Forward amplitude analyses in elastic hadron scattering and the LHC Data

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QCD challenges in pp, pA and AA collisions at high energies
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One Challenge

- One Challenge: Energy dependence of *forward* hadronic quantities:

$$\sigma_{\text{tot}}(s) = \frac{1}{s}\,\text{Im}\,F(s,t=0) \quad \text{and} \quad \rho(s) = \frac{\text{Re}\,F(s,t=0)}{\text{Im}\,F(s,t=0)}$$

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- Why?
  - Hadronic scattering $\to$ QCD
  - Optical theorem: $\sigma_{\text{tot}}(s) \leftrightarrow$ Elastic Scattering $[F(s,t)]$
  - Elastic scattering (high s and small t) $\to$ pQCD doesn't apply (large α_s)
  - Full non-pQCD description not yet available

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- Our focus here:
  - Fits to σ_{tot} and ρ data with Regge pole inspired parametrizations:

Amplitude Analysis

Outline

- Parametrizations to $\sigma_{\rm tot}$: Regge Poles (Reggeon and Pomeron)
- Determination of Real part of the amplitude (DDR)
- Summary of Models used here
- Datasets
- Fit Results
- Summary and Conclusions

Regge Pole Phenomenology

Let's consider pp and $\bar{p}p$ scattering.

$$\sigma_{\rm tot}(s) = \sigma_R(s) + \sigma_P(s)$$

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- Reggeon contribution: simple pole in complex l plane

$$\sigma_R(s) = a_1 \left(\frac{s}{s_0}\right)^{-b_1} + \tau a_2 \left(\frac{s}{s_0}\right)^{-b_2}$$

$$b_i = 1 - \alpha_i(0)$$

- $\alpha(t)$: Reggeon trajectory
- $\tau = -1(+1)$ for pp ($\bar{p}p$)

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$$\sigma_{\text{tot}}(s) = \sigma_R(s) + \sigma_P(s)$$

- Pomeron contribution: Exchange of object with vacuum quantum numbers

  - Critical $[\alpha_P(0) = 1]$: A_P
  - Supercritical $[\alpha_P(0) = 1 + \epsilon]$: $\delta \left(\frac{s}{s_0}\right)^{\epsilon}$
  - Higher-order poles $[\alpha_P(0) = 1]$:

$$Z_1 + B_1 \ln(s/s_0) \text{ (2nd order)}$$
$$Z_2 + B_2 \ln^2(s/s_0) \text{ (3rd order)}$$

COMPETE Collab. (2002)

- Comparative analysis in order to determine the best parametrization
- Parametrizations based in Regge pole Phenomenology

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- Plenty agreement with σ_{tot} by TOTEM (LHC7) 10 years later

PDG Results

- Parametrization corresponds to COMPETE best result, w/ small changes over the years:

$$\text{Reggeons} + Z + H \ln^2\left(\frac{s}{s_M}\right)$$

$$\text{2016 edition: } H = \frac{\hbar c}{M^2} \quad \text{and} \quad s_M = (2m_p + M)^2$$

- Fits to σ_{tot} and ρ from several reactions (with DDR)
- COMPETE and PDG: Leading term has a "Froissart"-like behavior

Other option: Log-raised-to-γ

Of interest here:

- First proposed by Amaldi et al. in the 1970s:

$$\text{Reggeons} + A + B\ln^{\gamma}\left(\frac{s}{s_0}\right), \quad s_0 = 1\ \text{GeV}^2$$

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- Fits to σ_{tot} and ρ for pp and $\bar{p}p$, $\sqrt{s_{max}} = 62$ GeV
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- Update by UA4/2 Collab. (1990s) ($\sqrt{s_{max}} = 546$ GeV):

$$\gamma = 2.25^{+0.35}_{-0.31}$$

FMS Approach

Fagundes, Menon and Silva (2013)

- Uses Amaldi et al. parametrization

$$\sigma_{\mathrm{tot}}(s) = a_1 \left(\frac{s}{s_0}\right)^{-b_1} + \tau a_2 \left(\frac{s}{s_0}\right)^{-b_2} + \alpha + \beta \ln^{\gamma}\left(\frac{s}{s_0}\right)$$

with $s_0 = 4m_p^2 \approx 3.521$ GeV^2 fixed, $\tau = -1(+1)$ for $pp(\bar{p}p)$

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- Real part calculated via Derivative Dispersion Relations (DDR)

FMS Approach

Even (+) and Odd (-) amplitudes: $F_{\pm}(s) = \dfrac{F_{pp} \pm F_{\bar{p}p}}{2}$

Derivative Dispersion Relations: Operator expansion by Kang and Nicolescu (PRD, 1975)

$$\frac{\mathrm{Re}\ F_{+}(s)}{s} = \frac{K}{s} + \underbrace{\left[\frac{\pi}{2}\frac{d}{d\ln s} + \frac{1}{3}\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)^{3} + \frac{2}{15}\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)^{5} + \ldots\right]}_{\tan\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)} \frac{\mathrm{Im}\ F_{+}(s)}{s}$$

$$\frac{\mathrm{Re}\ F_{-}(s)}{s} = -\int \left\{\frac{d}{d\ln s}\left[\cot\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)\right]\frac{\mathrm{Im}\ F_{-}(s)}{s}\right\} d\ln s$$

$$= -\frac{2}{\pi}\int \left\{\left[1 - \frac{1}{3}\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)^{2} - \frac{1}{45}\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)^{4} - \ldots\right]\frac{\mathrm{Im}\ F_{-}(s)}{s}\right\} d\ln s$$

K: subtraction constant (free parameter)

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- For the high energy term $[\ln^{\gamma}(s/s_0)]$ we don't have a closed form
- From fits to σ_{tot} data, $\gamma < 3 \to$ DDR $(+)$ up to third order: [FMS, Braz. J. Phys. 2012]

$$\mathcal{A}\,\ln^{\gamma-1}\left(\frac{s}{s_0}\right) + \mathcal{B}\,\ln^{\gamma-3}\left(\frac{s}{s_0}\right) + \mathcal{C}\,\ln^{\gamma-5}\left(\frac{s}{s_0}\right)$$

ρ Parametrization

Adding all terms

$$\rho(s) = \frac{1}{\sigma_{\rm tot}(s)} \left\{ \frac{K}{s} - a_1 \tan\left(\frac{\pi\, b_1}{2}\right) \left[\frac{s}{s_0}\right]^{-b_1} + \tau\, a_2 \cot\left(\frac{\pi\, b_2}{2}\right) \left[\frac{s}{s_0}\right]^{-b_2} \right.$$
$$\left. + \quad \mathcal{A}\, \ln^{\gamma-1}\left(\frac{s}{s_0}\right) + \mathcal{B}\, \ln^{\gamma-3}\left(\frac{s}{s_0}\right) + \mathcal{C}\, \ln^{\gamma-5}\left(\frac{s}{s_0}\right) \right\}.$$

$$\mathcal{A} = \frac{\pi}{2}\beta\,\gamma, \qquad \mathcal{B} = \frac{1}{3}\left[\frac{\pi}{2}\right]^3 \beta\,\gamma\,[\gamma-1][\gamma-2],$$

$$\mathcal{C} = \frac{2}{15}\left[\frac{\pi}{2}\right]^5 \beta\,\gamma\,[\gamma-1][\gamma-2][\gamma-3][\gamma-4]$$

- Analytical result. IDR demands numerical integration
- If $\gamma = 2 \rightarrow$ Recover $\ln^2 s$ parametrization (COMPETE/PDG)

Models and Dataset

- **Models**

  (1) FMS-$L\gamma$ model: γ is a free parameter

  (2) FMS-$L2$ model: $\gamma = 2$ fixed $\rightarrow$ Same analytical form as COMPETE and PDG par [Froissart-like leading term].

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  - In all cases: ρ measurement at 8 TeV by TOTEM
  - Cosmic Ray data not included

Initial Values

- For each ensemble (T and T+A)

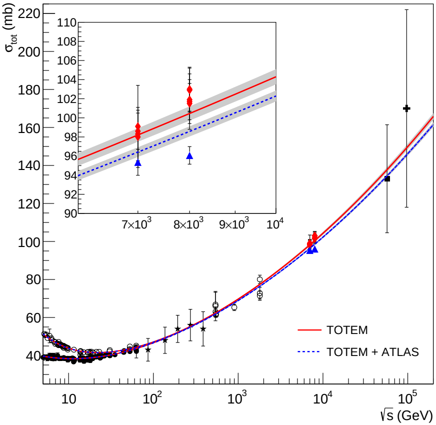
$$\text{PDG 2016} \longrightarrow \underset{(\ln^2 s)}{L2} \longrightarrow \underset{(\ln^\gamma s)}{L\gamma}$$

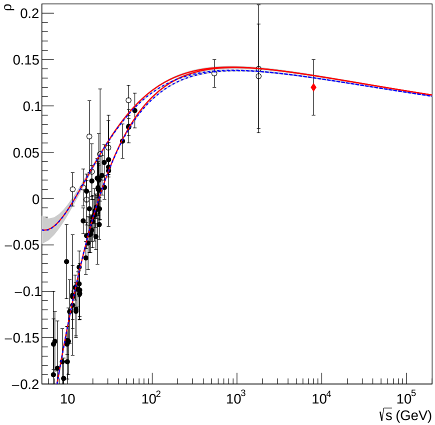
- Fits done with 1σ C.L.
- Error bands in figures calculated with standard error propagation.

Results: FMS-$L2$ model

$$\text{Reggeons} + \alpha + \beta \ln^2(s/s_0)$$

- $\chi^2/\text{DOF} = 1.09$ (T)
- $\chi^2/\text{DOF} = 1.15$ (T+A)





Results: FMS-$L\gamma$ model

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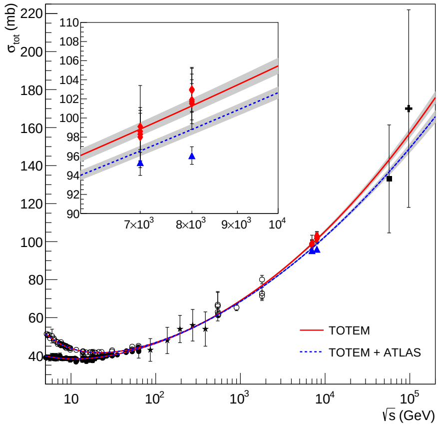

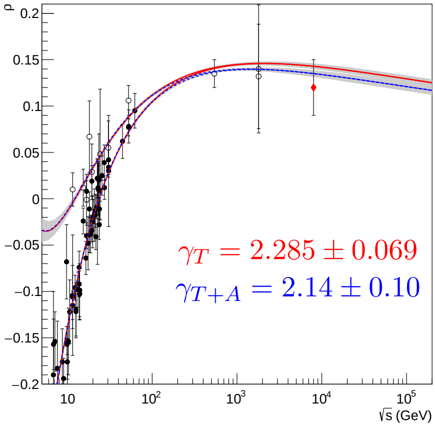

Summary and Conclusions

T $\rightarrow \chi^2$/DOF $\sim$ 1.07 - 1.09
T+A $\rightarrow \chi^2$/DOF $\sim$ 1.14 - 1.15

- Ensemble T: good description of data
- Ensemble T+A: good description of data, but with slightly larger χ^2

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  - T+A: $\gamma = 2.14 \pm 0.10 \rightarrow$ qualitative saturation of Froissart bound close do Amaldi et al. value (1970s)

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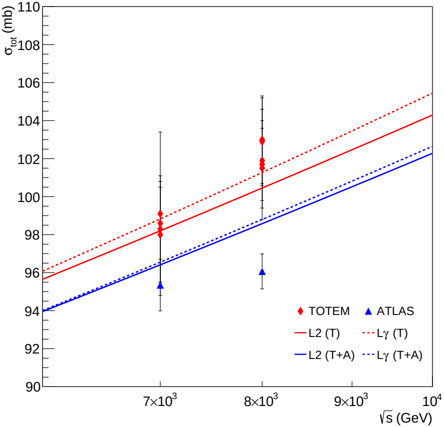
- TOTEM and ATLAS data suggest different scenarios for $\sigma_{\rm tot}$
- Analogous to 1.8 TeV situation



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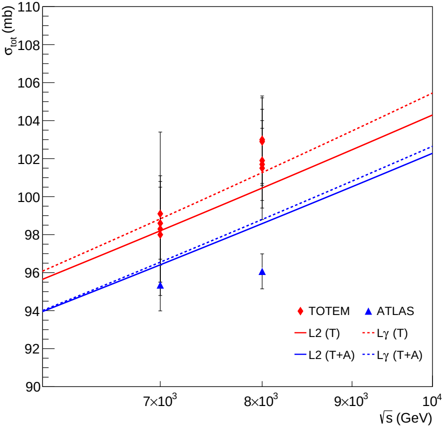

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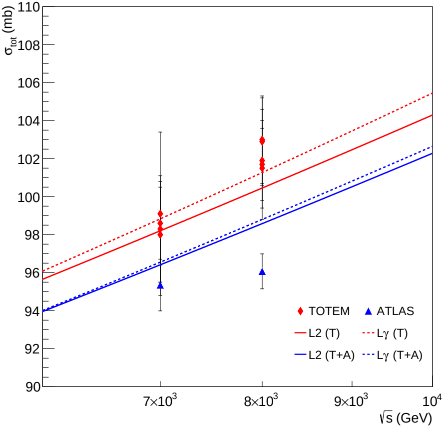

- $L\gamma$ can handle the variation of the rate of increase of σ_{tot}

Sponsor

FAPESP

THANK YOU!!

Backup Slides

Results: FMS Approach

	TOTEM		TOTEM + ATLAS	
	L2	Lγ	L2	Lγ
a_1	32.11 $\pm$ 0.60	31.5 $\pm$ 1.2	32.16 $\pm$ 0.67	31.62 $\pm$ 0.92
b_1	0.381 $\pm$ 0.017	0.521 $\pm$ 0.41	0.406 $\pm$ 0.016	0.476 $\pm$ 0.056
a_2	16.98 $\pm$ 0.72	17.17 $\pm$ 0.72	17.01 $\pm$ 0.72	17.10 $\pm$ 0.72
b_2	0.545 $\pm$ 0.013	0.548 $\pm$ 0.013	0.545 $\pm$ 0.013	0.546 $\pm$ 0.013
α	29.25 $\pm$ 0.44	33.83 $\pm$ 0.80	30.06 $\pm$ 0.34	32.5 $\pm$ 1.5
β	0.2546 $\pm$ 0.0039	0.108 $\pm$ 0.022	0.2451 $\pm$ 0.0028	0.158 $\pm$ 0.048
γ	2 (fixed)	2.285 $\pm$ 0.069	2 (fixed)	2.14 $\pm$ 0.10
s_0	3.521 (fixed)	3.521 (fixed)	3.52 (fixed)	3.521 (fixed)
K	50 $\pm$ 17	110 $\pm$ 30	61 $\pm$ 17	89 $\pm$ 32
χ^2/ν	1.09	1.07	1.15	1.14
$P(\chi^2/\nu)$	0.150	0.203	0.059	0.061