

# Central Exclusive Production of $2\pi$ and $4\pi$ systems

—an overview from an experimental physicist—

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- However, this talk is given by an experimental physicist, i.e. the theory part is brief and necessarily rudimentary.
- We shall define **four reference frames**—necessary for 2- to 3-body processes.

## Plan of Talk

- Kinematics for Central Production: Two- to Three-Body Processes

*“Pomeron Physics and QCD,”*

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- Production Amplitudes:

Decay amplitudes for  $X^0 \rightarrow 2\text{-body}$

$J \rightarrow s + \sigma$  where  $J$ ,  $s$  and  $\sigma$  are the spins.

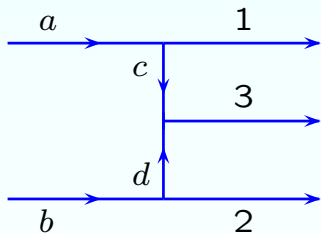


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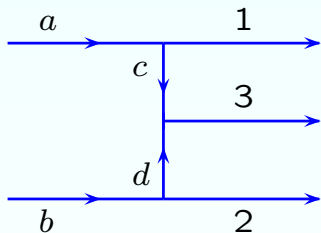
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- Conclusions and Future Prospects

The basic coordinate system:

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The basic coordinate system:



## Overall CM system

$$\vec{a} + \vec{b} = 0$$

( $\vec{a}$  along the z axis)

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

(a plane going through the origin)

Three Euler angles needed  
to define the plane.

$$d\phi_3(a + b \rightarrow 1 + 2 + 3) = \frac{4}{(4\pi)^5} (dR)_{\text{CM}} \left( \frac{1}{4s} \right) dm_{13}^2 dm_{23}^2$$

$$dR = d\alpha(d \cos \beta) d\gamma$$

There are **four** coordinate systems needed for the process

$a + b \rightarrow 1 + 2 + 3$ :

- The Detector system in the LAB (**The first coordinate system**).

$$\hat{z} = (0, 0, 1) \propto \text{the beam direction}$$

$$\hat{y} = (0, 1, 0) \propto \text{vertical in LAB}$$

$$\hat{x} = (1, 0, 0) = \hat{y} \times \hat{z}$$

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- We stay in the overall CM frame, i.e.  $\vec{a} + \vec{b} = 0$ ,  $\vec{a} = -\vec{b} \propto \hat{z}$

The plane defined by  $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$  where  $(\beta, \alpha)$  stands for the **normal** (defined by  $\hat{y}_n \propto \vec{p}_1 \times \vec{p}_2$ ) to the reaction plan, i.e.

$$\hat{z}_n = (-\sin \alpha, \cos \alpha, 0), \quad \text{this is the node}$$

$$\hat{y}_n = (\sin \beta \cos \alpha, \sin \beta \sin \alpha, \cos \beta)$$

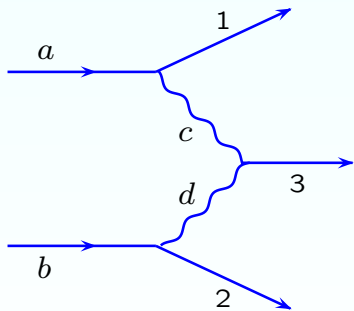
$$\hat{x}_n = \hat{y}_n \times \hat{z}_n$$

The vectors  $\vec{p}_i$ , ( $i = 1, 2, 3$ ), lie in the  $x_n$ - $z_n$  plane.

We define  $\vec{p}_1$  to be in this plane, rotating around the  $\hat{y}_n$  axis by  $\gamma_1 = \pi/2 + \gamma$  from the  $\hat{z}_n$  axis. If  $\gamma \simeq 0$ , then we see that  $\hat{z}_n \simeq \hat{z}$ , which simply means that  $\vec{a}$  and  $\vec{p}_1$  are **nearly parallel**. The  $\vec{p}_2$  is obtained by a similar rotation by  $\gamma_2 = 3\pi/2 - \delta + \gamma$  where  $\vec{p}_1 \cdot \vec{p}_2 = p_1 p_2 \cos(\pi - \delta)$ . **This is the second coordinate system.**

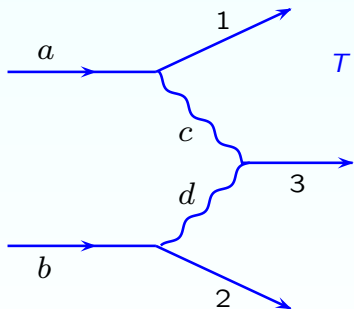
# Production Amplitudes

## Production Amplitudes





## Production Amplitudes



$$T \propto A(a \rightarrow 1 + c) * A(b \rightarrow 2 + d) \\ A(c + d \rightarrow 3) * A(3 \rightarrow 4 + 5 + 6 + \dots)$$

$A(a \rightarrow 1 + c)$  in the  $a$  rest frame

$A(b \rightarrow 2 + d)$  in the  $b$  rest frame

$A(c + d \rightarrow 3)$  in the  $3$  rest frame

$A(3 \rightarrow 4 + \dots)$  in the  $3$  rest frame

## Decay Amplitudes

Let the particle  $3(X^0)$  be in the state of  $|jm\rangle$   
and let it decay into two particles

$$|jm\rangle \rightarrow |s_4 \lambda_4\rangle + |s_5 \lambda_5\rangle$$

(particles 4 and 5 are new— introduced to indicate decay products)

$$A(j \rightarrow s_4 + s_5) = \langle (4+5) | \mathcal{M}_d | jm \rangle \propto jF_{\lambda_4 \lambda_5}^{s_4 s_5} D_{m, (\lambda_4 - \lambda_5)}^{j*}(\phi_4, \theta_4, 0)$$

$$jF_{-\lambda_4 - \lambda_5}^{s_4 s_5} = \nu_j \nu_4 \nu_5 jF_{\lambda_4 \lambda_5}^{s_4 s_5}$$

where  $\nu_j$  is the 'naturalness' of the particle  $j$

$$\nu_j = \eta(-)^j \quad \text{for bosons} \quad = \eta(-)^{j-1/2} \quad \text{for fermions}$$

where  $\eta$  is the intrinsic parity. Similarly for  $s_4$  and  $s_5$ .

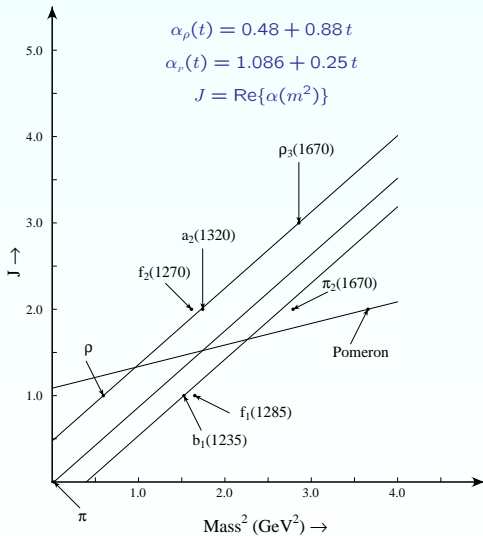
# Pomerons

## Regge Trajectories

$$\alpha_\rho(t) = 0.48 + 0.88 t$$

$$\alpha_p(t) = 1.086 + 0.25 t$$

$$J = \text{Re}\{\alpha(m^2)\}$$

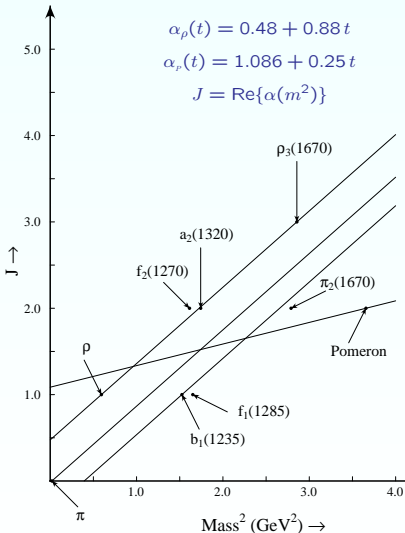


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Pomeron comes with an even signature<sup>a</sup>.

$$J^{PC}(\mathbb{P}) = 2^{++}, 4^{++}, 6^{++}, \dots$$

$\mathbb{P}(J^{PC} = 2^{++})$  at mass = 1.91 GeV.

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<sup>a</sup>See, however, a paper by F. Close and A. Kirk, PLB **397**, 333 (1997).

$$|\ell S JM\rangle = \sum_{m_1 m_2} (s_1 m_1 s_2 m_2 | S m_s) (S m_s \ell m | JM) \\ \times \int d\vec{k} Y_m^\ell(\vec{k}) |\mathbb{G}, +\vec{k}, s_1 m_1\rangle |\mathbb{P}, -\vec{k}, s_2 m_2\rangle$$

so that

$$2|\ell S JM\rangle \Big|_{\text{symm}} = \sum_{m_1 m_2} (s_1 m_1 s_2 m_2 | S m_s) (S m_s \ell m | JM) \\ \times \int d\vec{k} Y_m^\ell(\vec{k}) |\mathbb{G}, +\vec{k}, s_1 m_1\rangle |\mathbb{P}, -\vec{k}, s_2 m_2\rangle \\ + \sum_{m_1 m_2} (s_2 m_2 s_1 m_1 | S m_s) (S m_s \ell m | JM) \\ \times \int d\vec{k} Y_m^\ell(\vec{k}) |\mathbb{P}, +\vec{k}, s_2 m_2\rangle |\mathbb{G}, -\vec{k}, s_1 m_1\rangle$$

The result:

$$s_1 + s_2 + S + \ell = \text{even} \quad \longrightarrow \quad (\mathbb{P} + \mathbb{P}) \oplus (\mathbb{P} + \mathbb{P}); \quad \boxed{S + \ell = \text{even}}$$

$$(\gamma + \mathbb{P}) \oplus (\mathbb{P} + \gamma); \quad \boxed{S + \ell = \text{odd}}$$

Exotic(non- $q\bar{q}$ )  $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, 4^{+-}, 5^{-+}$

Likely  $J^{PC}$  states allowed are:

$(\mathbb{P} + \mathbb{P}) \oplus (\mathbb{P} + \mathbb{P});$   $S + \ell = \text{even}$   $\longrightarrow$

$$I^G = 0^+, \quad S = 0, 1, 2, 3, 4; \quad \vec{J} = \vec{S} + \vec{\ell}$$

$$J^{PC}(\ell = 0; S = 0, 2, 4) \longrightarrow 0^{++}, 2^{++}, 4^{++}$$

$$(\ell = 1; S = 1, 3) \longrightarrow 0^{-+}, 1^{-+}, 2^{-+}, 3^{-+}, 4^{-+}$$

$$(\ell = 2; S = 0, 2, 4) \longrightarrow 0^{++}, 1^{++}, 2^{++}, 3^{++}, 4^{++}, 5^{++}, 6^{++}$$

$(\gamma + \mathbb{P}) \oplus (\mathbb{P} + \gamma);$   $S + \ell = \text{odd}$   $\longrightarrow$

$$I^G = (0^-, 1^+), \quad S = 1, 2, 3; \quad \vec{J} = \vec{S} + \vec{\ell}$$

$$J^{PC}(\ell = 0; S = 1, 3) \longrightarrow 1^{+-}, 3^{+-}$$

$$(\ell = 1; S = 2) \longrightarrow 1^{--}, 2^{--}, 3^{--}$$

$$(\ell = 2; S = 1, 3) \longrightarrow 1^{+-}, 2^{+-}, 3^{+-}, 4^{+-}, 5^{+-}$$

Three Processes for **Central** Production of  $\pi\pi$ :

$$l + \ell = \text{even}$$

- $\mathbb{P}(2^{++}) + \mathbb{P}(2^{++}) \rightarrow \pi\pi (I^G = 0^+; J^{PC} = 0^{++}, 2^{++}, 4^{++}, \dots)$

Two Natural-parity exchanges  $\rightarrow \pi\pi$  ( $\ell = 0, 2, 4$ )

Dominant Process:  $S + D$  waves



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- $\rho(770)(1^{--}) + \mathbb{P}(2^{++}) \rightarrow \pi\pi (I^G = 1^+; J^{PC} = 1^{--}, 3^{--}, 5^{--}, \dots)$   
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 $P$ -wave dominant

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- $b_1(1235)(1^{+-}) + \mathbb{P}(2^{++}) \rightarrow \pi\pi(I^G = 1^+; J^{PC} = 1^{--}, 3^{--}, 5^{--}, \dots)$   
Unnatural-parity and Natural-parity exchanges  $\rightarrow \pi\pi$  ( $\ell = 1, 3, 5$ )  
 $P$ -wave dominant

- At ALICE, we have an additional production mechanism for Particle 3: photon-Pomeron processes,

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- $\rho + \text{Pomeron} \rightarrow \pi\pi$  ( $\ell = \text{odd}$ ) or  $4\pi$  all with  $I^G = 1^+$   
 Non-strange light-quark ( $u, \bar{u}, d$  or  $\bar{d}$ ) bound systems  
 Mass  $\simeq 0.8$  GeV or higher

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 Mass  $\simeq 1.0$  GeV or higher
- $J/\psi + \text{Pomeron} \rightarrow D\bar{D}$  or  $J/\psi + 2\pi$  all with  $I^G = 0^-$   
 $c\bar{c}$  bound systems—charmonia  
 Mass  $\simeq 3.0$  GeV or higher

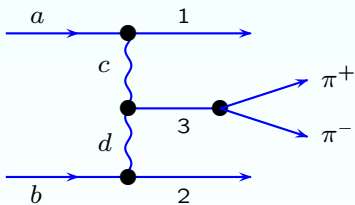
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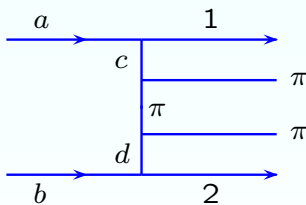
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 $c\bar{c}$  bound systems—charmonia  
 Mass  $\simeq 3.0$  GeV or higher
- $\Upsilon + \text{Pomeron} \rightarrow B\bar{B}$  or  $\Upsilon + 2\pi$  all with  $I^G = 0^-$   
 $b\bar{b}$  bound systems—bottomonia  
 Mass  $\simeq 9.5$  GeV or higher

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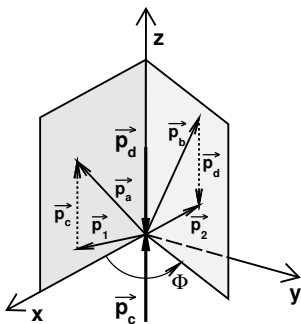
$2\pi$  Resonance



$2\pi$  Continuum



$a + b \rightarrow 1 + 3 + 2$  in the 3RF



$$p_a = p_1 + p_c, \quad p_b = p_2 + p_d$$

$$p_c + p_d = p_3$$

$$\vec{p}_c + \vec{p}_d = \vec{p}_3 = 0$$

$$\hat{z}_m \propto \vec{p}_c \text{ and } -\hat{z}_m \propto \vec{p}_d$$

$\vec{p}_a$  and  $\vec{p}_1$  in the  $x_m$ - $z_m$  plane by definition. So we must have

$$\hat{y}_m \propto \vec{z}_m \times \vec{p}_a \quad \text{and} \quad \hat{x}_m = \hat{y}_m \times \hat{z}_m$$

The plane formed by  $\vec{p}_b$  and  $\vec{p}_2$  is denoted by the plane rotated by  $\Phi$  around the  $z_m$ -axis.

This completes the construction of the **third** coordinate system.

## Amplitudes for $a + b \rightarrow 1 + 3 + 2$

The reaction

$$a \rightarrow 1 + c, \quad b \rightarrow 2 + d, \quad c + d \rightarrow 3$$

The corresponding amplitudes

$$A = \sum_{ij} A(a \rightarrow 1 + c_i) * \Delta(c_i) * A(b \rightarrow 2 + d_j) * \Delta(d_j) \\ * A(c_j + d_j \rightarrow 3)$$

where

$$\{i, j\} = \{\text{Pomeron+Pomeron, photon+Pomeron, Pomeron+photon}\}$$

This completes the construction of all the relevant amplitudes in the problem.

## Conclusions:

- The reaction

$$a + b \rightarrow 1 + 3 + 2 \quad 3 \rightarrow 4 + 6 + \dots$$
$$a \rightarrow 1 + c \quad b \rightarrow 2 + d \quad c + d \rightarrow 3$$

is a 2- to 3-body process, if and only if the Regge domain formula holds

$$s_{13} s_{23} \simeq s w_3^2, \quad \text{transverse mass: } w_3^2 = m_3^2 + \kappa_3^2$$

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- The reaction requires **three** different rest frames (RF's):

(a) The detector frame:  $\{\hat{x}, \hat{y}, \hat{z}\}$

(b) The overall CM system where the normal  $\hat{y} \propto \vec{p}_1 \times \vec{p}_2$   
 the plane  $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$  lies the x-z plane:  $\{\hat{x}_n, \hat{y}_n, \hat{z}_n\}$

The Euler angles  $R = (\alpha, \beta, \gamma)$  are used to transform  
 from the frame (a) to the frame (b).

(c) In the 3RF,  $\hat{z}_m \propto \vec{c}$ ,  $\hat{y}_m \propto \vec{a} \times \vec{p}_1$   $\hat{x}_m = \hat{y}_m \times \hat{z}_m$ :  $\{\hat{x}_m, \hat{y}_m, \hat{z}_m\}$

$\vec{a}$  and  $\vec{p}_1$  lies in the  $x_m$ - $z_m$  plane; the plane formed by  $\vec{b}$  and  $\vec{p}_2$   
 rotated by angle  $\Phi$  around the  $\hat{z}_m$  axis.

- Double-Pomeron process:  
Ground state are  $f_0(500)$ ,  $f_2(1275)$  and  $f_4(2050)$ .  
Exotic mesons are possible  $I^G(J^{PC}) = 0^+(1^{-+}), 0^+(3^{-+}) \dots$

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- Photon-Pomeron process:  
 Ground state are  $h_1(1170)$  and  $b_1(1235)$ .  
 The Regge recurrence NOT observed.  
 Exotic mesons are possible:  
 $I^G(J^{PC}) = (0^-, 1^+)(2^{+-}, 4^{+-} \dots)$

- Double-Pomeron process:  
Ground state are  $f_0(500)$ ,  $f_2(1275)$  and  $f_4(2050)$ .  
Exotic mesons are possible  $I^G(J^{PC}) = 0^+(1^{-+}), 0^+(3^{-+}) \dots$
- Photon-Pomeron process:  
Ground state are  $h_1(1170)$  and  $b_1(1235)$ .  
The Regge recurrence NOT observed.  
Exotic mesons are possible:  
 $I^G(J^{PC}) = (0^-, 1^+)(2^{+-}, 4^{+-} \dots)$
- ALICE can produced very exciting new results on mesons—  
in the next few years

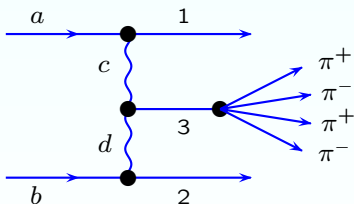
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- We need a new PhD candiate to work on the  $4\pi$  channel



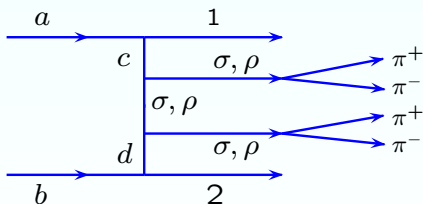
- $4\pi$  production via  $\sigma\sigma$  and  $\rho\rho$  intermediate states:

*Exclusive diffractive production of  $\pi^+\pi^-\pi^+\pi^-$   
via the intermediate  $\sigma\sigma$  and  $\rho\rho$  intermediate states  
in proton-proton collisions within tensor Pomeron approach*

Piotr Lebiedowicz, Otto Nachtmann and Antoni Szczurek  
Phy. Rev. **D94**, 034017(2016)



$4\pi$  Resonance



$4\pi$  Continuum

## Distribution with an arbitrary degree of coherence

For both  $2\pi$  and  $4\pi$  final states, Write

$$I(\tau) = \sum_{\text{external spins}} \left| r_0 A_{\text{Res}}(\tau) + r \{\exp \phi\} A_{\text{Con}}(\tau) \right|^2$$

where  $\tau$  is the phase-space variable for the production of the  $2\pi$  and  $4\pi$  final states and  $r_0 (0 \rightarrow 1)$ ,  $r (-\infty \rightarrow +\infty)$  and  $\phi (0 \rightarrow 2\pi)$  are real. *We further assume that  $r_0$ ,  $r$  and  $\phi$  are real and independent of  $\tau$ , i.e. they are taken to be constants in the problem.* We see that

$$I(\tau) = r_0^2 \left| A_{\text{Res}}(\tau) \right|^2 + r^2 \left| \{\exp \phi\} A_{\text{Con}}(\tau) \right|^2 + 2r_0 r \Re \left\{ A_{\text{Res}}^*(\tau) \{\exp \phi\} A_{\text{Con}}(\tau) \right\}$$

where  $(r_0, r_1) = (1, 0)$  and  $r = \phi = 0$  for the **Resonance** term only and  $(r_0, r_1) = (0, 1)$  and  $\phi = 0$  for the **Continuum** term only.

## The Collins-Soper Reference Frame

*Angular distribution of dileptons in high-energy hadron collisions,*

J. C. Collins and D. E. Soper  
Phys. Rev. **D16**, 2219 (1977)

Let  $\hat{e}_i$ , ( $i = 1, 2, 3$ ) be the unit vectors which define the reference frame. Then we have, in the 3RF,

Note that  $\angle(\hat{p}_a \cdot \hat{e}_3) = \angle(-\hat{p}_b \cdot \hat{e}_3)$  so that the vector  $\hat{e}_3$  bisects the two vectors  $\hat{p}_a$  and  $-\hat{p}_b$ . And  $\hat{e}_2$  is the unit vector normal to the plane formed by  $\hat{p}_a$  and  $-\hat{p}_b$ . And  $\hat{e}_2$  lies in the plane formed by  $\hat{p}_a$  and  $-\hat{p}_b$ . **This is the fourth reference frame.**

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$$\hat{e}_2 = \frac{\hat{p}_a \times \hat{p}_b}{|\hat{p}_a \times \hat{p}_b|}$$

$$\hat{e}_3 = \frac{\hat{p}_a - \hat{p}_b}{|\hat{p}_a - \hat{p}_b|}$$

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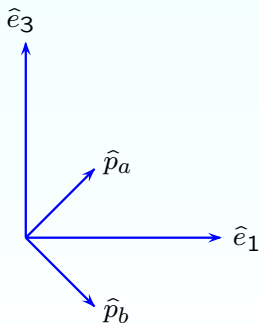
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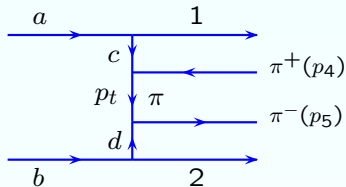
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# $\pi + \pi^-$ continuum

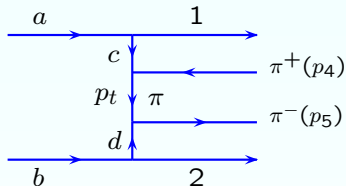
## $\pi + \pi^-$ continuum



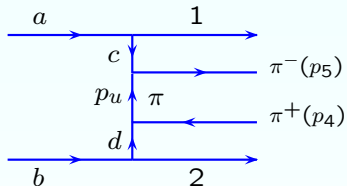
$$\begin{aligned} p_c &= p_1 - p_a \\ &= p_t - p_4 \end{aligned}$$

$$\begin{aligned} p_d &= p_2 - p_b \\ &= p_5 - p_t \end{aligned}$$

## $\pi^+ \pi^-$ continuum



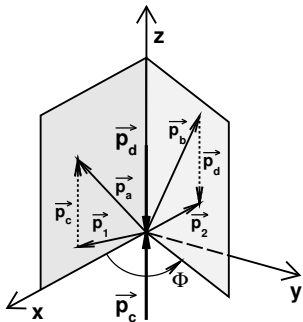
$$\begin{aligned} p_c &= p_1 - p_a \\ &= p_t - p_4 \\ p_d &= p_2 - p_b \\ &= p_5 - p_t \end{aligned}$$



$$\begin{aligned} p_c &= p_1 - p_a \\ &= p_5 - p_u \\ p_d &= p_2 - p_b \\ &= p_u - p_4 \end{aligned}$$

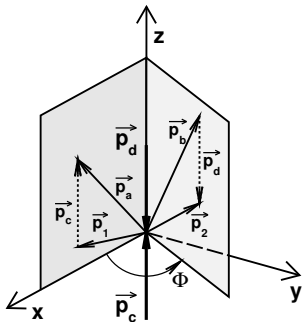


## The Last Slide



The 'correct' reference frame  
for a study of particle 3.

## The Last Slide



The 'correct' reference frame  
for a study of particle 3.

Thank you for your attention

## $\pi^+ \pi^-$ continuum

The propagator for a tensor Pomeron

$$i \Delta_{\mu\nu, \kappa\lambda}^{(\mathbb{P})}(s, t) = \frac{1}{4s} \left( g_{\mu\kappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\kappa} - \frac{1}{2} g_{\mu\lambda} g_{\nu\kappa} \right) (-i s \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

where  $\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}} t = 1.0808 + 0.25 t$ .

We obtain, for the reaction  $\lambda_a \lambda_b \rightarrow \lambda_1 \lambda_2 \pi^+ \pi^-$ ,

with  $e \cdot f = e^\mu f_\mu = e^\mu g_{\mu\nu} f^\nu = e_\nu f^\nu$ ,

$$\mathcal{M}(\mathbb{P}\mathbb{P} \rightarrow \pi^+ \pi^- \text{continuum}) =$$

$$\begin{aligned} & (p_1 - p_a) \cdot (p_t - p_4) \times (p_2 - p_b) \cdot (p_5 - p_t) \delta_{\lambda_1 \lambda_a} \delta_{\lambda_2 \lambda_b} \\ & \times \left( \frac{1}{4s_{14}} \right) (-i s_{14} \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t_c)-1} \times \left( \frac{1}{4s_{25}} \right) (-i s_{25} \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t_d)-1} \end{aligned}$$

$$\mathcal{M}(\mathbb{P}\mathbb{P} \rightarrow \pi^- \pi^+ \text{continuum}) =$$

$$\begin{aligned} & (p_1 - p_a) \cdot (p_5 - p_u) \times (p_2 - p_b) \cdot (p_u - p_4) \delta_{\lambda_1 \lambda_a} \delta_{\lambda_2 \lambda_b} \\ & \times \left( \frac{1}{4s_{15}} \right) (-i s_{15} \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t_c)-1} \times \left( \frac{1}{4s_{24}} \right) (-i s_{24} \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t_d)-1} \end{aligned}$$