# Leading neutron production in *ep*, *pp* and *pA* collisons

**Diego Spiering**

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CAPES

*Based on* PLB 752 (2016) 76, PRD 93 (2016) 054025 *and* PRD 94 (2016) 014009.

*In collaboration with V. P. Gonçalves, F. S. Navarra, B. D. Moreira and F. Carvalho.*

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- The high energy leading neutron production can prove the low-$x$ regime, where non-linear effects are expected on the QCD dynamics.

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- Describe the HERA data of inclusive and exclusive processes with leading neutron production on *ep* colliders.

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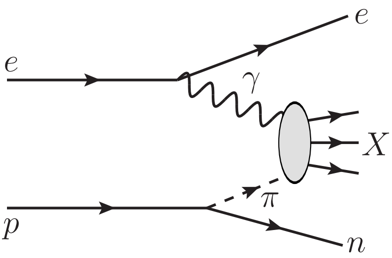
- Estimate the impact of non-linear effects.
- Extend the study of leading neutron production to $\gamma h$ interactions at RHIC and LHC.

# Electron-proton collisions

# Leading neutron production at HERA

**inclusive**

$e + p \to e + n + X$



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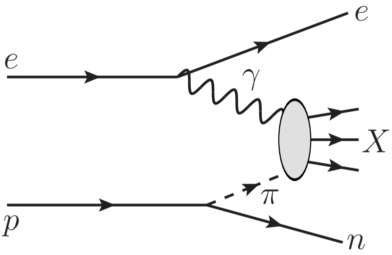


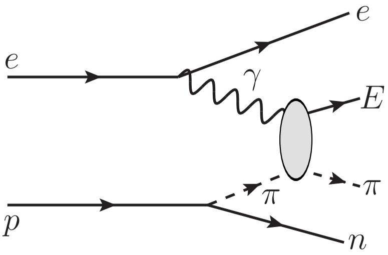

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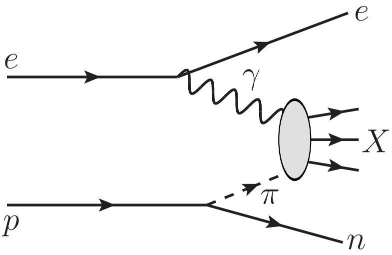


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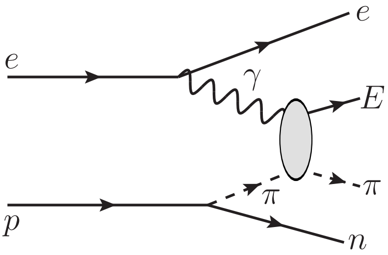


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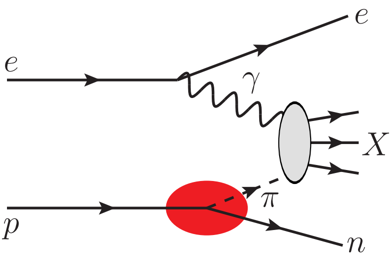


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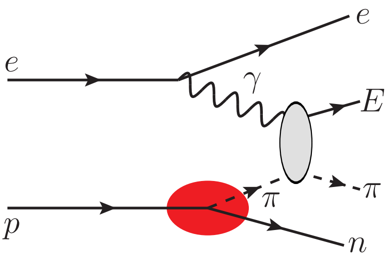


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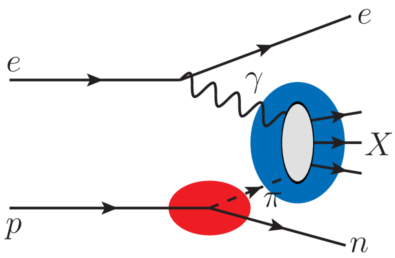


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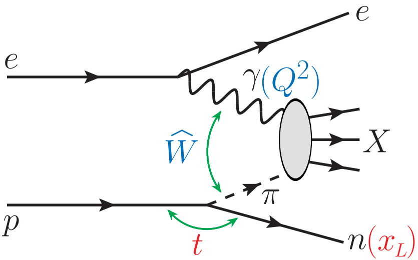


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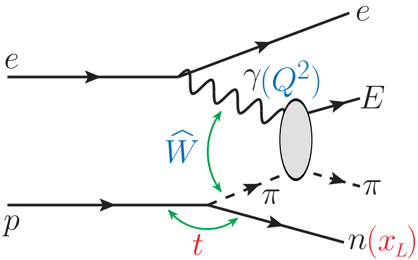


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# Flux of pions / Pion splitting function

$$f_{\pi/p}(x_L,t) = \frac{1}{4\pi}\frac{2g^2_{p\pi p}}{4\pi}\frac{-t}{(t-m^2_\pi)^2}(1-x_L)^{1-2\alpha(t)}[F(x_L,t)]^2$$

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$$F_1(x_L,t) = \exp\left[R^2\frac{t-m_\pi^2}{1-x_L}\right], \quad \alpha(t)=0$$

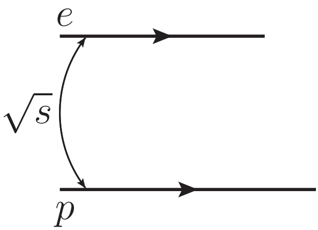
$$F_2(x_L,t) = 1\,, \quad \alpha(t)=t$$

$$F_3(x_L,t) = \exp\left[b(t-m_\pi^2)\right], \quad \alpha(t)=t$$

$$F_4(x_L,t) = \frac{\Lambda_m^2-m_\pi^2}{\Lambda_m^2-t}, \quad \alpha(t)=0$$

$$F_5(x_L,t) = \left[\frac{\Lambda_d^2-m_\pi^2}{\Lambda_d^2-t}\right]^2, \quad \alpha(t)=0$$

# Leading neutron production in inclusive processes (dipole formalism)



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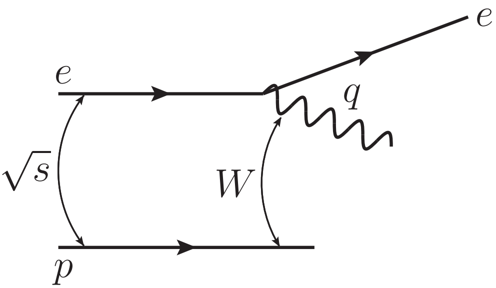

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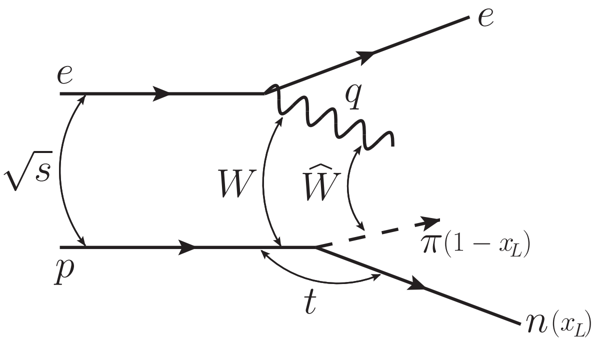

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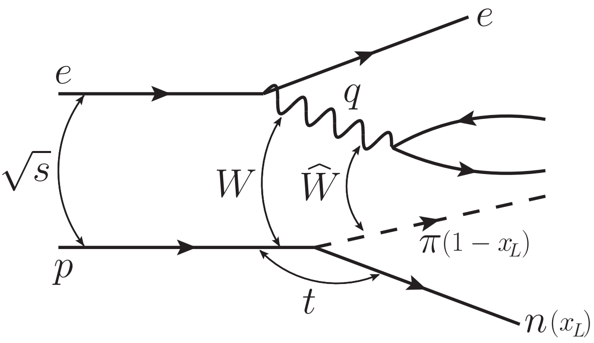

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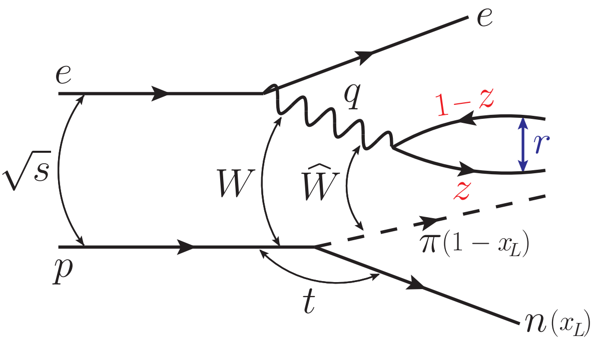

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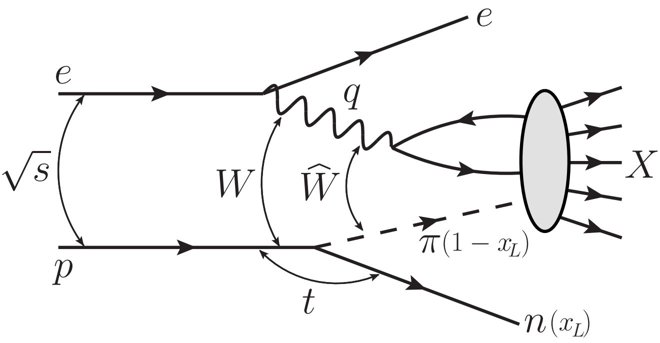

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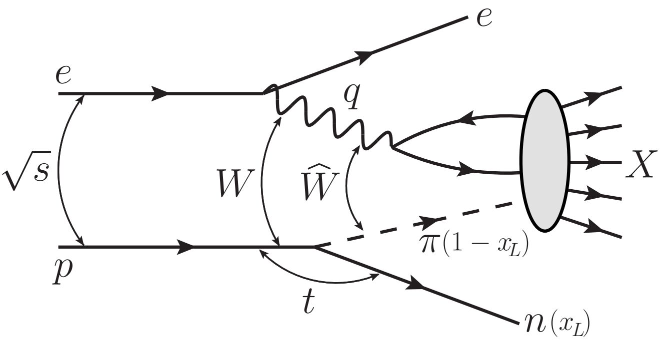


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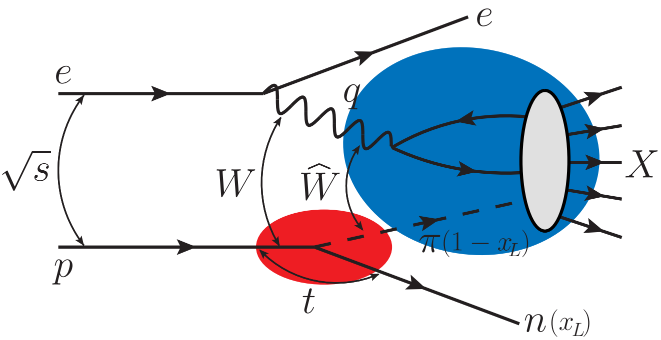


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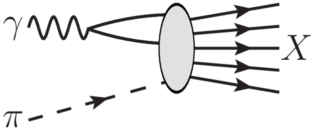
$$\sigma^{inc}_{\gamma^*\pi}(\hat{x}, Q^2) = \int dz \int d^2\vec{r} \sum_{L,T} \left|\psi_{L,T}(z, \vec{r}, Q^2)\right|^2 \sigma_{d\pi}(\hat{x}, \vec{r})$$

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Photon wave function:

$$\left|\psi_L(\vec{r}, Q^2)\right|^2 = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 4Q^2 z^2 (1-z)^2 K_0^2(\epsilon r)$$

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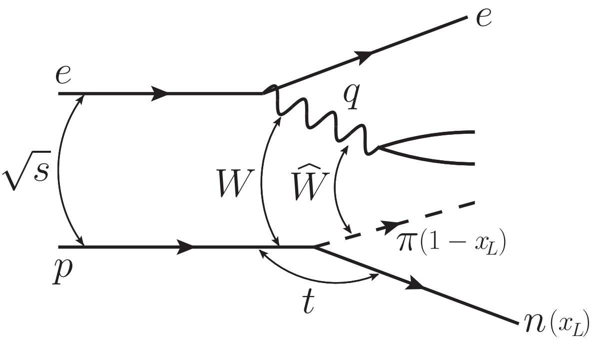

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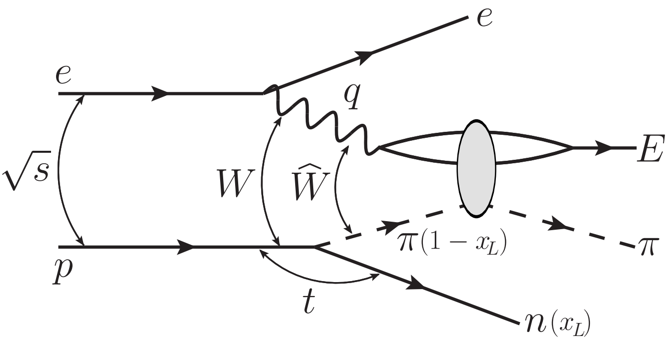

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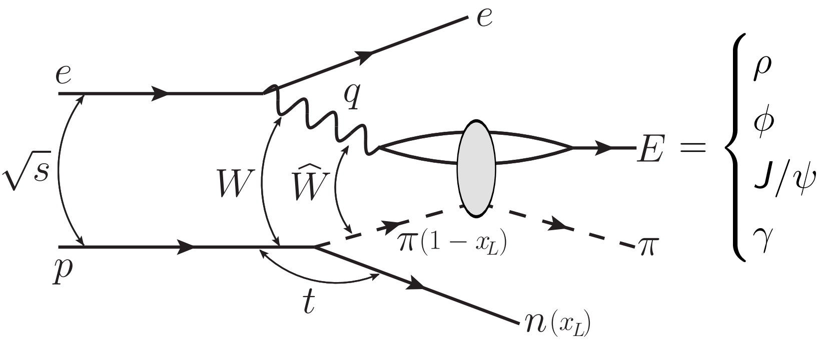

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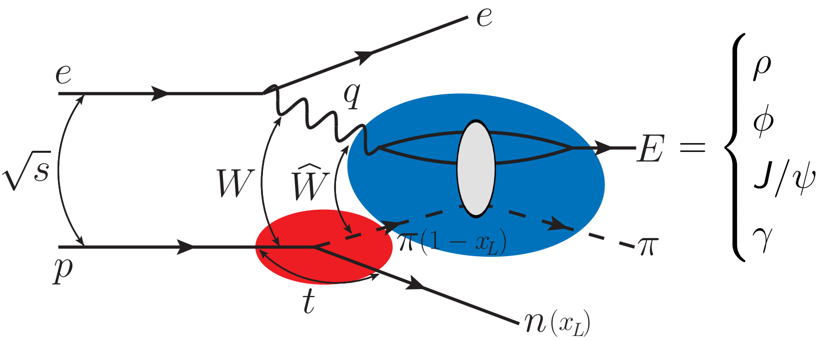


$$\frac{d^2\sigma(W,Q^2,x_L,t)}{dx_L dt} = f_{\pi/p}(x_L,t)\,\sigma^{exc}_{\gamma^*\pi}(\hat{W}^2,Q^2)$$

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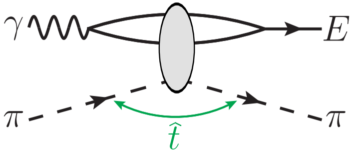
$$\sigma^{exc}_{\gamma^*\pi}(\hat{x}, Q^2) = \sum_{i=L,T} \int_{-\infty}^{0} \frac{d\sigma_i}{d\hat{t}} d\hat{t} = \sum_{i=L,T} \int_{-\infty}^{0} \left| \mathcal{A}_i^{\gamma^*\pi\to E\pi}(\hat{x}, Q^2, \Delta) \right|^2 d\hat{t}$$

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Scattering amplitude:

$$\mathcal{A}_{L,T}^{\gamma^*\pi \to E\pi} = i \int dz\, d^2\vec{r}\, d^2\vec{b}\, e^{-i[\vec{b}-(1-z)\vec{r}]\cdot\vec{\Delta}} \left[\Psi_E^*\Psi(z, \vec{r}, Q^2)\right]_{L,T} 2\mathcal{N}^\pi(\hat{x}, \vec{r}, \vec{b})$$

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Overlap functions for Vector Mesons ($V = \rho,\ \phi,\ J/\psi$): $\phi_{L,T} \equiv \phi_{L,T}(z, r)$

$$(\Psi_V^* \Psi)_L = \frac{\hat{e}_f e}{4\pi} \frac{N_c}{\pi} 2Qz(1-z) K_0(\epsilon r) \left[ M_V \phi_L + \delta \frac{m_f^2 - \nabla_r^2}{M_V z(1-z)} \phi_L \right]$$

$$(\Psi_V^* \Psi)_T = \frac{\hat{e}_f e}{4\pi} \frac{N_c}{\pi z(1-z)} \left\{ m_f^2 K_0(\epsilon r) \phi_T - \left[ z^2 + (1-z)^2 \right] \epsilon K_1(\epsilon r) \partial_r \phi_T \right\}$$

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# Dipole-pion scattering amplitude

Main assumption $\Longrightarrow$ $\underbrace{\mathcal{N}^{\pi}(\hat{x}, \vec{r}, \vec{b})}_{\text{pion}} = R_q \cdot \underbrace{\mathcal{N}^{p}(\hat{x}, \vec{r}, \vec{b})}_{\text{proton}}$

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$\mathcal{N}^{p}(\hat{x}, \vec{r}, \vec{b}) \Rightarrow$ contains all the information about the hadron wave function

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Dipole-proton scattering amplitude

bCGC

$$\mathcal{N}^{p}(\hat{x},\vec{r},\vec{b}) = \begin{cases} \mathcal{N}_0 \left(\frac{r\,Q_s(b)}{2}\right)^{2\left(\gamma_s + \frac{\ln(2/r\,Q_s(b))}{\kappa\,\lambda\,Y}\right)} & rQ_s(b) \leq 2 \\ 1 - e^{-A\,\ln^2(B\,r\,Q_s(b))} & rQ_s(b) > 2 \end{cases}$$

# Dipole-pion scattering amplitude

Main assumption $\Longrightarrow$ $\mathcal{N}^{\pi}(\hat{x}, \vec{r}, \vec{b}) = R_q \cdot \mathcal{N}^{p}(\hat{x}, \vec{r}, \vec{b})$

$$R_q = \text{cte}$$

$\mathcal{N}^{p}(\hat{x}, \vec{r}, \vec{b}) \Rightarrow$ constrained by HERA data for inclusive and exclusive processes

Dipole-proton scattering amplitude

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BFKL close to the saturation regime

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BK deeply in the saturation regime

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## Saturation scale

$$Q_s(b) \equiv Q_s(\hat{x},b) = \left(\frac{x_0}{\hat{x}}\right)^{\frac{\lambda}{2}} \left[\exp\left(-\frac{b^2}{2B_{CGC}}\right)\right]^{\frac{1}{2\gamma_s}}$$

# Dipole-pion scattering amplitude

Main assumption $\Longrightarrow$ $\mathcal{N}^{\pi}(\hat{x},\vec{r},\vec{b}) = R_q \cdot \mathcal{N}^{p}(\hat{x},\vec{r},\vec{b})$

$$R_q = \text{cte}$$

---

$\mathcal{N}^{p}(\hat{x},\vec{r},\vec{b}) \Rightarrow$ constrained by HERA data for inclusive and exclusive processes

- GBW $\Leftrightarrow$ Golec-Biernat−Wüsthoff
- IIMS $\Leftrightarrow$ Iancu−Itakura−Munier−Soyez
- bCGC $\Leftrightarrow$ Kowalski−Motyka−Watt
- rcBK $\Leftrightarrow$ running coupling Balitsky−Kovchegov equation
- DGLAP $\Leftrightarrow$ Dokshitzer−Gribov−Lipatov−Altarelli−Parisi equation (CTEQ)

# Absorptive corrections

$$\sigma(\gamma^*\pi \to X) = \mathcal{K}_{inc} \cdot \int dz \int d^2\vec{r} \sum_{L,T} \left|\psi_{L,T}(z,\vec{r},Q^2)\right|^2 \sigma_{d\pi}(\hat{x},\vec{r})$$

$$\sigma(\gamma^*\pi \to E\pi) = \mathcal{K}_{exc} \cdot \frac{1}{16\pi} \sum_{L,T} \int_{-\infty}^{0} \left|\mathcal{A}_{L,T}^{\gamma^*\pi \to E\pi}(\hat{x},\Delta)\right|^2 d\hat{t}$$

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Open questions

# Absorptive corrections

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Open questions $\Rightarrow$ $\left\{ \mathcal{K}_{inc} \stackrel{?}{=} \mathcal{K}_{exc} \right.$

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Open questions $\Rightarrow$ $\left\{ \mathcal{K}_{inc} \stackrel{?}{=} \mathcal{K}_{exc} \Leftrightarrow \exists \mathcal{K}\,? \right.$

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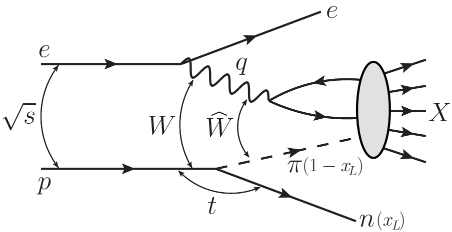
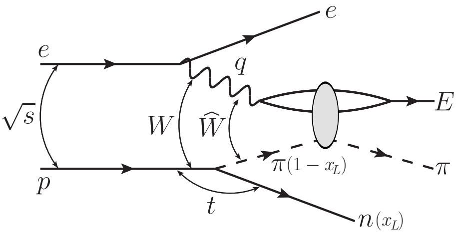
Our assumption:
$\mathcal{K} = \text{cte}$
$(0.5 \leq \mathcal{K} \leq 1.0)$

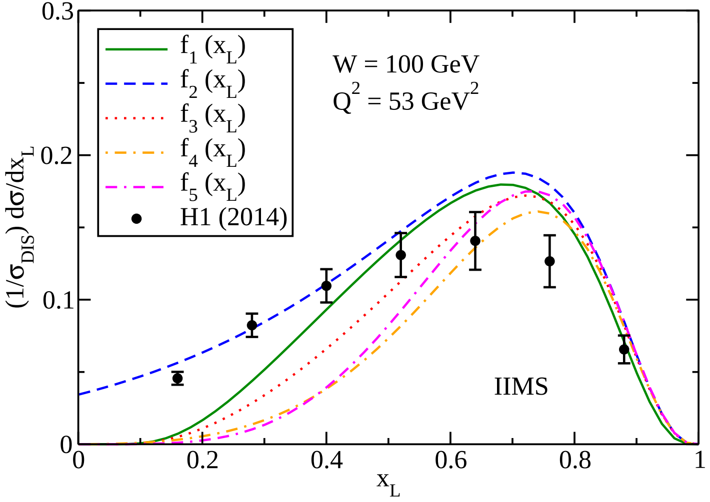
# Results for electron-proton collisions



# Results for **inclusive** processes: flux of pions

$\mathcal{K} = 1$
$R_q = 2/3$



# Results for **inclusive** processes: $\gamma^*\pi$ interaction

$\mathcal{K} = 1$
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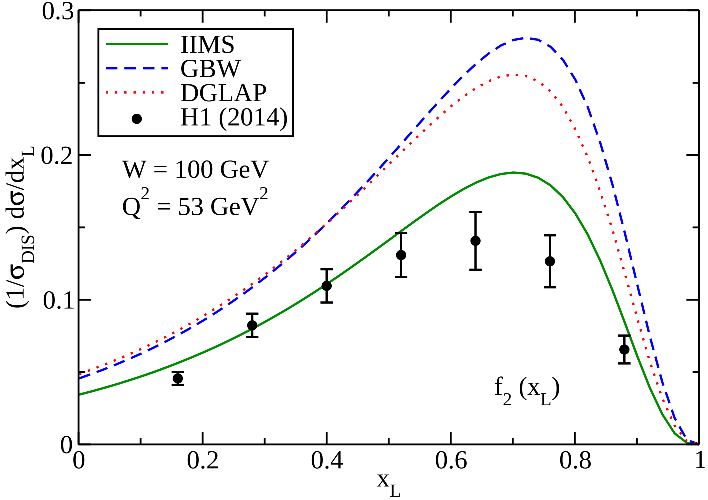
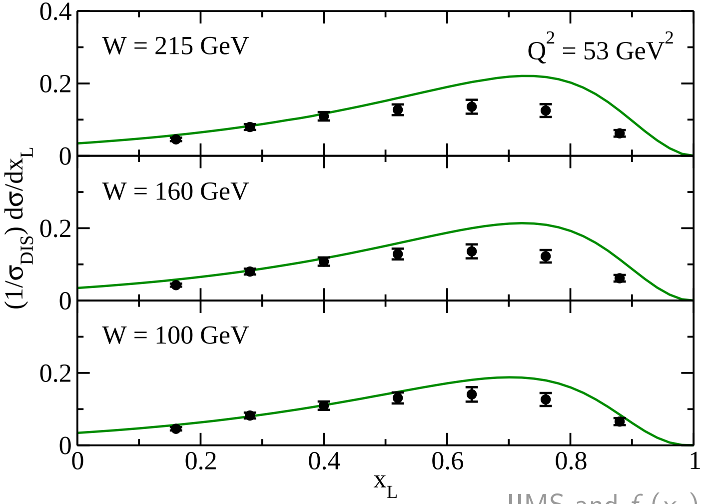


# Results for **inclusive** processes: $\mathcal{K}$ and $R_q$ dependence

$\mathcal{K} = 1$
$R_q = 2/3$




IIMS and $f_2(x_L)$

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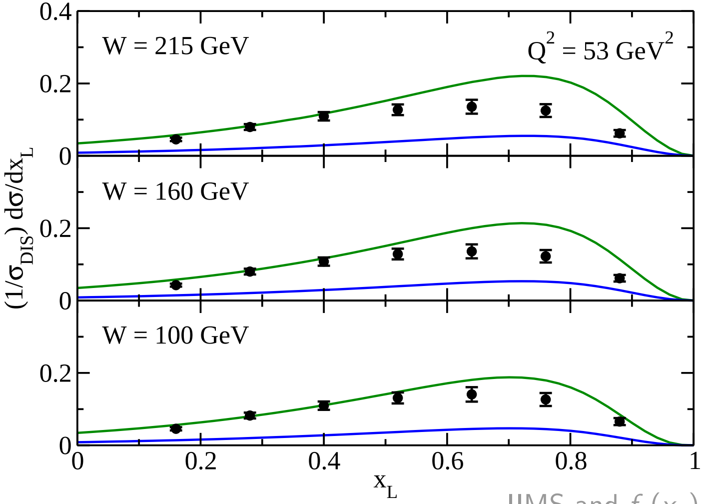


$\mathcal{K} = 1/2$
$R_q = 1/3$

IIMS and $f_2(x_L)$

# Results for **inclusive** processes: $\mathcal{K}$ and $R_q$ dependence

$\mathcal{K} = 1$
$R_q = 2/3$

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or
$\mathcal{K} = 1/2$
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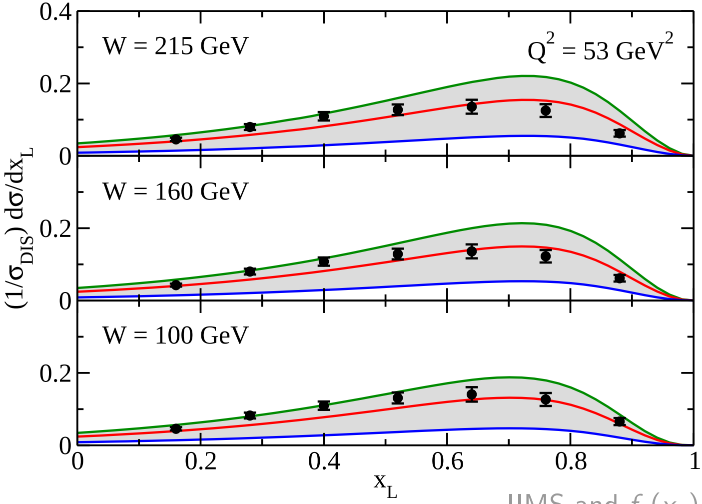


IIMS and $f_2(x_L)$

# Results for **exclusive** processes: constraining the $\mathcal{K}$-factor

Initially, we have assumed that: $R_q = 2/3$ (upper limit)

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Our strategy to constrain the $\mathcal{K}$-factor: For a given model of the pion flux, $R_q$ and dipole scattering amplitude, we estimate the total cross section. The value of $\mathcal{K}$ will be the value necessary to make our predictions consistent with the HERA data.

$$\text{HERA data} \quad \Longrightarrow \quad \sigma = a \pm b \quad \Longrightarrow \quad \begin{cases} a - b & \text{defines } \mathcal{K}_{min} \\ a & \text{defines } \mathcal{K}_{med} \\ a + b & \text{defines } \mathcal{K}_{max} \end{cases}$$

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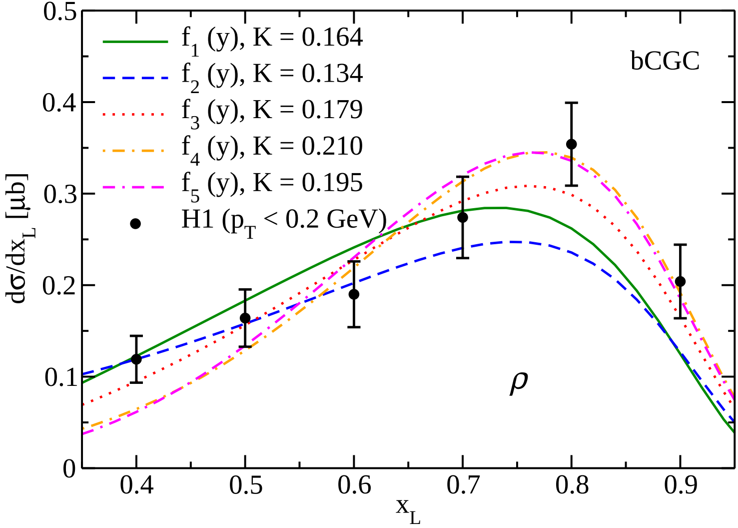
Important to remember that:

$$\sigma(\gamma^* \pi \to E\pi) \propto \mathcal{K} \cdot R_q^2$$

So, if for example $R_q = 1/3$, then:

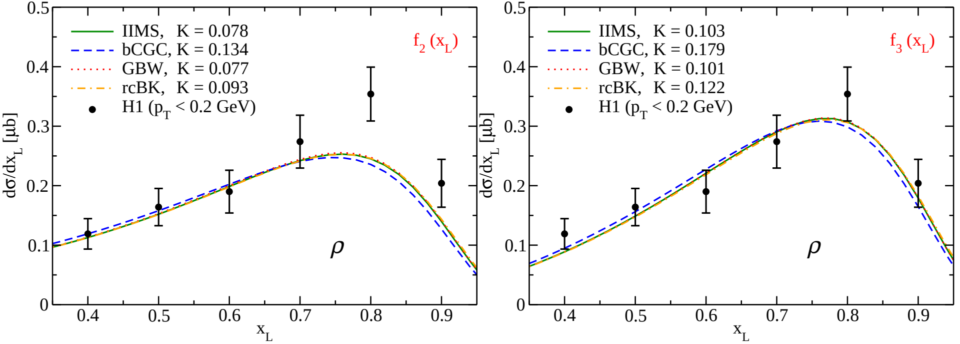
$$\mathcal{K}(R_q{=}1/3) = 4 \cdot \mathcal{K}(R_q{=}2/3)$$

# Results for **exclusive** processes: flux of pions




$W = 60$ GeV and $Q^2 = 0.04$ GeV$^2$.

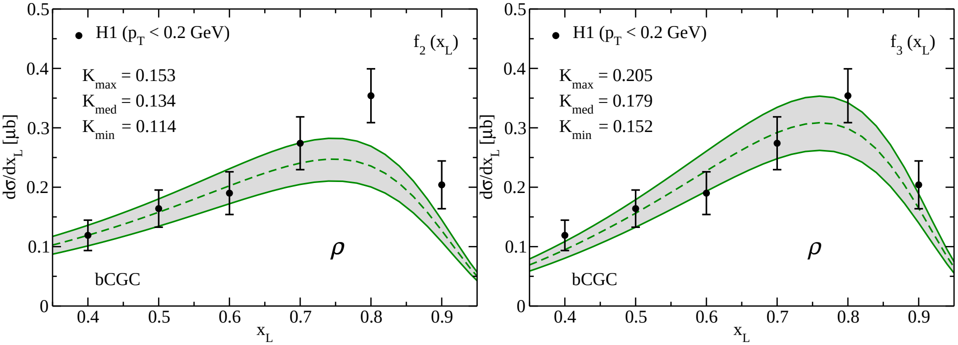
# Results for **exclusive** processes: $\gamma^{*}\pi$ interaction




$W = 60$ GeV and $Q^2 = 0.04$ GeV$^2$.

# Results for **exclusive** processes: upper and lower bounds

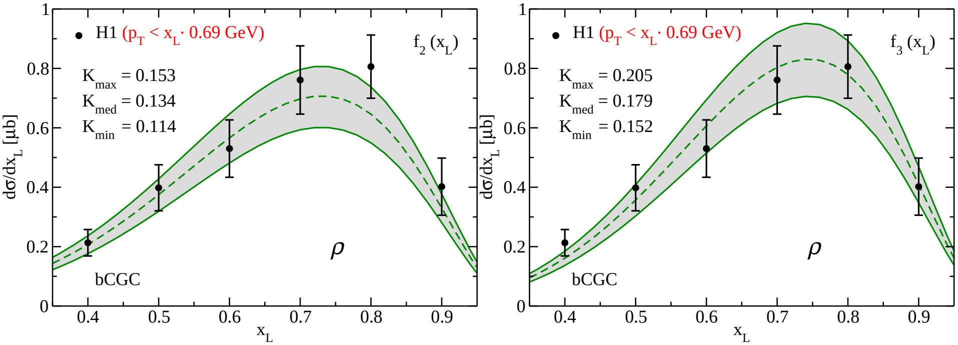
Bounds associated with the experimental uncertainty present in the H1 data.




$W = 60$ GeV and $Q^2 = 0.04$ GeV$^2$.

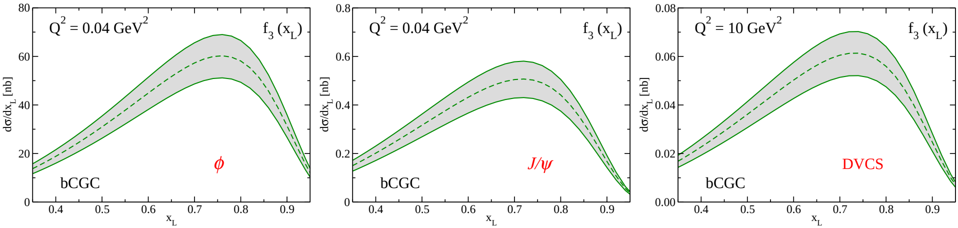
# Results for **exclusive** processes: testing $\mathcal{K}$-factor

A cross-check: the same range of $\mathcal{K}$ to describe different H1 data.




$W = 60$ GeV and $Q^2 = 0.04$ GeV$^2$.

# Results for **exclusive** processes: predictions for HERA




$W = 60$ GeV and $p_T < 0.2$ GeV:

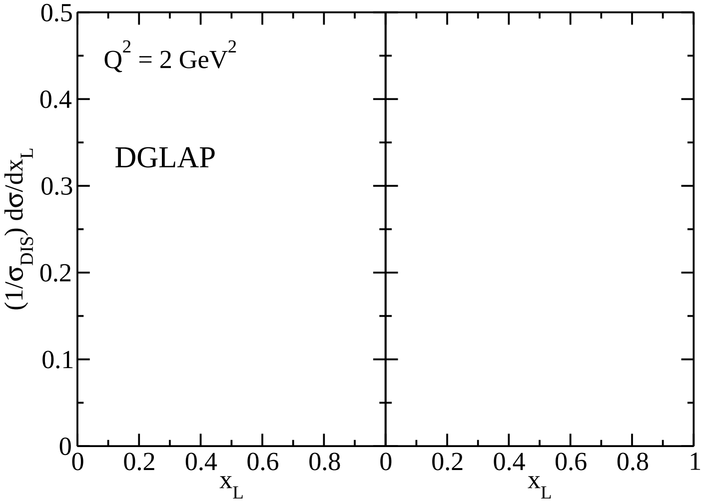
$$\sigma(\gamma p \to \phi \pi n) = 25.47 \pm 3.70 \text{ nb} \qquad (Q^2 = 0.04 \text{ GeV}^2)$$

$$\sigma(\gamma p \to J/\psi \pi n) = 0.22 \pm 0.03 \text{ nb} \qquad (Q^2 = 0.04 \text{ GeV}^2)$$

$$\sigma(\gamma p \to \gamma \pi n) = 0.008 \pm 0.001 \text{ nb} \qquad (Q^2 = 10.0 \text{ GeV}^2)$$

# Predictions for future *ep* colliders: Feynman scaling in inclusive processes

**Linear**



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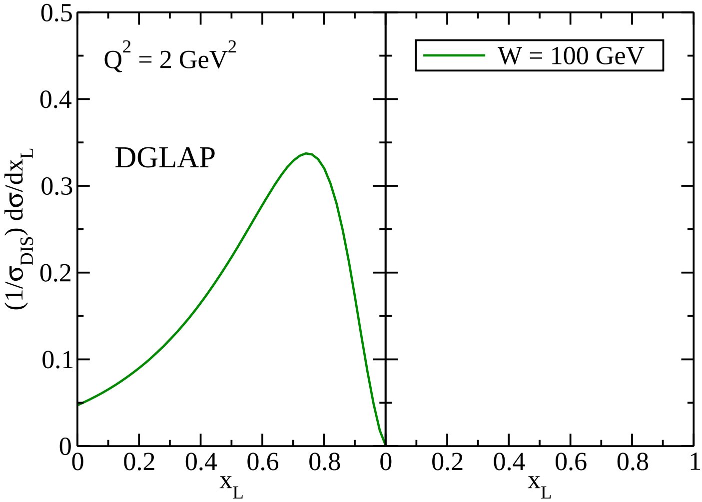

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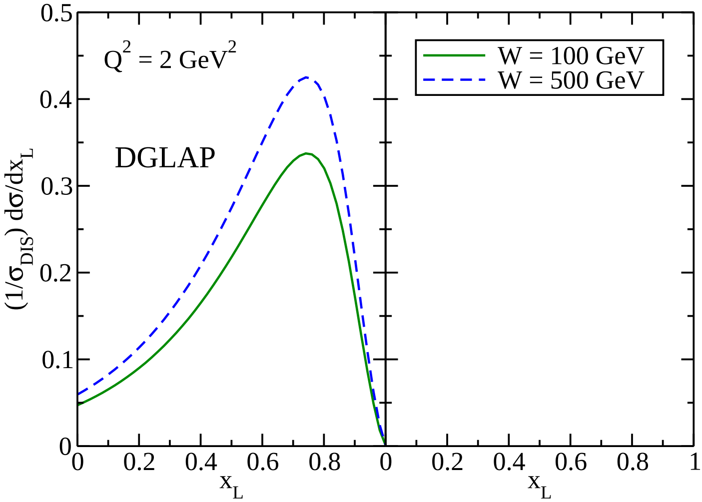

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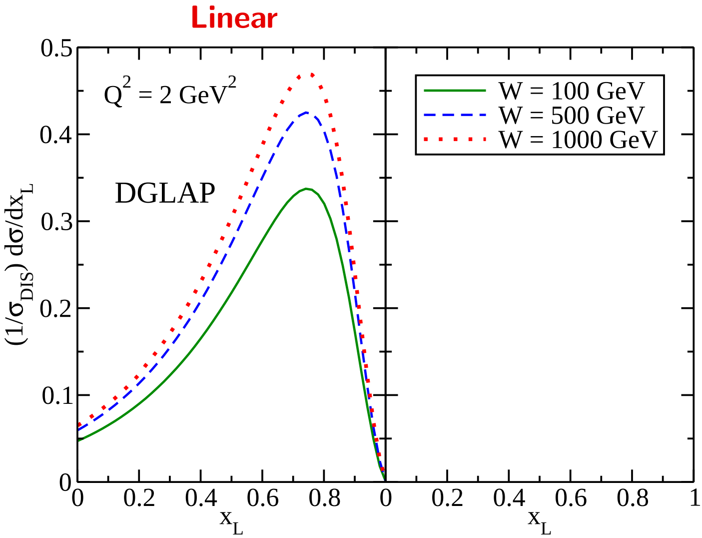

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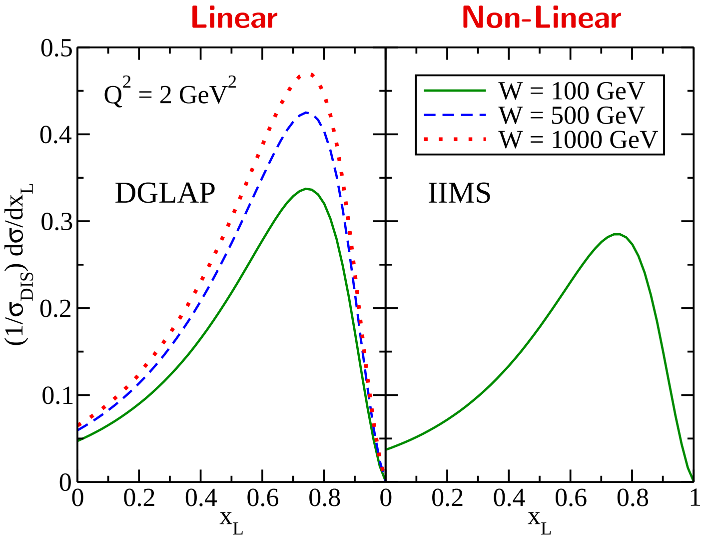

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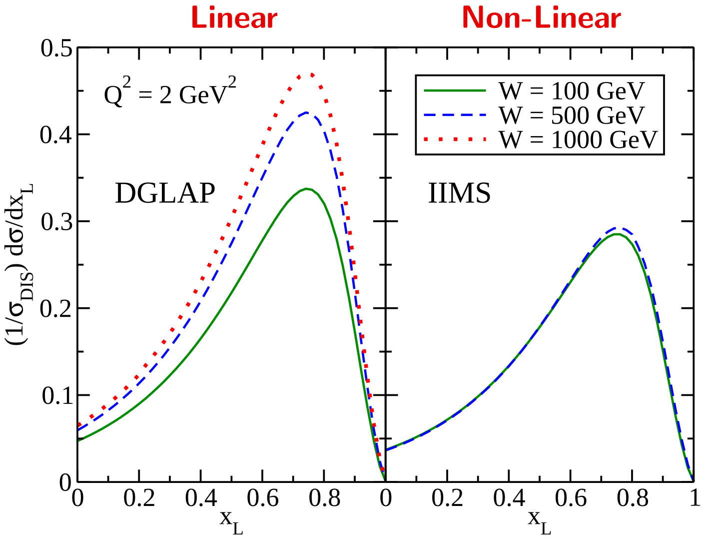

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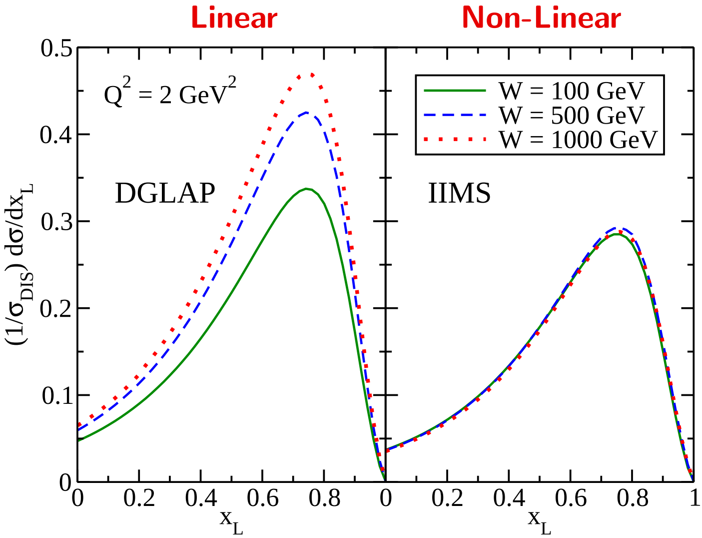

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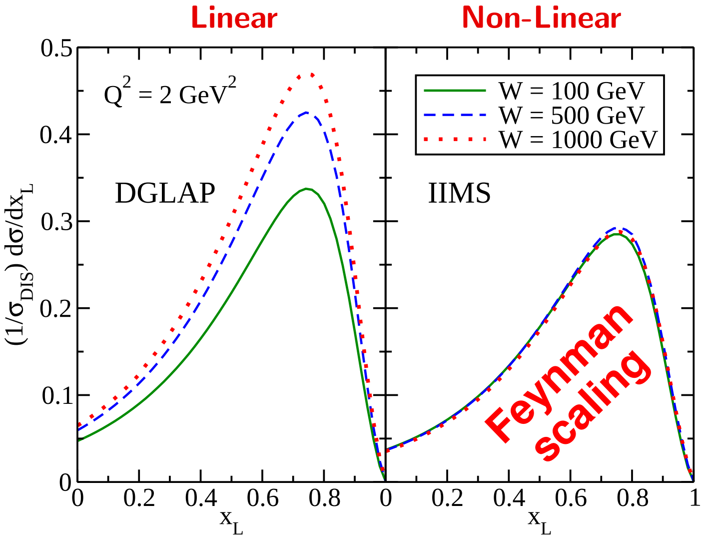
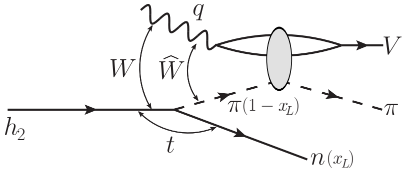


# Hadron-hadron collisions at large impact parameters

# Photon-induced interactions in hadron-hadron collisions (exclusive production)




$$\sigma(\gamma h_2 \to V + \pi + n)$$

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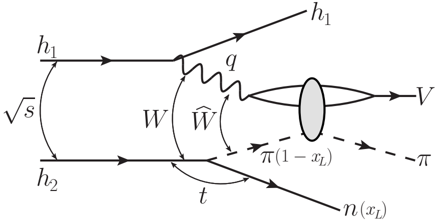


$$\frac{d\sigma(h_1 + h_2 \to h_3 + V + \pi + n)}{dY} = \left[\omega \left.\frac{dN}{d\omega}\right|_{h_1} \sigma(\gamma h_2 \to V + \pi + n)\right]_{\omega_L}$$

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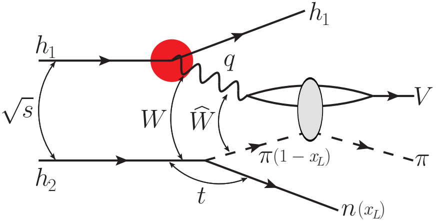
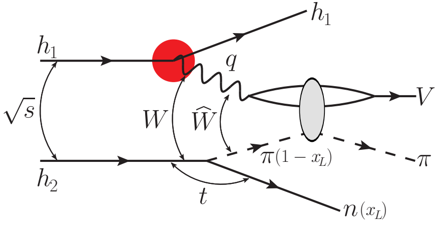
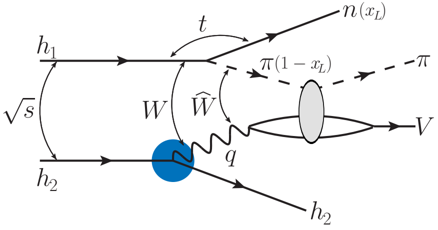



photon flux

$$\frac{d\sigma(h_1 + h_2 \to h_3 + V + \pi + n)}{dY} = \left[ \overbrace{\omega \left. \frac{dN}{d\omega} \right|_{h_1}}^{\text{photon flux}} \sigma(\gamma h_2 \to V + \pi + n) \right]_{\omega_L}$$

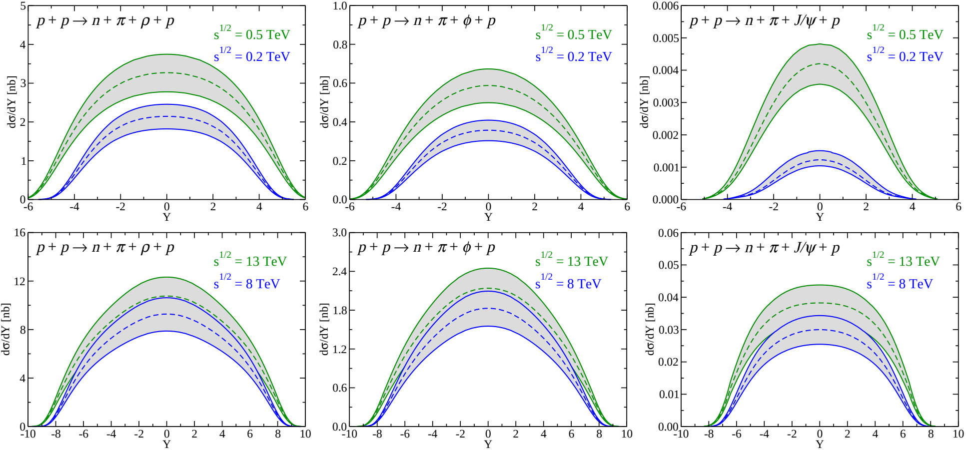
# Photon-induced interactions in hadron-hadron collisions (exclusive production)




$$\frac{d\sigma(h_1+h_2\to h_3+V+\pi+n)}{dY} = \Big[\overbrace{\omega\,\frac{dN}{d\omega}\Big|_{h_1}}^{\text{photon flux}}\sigma(\gamma h_2\to V+\pi+n)\Big]_{\omega_L} + \Big[\omega\,\frac{dN}{d\omega}\Big|_{h_2}\sigma(\gamma h_1\to V+\pi+n)\Big]_{\omega_R}$$

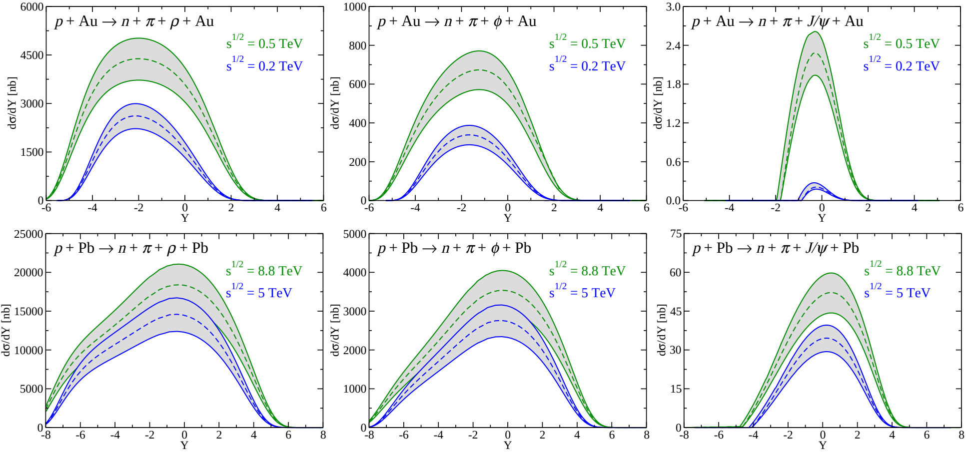
# Results for exclusive VM production associated with a leading neutron at *pp* collisions




C.M. energies: RHIC (upper panels) and LHC (lower panels)

# Results for exclusive VM production associated with a leading neutron at *pA* collisions

There is a $Z^2$ enhancement on the photon flux of the nucleus.




C.M. energies: RHIC (upper panels) and LHC (lower panels)

# Results for exclusive VM production associated with a leading neutron at *pp* and *pA* collisions

$$p + p \rightarrow n + p + V + \pi$$

| $\sigma$ [nb] | $\sqrt{s} = 0.2$ TeV | $\sqrt{s} = 0.5$ TeV | $\sqrt{s} = 8$ TeV | $\sqrt{s} = 13$ TeV |
|---|---|---|---|---|
| $\rho$ | 14.34 | 25.98 | 106.12 | 130.14 |
| $\phi$ | 2.15 | 4.21 | 19.63 | 24.42 |
| $J/\psi$ | 0.0049 | 0.022 | 0.30 | 0.42 |

$$p + A \rightarrow n + A + V + \pi$$

| $\sigma$ [nb] | $\sqrt{s} = 0.2$ TeV | $\sqrt{s} = 0.5$ TeV | $\sqrt{s} = 5.0$ TeV | $\sqrt{s} = 8.8$ TeV |
|---|---|---|---|---|
| $\rho$ | 10807.00 | 24518.30 | 121043.00 | 163821.00 |
| $\phi$ | 1278.41 | 3386.65 | 20403.80 | 28445.60 |
| $J/\psi$ | 0.23 | 4.65 | 159.61 | 276.16 |

Where we have used the central value $\mathcal{K}_{\text{med}}$.

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$\rho$ production on LHC (run 2): $\sim 10^5$ nb.

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| $J/\psi$ | 0.0049 | 0.022 | 0.30 | 0.42 |

$$p + A \rightarrow n + A + V + \pi$$

| $\sigma$ [nb] | $\sqrt{s} = 0.2$ TeV | $\sqrt{s} = 0.5$ TeV | $\sqrt{s} = 5.0$ TeV | $\sqrt{s} = 8.8$ TeV |
|---|---|---|---|---|
| $\rho$ | 10807.00 | 24518.30 | 121043.00 | 163821.00 |
| $\phi$ | 1278.41 | 3386.65 | 20403.80 | 28445.60 |
| $J/\psi$ | 0.23 | 4.65 | 159.61 | 276.16 |

The case with a leading neutron is smaller by $\sim 10^2$ of magnitude ($pp/pA$).

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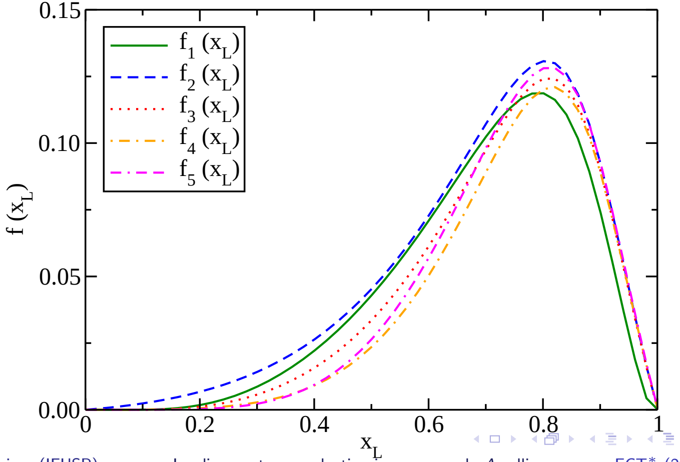
Thank you for your attention!

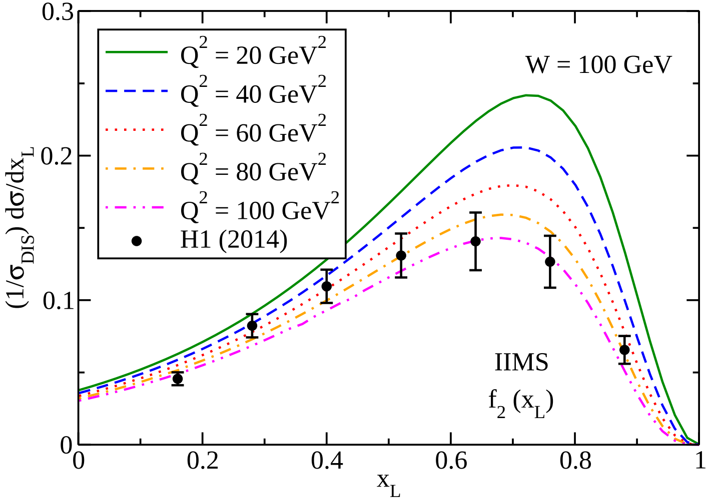
# Flux of pions / Pion splitting function

$$f_{\pi/p}(x_L, t) = \frac{1}{4\pi}\frac{2g^2_{p\pi p}}{4\pi}\frac{-t}{(t-m^2_\pi)^2}(1-x_L)^{1-2\alpha(t)}\,[F(x_L, t)]^2$$

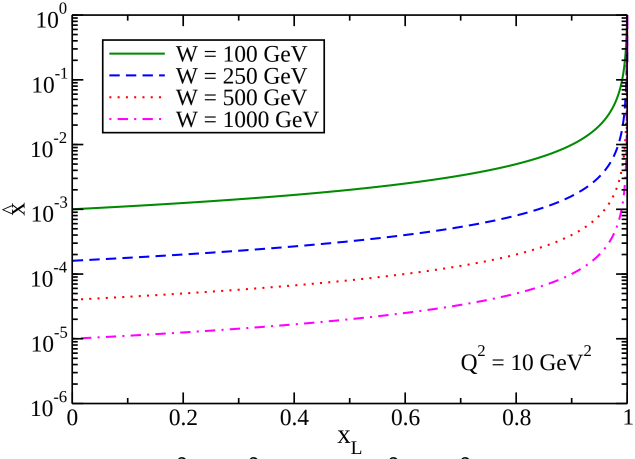


# Results for **inclusive** processes: $Q^2$ dependence

$\mathcal{K} = 1$
$R_q = 2/3$



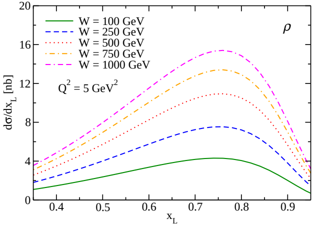
# Predictions for future *ep* colliders: Typical values of Bjorken-$x$

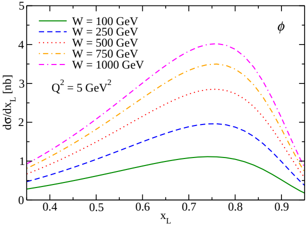


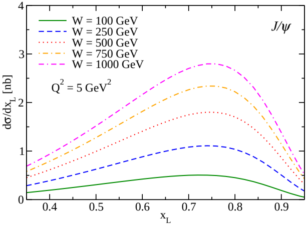

$$\hat{x} = \frac{Q^2 + m_f^2}{\hat{W}^2 + Q^2} = \frac{Q^2 + m_f^2}{(1 - x_L)W^2 + Q^2}$$

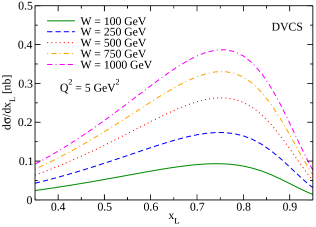
# Predictions for future *ep* colliders

Dependence on the energy for exclusive processes:




[bCGC and $f_3(y)$]

$Q^2 = 5$ GeV$^2$, and $p_T < 0.2$ GeV:

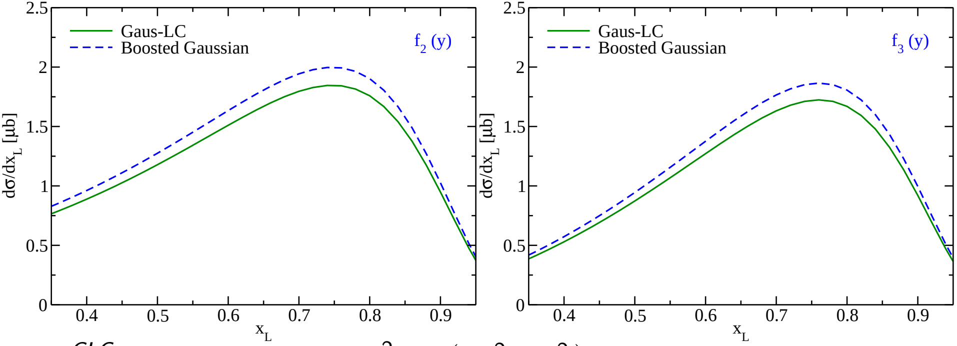
$$\sigma(\gamma p \rightarrow \rho \pi n) = 6.55 \pm 0.95 \text{ nb}$$

$$\sigma(\gamma p \rightarrow \phi \pi n) = 1.71 \pm 0.25 \text{ nb}$$

$$\sigma(\gamma p \rightarrow J/\psi \pi n) = 1.20 \pm 0.17 \text{ nb}$$

$$\sigma(\gamma p \rightarrow \gamma \pi n) = 0.16 \pm 0.002 \text{ nb}$$

# Results for **exclusive** processes: wave function dependence

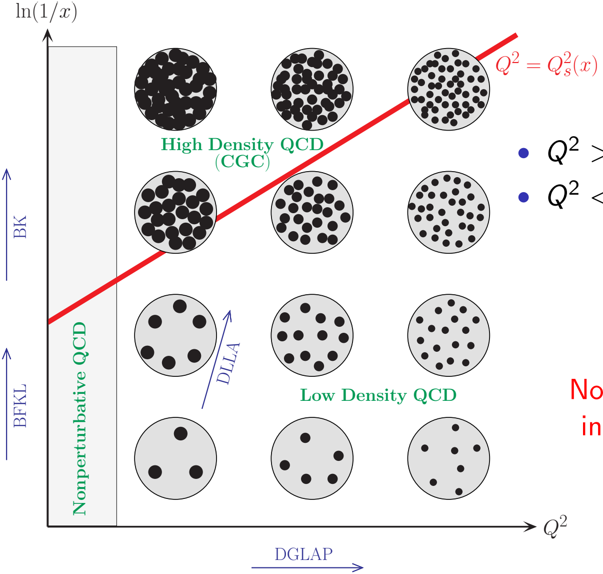



$$\phi_T^{GLC}(r,z) = N_T\,[z(1-z)]^2 \exp\left(-r^2/2R_T^2\right)$$

$$\phi_L^{GLC}(r,z) = N_L z(1-z) \exp\left(-r^2/2R_L^2\right)$$

$$\phi_{T,L}^{BoG}(r,z) = \mathcal{C}_{T,L}\, z(1-z) \exp\left[-\frac{m_f^2R^2}{8z(1-z)} - \frac{2z(1-z)r^2}{R^2} + \frac{m_f^2R^2}{2}\right]$$

# QCD (regimes)




- $Q^2 > Q_s^2 \Longrightarrow$ DGLAP, BFKL
- $Q^2 < Q_s^2 \Longrightarrow$ BK

$$\underbrace{Q_s^2 = Q_s^2(x)}$$

$$\Downarrow$$

Non-linear contribution increases with energy