Exclusive diffractive processes with saturation at NLO accuracy

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RB, A.V.Grabovsky, L.Szymanowski, S.Wallon
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RB, A.V.Grabovsky, D.Yu.Ivanov, L.Szymanowski, S.Wallon
arXiv:1612.08026 [hep-ph]

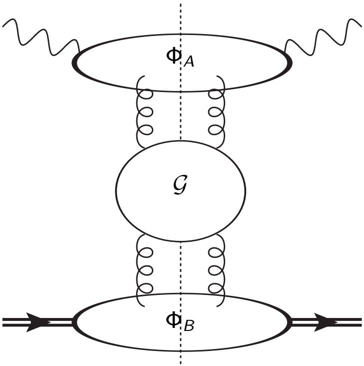
Probing QCD in the Regge limit and towards saturation

Observables to probe small$-x$ QCD and saturation physics

- Perturbation theory should apply : a hard scale Q^2 is required
- One needs semihard kinematics : $s \gg p_T^2 \gg \Lambda_{QCD}^2$ where all the typical transverse scales p_T are of the same order
- Saturation is reached when $Q^2 \sim Q_s^2 \propto \left(\frac{A}{x}\right)^{\frac{1}{3}}$: the smaller $x \sim \frac{Q^2}{s}$ is and the heavier the target ion, the easier saturation is reached.
- Typical processes : DIS, Mueller-Navelet double jets, ultraperipheral events at the LHC...

Precision tests of high-energy dynamics

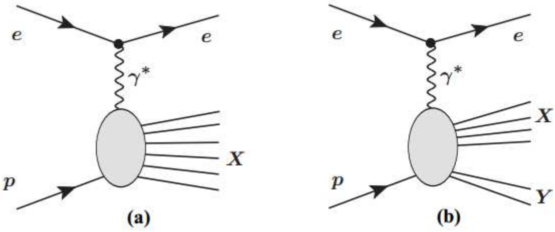
- BFKL and B-JIMWLK kernels are known at NLL accuracy, resumming $\alpha_s(\alpha_s \log s)^n$ corrections (Lipatov, Fadin ; Camici, Ciafaloni ; Balitsky, Chirilli ; Kovner, Lublinsky, Mulian)
- Very few impact factors are known at NLO accuracy
  - $\gamma^* \to \gamma^*$ (Bartels, Colferai, Gieseke, Kyrielis, Qiao; Balitsky, Chirilli; Beuf?)
  - Forward jet production (Bartels, Colferai, Vacca ; Caporale, Ivanov, Murdaca, Papa, Perri ; Chachamis, Hentschinski, Madrigal, Sabio Vera)
  - Inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
  - $\gamma_L^* \to V_L$ in the forward limit (Ivanov, Kostsky, Papa)



Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a rapidity gap



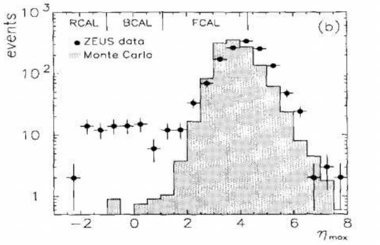
DIS events | DDIS events

DIS : Deep Inelastic Scattering, DDIS : Diffractive DIS

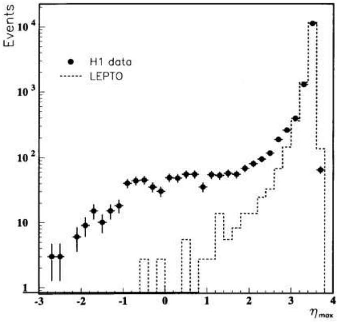
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of events reveal a rapidity gap



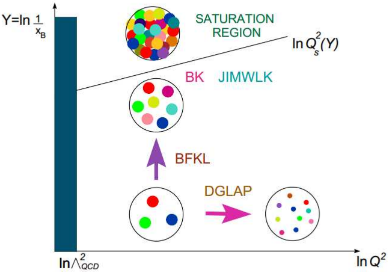
ZEUS, 1993



H1,1994

Diffractive DIS

Theoretical approaches for DDIS using pQCD

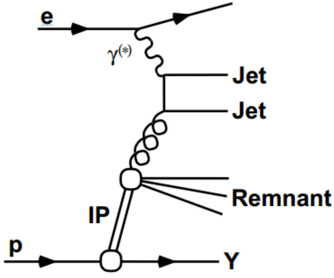


- Collinear factorization approach
  - Relies on a QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state

Theoretical approaches for DDIS using pQCD

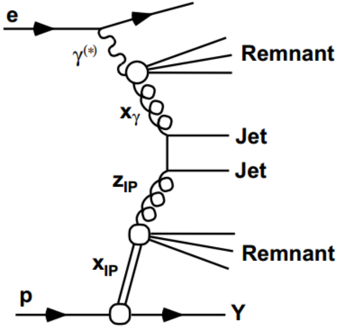
Collinear factorization approach

a)



Direct

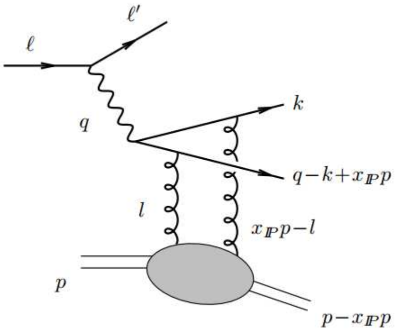
b)



Resolved

Theoretical approaches for DDIS using pQCD

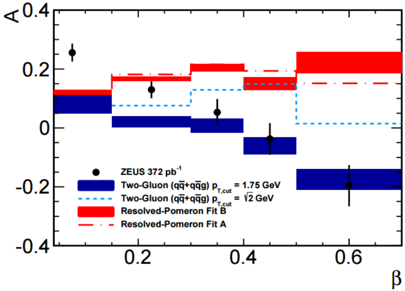
k_T-factorization approach : two gluon exchange



Bartels, Ivanov, Jung, Lotter, Wüsthoff
Braun and Ivanov developed a similar model in collinear factorization

Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data



ZEUS collaboration, 2015

NLO computations

in the shockwave (CGC) framework

The shockwave approach

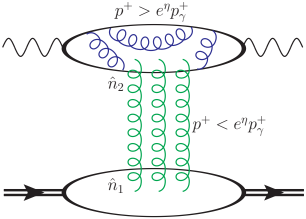
One decomposes the gluon field $\mathcal{A}$ into an internal field and an external field :

$$\mathcal{A}^{\mu} = A^{\mu} + b^{\mu}$$

The internal one contains the gluons with $p_g^+ > e^{\eta} p_\gamma^+$ and the external one contains the gluons with $p_g^+ < e^{\eta} p_\gamma^+$. One writes :

$$b_\eta^\mu(z) = \delta(z^+) B_\eta(\vec{z}) n_2^\mu$$

Intuitively, large boost Λ along the $+$ direction :



$$b^+(x^+, x^-, \vec{x}) \to \frac{1}{\Lambda} b^+\left(\Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x}\right)$$

$$b^-(x^+, x^-, \vec{x}) \to \Lambda b^-\left(\Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x}\right)$$

$$b^i\ (x^+, x^-, \vec{x}) \to\ b^i\left(\Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x}\right)$$

Lightcone variables $x^+ \equiv \frac{x^0 + x^3}{\sqrt{2}}, \quad x^- \equiv \frac{x^0 - x^3}{\sqrt{2}}$

Propagator through a shockwave

$$G(z_2, z_0) = -\int d^4 z_1 \theta(z_2^+)\,\delta(z_1^+)\,\theta(-z_0^+)\,G(z_2 - z_1)\,\gamma^+ G(z_1 - z_0)\,U_1$$

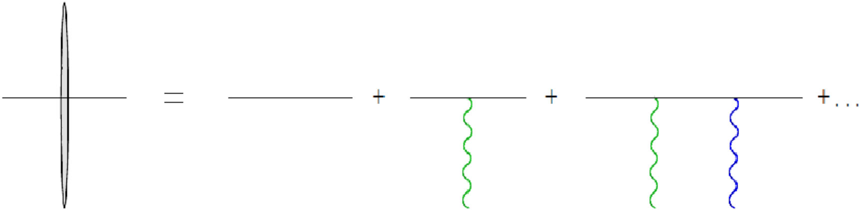
$$G(q, p) = (2\pi)\,\theta(p^+)\int d^D q_1 \delta(q + q_1 - p)\,\delta(q_1^+) G(q)\,\gamma^+ \tilde{U}_{\vec{q}_1} G(p)$$

Wilson lines :

$$U_i^\eta = U_{\vec{z}_i}^\eta = P\exp\left[ig\int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i)\,dz_i^+\right]$$

$$U_i^\eta = 1 + ig\int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i)dz_i^+ + (ig)^2\int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) b_\eta^-(z_j^+, \vec{z}_j)\theta(z_{ji}^+)dz_i^+ dz_j^+$$

...



Balitsky's hierarchy of equations

Dipole operator

$$\mathbf{U}_{12}^{\eta} = \frac{1}{N_c}\mathrm{Tr}\left(U_1^{\eta} U_2^{\eta\dagger}\right) - 1$$

B-JIMWLK equation

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

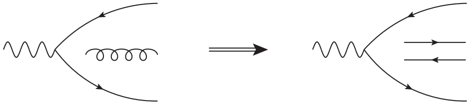
$$\frac{\partial \mathbf{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathbf{U}_{13}^{\eta} + \mathbf{U}_{32}^{\eta} - \mathbf{U}_{12}^{\eta} - \mathbf{U}_{13}^{\eta}\mathbf{U}_{32}^{\eta}\right]$$

$$\frac{\partial \mathbf{U}_{13}^{\eta}\mathbf{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft limit $N_c \to \infty$ in Balitsky's equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \langle \mathbf{U}^{\eta}_{12} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\langle \mathbf{U}^{\eta}_{13} \rangle + \rangle \mathbf{U}^{\eta}_{32} \rangle - \langle \mathbf{U}^{\eta}_{12} \rangle - \langle \mathbf{U}^{\eta}_{13} \rangle \langle \mathbf{U}^{\eta}_{32} \rangle \right]$$

Non-linear term : saturation

First step

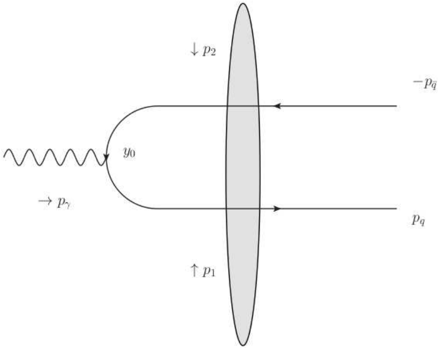
NLO open $q\bar{q}$ production and LO $q\bar{q}g$ production

Assumptions

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$p_g^+ < \alpha p_\gamma^+$$

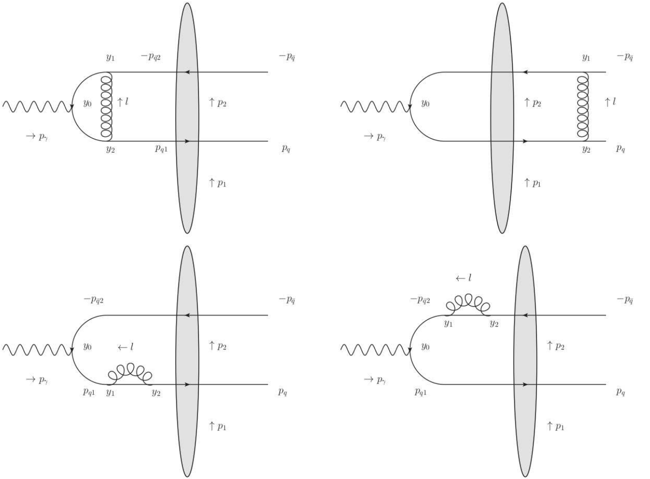
Leading Order



$$\mathcal{A}_0 = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_0^\alpha \left(\vec{p}_1, \vec{p}_2\right) \delta\left(\vec{p}_{q1} + \vec{p}_{\bar{q}2}\right) \tilde{\mathbf{U}}_{12}^\alpha$$

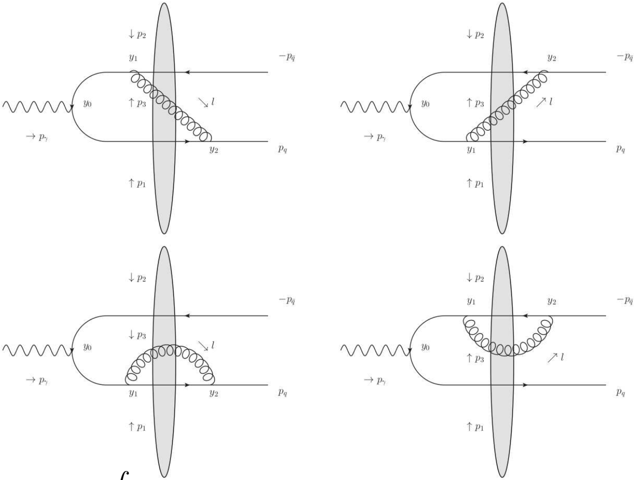
$p_{ij} \equiv p_i - p_j$

First kind of virtual corrections



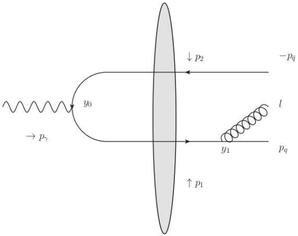
$$\mathcal{A}_{V1} \propto \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi^{\alpha}_{V1}\left(\vec{p}_1,\ \vec{p}_2\right) \delta\left(\vec{p}_{q1} + \vec{p}_{\bar{q}2}\right) \left(\frac{N_c^2 - 1}{N_c}\right) \tilde{\mathbf{U}}^{\alpha}_{12}$$

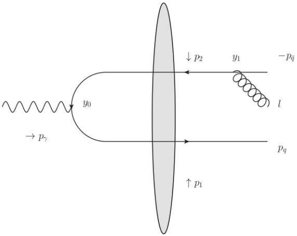
Second kind of virtual corrections



$$\mathcal{A}_{V2} \propto \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \Phi^\alpha_{V2}(\vec{p}_1,\, \vec{p}_2,\, \vec{p}_3)\delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3)$$
$$\left[\delta(\vec{p}_3)\left(\frac{N_c^2-1}{N_c}\right)\tilde{\mathbf{U}}^\alpha_{12} + N_c\left(\tilde{\mathbf{U}}^\alpha_{13}\tilde{\mathbf{U}}^\alpha_{32} + \tilde{\mathbf{U}}^\alpha_{13} + \tilde{\mathbf{U}}^\alpha_{32} - \tilde{\mathbf{U}}^\alpha_{12}\right)\right]$$

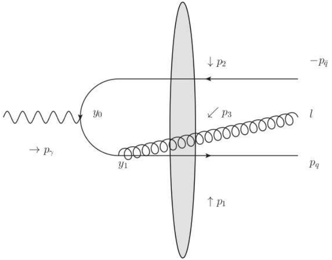
First kind of real corrections

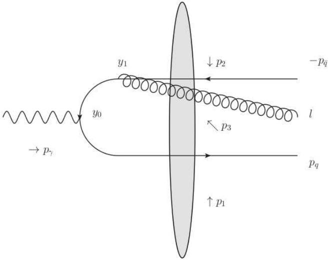




$$\mathcal{A}_{R1} \quad = \quad \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \, \Phi^\alpha_{R1}\left(\vec{p}_1,\ \vec{p}_2\right) \delta\left(\vec{p}_{q1} + \vec{p}_{\bar{q}2} + \vec{p}_g\right) \left(\frac{N_c^2 - 1}{N_c}\right) \tilde{\mathbf{U}}^\alpha_{12}$$

Second kind of real corrections



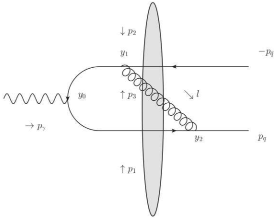


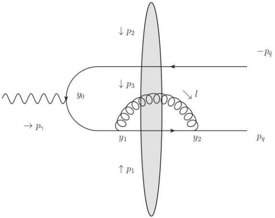
$$\mathcal{A}_{R2} = \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \Phi^\alpha_{R2}(\vec{p}_1,\ \vec{p}_2,\ \vec{p}_3)\delta(\vec{p}_{q1}+\vec{p}_{\bar{q}2}+\vec{p}_{g3})$$
$$\left[\left(\frac{N_c^2-1}{N_c}\right)\tilde{\mathbf{U}}^\alpha_{12}\delta(\vec{p}_3) + N_c\left(\tilde{\mathbf{U}}^\alpha_{13}\tilde{\mathbf{U}}^\alpha_{32} + \tilde{\mathbf{U}}^\alpha_{13} + \tilde{\mathbf{U}}^\alpha_{32} - \tilde{\mathbf{U}}^\alpha_{12}\right)\right]$$

Divergences

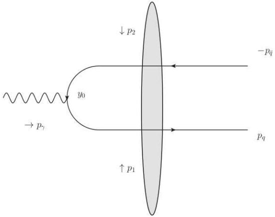
- Rapidity divergence $p_g^+ \to 0$ $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Rapidity divergence





Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Rapidity divergence

B-JIMWLK equation

$$\frac{\partial \tilde{\mathbf{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathbf{U}}_{13}^{\alpha} \tilde{\mathbf{U}}_{32}^{\alpha} + \tilde{\mathbf{U}}_{13}^{\alpha} + \tilde{\mathbf{U}}_{32}^{\alpha} - \tilde{\mathbf{U}}_{12}^{\alpha} \right)$$
$$\times \left[2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2 \right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2 \right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\tilde{\mathbf{U}}_{12}^{\alpha} \Phi_0 \rightarrow \Phi_0 \tilde{\mathbf{U}}_{12}^{\eta} + \log \left(\frac{e^{\eta}}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \left(\tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right)$$

Rapidity divergence

Virtual contribution

$$(\Phi^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{\alpha^2}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

BK contribution

$$(\Phi^{\mu}_{BK})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{\alpha^2}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] \right\}$$

Sum : the α dependence cancels

$$(\Phi'^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

Rapidity divergence

Cancellation of the remaining $1/\epsilon$ divergence

Convolution

$$\begin{aligned}(\Phi'^{\mu}_{V2} \otimes \mathbf{UU}) &= \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\} \\ &\times \quad \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3)\left[\tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} - \tilde{\mathbf{U}}_{13}\tilde{\mathbf{U}}_{32}\right]\Phi^{\mu}_0(\vec{p}_1, \vec{p}_2)\end{aligned}$$

Rq :

- $\Phi_0(\vec{p}_1, \vec{p}_2)$ only depends on one of the t-channel momenta.
- The double-dipole operators cancels when $\vec{z}_3 = \vec{z}_1$ or $\vec{z}_3 = \vec{z}_2$.

This permits one to show that the convolution cancels the remaining $\frac{1}{\varepsilon}$ divergence.

Then $\tilde{\mathbf{U}}^{\alpha}_{12}\Phi_0 + \Phi_{V2}$ is finite

Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Constructing a finite cross section

Exclusive diffractive production of a forward dijet

From partons to jets

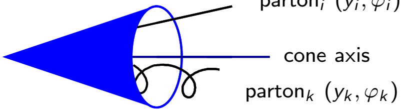
Soft and collinear divergence

Jet cone algorithm

We define a cone width for each pair of particles with momenta p_i and p_k, rapidity difference ΔY_{ik} and relative azimuthal angle $\Delta\varphi_{ik}$

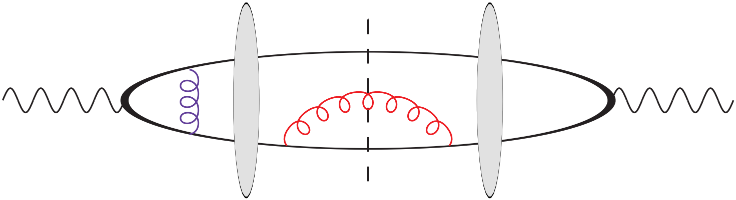
$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.



Applying this in the small R^2 limit cancels our soft and collinear divergence.

Remaining divergence



- UV divergence $\vec{p}_g^{\,2} \to +\infty$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$$

- Soft divergence $p_g \to 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

Remaining divergence

Soft real emission

$$(\Phi_{R1}\Phi_{R1}^{*})_{soft} \propto (\Phi_0\Phi_0^{*}) \int_{\text{outside the cones}} \left| \frac{p_q^{\mu}}{(p_q.p_g)} - \frac{p_{\bar{q}}^{\mu}}{(p_{\bar{q}}.p_g)} \right|^2 \frac{dp_g^+}{p_g^+} \frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$(\Phi_{R1}\Phi_{R1}^{*})_{col} \propto (\Phi_0\Phi_0^{*}) \left(\mathcal{N}_q + \mathcal{N}_{\bar{q}}\right)$$

Where $\mathcal{N}$ is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2-\frac{d}{2})} \int_{\alpha p_\gamma^+}^{p_{jet}^+} \frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+} \int_{\text{in cone k}} \frac{d^d \vec{p}_g d^d \vec{p}_k}{(2\pi)^d \mu^{d-2}} \frac{\mathrm{Tr}\left(\hat{p}_k \gamma^{\mu} \hat{p}_{jet} \gamma^{\nu}\right) d_{\mu\nu}(p_g)}{2p_{jet}^+ \left(p_k^- + p_g^- - p_{jet}^-\right)^2}$$

Those two contributions cancel exactly the virtual divergences (both UV and soft)

Cancellation of divergences

Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left(\frac{N_c^2-1}{2N_c}\right) \left(S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}\right) d\sigma_0$$

Virtual contribution

$$\begin{aligned} S_V &= \left[2\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - 3\right]\left[\ln\left(\frac{x_j x_{\bar{j}} \mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}} \vec{p}_j)^2}\right) - \frac{1}{\epsilon}\right] \\ &+ 2i\pi \ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) + \ln^2\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - \frac{\pi^2}{3} + 6 \end{aligned}$$

Real contribution

$$\begin{aligned} S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} &= 2\left[\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j \vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2} \vec{p}_j^{\,2}}\right) \ln\left(\frac{4E^2}{x_{\bar{j}} x_j (p_\gamma^+)^2}\right)\right. \\ &+ 2\ln\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon} - \ln\left(\frac{x_{\bar{j}} x_j \mu^2}{(x_{\bar{j}}\vec{p}_j - x_j \vec{p}_{\bar{j}})^2}\right)\right) - \ln^2\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right) \\ &+ \left.\frac{3}{2}\ln\left(\frac{16\mu^4}{R^4 \vec{p}_j^{\,2} \vec{p}_{\bar{j}}^{\,2}}\right) - \ln\left(\frac{x_j}{x_{\bar{j}}}\right) \ln\left(\frac{x_j \vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}} \vec{p}_j^{\,2}}\right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7\right] \end{aligned}$$

Cancellation of divergences

Total divergence

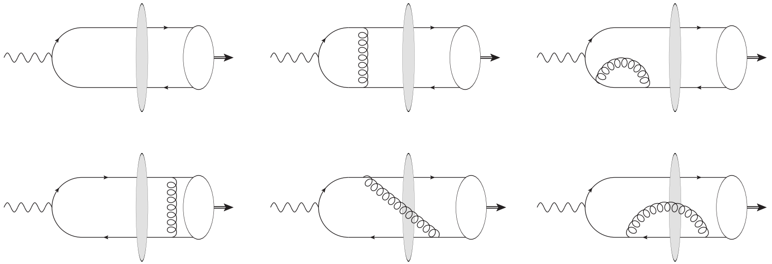
$$\begin{aligned} div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\ &= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)+\frac{3}{2}\right)\right. \\ &+ \left.\ln(8) - \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) + \frac{13-\pi^2}{2}\right] \end{aligned}$$

Our cross section is thus finite

Constructing a finite amplitude

Exclusive diffractive production of a light neutral vector meson

Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A}_0 = -\frac{e_V f_V \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel(x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d}$$
$$\times (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right)$$
$$\times \Phi_0^\beta(x, \vec{p}_1, \vec{p}_2) \tilde{\mathbf{U}}_{12}^\eta .$$

Leading twist for a longitudinally polarized meson

Otherwise general kinematics, including transverse virtual photon (twist 3) contributions, and the photoproduction limit (for large $t-$channel momentum transfer)

ERBL evolution equation

Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial \varphi(x,\mu_F^2)}{\partial \ln \mu_F^2} = \frac{\alpha_s C_F}{2\pi} \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon} \left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon \int_0^1 dz \varphi(z,\mu_F^2) \mathcal{K}(x,z),$$

where we parameterize the ERBL kernel for consistency as

$$\begin{aligned} \mathcal{K}(x,z) &= \frac{x}{z}\left[1+\frac{1}{z-x}\right]\theta(z-x-\alpha) \\ &+ \frac{1-x}{1-z}\left[1+\frac{1}{x-z}\right]\theta(x-z-\alpha) \\ &+ \left[\frac{3}{2}-\ln\left(\frac{x(1-x)}{\alpha^2}\right)\right]\delta(z-x). \end{aligned}$$

It is equivalent to the usual ERBL kernel

Infrared finiteness

The amplitude we obtain is finite. For example the dipole $\gamma_L^* \to V_L$ contribution reads

$$\begin{aligned}
\Phi_1^+(x) &= \int_0^x dz \left(\frac{x-z}{x}\right) \Phi_0^+(x-z) \\
&\times \left[1 + \left(1 + \left[\frac{1}{z}\right]_+\right) \ln\left(\frac{\left(((\bar{x}+z)\vec{p}_1 - (x-z)\vec{p}_2)^2 + (x-z)(\bar{x}+z)Q^2\right)^2}{\mu_F^2 (x-z)(\bar{x}+z)Q^2}\right)\right] \\
&+ \frac{1}{2}\Phi_0^+(x)\left[\frac{1}{2}\ln^2\left(\frac{\bar{x}}{x}\right) + 3 - \frac{\pi^2}{6} - \frac{3}{2}\ln\left(\frac{\left((\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2\right)^2}{x\bar{x}\mu_F^2 Q^2}\right)\right] \\
&+ \frac{(p_\gamma^+)^2}{2x\bar{x}} \int_0^x dz \left[(\phi_5)_{LL}|_{\vec{p}_3=\vec{0}} + (\phi_6)_{LL}|_{\vec{p}_3=\vec{0}}\right]_+ + (x \leftrightarrow \bar{x}, \vec{p}_1 \leftrightarrow \vec{p}_2).
\end{aligned}$$

No end point singularity, even for a transverse photon and even in the photoproduction limit.

Practical use of such results for phenomenology

Practical use of such results

- Build non-perturbative initial conditions for the scattering matrix elements

$$\langle P'|\tilde{\mathbf{U}}_{12}^{\eta}|P\rangle, \quad \langle P'|\tilde{\mathbf{U}}_{13}^{\eta}\tilde{\mathbf{U}}_{32}^{\eta}|P\rangle$$

- Solve the NLO JIMWLK evolution equation for the dipole and double-dipole operators with these initial conditions at a typical target rapidity $\eta = Y_0$

- Evaluate the resulting matrix elements at a typical projectile rapidity $\eta = Y$ and convolute them with the impact factor.

Residual parameter dependence

Required parameters

- Renormalization scale μ_R
- Factorization scale μ_F in the case of meson production
- Typical target rapidity Y_0
- Typical projectile rapidity Y

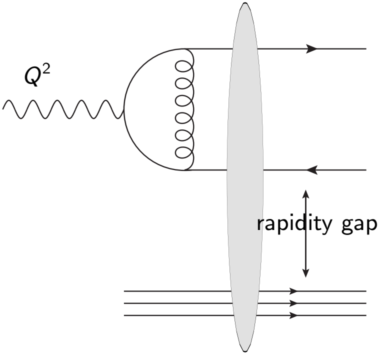
In the linear BFKL limit, the cross section only depends on $Y - Y_0$, so only one arbirary parameter s_0 defined by

$$Y - Y_0 = \ln\left(\frac{s}{s_0}\right)$$

is required.

General amplitude

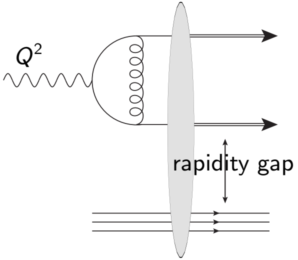
- Most general kinematics
- The hard scale can be Q^2, t, M_X^2...
- The target can be either a proton or an ion, or another impact factor.
- Finite results for $Q^2 = 0$
- One can study ultraperipheral collision by tagging the particle which emitted the photon, in the limit $Q^2 \to 0$.



The general amplitude

Phenomenological applications : exclusive dijet production at NLO accuracy

- HERA data for exclusive dijet production in diffractive DIS can be fitted with our results
- We can also give predictions for the same process in a future electron-ion or electron-proton collider (EIC, LHeC...)
- For $Q^2 = 0$ we can give predictions for ultraperipheral *pp* and *pA* collisions at the LHC



Amplitude for diffractive dijet production

Phenomenological applications : exclusive trijet production at LO accuracy

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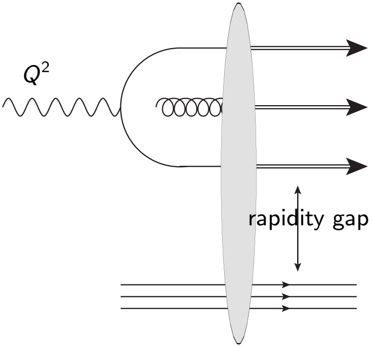
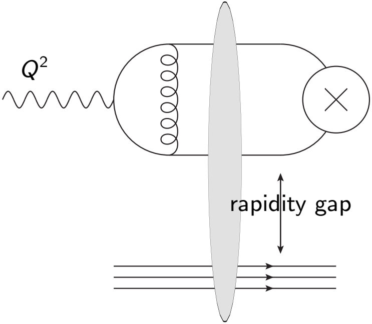


Amplitude for diffractive trijet production

[Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans]

Phenomenological applications

- Most general kinematics
- The hard scale can be Q^2, t or m^2.
- The target can be either a proton or an ion, or another impact factor.
- Finite results for $Q^2 = 0$
- One can study ultraperipheral collision by tagging the particle which emitted the photon, in the limit $Q^2 \to 0$.



Amplitude for diffractive V production

Comparison with previous results [Work in progress]

- The $\gamma_L^* \to V_L$ contribution in the forward limit should coincide with previous results of Ivanov, Kotsky, Papa
- The comparison is non-trivial due to additional contributions from the formal BFKL/BK transition

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi'_{BK} = (\Phi_{BFKL} \otimes \mathcal{O})(\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL}\mathcal{O})(\mathcal{O}^{-1} \otimes \Phi'_{BFKL})$$

$\mathcal{O}$ was obtained to prove the kernel equivalence, but never checked on an impact factor

Conclusion

- We provided the full computation of the impact factor for the exclusive diffractive production of a forward dijet and of a light neutral vector meson with NLO accuracy in the shockwave approach
- It leads to an enormous number of possible phenomenological applications to test QCD in its Regge limit and towards saturation in past, present and future ep, eA, pp and pA colliders
- Several theoretical extensions could be obtained with slight modifications to our result