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Analytical Calculation for the Gluon Fragmentation into Spin-triplet S-wave Quarkonium

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Outline

- Introduction
- IBP reduction
- Analytical calculation
- Discussion
- Conclusion and Summary

Introduction

Heavy quarkonium

- Simple bound states
  - Non-relativistic system: $v \ll 1$

    Charmonium ($c\bar{c}$): $v^2 \approx 0.3$

    Bottomonium ($b\bar{b}$): $v^2 \approx 0.1$
  - Heavy mass: $m_Q \gg \Lambda_{QCD}$
- Clearly separated momentum scales:

$$m_Q、m_Q v、m_Q v^2、\Lambda_{QCD}$$

  - Hard part

    Production of $Q\bar{Q}$: pQCD
  - Soft part

    Relative Momentum: NRQCD

Introduction

Production mechanism

- Color singlet model
- Color evaporation model
- NRQCD model

$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}+X}\langle \mathcal{O}^H(n)\rangle$$

  - color-octet mechanism: Fock state expansion
  - Short-distance coefficients (SDCs)
    - perturbation calculable
    - Organized in powers of α_s
  - Long-distance matrix elements (LDMEs)
    - parametrize the non-perturbative part
    - Organized in powers of v
    - Universal (process independent)

Introduction

Motivation

- At high p_T, the hadron production cross section is dominated by the parton fragmentation.
- Polarization puzzle
  - $\psi(nS)$ and $\Upsilon(nS)$ production at LO are calculated to be transversely polarized
  - Experimental measurements found them almost unpolarized
- Need analytical results for the fragmentation functions (FFs) at higher order of α_s
  - Analyze the asymptotic behavior
  - Convenient to convolve with the splitting function and the hard part

Introduction

QCD Collinear Factorization

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\hat{\sigma}_{A+B\to i+X}\left(\frac{p_T}{z},\mu\right) \otimes D_{i\to H}(z,\mu) + \mathcal{O}(\frac{1}{p_T^2})$$

- Evolution by the DGLAP equation

$$\mu\frac{d}{d\mu}D_{g\to H}(z,\mu) = \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ig}\left(\frac{z}{\xi};\alpha_s(\mu)\right) D_{i\to H}(\xi,\mu)$$

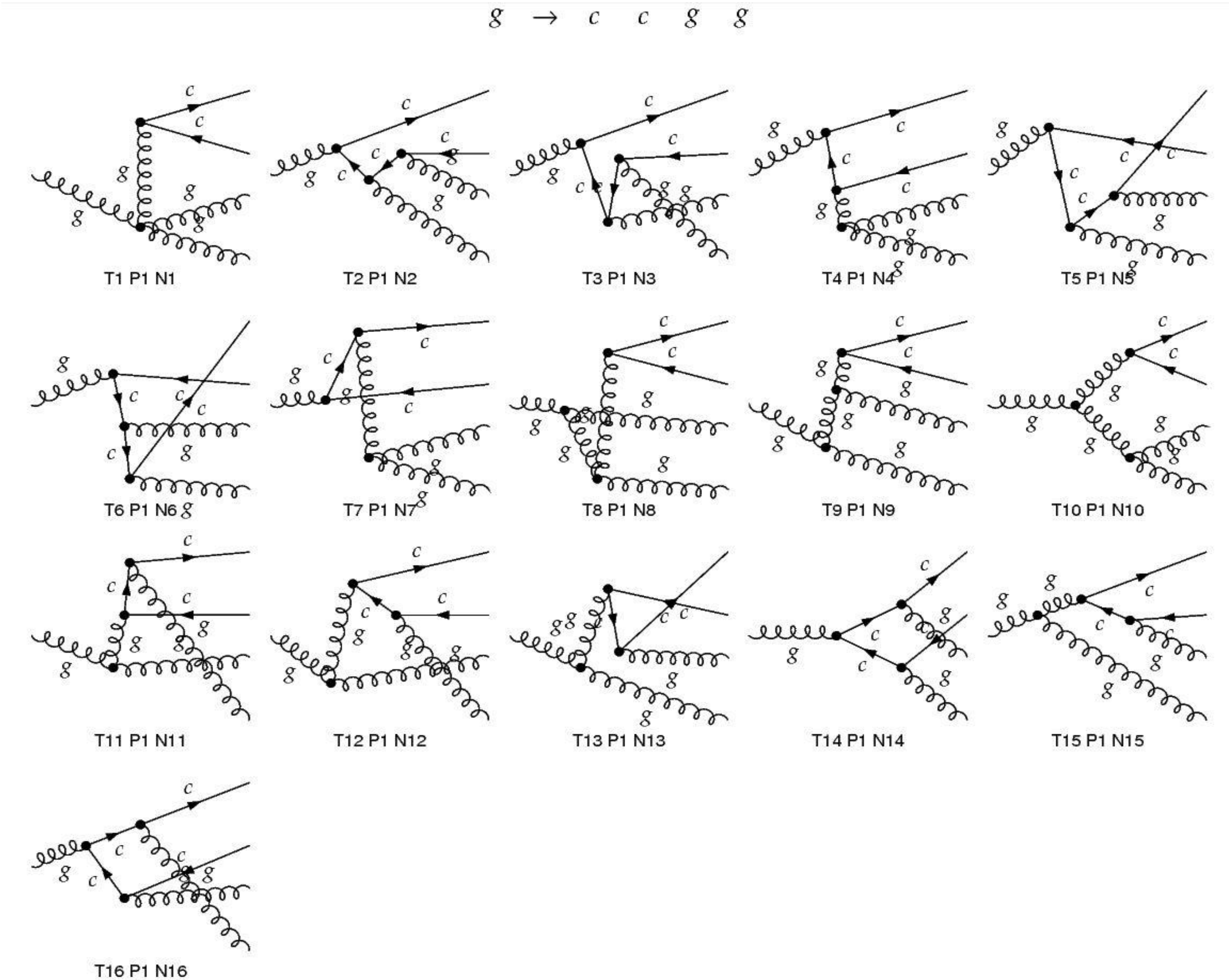
- Compare with the NRQCD factorization

$$D_{i\to H}(z,\mu_0) = \sum_n d_n(z,\mu_0,\mu_f)\langle \mathcal{O}^H(n)\rangle$$

  - LDMEs: lattice, potential model, experiment
  - SDCs: match from the Full QCD calculation
    - Parton fragment into a free $Q\bar{Q}$ state.
    - Calculate the Feynman diagrams

Introduction

Feynman diagrams of $g \to c\bar{c}(^3S_1^{[1]})$



Introduction

Present calculation

Numerical results

1. E. Braaten and T.C. Yuan, Phys. Rev. Lett. 71, 1673 (1993) [hep-ph/9303205].
2. E. Braaten and T.C. Yuan, Phys. Rev. D 52, 6627 (1995) [hep-ph/9507398].
3. G.T. Bodwin and J. Lee, Phys. Rev. D 69 (2004) 054003 [hep-ph/0308016]

Analytical results absent!!!

No polarized results

Difficulties of analytical calculation

- the Light cone coordinate
- Complex phase space with two soft gluon emitting
- High loop formulas

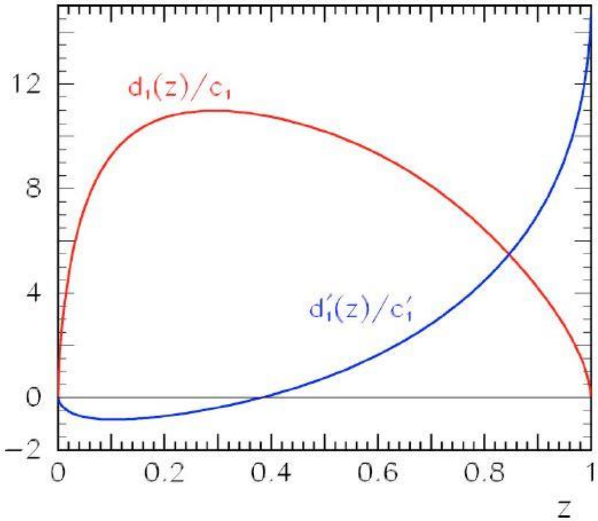


FIG. 3. The color-singlet short-distance coefficients $d_1(z)$ and $d_1'(z)$, which are defined in Eq. (30). The scaling factors in this figure are $c_1 = 10^{-4} \times \alpha_s^3/m^3$ and $c_1' = 10^{-3} \times \alpha_s^3/m^3$.

IBP reduction

- A useful method in high loop calculation
- For a family of Feynman integrals

$$F(a_1, \dots, a_n) = \int \frac{d^D k_1 \dots d^D k_h}{E_1^{a_1} \dots E_n^{a_n}}$$

  E_i : linear functions with respect to the scalar product of loop momenta k_i and external momenta p_i.
- The integration by parts (IBP) relations:

$$\int d^D k_1 \dots d^D k_h \frac{\partial}{\partial k_i}\left(p_j \frac{1}{E_1^{a_1} \dots E_n^{a_n}}\right) = 0$$

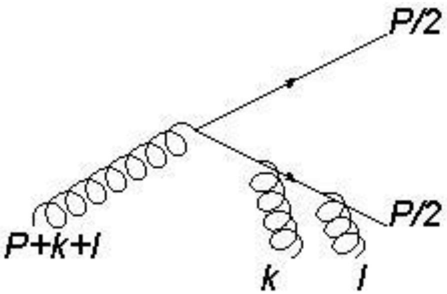
- If E_i is independent and complete, these relations can be rewritten in the following form:

$$\sum_i \alpha_i \, F\big(a_1 + b_{i,1}, \dots, a_n + b_{i,n}\big) = 0$$

  - $b_{i,j} \in \{-1,0,1\}$
  - α_i : linear function of a_j
- Reduce to a finite number of integrals with the denominators lesser and of lower power

Analytical Calculation

- For the unpolarized quarkonium, the FF can be written by the sum of a series of



$$\int d\Phi\, f(z,z_1)\frac{(k\cdot l)^{n_5}}{A^{n_1}B^{n_2}C^{n_3}D^{n_4}}$$

Where, $A = k\cdot P,\ B = l\cdot P,\ C = 2k\cdot l + k\cdot P + l\cdot P,\ D = 1 + 2k\cdot l + 2k\cdot P + 2l\cdot P,$ and

$$d\Phi = \frac{d^D k}{(2\pi)^D}\frac{d^D l}{(2\pi)^D}\frac{P\cdot n}{z^2 S}\delta_+(k^2)\delta_+(l^2)\delta(k\cdot n + l\cdot n - \frac{1-z}{z}P\cdot n)$$

- IBP requires the loop momenta to be integrated boundlessly
- The delta function transform as

$$\delta(k^2) = \frac{i}{2\pi}\lim_{\varepsilon\to 0}\left(\frac{1}{k^2+i\varepsilon} - \frac{1}{k^2-i\varepsilon}\right)$$

- We take

$$E = k^2 + i\varepsilon, F = l^2 + i\varepsilon, G = k\cdot n + l\cdot n - \frac{1-z}{z}P\cdot n + i\varepsilon, H = k\cdot n$$

Analytical Calculation

$EFG: i\varepsilon \to -i\varepsilon$

- The FF transforms to

$$\int \frac{d^D k}{(2\pi)^D} \frac{d^D l}{(2\pi)^D} \frac{(k\cdot l)^{n_5} H^0}{A^{n_1} B^{n_2} C^{n_3} D^{n_4} EFG} + \cdots \text{(7 other similar expressions)}$$

- These 8 similar expressions may be canceled if any of "E, F, G" vanish after the IBP reduction
- Change them back to the integrals with the delta function, we get the master integrals (MIs)(I_a)

$$D_{g\to H}(z) = \sum_{a=1}^{13} f_a(z,\epsilon) I_a$$

where $I_a = \int d\Phi\, F_a$

and F_a is

$$\frac{1}{C}, \frac{1}{D}, \frac{1}{AB}, \frac{1}{AC}, \frac{1}{AD}, \frac{B}{AC}, \frac{D}{AC}, \frac{B}{AD}, \frac{C}{AD}, \frac{1}{AC^2}, \frac{1}{A^2 D}, \frac{1}{CD}, \frac{1}{ABD}$$

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Analytical Calculation

Differential Equation

- The first 12 MIs can be calculated in traditional method
- Take $g(z) = \int d\Phi \frac{z^2}{ABD} = \int \frac{d^D k}{(2\pi)^D} \frac{d^D l}{(2\pi)^D} \left(\frac{i}{2\pi}\right)^3 \frac{P \cdot n}{S} \frac{1}{ABDEFG}$
- Then take the derivative of $g(z)$:

$$\frac{dg(z)}{dz} = \int \frac{d^D k}{(2\pi)^D} \frac{d^D l}{(2\pi)^D} \left(\frac{i}{2\pi}\right)^3 \frac{-(P \cdot n)^2}{z^2 S} \frac{1}{ABDEFG^2}$$

- Use the IBP reduction again

$$\frac{dg(z)}{dz} = \frac{2(z-1)z}{2z-1} \epsilon\, g(z) + h(z)$$

  - $g(z)$: finite
  - $h(z)$: linear combination of the first 12 MIs
  - Initial value: $g(1) = 0$

Analytical Calculation

- The final result for the unpolarized quarkonium:

$$D_{g\to H}(z) = \frac{16(N^2-4)\pi^3\alpha_s^3}{3N^2m^3}\left(\frac{(4z+1)z^2}{4}I_{13} + d_{\rm oth}(z)\right)\left\langle \mathcal{O}\left({}^3S_1^{[1]}\right)\right\rangle$$

$$I_{13} = -\frac{\mathrm{Li}_3\left(\frac{2z-1}{z}\right)}{128\pi^4z^2} - \frac{\mathrm{Li}_3\left(\frac{z}{z-1}\right)}{128\pi^4z^2} - \frac{\mathrm{Li}_3\left(\frac{2z-1}{z-1}\right)}{128\pi^4z^2} + \frac{\mathrm{Li}_2(z)\ln\left(\frac{1-z}{z}\right)}{128\pi^4z^2} - \frac{\mathrm{Li}_2\left(\frac{2z-1}{z-1}\right)\ln\left(\frac{1-z}{z}\right)}{128\pi^4z^2}$$
$$- \frac{\ln^3\left(\frac{1-z}{z}\right)}{768\pi^4z^2} + \frac{\ln z\,\ln(1-z)\,\ln\left(\frac{1-z}{z}\right)}{256\pi^4z^2} + \frac{\zeta(3)}{128\pi^4z^2}$$

$$d_{\rm oth}(z) = \frac{5(z-1)}{96\pi^4}\mathrm{Li}_2(1-z) + \frac{5(z-1)}{48\pi^4}\mathrm{Li}_2\left(\frac{z-1}{z-2}\right) - \frac{(z-1)(2z+3)}{48\pi^4}\mathrm{Li}_2\left(\frac{2(z-1)}{z-2}\right)$$
$$- \frac{\left(32z^7-160z^6+434z^5-763z^4+734z^3-376z^2+96z-9\right)z}{1536\pi^4(z-1)^3(2z-1)^3}\ln^2 z$$
$$- \frac{(z-1)\left(16z^6-168z^5-12z^4+474z^3-528z^2+243z-40\right)}{1536\pi^4(2z-1)^3}\ln^2(1-z)$$
$$+ \frac{\left(z^7-13z^6+25z^5+43z^4-206z^3+282z^2-166z+40\right)}{768\pi^4(z-1)^3}\ln^2(2-z)$$
$$+ \frac{(z-1)\left(16z^5-168z^4-12z^3+154z^2-48z+3\right)z}{768\pi^4(2z-1)^3}\ln z\,\ln(1-z)$$
$$- \frac{\left(z^6-13z^5+25z^4+3z^3-46z^2+42z-6\right)z}{384\pi^4(z-1)^3}\ln z\,\ln(2-z) \qquad ($$
$$- \frac{\left(8z^6+36z^5-338z^4+741z^3-599z^2+195z-19\right)z}{768\pi^4(z-1)^2(2z-1)^2}\ln z$$
$$- \frac{(z-1)\left(8z^4-76z^3+10z^2+73z-24\right)}{768\pi^4(2z-1)^2}\ln(1-z)$$
$$+ \frac{(z-2)\left(z^3-z^2-7z+1\right)z}{192\pi^4(z-1)^2}\ln(2-z)$$
$$+ \frac{32\pi^2z^4+330z^4-112\pi^2z^3-861z^3+144\pi^2z^2+876z^2-80\pi^2z-273z+16\pi^2}{2304\pi^4(z-1)(2z-1)}$$

Analytical Calculation

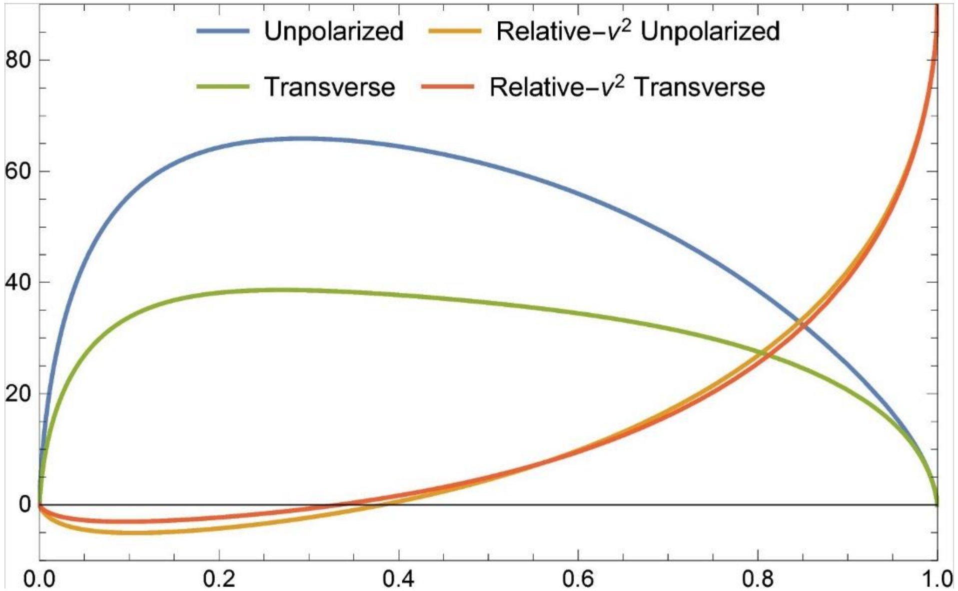
Other results (similar structure with the unpolarized one)

- The LO relative-v^2 correction
- These two results are in agreement with the previous numerical results

- The LO transverse and longitudinal polarized results
- The LO transverse and longitudinal polarized relative-v^2 correction
- These results have never been calculated before

Discussion

- Compare with relativistic corrections

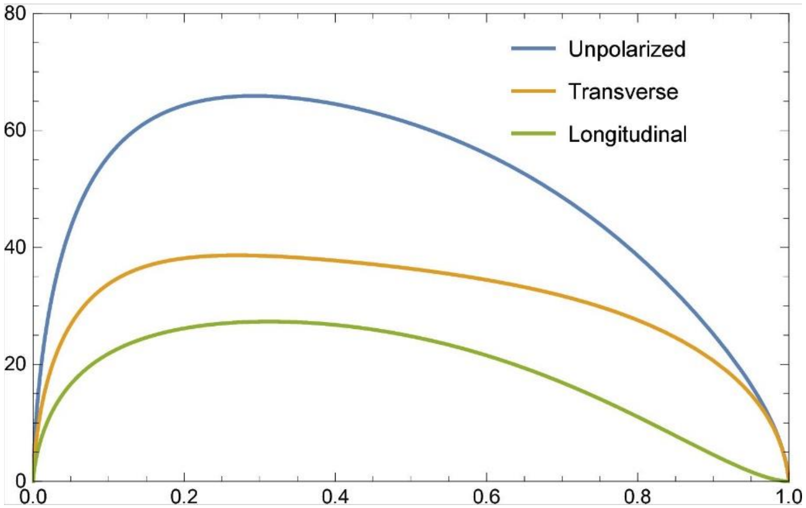


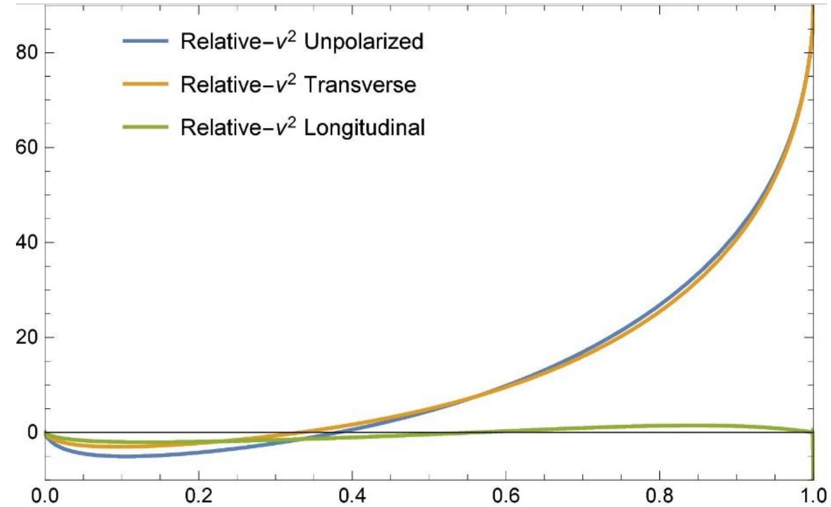
  - The relativistic correction may be negative at $z \to 0$.
  - The integral over z give the result that the relativistic corrections will give large contributions.
  - Doubt on the validity of the v expansion

    ---- See Ma Yan-Qing's talk

Discussion

- Compare with polarized results





- In the limit of $z \to 1$, the transverse polarized quarkonium is dominant.
- LO of α_s and v, the transverse contribution is almost twice of the longitudinal contribution: quarkonium almost unpolarized
- Relativistic correction is dominated by the transverse contribution, while the longitudinal contribution is almost zero through z.

Conclusion and Summary

- With IBP reduction method, we can simplify the high loop integrals into finite master integrals, which are easier to be calculated.
- The master integrals which can not be easily calculated can be considered to construct differential equations.
- For gluon fragmentation to color-octet ${}^3S_1$, the relativistic corrections make a large contribution to the LO results.
- At LO of α_s and v, the quarkonium is almost unpolarized, while for the relativistic corrections, the quarkonium is almost transverse.

Thank you!