All-Heavy Tetraquarks

Jean-Marc Richard

Institut de Physique Nucléaire de Lyon
Université de Lyon–IN2P3-CNRS
Villeurbanne, France

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Based on recent work with J. Vijande, A. Valcarce,
older work with Cafer Ay et Hyam Rubinstein
and ongoing work with Emiko Hiyama, Makoto Oka and Atsushi Hosaka.

History

- $(Q_1 Q_2 \bar{Q}_3 \bar{Q}_4)$: many speculations
- Encouraged by the success of quarkonium and quarkonium-like (XYZ) spectroscopy
- Can we get a bound or resonant double quarkonium?
- $(cc\bar{c}\bar{c})$, $(bc\bar{c}\bar{c})$, $(bb\bar{c}\bar{c})$, ...
- Exp. J/ψ trigger seems easier
- Th. How well can we extrapolate quarkonium dynamics to higher configurations
- New colour substructures, 3- or 4-body forces, etc.
- A wave of encouraging results on the possibility of $(cc\bar{c}\bar{c})$ or $(bb\bar{b}\bar{b})$, Vary, Karliner et al. Bai et al., Zhu et al., etc.
- Critically examined int he case of constituent models (Valcarce, Vijnade, R., PRD in press)

Chromoelectric limit

- Chromomagnetic terms $\propto (m_i\, m_j)^{-1}$ less important in the heavy quark sector
- Simplest chromoelectric prototype in the literature

$$H = \sum \frac{\boldsymbol{p}_i}{2\,m_i} - \frac{16}{3} \sum \tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij})\ ,$$

- with 2-body forces and colour as a global operator
- where $v(r)$ is the quarkonium potential
- has some analogy with 4-unit-charges systems in QED

$$H = \sum \frac{\boldsymbol{p}_i}{2\,m_i} + \sum e_i\, e_j / r_{ij}\ .$$

Chromoelectric limit-2

- Striking mass dependence, $(QQ\bar{q}\bar{q})$ favored in the CE limit, $(M^+M^+m^-m^-)$ more stable than $(m^+m^+m^-m^-)$
- $(M^+m^+M^-m^-)$ unstable in QED if $M/m \gtrsim 2.2$
- Naive CE limit and QED differ in the equal-mass case
  - $(e^+e^+e^-e^-) = \mathrm{Ps}_2$ stable (Wheeler, Hylleraas and Ore, …)
  - $(QQ\bar{Q}\bar{Q})$ unstable in serious 4-body estimates in naive CE model.
- Delicate 4-body problem, as e.g., the tetraneutron
- Some approximations favor binding artificially, e.g.

$$a\,(r_{12}^2 + r_{34}^2) + b \sum_{\substack{i=1,2\\ j=3,4}} r_{ij}^2 = \begin{cases} (a + b)(x^2 + y^2) + 2\,b\,z^2 (\text{exact}) \\ (a \qquad)(x^2 + y^2) + 2\,b\,z^2 (\text{diquarks}) \end{cases}$$

if $\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{y} = \boldsymbol{r}_4 - \boldsymbol{r}_3$ and $\boldsymbol{z} = (\boldsymbol{r}_3 + \boldsymbol{r}_4 - \boldsymbol{r}_1 - \boldsymbol{r}_2)/\sqrt{2}$

More on symmetry breaking

- Role of symmetry breaking
- $\min(p^2 + x^2 + \lambda\, x) < \min(p^2 + x^2)$ in elementary QM
- $\min(H_{\text{even}} + H_{\text{odd}}) < \min H_{\text{even}}$
- $\min(M^+ M^+ m^- m^-) < \min(\mu^+ \mu^+ \mu^- \mu^-)\,, \quad 2\,\mu^{-1} = M^{-1} + m^{-1}$
- $(M^+ M^+ m^- m^-)$ and $(\mu^+ \mu^+ \mu^- \mu^-)$ have the same threshold

$$\frac{\boldsymbol{p}_1^2}{2M} + \frac{\boldsymbol{p}_2^2}{2M} + \frac{\boldsymbol{p}_3^2}{2m} + \frac{\boldsymbol{p}_4^2}{2m} + V = \left[\sum \frac{\boldsymbol{p}_i^2}{2\mu} + V\right] + \left(\frac{1}{4M} - \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right]$$

$$\Rightarrow \qquad \min(H_{C-\text{even}} + H_{C-\text{odd}}) < \min H_{C-\text{even}}$$

and this explains why H_2 is more stable than Ps_2.

Breaking particle identity?

- same reasoning?

$$\frac{\mathbf{p}_1^2}{2M} + \frac{\mathbf{p}_2^2}{2m} + \frac{\mathbf{p}_3^2}{2M} + \frac{\mathbf{p}_4^2}{2m} + V = \left[\sum \frac{\mathbf{p}_i^2}{2\mu} + V\right] + \left(\frac{1}{4M} - \frac{1}{4m}\right)\left[\mathbf{p}_1^2 + \mathbf{p}_3^2 - \mathbf{p}_2^2 - \mathbf{p}_4^2\right]$$

- Thus $(M^+m^+M^-m^-)$ more stable than $(\mu^+\mu^+\mu^-\mu^-)$????
- No!
- Since symmetry breaking benefits more to $(M^+M^-) + (m^+m^-)$!!!
- But some kind of metastability below the other threshold, $(M^+m^-) +$ c.c.
- In short (un)favorable symmetry breaking can (spoil) generate stability.

More on the equal-mass case-2

- So far, asymmetry in the kinetic energy
- Similar for the potential energy

$$H = \sum \boldsymbol{p}_i/(2m) + \sum g_{ij} v(r_{ij}) \ , \quad \sum g_{ij} = 2 \ .$$

If g_{ij} are equal: highest energy, and, roughly speaking, the broader the distribution of g_{ij}, the lower the energy.

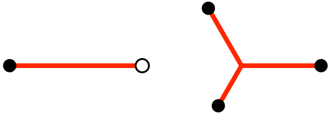
- Now, if you compare Ps_2 and quark models: Ps_2 favored

$(abcd)$	$v(r)$	g_{ij}	$\bar{g}$	Δg
Thr (1,3)+(2,4)	$-1/r, r$	$\{0, 0, 1, 0, 1, 0\}$	1/3	0.22
Ps_2	$-1/r$	$\{-1, -1, 1, 1, 1, 1, 1\}$	1/3	0.89
$[(qq)_{\bar{3}}(\bar{q}\bar{q})_3]$	$-1/r, r$	$\{1/2, 1/2, 1/4, 1/4, 1/4, 1/4\}$	1/3	0.01
$[(qq)_6(\bar{q}\bar{q})_{\bar{6}}]$	$-1/r, r$	$\{-1/4, -1/4, 5/8, 5/8, 5/8, 5/8\}$	1/3	0.17

Mixing effects small

Improved chromoelectric model

- Based on the string model
- Linear confinement interpreted as

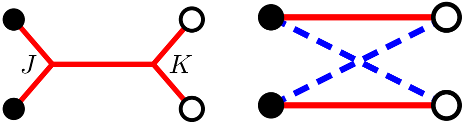


- Not very visible in baryon spectroscopy as compared to

$$V_{\text{conf}} = \frac{1}{2}(r_{12} + r_{23} + r_{31})$$

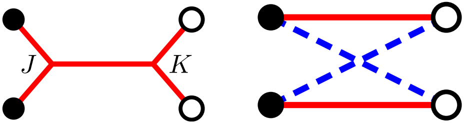
of the naive additive model.

- For tetraquarks, the minimum of



provides some extra attraction (Vijande et aL;, 2007, Bai et al 2017). The connected diagram alone binds for $M \gg m$.

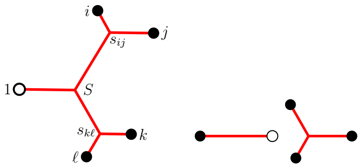
Adiabaticity and color-mixing



- The string potential corresponds to a Born-Oppenheimer treatment of the gluon field.
- With free rotations of the color wave function
- This is possible for $(bc\bar{b}\bar{c})$, not for $(bb\bar{b}\bar{b})$
- So the result by Bai et al corresponds to a fictitious $(bb'\bar{b}\bar{b}')$ state with $b' \neq b$, though same mass.

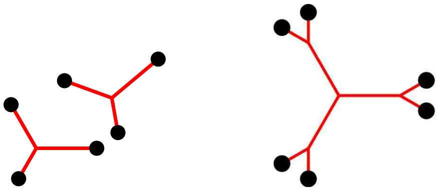
Higher configurations: pentaquark

- Same finding for pentaquark. In absence of constraints from antisymmetrization, pentaquark binding below the meson + baryon thresholds



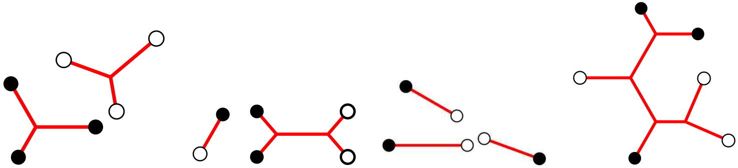
Higher configurations: dibaryon

- Same finding for pentaquark. In absence of constraints from antisymmetrization, dibaryon binding below the baryon+ baryon thresholds



Higher configurations: baryonium

- Same finding for $(3q, 3\bar{q})$. In absence of constraints from antisymmetrization, at least for some mass configurations, binding below the various thresholds (baryon-antibaryon, 3 mesons, meson + tetraquark)



Chromomagnetic binding

- In the 70s, the hyperfine splitting between hadrons ($J/\psi - \eta_c$, $\Delta - N$, etc.) explained à la Breit–Fermi, by a potential

$$V_{SS} = -A \sum_{i<j} \frac{\delta^{(3)}(\boldsymbol{r}_{ij})}{m_i\, m_j}\, \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\ ,$$

  a prototype being the magnetic part of one-gluon-exchange.
- Attractive coherences in the spin-color part: $\langle \sum \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \rangle$ sometimes larger for multiquarks than for the threshold.
- In particular $\langle \ldots \rangle$ twice larger (and attractive) in the best ($uuddss$) as compared to $\Lambda + \Lambda$.
- But $\langle \delta^{(3)}(\boldsymbol{r}_{ij}) \rangle$ much weaker for multiquarks than for ordinary hadrons, and needs to be computed. Hence uncertainties.
- Astonishing success with > 20 experiments on H and still lattice computations of H 40 years later!

Chromomagnetic binding-2

- Other configurations found, such as the heavy P (Gignoux et al., Lipkin, 1987) ($\bar{Q}qqqq$)
- Any correction repulsive: binding is not secured
- In particular $\mathrm{SU(3)_F}$ breaking
- Short-range factor $\langle\delta(\boldsymbol{r}_{ij})\rangle$ borrowed from baryons
- The model was improved by Høgassen et al., and later by Stancu, Zhu et al., …

$$H = -\sum_{i<j} C_{ij}\,\lambda_i^{(c)}.\lambda_j^{(c)}\,\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\ ,$$

  C_{ij} tuned to qq, cq, cs, …in ordinary hadrons
- Astonishing picture of the $X(3872)$
- Further studies, e.g., PKU
- b-sector, and/or all-heavy systems more problematic.
- as it requires interplay of chr.-elec. and magn. effects

Summary

- In charm sector, both chromoelecrric and magnetic effects to be included
- Naive chromoelectric model (additive with color factors) follow the trends of $(++--)$ in QED
- But less favorable, due to the non-Abelian algebra of charges
- For non-identical quarks and antiquarks, string potential offers good opportunities
- The 4-body (and higher!) problem is delicate. The diquark-antidiquark approximation is antivariational, and thus might produce artificial binding.

Production of T_{cc} from B_c or Ξ_{bc}

Figs from the Roma group