A new factorization theories for heavy quarkonium production

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Based on work done with Kuang-Ta Chao: 1703.08402

FCPPL Quarkonium Production Workshop
Peking University, Beijing, March 30th, 2017

Outline

I. Introduction

II. Problems with NRQCD

III. Soft gluon factorization

IV. Comparison

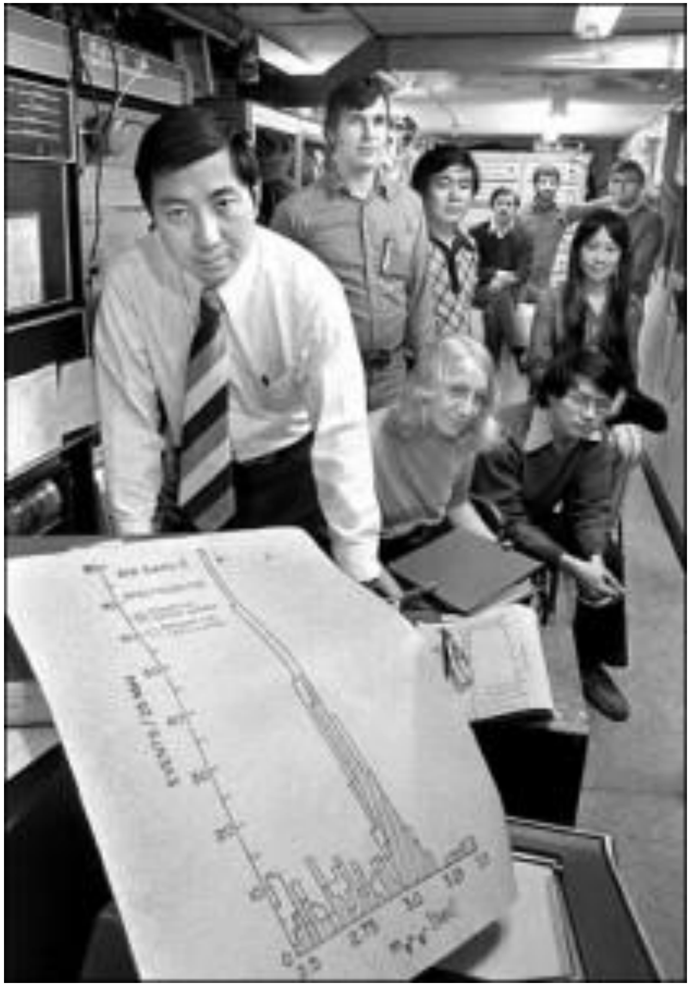
Discovery of the J/ψ: J particle

➢ GIM mechanism and charm quark

To suppress FCNC process, Glashow–Iliopoulos–Maiani mechanism required the existence of a fourth quark

➢ J particle discovered at BNL

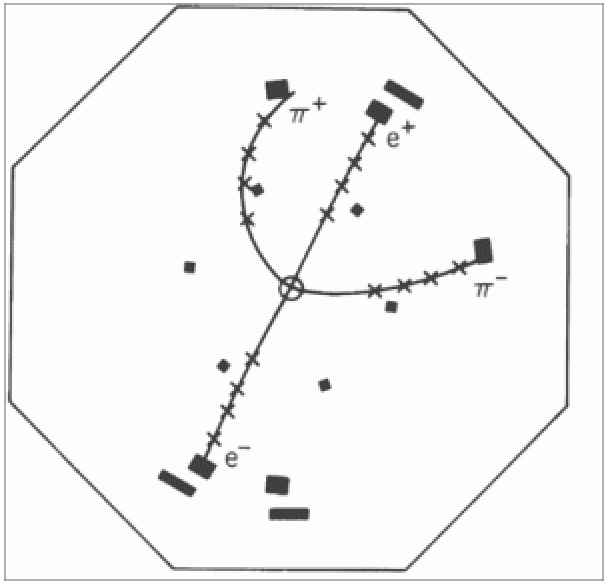
- In $p + Be \to e^+ + e^- + X$
- 3.1 GeV, about three times heavier than the proton
- With $J^{PC} = 1^{--}$

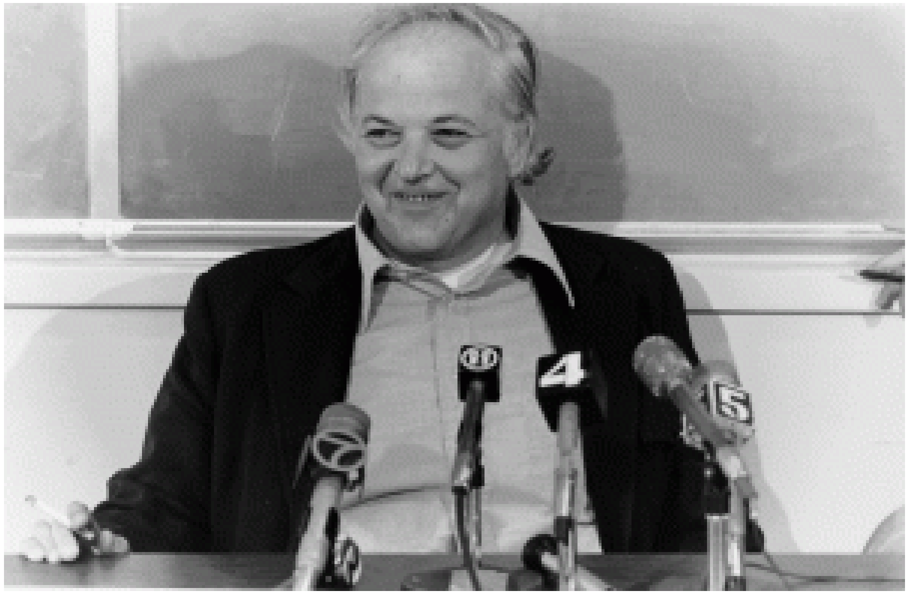


Samuel Ting and his BNL team.
Nobel Prize in 1976

- ψ **particle discovered at SLAC**

In $e^+ + e^- \to \pi^+ + \pi^-$

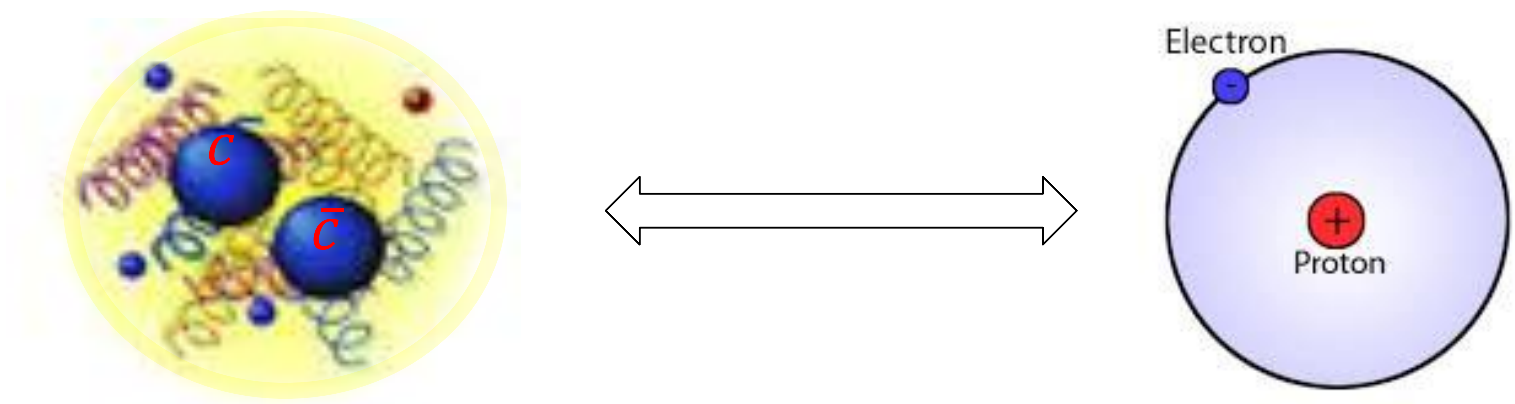




Burton Richter following the announcement of co-winning the 1976 Nobel Prize.

➢ Bound state of $Q\bar{Q}$ pair under strong interaction

Eg: $J/\psi\ \ \psi', \chi_{cJ}, \Upsilon(nS), \chi_{bJ}(nP) \cdots$



- ✓ **The simplest system in QCD: two-body problem**
- ✓ **"Hydrogen atom in QCD", "an ideal laboratory in QCD"**

Velocity of heavy quarks

- **Coulomb potential between color singlet heavy quark pair:** $V(r) = -C_F \frac{\alpha_s(1/r)}{r}$
- **Virial theorem:** $mv^2 \sim V(r) \sim \frac{\alpha_s(1/r)}{r}$
- **Uncertainty principle:** $r \sim \frac{1}{mv}$
- **Velocity is determined by quark mass**

$$\alpha_s(mv) \sim mv^2\, r \sim v$$

Property

- **A non-relativistic QCD system:** $v^2 \ll 1$

  Charmonium: $\mathrm{m}{\sim}1.3\mathrm{Ge}V,\ v^2 \approx 0.3$

  Bottomonium: $m{\sim}4.5GeV,\ v^2 \approx 0.1$

- **Multiple well-separated scales :**

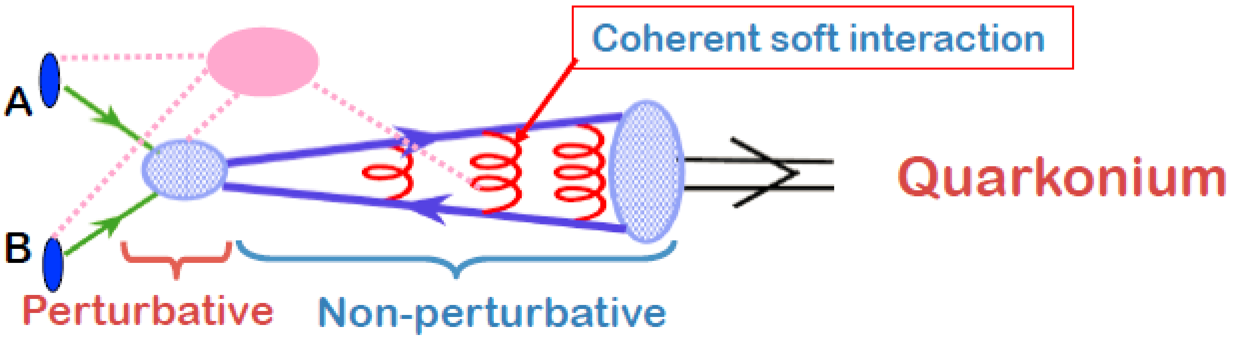
Quark mass:	M	
Momentum:	Mv	M >> Mv >> $Mv^2{\sim}\Lambda_{\mathrm{QCD}}$
Energy:	Mv^2	

- **Involving both perturbative and nonperturbative physics**

- **Production: ideal to understand hadronization, to study QGP**

Space-time picture for production

- **Hadronization followed by production of an off-shell heavy quark pair**



$\Delta r \sim \frac{1}{2m}, \quad \frac{1}{m\,v}, \quad \frac{1}{m\,v^2}$

- Time scale for producing heavy quark pair: $\frac{1}{2\,m}$
- Time scale for expansion: $\frac{1}{m\,v}$
- Time scale for forming bound state: $\frac{1}{m\,v^2}$

Approximation

➢ On-shell pair + hadronization

$$\sigma_{AB\to H+X} = \sum_n \int_n d\Gamma_{(Q\bar{Q})_n} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{(Q\bar{Q})_n}}\right] F_{(Q\bar{Q})_n\to H}(p_Q, p_{\bar{Q}}, P_H)$$

- Needs justification
- Corrections are at higher order in v
- Different assumptions/treatments on how the heavy quark pair becomes a heavy quarkonium: different factorization methods

Historical theories for quarkonium production

1. 1975 – CSM&CEM

Einhorn, Ellis (1975), Chang (1980) …

CSM: IR div., ψ' surplus

Fritzsch (1977), Halzen (1977) …

CEM: wrong for ratio

2. 1994 - NRQCD

Bodwin, Braaten, Lepage, 9407339

Polarization puzzle

Hierarchy problem

Universality problem

3. 2017 - SGF

YQM, Chao, 1703.08402

May resolve problems in NRQCD

Kang, Qiu, Sterman, 1109.1520
Fleming, Leibovich, Mehen, Rothstein 1207.2578
Kang, YQM, Qiu, Sterman, 1401.0923

2011 - Collinear factorization

Deal with scales $\gg m$

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NRQCD Factorization

➢ Factorization formula

Bodwin, Braaten, Lepage, 9407339

$$d\sigma_\psi = \sum_{i,j,n} \underbrace{\int dx_1 dx_2 \, G_{i/A} G_{j/B}}_{\Lambda_{QCD}} \times \underbrace{\hat{\sigma}[ij \to c\bar{c}[n] + X]}_{m_c} \times \underbrace{\langle O_n^\psi \rangle}_{m_c v}$$

Parton distribution function

Hadronization (LDMEs)

Production of heavy quark pair

- n: quantum numbers of the pair, spectroscopic notation ${}^{2S+1}L_J^{[c]}$.
- Color, spin, orbital angular momentum, total angular momentum

1) Polarization puzzle

➢ LO NRQCD

- Dominated by ${}^3S_1^{[8]}$, LO NRQCD predicts transversely polarized J/ψ, contradicts with CDF data

CDF, 0704.0638

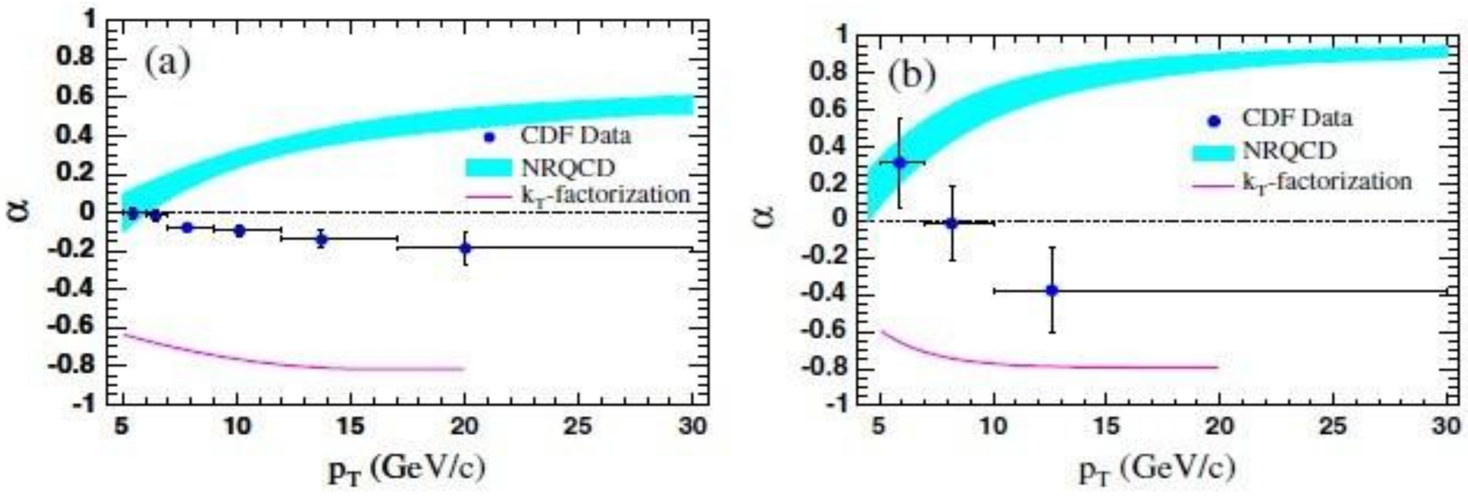
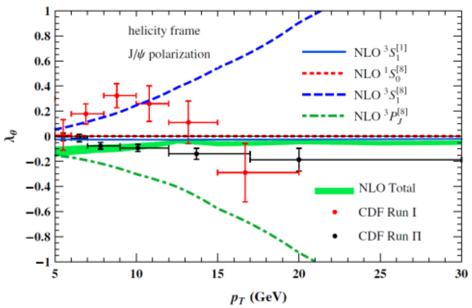


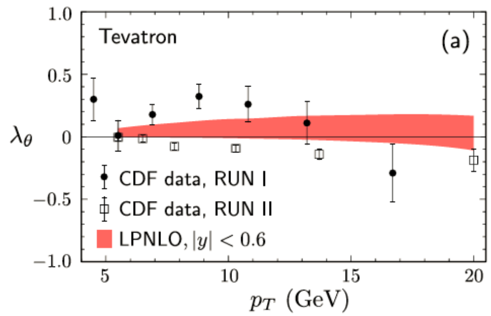
FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

1) Polarization puzzle con.

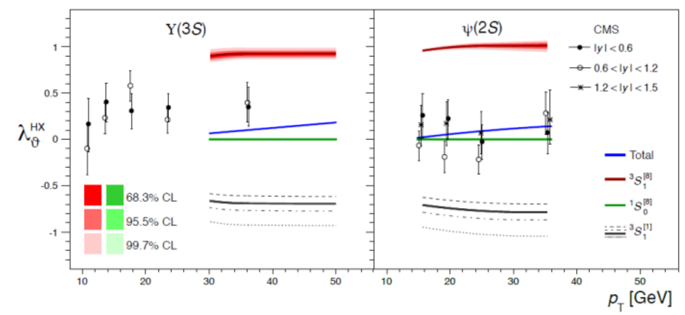
- J/ψ: transverse polarization cancelled between ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channel, ${}^1S_0^{[8]}$ may dominate



Chao,YQM,Shao,Wang, Zhang,1201.2675



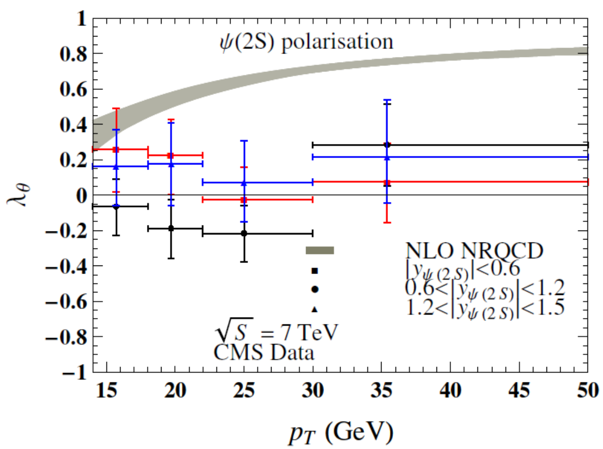
Bodwin, Chung, Kim, Lee, 1403.3612



Faccioli,Knunz,Lourenco, Seixas,Wohri,1403.3970

- $\psi(2S)$: still hard to understand

Shao, Han, YQM, Meng, Zhang, Chao, 1411.3300



2) Hierarchy problem

- **Best fit of J/ψ yield data at high p_T**

YQM, Wang, Chao, 1009.3655

$$\mathrm{M}_0 = \langle O(^1S_0^{[8]})\rangle + 3.9\langle O(^3\boldsymbol{P}_0^{[8]})\rangle/m_c^2 \approx 0.074\ \mathbf{GeV}^3$$

$$\mathrm{M}_1 = \langle O(^3S_1^{[8]})\rangle - 0.56\langle O(^3\boldsymbol{P}_0^{[8]})\rangle/m_c^2 \approx 0.0005\ \mathbf{GeV}^3$$

- **Velocity scaling rule of NRQCD**

$$\langle O(^1S_0^{[8]})\rangle \sim \langle O(^3S_1^{[8]})\rangle \sim \langle O(^3\boldsymbol{P}_0^{[8]})\rangle$$

Thus

$$M_0 \sim M_1$$

- **Two orders difference: unnatural**

3) Universality problem

- **Necessary condition for NRQCD**
  - LDMEs, like M_0 and M_1, are process independent

- **Upper bound of M_0 set by e^+e^- collision**

Zhang, YQM, Wang, Chao, 0911.2166

$$M_0 < 0.02\text{GeV}^3$$

  - Comparing with $\mathrm{M}_0 \approx 0.074$ GeV^3 from pp collison

- **Global fit of LDMEs**

Butenschoen, Kniehl, 1105.0820

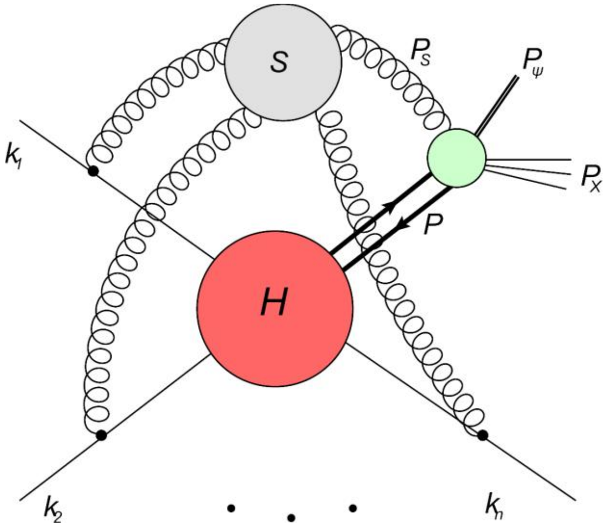
$$\chi^2_{\text{d.o.f.}} = 725/194 = 3.74$$

  - Data cannot described consistently!

Possible reason: soft gluon emission

➢ **Soft gluon emission in the hadronization process**

- P_ψ is different from P
- NRQCD approximate P by P_ψ
- Xsection approximately $\propto P^{-4}$



➢ **An over simplified model of NRQCD expansion**

$$\int_{-1}^{1} \frac{dx}{2(1+\lambda+\lambda\, x)^4} = 0.42$$

With $\lambda = 0.3$

$$= 1 - 4\lambda + 40/3\lambda^2 - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + +1.2 - 1.08 + 0.91 - 0.73 + \cdots.$$

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Factorization

➢ Factorization formalism for H production:

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n \int \frac{d^4P}{(2\pi)^4} d\hat{\sigma}_n(P) F_n^H(P, P_H) \qquad n = {}^{2S+1}L_J^{[c]}$$

$$p_Q = P/2 + q \qquad p_{\bar{Q}} = P/2 - q$$

$$P \cdot q = 0 \text{ and } q^2 = m_Q^2 - M^2/4, \text{ with } M = \sqrt{P^2}$$

- $d\hat{\sigma}$: perturbatively calculable

➢ Soft gluon distributions

$$F_n^H(P, P_H) = \int d^4x e^{iP\cdot x} \langle 0|\bar{\psi}(0)\Gamma_n' \Phi^\dagger(0)\psi(0) a_H a_H^+ \bar{\psi}(x)\Gamma_n \Phi(x)\psi(x)|0\rangle$$

- Expectation values of bilocal operators in QCD vacuum

What is new?

- **Factorization in full QCD but not NRQCD effective field theory**
  - More convenient to deal with power corrections in full QCD than EFT
- **Momentum difference between $Q\bar{Q}$ and H considered**
  - No additional large power corrections
- **External $Q\bar{Q}$ in hard part are on mass shell**
  - Gauge invariance is guaranteed
  - Different from shape function models
- **Proof of the factorization equivalent to NRQCD**
  - One loop proof is available

Simplification: 1d form

- **4d-factorization hard to use in practice**
  - Hard to extract four dimensional SGDs
  - Hard for perturbative calculation
- **Property of SGDs**
  - At the rest frame of H, dominant region

$$P^{\mu}_{rest} = (M + O(\lambda^2), O(\lambda), O(\lambda), O(\lambda))$$

- **Expanding $O(\lambda)$ terms in hard part:**

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum \int dz\, d\hat{\sigma}_n(P_H/z) F_n^H(z)$$

where $z = \frac{m_H}{M}$ $\quad F_n^H(z) = \int \frac{d^4P}{(2\pi)^4}\delta(z - \sqrt{P_H^2/P^2}) F_n^H(P, P_H)$

- **Comparison:** relating momentum of χ_{cJ} and its decaying J/ψ

YQM, Wang, Chao, 1002.3987

$$p_{J/\psi} \approx \frac{m_{J/\psi}}{m_{\chi_{cJ}}} p_{\chi_{cJ}}$$

- Deviation less than 8%

The over simplified model

- **"SGF-1d expansion"**

$$\int_{-1}^{1} \frac{dx}{2(1+\lambda+\lambda\, x)^4} = 0.42$$

With $\lambda = 0.3$

$$= \frac{1}{(1+\lambda)^4}\left(1 + \frac{10}{3}\lambda^2 - \frac{20}{3}\lambda^3 + 17\lambda^4 + \cdots\right)$$

$$= 0.350 + 0.105 - 0.063 + 0.048 - 0.035 + \cdots$$

- **Comparing with "NRQCD expansion"**

$$\int_{-1}^{1} \frac{dx}{2(1+\lambda+\lambda\, x)^4} = 0.42$$

$$= 1 - 4\lambda + 40/3\lambda^2 - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + +1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

Simplification: 0d form

➢ 0d expansion

- If $F_n^H(z)$ peaks around $z = z_n \sim 1 - O(\lambda/m_H)$
- Approximate $F_n^H(z) \approx \delta(z - z_n)\langle O_n^H \rangle$

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n d\hat{\sigma}_n(P_H/z_n)\langle \mathcal{O}_n^H \rangle$$

➢ Roughly recover NRQCD if choosing $z_n = 1$

- May result in large corrections

Simplification: expansion of m_Q

- **At least two hard scales in short distance**
  - Invariant mass of $Q\bar{Q}$ pair M and quark mass m_Q
  - Relation: $M = 2m_Q + O(\lambda)$
- **Expansion**
  - $d\hat{\sigma}(m_Q, M) = d\hat{\sigma}\left(\frac{M}{2}, M\right) + \left(m_Q - \frac{M}{2}\right) d\hat{\sigma}'\left(\frac{M}{2}, M\right) + \cdots$
  - Good convergence
- **Comparing with NRQCD expansion**
  - $d\hat{\sigma}(m_Q, M) = d\hat{\sigma}(m_Q, 2m_Q) + (M - 2m_Q) d\hat{\sigma}'(m_Q, 2m_Q) + \cdots$
  - Bad convergence: $d\hat{\sigma}(m_Q, M)$ may $\propto M^{-5}$

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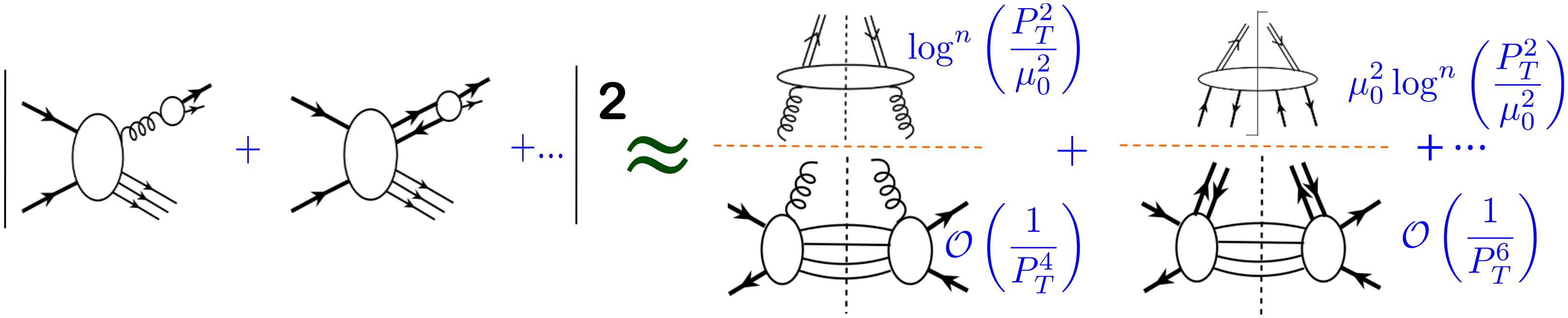
J/ψ production via gluon fragmentation

$E\frac{d\sigma_{J/\psi}}{d^3P}$:



- **Easy to calculate**

$$d\sigma_H(p_T) = \int dx\, d\hat{\sigma}_g(p_T/x) D_{g\to H}(x)$$

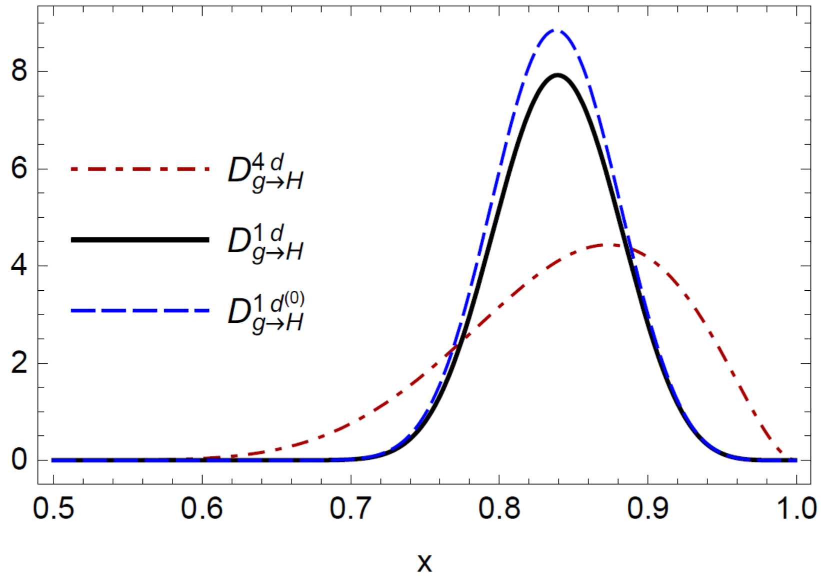
- $d\hat{\sigma}_g$: **well known**
- $D_{g\to H}$: **calculated by using NRQCD or SGF**

Fragmentation functions

➢ Model input

$$F^H_{^3S_1^{[8]}}(P, P_H) = a\,k^2 \exp(-\frac{k_0^2 + k^2}{\Lambda^2})$$

- $\Lambda \sim m_Q v^2$, **choose** $500 MeV$
- **Conclusion independent of the model**



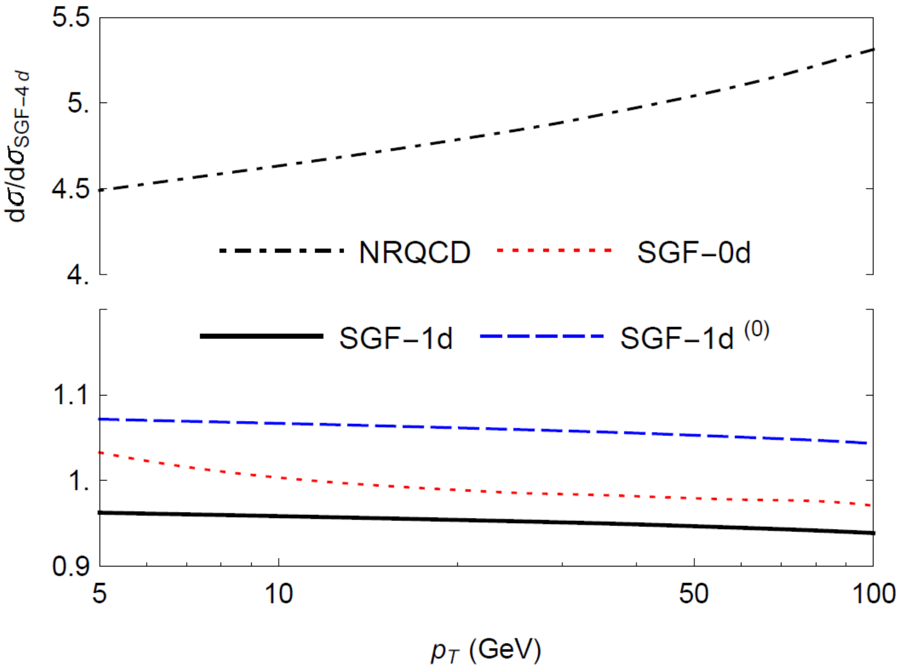
- **SGF-4d and -1d have different shape, but have the same accumulated value**

$$\int_0^1 dx\, D^{4d}_{g\to H}(x) = \int_0^1 dx\, D^{1d}_{g\to H}(x)$$

- **-1d$^{(0)}$: leading term of expansion m_Q around $M/2$, very close to -1d.**

Cross section ratio

➢ Assume SGF-4d is exact



- 1d very close to 4d, deviation less than 6%
- Expansion m_Q results in about 10% uncertainty
- 0d with $z_0 = 0.86$ well reproduce 4d
- NRQCD overshoots 4d by a factor of 4

➢ Rough explanation

$$0.86^9 \approx 1/4 \sim (1 - v^2/2)^9$$

Summary

- **NRQCD factorization: polarization puzzle, hierarchy problem, universality problem**
  - Possible reason: convergence of v^2 expansion is too bad because of soft gluon emission
- **Soft gluon factorization (SGF) can describe quarkonium production and decay**
  - Soft gluons effects are considered, should have much smaller v^2 correction
- **Two important expansions**
  - From 4d to 1d, with small v^2 correction
  - Expansion m_Q around $M/2$, good convergence

Outlook

➢ Proof of SGF to all order in perturbation theory

- Equivalent to proof NRQCD to all order in α_s and v^2
- One-loop proof is available; two-loop should not be hard

➢ Phenomenological study

- Complexity is similar to NRQCD, thanks to the two expansions
- Most established codes can use directly (FDC, Helac-Onia,…)
- All NRQCD results should be redone, a lot of works

➢ May resolve problems in NRQCD

- Universality problem: importance of v^2 correction depends on process
- Hierarchy problem: contributions from different channels changed in SGF
- Polarization puzzle: may also have large v^2 correction

Thank you!

History of high order calculation: pp collision

See also Jian-Xiong Wang's talk

- 0703113: Campbell, Maltoni, Tramontano

  NLO, cross section, S-wave

- 0802.3727: Gong, Wang

  NLO, polarization, S-wave

- 0806.3282: Artoisenet, Campbell, Lansberg, Maltoni, Tramontano

  $\mathrm{NNLO}^{\star}$, S-wave

- 1002.3987: YQM, Wang, Chao
- 1009.3655: YQM, Wang, Chao
- 1009.5662: Butenschöen, Kniehl

  Complete NLO (S- and P-wave), cross section

- 1201.1872: Butenschöen, Kniehl
- 1201.2675: Chao,YQM,Shao,Wang,Zhang
- 1205.6682: Gong,Wan,Wang,Zhang

  Complete NLO (S- and P-wave), with polarization

NOT fully comprehensive!!!

Cross section v.s. polarization

- Fit to J/ψ cross section requires a very small

$$M_1 = \langle O({}^3S_1^{[8]})\rangle - 0.56\,\langle O({}^3\boldsymbol{P}_0^{[8]})\rangle/m_c^2$$

YQM, Wang, Chao, 1009.3655

- Transverse polarization proportional to

$$M'_1 = \langle O({}^3S_1^{[8]})\rangle - 0.52\,\langle O({}^3\boldsymbol{P}_0^{[8]})\rangle/m_c^2$$

Chao,YQM,Shao,Wang,Zhang,1201.2675

- **Cross section requires small transverse polarization— consistent with data!!!**