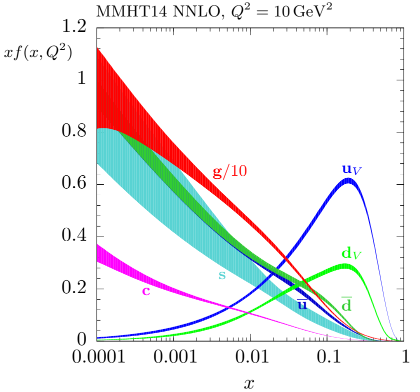
Motivation for threshold resummation

- Not close to hadronic absolute threshold
- PDFs large in low x region
- Close to partonic absolute threshold
- Allows us to go beyond the current scope of fixed order



[Harland-Lang, Martin, Motylinski, Thorne , '14]

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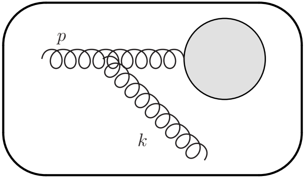
IR Divergences



Soft gluon emission from massless particles leads to:

$$\mathcal{M}_{emi.} \sim \frac{p \cdot \epsilon^*(k)}{p \cdot k} = \frac{p \cdot \epsilon^*(k)}{p_0 k_0 (1 - \cos\theta)}$$

Cross section has a collinear divergence for each initial state particle:

$$\sigma_{emi.} \sim \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} = \frac{2p_1 \cdot p_2}{p_{1,0} p_{2,0} k_0^2 (1 - \cos^2\theta)}$$

- **soft divergences:** $k_0 \to 0$
- **collinear divergences** $\cos\theta \to \pm 1$

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Phase space factorization

Factorization of the phase space needed:

$$dPS_2 = \frac{d^3k}{(2\pi)^3 2k_0}\frac{d^4Q}{(2\pi)^4}(2\pi)^4\delta^4(p_1+p_2-k-Q)\delta\Big((p_1+p_2-k)^2-Q^2\Big)$$

$$= \frac{d^3k}{(2\pi)^3 2k_0}\delta\Big((p_1+p_2-k)^2-Q^2\Big)$$

The remaining delta-function using $\hat{\tau} = Q^2/\hat{s}$ and $x_1 = 1 - 2k_0/\sqrt{\hat{s}}$:

$$\delta\big(\hat{s} - 2k\cdot(p_1+p_2) - Q^2\big) = \frac{1}{\hat{s}}\delta(x_1 - \hat{\tau}_Q)$$

For multiple emissions we obtain in the soft limit

$$\delta\Big(1 - \hat{\tau}_Q - 2(k_{0,1} + \ldots k_{0,n})/\sqrt{\hat{s}}\Big) = \delta(x_1 x_2 \ldots x_n - \hat{\tau}_Q)$$

Does not factorize in x-space

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N-space

Solution: Mellin transform

$$\int_0^1 d\hat{\tau}_Q \hat{\tau}_Q^{N-1} \delta(x_1 x_2 \ldots x_n - \hat{\tau}_Q) = x_1^{N-1} x_2^{N-1} \ldots x_n^{N-1}$$

or Laplace transform

$$\int_0^1 d\hat{\tau}_Q e^{(1-\hat{\tau}_Q)N} \delta\Big(1 - \hat{\tau}_Q - 2(k_{0,1} + \ldots k_{0,n})/\sqrt{\hat{s}}\Big)$$

$$= e^{(1-x_1)N} e^{(1-x_2)N} \ldots e^{(1-x_n)N}$$

$\frac{\log^n(1-\hat{\tau})}{1-\hat{\tau}} \Rightarrow \log^{n+1} N$ and threshold $\hat{\tau} \to 1 \sim N \to \infty$

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Threshold exponent

Resummation equation:

$$G_{a\bar{a}}(N) = \mathcal{C}_{a\bar{a}}\ \exp[\mathcal{G}_{\mathsf{thr}}(N)],$$

Sudakov exponent:

$$\mathcal{G}_{\mathsf{thr}}(N) = -\int_{1/\bar{N}}^{1} \frac{dy}{y}\left[2\int_{\mu_F^2}^{y^2 Q^2} \frac{dq^2}{q^2} A_a(\alpha_{\rm s}(q)) + \tilde{D}_a(\alpha_{\rm s}(yQ))\right]$$

$$= \int_{Q^2/\bar{N}^2}^{Q^2} \frac{dq^2}{q^2}\left[2A_a(\alpha_{\rm s}(q))\log\left(\frac{\bar{N}q}{Q}\right) - \frac{1}{2}\tilde{D}_a(\alpha_{\rm s}(q))\right]$$

$$-\,2\log\bar{N}\int_{\mu_{\rm F}^2}^{Q^2} \frac{dq^2}{q^2} A_a(\alpha_{\rm s}(q))$$

[Sterman, '87][Catani, Trentadue, '89][Vogt, '00][Catani, de Florian, Grazzini, '03]

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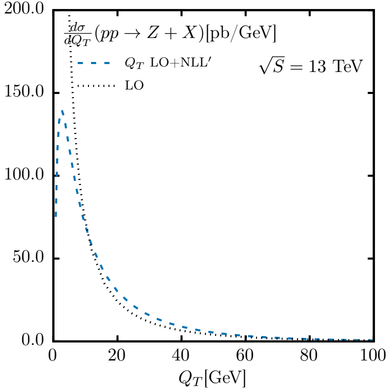
Q_T distribution

Q_T resummation needed for Q_T distributions in the small Q_T limit



Fourier transform: $\frac{\log^n(Q_T)}{Q_T} \Rightarrow \log^{n+1} b$ and threshold $Q_T \to 0 \sim b \to \infty$

Q_T exponent

Q_T resummation:

$$\mathcal{H}^F(N, Q, \alpha_s(\mu_R)) =$$

$$\sigma^{(0)}_{a\bar{a}\to F}(\alpha_s(Q)) H^F(\alpha_s(Q)) \Big(\tilde{C}(N, \alpha_s(Q))\Big)^2 \exp\left\{\int_{\mu_F^2}^{Q^2} \frac{dq^2}{q^2} 2\gamma(N, \alpha_s(q))\right\}$$

Sudakov exponent:

$$\mathcal{G}_{Q_T}(N, b) = -\int_{Q^2/\bar{b}^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q)) \log\frac{Q^2}{q^2} + \tilde{B}(N, \alpha_s(q))\right]$$

[Collins, Soper, Sterman, '85][Bozzi, Catani, de Florian, Grazzini, '03, '05, '07]

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NLO Eikonal integral

$$E_{\mathrm{DY}}^{\mathrm{eik,\ NLO}}(\mathbf{b},N) = \alpha_s 4\pi C_F \int \frac{d^4k}{(2\pi)^3}\theta(k_0)\delta\left(k^2\right)\exp\left[-\sqrt{2}\frac{k_+ + k_-}{2Q}N - i\mathbf{b}\cdot\mathbf{k_T}\right]\frac{2}{k_T^2}$$

$$= \alpha_s 8\pi C_F \int \frac{d^2k_T}{(2\pi)^3} e^{-i\mathbf{b}\cdot\mathbf{k_T}}\frac{2}{k_T^2}K_0\left(\frac{2Nk_T}{Q}\right)$$

Including virtual and collinear counter term:

$$E_{\mathrm{DY}}^{\mathrm{eik,\ NLO}}(b,N) = 2\frac{\alpha_s}{\pi}C_F\int_0^{Q^2}\frac{dk_T^2}{k_T^2}\left[J_0(bk_T)K_0\left(\frac{2Nk_T}{Q}\right) + \log\left(\frac{\bar{N}k_T}{Q}\right)\right]$$

$\bar{N} = Ne^{\gamma_E}$

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NLL exponent

Generalizes to:

$$E_{ab}(b,N,Q,\mu_F) = \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \left\{ \sum_{i=a,b} A_i(\alpha_s(k_T)) \left[J_0(bk_T) K_0\left(\frac{2Nk_T}{Q}\right) + \log\left(\frac{\bar{N}k_T}{Q}\right)\right]\right\}$$

$$- \log \bar{N} \int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i(\alpha_s(k_T))$$

Approximates to:

$$E_{a\bar{a}}(\chi,N,Q,\mu_F) = 2\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T)) \log\left(\frac{\bar{N}k_T}{Q}\right)$$

$$- 2\log \bar{N} \int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T))$$

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Joint function χ

- χ reproduces threshold limit: $\lim_{N\to\infty}\chi(\bar{N},\bar{b}) = \bar{N}$
- χ reproduces q_T limit: $\lim_{b\to\infty}\chi(\bar{N},\bar{b}) = \bar{b} = Qbe^{\gamma_E}/2$
- Function determines power suppressed terms
- Contour needs to avoid poles and branch-cuts

Examples:

- $\chi = \bar{b} + \bar{N}$, simple pole and branch-cut structure, $\bar{N}/\bar{b}$ and $\bar{b}/\bar{N}$ power suppressed terms
- $\chi = \bar{b} + \frac{\bar{N}}{1+\bar{b}\eta/\bar{N}}$, $\eta > 0$, $(\bar{N}/\bar{b})^2$ and $\bar{b}/\bar{N}$ power suppressed terms, more complicated pole and branch-cut structure, angular restrictions for $\eta \neq 1/4$

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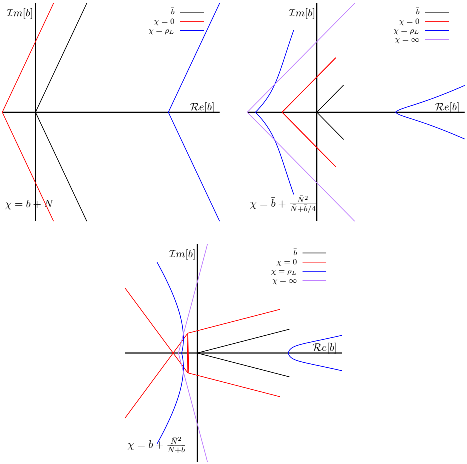
Contour



$\chi = 0$, $\chi = \exp[1/(2\alpha_s b_0)]$, $\chi = \infty$, $\bar{b}$

Comparison to threshold and q_T

$$E_{a\bar{a}}(\chi, N, Q, \mu_F) = 2\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T)) \log\left(\frac{\bar{N} k_T}{Q}\right)$$
$$- 2\log\bar{N} \int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T))$$

Becomes threshold exponential for $\chi \to \bar{N}$

$$E_{a\bar{a}}(\chi, N, Q, \mu_F) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[A_a(\alpha_s(k_T)) \log\left(\frac{Q^2}{k_T^2}\right) + B(\alpha_s(k_T))\right]$$
$$+ \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \left[-2\log\bar{N} A_a(\alpha_s(k_T)) - B(\alpha_s(k_T))\right]$$

Becomes q_T exponential for $\chi \to \bar{b}$

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Overview at NLL

Written similar to *[Bozzi, Catani, de Florian, Grazzini, '06]*

$$\frac{d\sigma_F^{(\mathrm{res})}}{dQ_T^2} = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \int_{C_T} \frac{dN}{2\pi i} \left(\frac{Q^2}{s}\right)^{-N+1} \tilde{W}^F(b, N, Q)$$

$$\tilde{W}^F(N, b, Q) = \sum_{c,d} \sum_{\{I\}} \mathcal{H}_{cd}^{\{I\},\,F}(N, Q, \alpha_s(\mu_R), Q^2/\mu_R^2, Q^2/\mu_F^2, Q^2/\mu_Q^2)$$

$$\times\ \tilde{f}_{c/h_1}(N, \mu_F^2)\ \tilde{f}_{d/h_2}(N, \mu_F^2)$$

$$\times\ \exp\left\{E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2)\right\}$$

$$\mathcal{H}^{\,F}(N, Q, \alpha_s(\mu_R), Q^2/\mu_R^2, Q^2/\mu_F^2, Q^2/\mu_Q^2) = \sigma^{(0)}_{a\bar{a}\to F}(\alpha_s(Q)) H_a^F(\alpha_s(Q)) \left(\tilde{C}(N, \alpha_s(Q))\right)^2$$

$$E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q)) \log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right]$$

$$+ \int_{\mu_F^2}^{Q^2} \frac{dq^2}{q^2} 2\gamma(N, \alpha_s(q))$$

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$$\times\ \exp\left\{E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2)\right\}$$

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$$+ \int_{\mu_F^2}^{Q^2} \frac{dq^2}{q^2} 2\gamma(N, \alpha_s(q))$$

Hard contribution

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Overview at NLL

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$$\frac{d\sigma_F^{(\mathrm{res})}}{dQ_T^2} = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \int_{C_T} \frac{dN}{2\pi i} \left(\frac{Q^2}{s}\right)^{-N+1} \tilde{W}^F(b, N, Q)$$

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$$\times\ \tilde{f}_{c/h_1}(N, \mu_F^2)\ \tilde{f}_{d/h_2}(N, \mu_F^2)$$
$$\times\ \exp\Big\{E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2)\Big\}$$

$$\mathcal{H}^{\,F}(N, Q, \alpha_s(\mu_R), Q^2/\mu_R^2, Q^2/\mu_F^2, Q^2/\mu_Q^2) = \sigma^{(0)}_{a\bar{a}\to F}(\alpha_s(Q)) H_a^F(\alpha_s(Q)) \Big(\tilde{C}(N, \alpha_s(Q))\Big)^2$$

$$E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q)) \log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] + \int_{\mu_F^2}^{Q^2} \frac{dq^2}{q^2} 2\gamma(N, \alpha_s(q))$$

Sudakov factor

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Interlude: $\tilde{B}_N$

Usually $\tilde{C}(N, \alpha_s(Q/\chi))$

All hard contributions computed at the same scale (Q), can be described as:

$$\tilde{B}_N(\alpha_s) = B(\alpha_s) + 2\beta(\alpha_s)\frac{d\log C_N(\alpha_s)}{d\log\alpha_s} + 2\gamma(N, \alpha_s)$$

Collinear-anomaly

Take into account difference in threshold and q_T values of $A^{(3)}$

[Becher, Neubert, '10]

$$E^{\text{col-anom, NNLL}} = -\int_{Q^2/\chi^2}^{Q^2/\bar{N}^2} \frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\text{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3 \log\frac{Q^2}{q^2}$$

$$= \int_{Q^2/\chi^2}^{Q^2/\bar{N}^2} \frac{dq^2}{q^2}\beta_0\tilde{D}^{(2)}\left(\frac{\alpha_s(q)}{\pi}\right)^3 \log\frac{Q^2}{q^2}$$

Hard contribution

Difference hard contribution threshold and q_T.

Can be computed based on eikonal integral

$$\Delta\mathcal{H}^{(1)} = A^{(1)}\left[2\log^2\bar{N} + \mathrm{Li}_2\left(-\frac{\bar{b}^2}{\bar{N}^2}\right) + \zeta_2 + 2\log^2\chi - 4\log\chi\log\bar{N}\right]$$

$$= A^{(1)}\left[\zeta_2 + \mathrm{Li}_2\left(-\frac{\bar{b}^2}{\bar{N}^2}\right) + 2\log^2(\chi/\bar{N})\right] \simeq A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]$$

Changes C_{qq} affects $\tilde{B}$

$$\tilde{B}^{(2)} \to \tilde{B}^{(2)} - 2\beta_0\Delta\mathcal{H}^{(1)}$$

Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$\begin{aligned}
E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_F^2) = &-\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] \\
&+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
&-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\
&-\int_{Q^2/\chi^2}^{Q^2/\bar{N}^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_F^2) = -\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right]$$

$$+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T))$$

$$-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2$$

$$-\int_{Q^2/\chi^2}^{Q^2/\bar{N}^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}$$

Change Hard contribution

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

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&+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
&-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\
&-\int_{Q^2/\chi^2}^{Q^2/\bar{N}^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

Exponentiation soft wide-angle

Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_F^2) = -\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right]$$
$$+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T))$$
$$-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2$$
$$-\int_{Q^2/\chi^2}^{Q^2/\bar{N}^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}$$

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$\begin{aligned}
E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_F^2) = &-\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] \\
&+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
&-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\
&-\int_{Q^2/\chi^2}^{Q^2/\bar{N}^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

Only N^{3}LL solution of integral

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_F^2) = -\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right]$$

$$+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T))$$

$$-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2$$

$$-\int_{Q^2/\chi^2}^{Q^2/\bar{N}^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}$$

Collinear anomaly

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Overview NNLL

Reduces to:

$$E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_F^2) =$$

$$-\int_{Q^2/\chi^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}(N, b, \alpha_s(q)) + \frac{1}{2}\tilde{D}(\alpha_s(q))\right]$$

$$-\frac{1}{2}\log\left(\frac{\bar{N}^2}{\chi^2}\right)\tilde{D}\left(\alpha_s\left(\frac{Q}{\chi}\right)\right) + 2\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}\gamma_{\textsf{soft}}(N, \alpha_s(q)),$$

$$\tilde{B}(N, b, \alpha_s) = B(\alpha_s) + 2\beta(\alpha_s)\frac{d\log\tilde{C}(N, b, \alpha_s)}{d\log\alpha_s} + 2\gamma(N, \alpha_s).$$

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Orders of Resummation

Perturbation reordered in α_s and L:

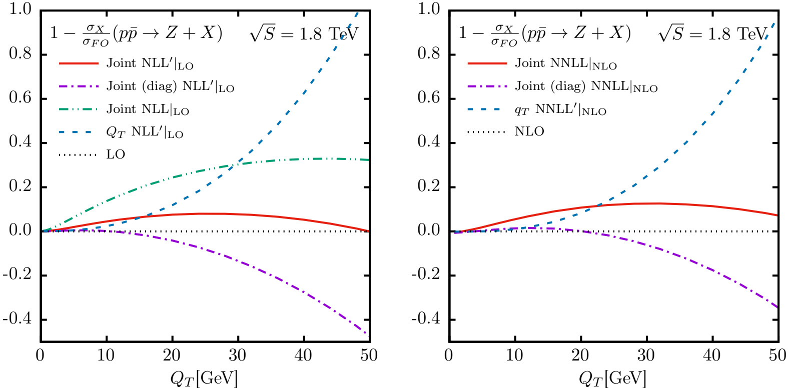
$$\tilde{\sigma} \sim \tilde{\sigma}_{LO} \times \mathcal{C}(\alpha_s) \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots]$$

	⇓	⇓	⇓
With orders of precision:	LL	NLL	NNLL
	⇓	⇓	⇓
	$\alpha_s^n L^{n+1}$	$\alpha_s^n L^n$	$\alpha_s^{n+1} L^n$

Results

[Marzani, VT, 1612.01432]

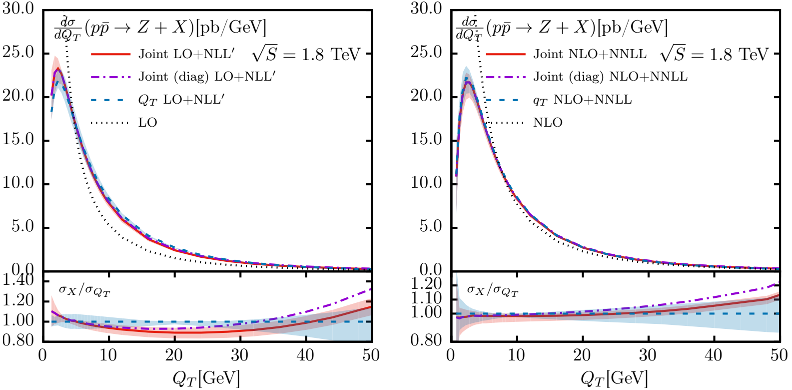
PDFs used: CT14



Results

[Marzani, VT, 1612.01432]

PDFs used: CT14



Results

[Marzani, VT, 1612.01432]

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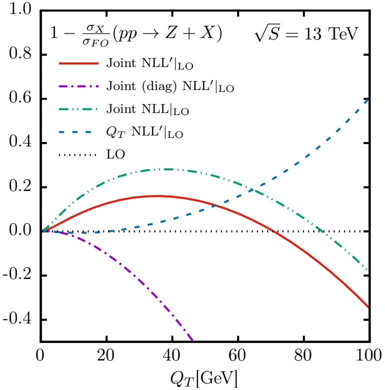

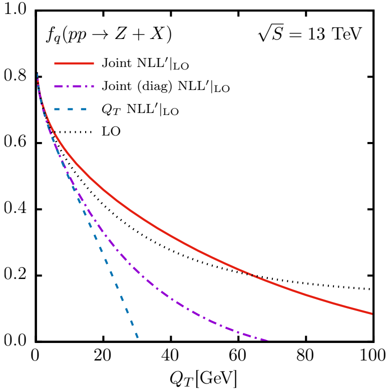

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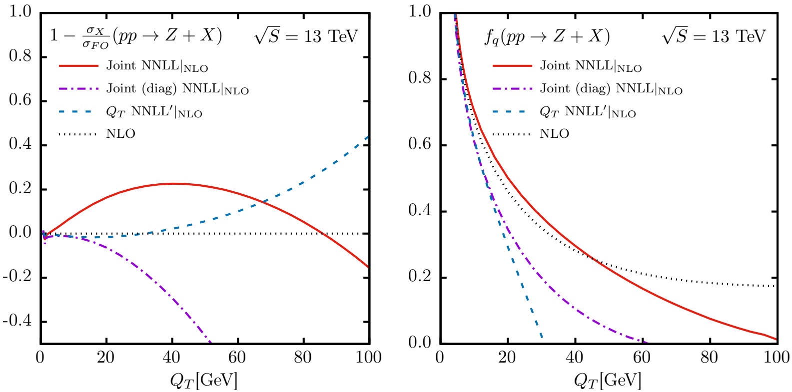
Results

[Marzani, VT, 1612.01432]

PDFs used: CT14



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Results

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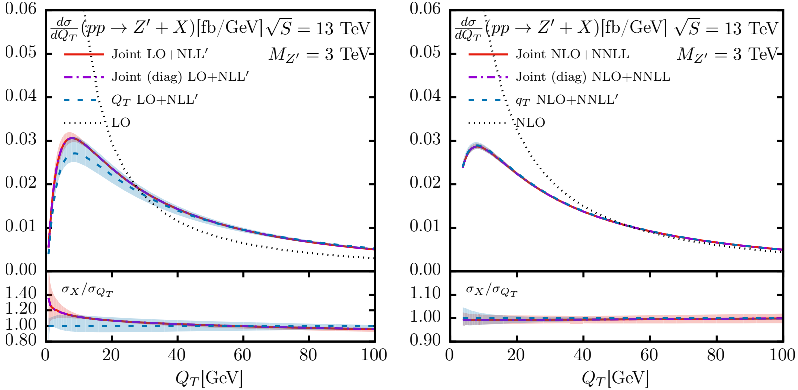

Summary

Conclusions

- Application of joint threshold, Q_T extended to NNLL
- Does not work well for LHC for Z-boson production
- Better agreement to Fixed order
- Lower scale uncertainty mid to high Q_T

Outlook

- Potentially works better for Higgs production
- Application to BSM processes

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Summary

Conclusions

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Thank you for your attention

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Backup

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