

Saturation of gluon TMDs at small x , probed with forward di-jets in $p+A$ collisions

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Contents of the talk

- Known issues with TMD factorization
 - process dependence, loss of universality, factorization breaking
- Case study for gluon TMDs at small- x : forward di-jets
 - TMD factorization is contained in the CGC framework, as the leading power in the hard scale
- Small- x evolution and saturation of the gluon TMDs
 - obtained from the JIMWLK equation
 - numerical results
- Forward di-jets in p+Pb collisions at the LHC
 - some predictions to motivate a measurement

Factorization breaking

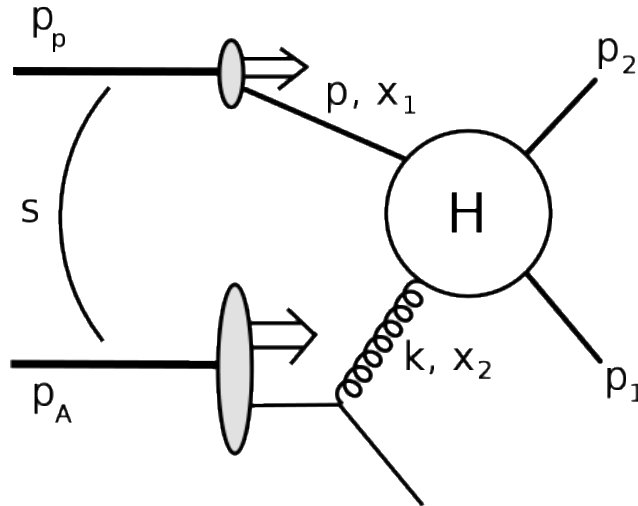
consider a hadronic collision $A + B \rightarrow C + D + \dots + X$

- "hard" factorization breaking: Collins and Qiu (2007)
a maximum of two transverse-momentum-dependent "hadrons" (parton distributions or fragmentation functions) may be considered for 3 or more TMD hadrons, factorization cannot be established
 - "soft" factorization breaking: Boer and Mulders (2000), Belitsky, Ji and Yuan (2003)
for one or two transverse-momentum-dependent hadrons, TMD factorization can be obtained, but different processes involve different TMDs
universality is lost
- note: even then TMD factorization is broken at some order in perturbation theory, here I am only discussing the validity at leading-order
- in our forward di-jet study, due to the asymmetry, only the target nucleus will be described with TMDs

$$\langle k_T \rangle \sim \Lambda_{QCD} \quad \text{[nucleus diagram]} \quad \longrightarrow \quad \text{[jet diagram]} \quad \text{[nucleus diagram]} \quad \langle k_T \rangle \sim Q_s$$

Dilute-dense kinematics

- large-x projectile (proton) on small-x target (proton or nucleus)



$$\hat{s} = (p + k)^2$$

$$\hat{t} = (p_2 - p)^2$$

$$\hat{u} = (p_1 - p)^2$$

Incoming partons' energy fractions:

$$x_1 = \frac{p^1}{s} (|p_{1t}| e^{y_1} + |p_{2t}| e^{y_2}) \quad \underline{y_{1,2} \gg 0} \quad x_1 \leftarrow 1$$

$$x_2 = \frac{p^1}{s} (|p_{1t}| e^{-y_1} + |p_{2t}| e^{-y_2}) \quad x_2 \boxed{x} 1$$

Gluon's transverse momentum (p_{1t} , p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos\Delta\varphi$$

The back-to-back regime

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

this is the regime of validity of TMD factorization:

$$\frac{d\sigma^{pA \rightarrow \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \sum_{i=1}^2 K_{ag \rightarrow cd}^{(i)} \Phi_{ag \rightarrow cd}^{(i)} \frac{1}{1 + \delta_{cd}}$$

it involves six unpolarized gluon TMDs $\Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2)$ (2 per channel)

their associated hard matrix elements $K_{ag \rightarrow cd}^{(i)}$ are on-shell (i.e. $k_t = 0$)

it can be derived in two ways:

from the generic TMD factorization framework (valid up to power corrections):
by taking the small- x limit

Bomhof, Mulders and Pijlman (2006)

Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

from the CGC framework (valid at small- x): by extracting the leading power

Dominguez, CM, Xiao and Yuan (2011)

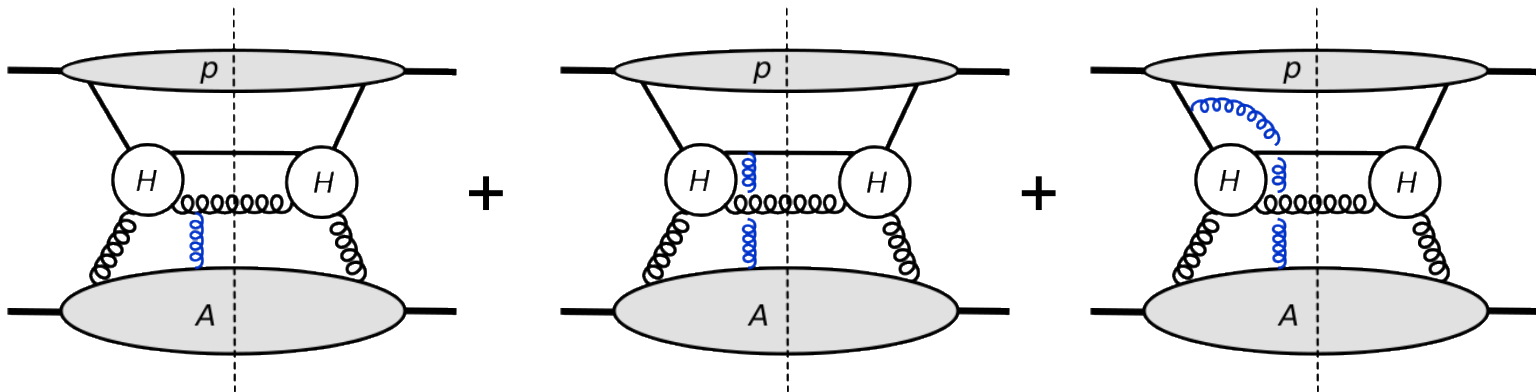
CM, Petreska, Roiesnel (2016)

TMD gluon distributions

- the naive operator definition is not gauge-invariant

$$F_{g/A}(x_2, k_t) \stackrel{\text{naive}}{=} \int \frac{d^{\leftarrow\pm} d^2 \leftarrow\bar{t}}{(2\hat{i})^3 p_A^-} e^{ix_2 p_A^- \leftarrow\pm - ik_t \cdot \leftarrow\bar{t}} \overline{A} / \text{Tr} \left[F^{i-} \left(\leftarrow\pm, \leftarrow\bar{t} \right) F^{i-} \left(0 \right) \right] A$$

- a theoretically consistent definition requires to include more diagrams



+ similar diagrams with 2, 3, ... gluon exchanges

They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

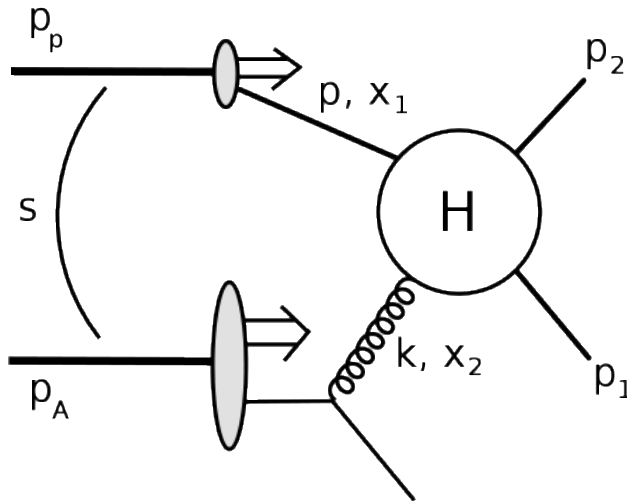
Process-dependent TMDs

- the proper operator definition(s)

some gauge link $P \exp -ig \int_{\leftarrow}^{\rightarrow} d\mathcal{H}A^a(\mathcal{H})T^a$

$$F_{g/A}(x_2, k_t) = 2 \frac{d^{\leftarrow\pm} d^2 \leftarrow\vec{t}}{(2\hat{t})^3 p_A^-} e^{ix_2 p_A^- \leftarrow\pm - ik_t \cdot \leftarrow\vec{t}} \overline{A} / \text{Tr} \left[F^{i-}(\leftarrow\pm, \leftarrow\vec{t}) U_{[\leftarrow, 0]} F^{i-}(0) \right] A^{\leftarrow}$$

$U_{[\leftarrow, \beta]}$ renders gluon distribution gauge invariant



however, the precise structure of the gauge link is process-dependent:

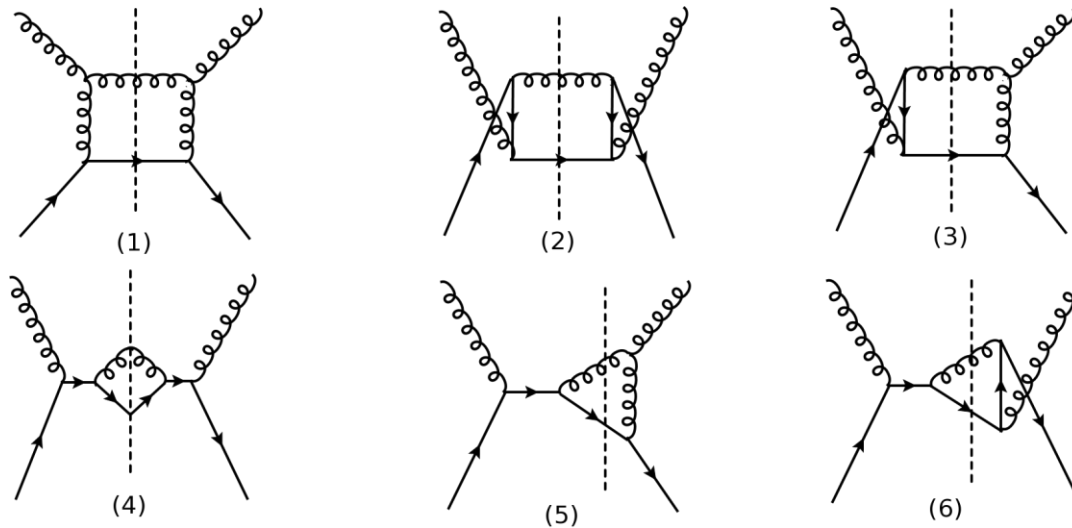
it is determined by the color structure of the hard process H

- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

TMDs for forward di-jets

- several gluon distributions are needed already for a single partonic sub-process

example for the $qg^* \rightarrow qg$ channel



each diagram generates a different gluon distribution

2 unintegrated gluon distributions per channel ($i=1,2$): $\Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2)$

$$qg^* \rightarrow qg \quad gg^* \rightarrow q\bar{q} \quad gg^* \rightarrow gg$$

The six TMD gluon distributions

- correspond to a different gauge-link structure

$$F_{g/A}(x_2, k_t) = 2 \frac{\int d\zeta^{\pm} d^2\zeta_{\perp} e^{ix_2 p_A^{\pm} \zeta^{\pm} - ik_t \cdot \zeta_{\perp}} \text{Tr} \left[F^{i-}(\zeta^{\pm}, \zeta_{\perp}) U_{[\zeta^{\pm}, 0]} F^{i-}(0) \right]}{(2\pi)^3 p_A^{\pm}}$$

several paths are possible for the gauge links

examples :



- when integrated, they all coincide

$$\int^{\mu^2} d^2 k_t \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2) = x_2 f(x_2, \mu^2)$$

- they are independent and in general they all should be extracted from data

only one of them has the probabilistic interpretation of the number density of gluons at small x_2

TMDs from the CGC

- the gluon TMDs involved in the di-jet process are:

(showing here the $qg^* \rightarrow qg$ channel TMDs only)

$$\mathcal{F}_{qg}^{(1)} = 2 \int \frac{d\xi^+ d^2 \boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \boldsymbol{\xi}} \left\langle \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(2)} = 2 \int \frac{d\xi^+ d^2 \boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \boldsymbol{\xi}} \left\langle \text{Tr} \left[F^{i-}(\xi) \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

- at small \mathbf{x} they can be written as: $U_{\mathbf{x}} = \mathcal{P} \exp \left[ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$

$$\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2 x d^2 y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \text{Tr} [(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)] \right\rangle_{x_2}$$

$$\mathcal{F}_{qg}^{(2)}(x_2, |k_t|) = -\frac{4}{g^2} \int \frac{d^2 x d^2 y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \frac{1}{N_c} \left\langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \text{Tr} [U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \right\rangle_{x_2}$$

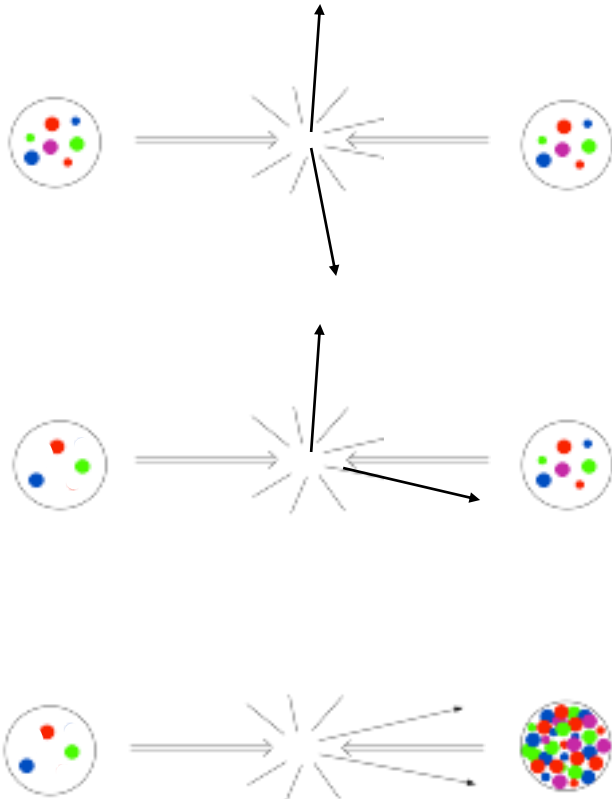
these Wilson line correlators also emerge directly of the CGC formulae

Di-jet final-state kinematics

final state : k_1, y_1 k_2, y_2

$$x_p = \frac{k_1 e^{y_1} + k_2 e^{y_2}}{\sqrt{s}} \quad x_A = \frac{k_1 e^{-y_1} + k_2 e^{-y_2}}{\sqrt{s}}$$

scanning the wave functions:



$$x_p \sim x_A < 1$$

central rapidities probe moderate x

$$x_p \text{ increases} \quad x_A \sim \text{unchanged}$$

$$x_p \sim 1, x_A < 1$$

forward/central doesn't probe much smaller x

$$x_p \sim \text{unchanged} \quad x_A \text{ decreases}$$

$$x_p \sim 1, x_A \ll 1$$

forward rapidities probe small x

Outline of the derivation

- using $\langle p|p' \rangle = (2\pi)^3 2p^- \delta(p^- - p'^-) \delta^{(2)}(p_t - p'_t)$ and translational invariance

$$\int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A|O(0, \xi)|A \rangle = \frac{2}{\langle A|A \rangle} \int \frac{d^3\xi d^3\xi'}{(2\pi)^3} e^{ix_2 p_A^- (\xi^+ - \xi'^+) - ik_t \cdot (\xi - \xi')} \langle A|O(\xi', \xi)|A \rangle .$$

- setting $\exp[ix_2 p_A^- (\xi^+ - \xi'^+)] = 1$ and denoting $\frac{\langle A|O(\xi', \xi)|A \rangle}{\langle A|A \rangle} = \langle O(\xi', \xi) \rangle_{x_2}$

we obtain e.g.

$$\mathcal{F}_{qg}^{(1)}(x_2, k_t) = 4 \int \frac{d^3x d^3y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \text{Tr} \left[F^{i-}(x) \mathcal{U}^{[-]\dagger} F^{i-}(y) \mathcal{U}^{[+]} \right] \right\rangle_{x_2}$$

- then performing the \mathbf{x} and \mathbf{y} integrations using

$$\partial_i U_{\mathbf{y}} = ig \int_{-\infty}^{\infty} dy^+ U[-\infty, y^+; \mathbf{y}] F^{i-}(y) U[y^+, +\infty; \mathbf{y}]$$

we finally get $\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \text{Tr} \left[(\partial_i U_{\mathbf{y}}) (\partial_i U_{\mathbf{x}}^\dagger) \right] \right\rangle_{x_2}$

The other (unpolarized) TMDs

- involved in the $gg^* \rightarrow q\bar{q}$ and $gg^* \rightarrow gg$ channels

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \frac{1}{N_c} \langle A | \text{Tr} [F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[+]}] \text{Tr} [\mathcal{U}^{[\square]\dagger}] | A \rangle ,$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \frac{1}{N_c} \langle A | \text{Tr} [F^{i-}(\xi) \mathcal{U}_\xi^{[\square]\dagger}] \text{Tr} [F^{i-}(0) \mathcal{U}_0^{[\square]}] | A \rangle ,$$

$$\mathcal{F}_{gg}^{(3)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A | \text{Tr} [F^{i-}(\xi) \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]}] | A \rangle ,$$

$$\mathcal{F}_{gg}^{(4)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A | \text{Tr} [F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[-]}] | A \rangle ,$$

$$\mathcal{F}_{gg}^{(5)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A | \text{Tr} [F^{i-}(\xi) \mathcal{U}_\xi^{[\square]\dagger} \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}_0^{[\square]} \mathcal{U}^{[+]}] | A \rangle ,$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \frac{1}{N_c^2} \langle A | \text{Tr} [F^{i-}(\xi) \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]}] \text{Tr} [\mathcal{U}^{[\square]}] \text{Tr} [\mathcal{U}^{[\square]\dagger}] | A \rangle .$$

The other TMDs at small-x

- involved in the $gg^* \rightarrow q\bar{q}$ and $gg^* \rightarrow gg$ channels

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t) = \frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \langle \text{Tr} [(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)] \text{Tr} [U_{\mathbf{x}} U_{\mathbf{y}}^\dagger] \rangle_{x_2} ,$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger] \text{Tr} [(\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \rangle_{x_2} ,$$

$$\mathcal{F}_{gg}^{(4)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{x}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{y}}^\dagger] \rangle_{x_2} ,$$

$$\mathcal{F}_{gg}^{(5)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger U_{\mathbf{x}} U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \rangle_{x_2} ,$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c^2} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \text{Tr} [U_{\mathbf{x}} U_{\mathbf{y}}^\dagger] \text{Tr} [U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \rangle_{x_2} .$$

with a special one singled out: the Weizsäcker-Williams TMD

$$\mathcal{F}_{gg}^{(3)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \rangle_{x_2}$$

x evolution of CGC correlators

the evolution of the gluon TMDs with decreasing x can be computed from the so-called JIMWLK equation

$$\frac{d}{d \ln(1/x_2)} \langle O \rangle_{x_2} = \langle H_{JIMWLK} O \rangle_{x_2}$$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

a functional RG equation that resums the leading logarithms in

$$y = \ln(1/x_2)$$

- the JIMWLK “Hamiltonian” reads:

$$H_{JIMWLK} = \frac{d}{d \log(1/x_2)} = \int \frac{d^2 \mathbf{x}}{2\pi} \frac{d^2 \mathbf{y}}{2\pi} \frac{d^2 \mathbf{z}}{2\pi} \frac{(\mathbf{x}-\mathbf{z}) \cdot (\mathbf{y}-\mathbf{z})}{(\mathbf{x}-\mathbf{z})^2 (\mathbf{z}-\mathbf{y})^2} \frac{\delta}{\delta A_c^-(\mathbf{x})} [1 + V_{\mathbf{x}}^\dagger V_{\mathbf{y}} - V_{\mathbf{x}}^\dagger V_{\mathbf{z}} - V_{\mathbf{z}}^\dagger V_{\mathbf{y}}]^{cd} \frac{\delta}{\delta A_d^-(\mathbf{y})}$$

with the adjoint Wilson line $V_{\mathbf{x}} = \mathcal{P} \exp \left[ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) T^a \right]$

Evolution of the "dipole" TMD

(in a mean-field type approximation)

- the Balitsky-Kovchegov (BK) evolution

Balitsky (1996), Kovchegov (1998)

$$\frac{d}{dY} f_Y(k) = \bar{\alpha} \int \frac{dk'^2}{k'^2} \left[\frac{k'^2 f_Y(k') - k^2 f_Y(k)}{|k^2 - k'^2|} + \frac{k^2 f_Y(k)}{\sqrt{4k'^4 + k^4}} \right] - \bar{\alpha} f_Y^2(k)$$

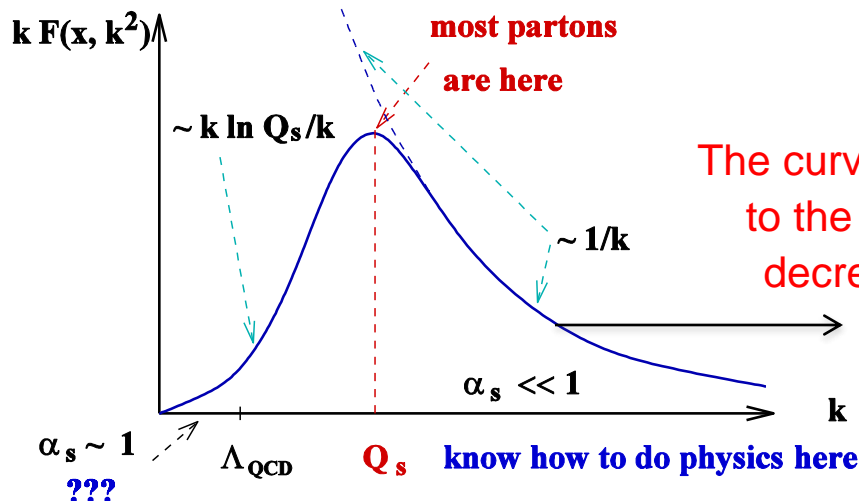
$$Y = \ln\left(\frac{1}{x}\right)$$

$$\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

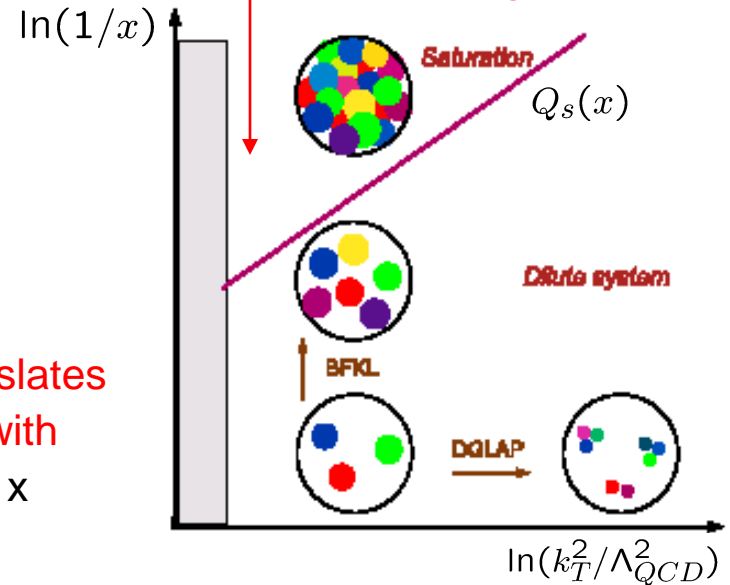
BFKL

non-linearity important when the gluon density becomes large

- solutions: qualitative behavior



The curve translates to the right with decreasing x



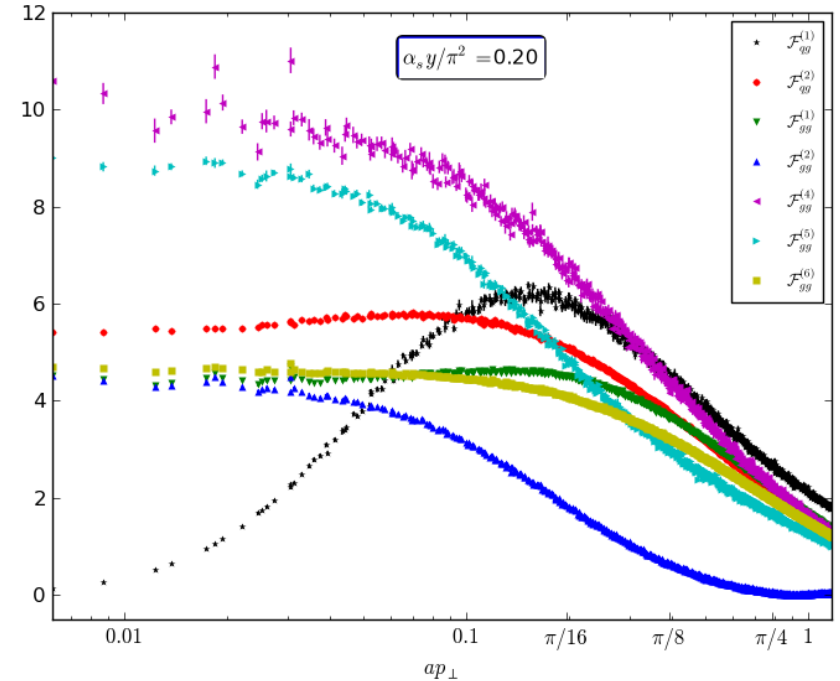
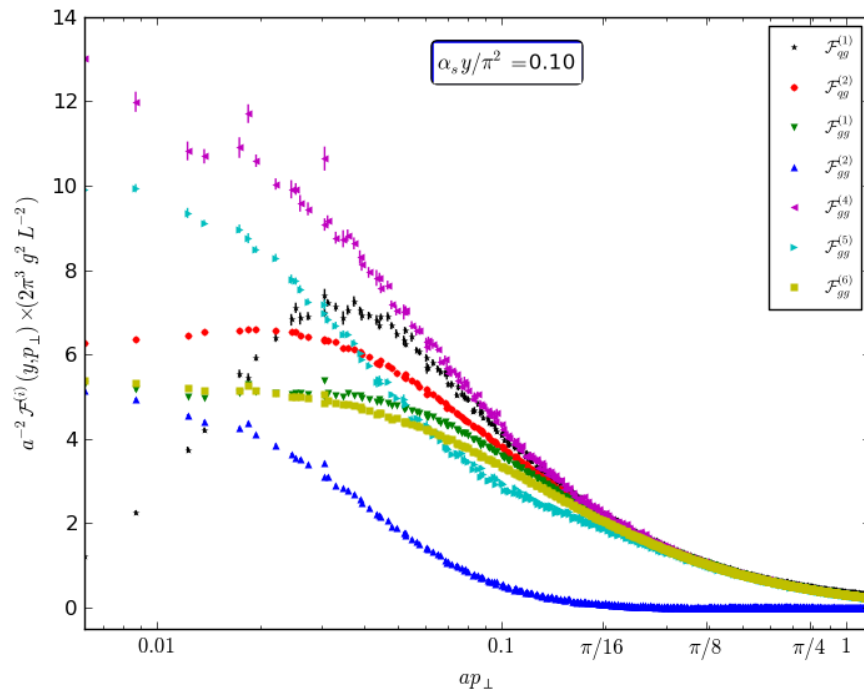
the distribution of partons as a function of x and k_T

JIMWLK numerical results

using a code written by Claude Roiesnel

initial condition at $y=0$: MV model
evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)



saturation effects impact the various gluon TMDs in very different ways

Phenomenology in p+Pb collisions at the LHC

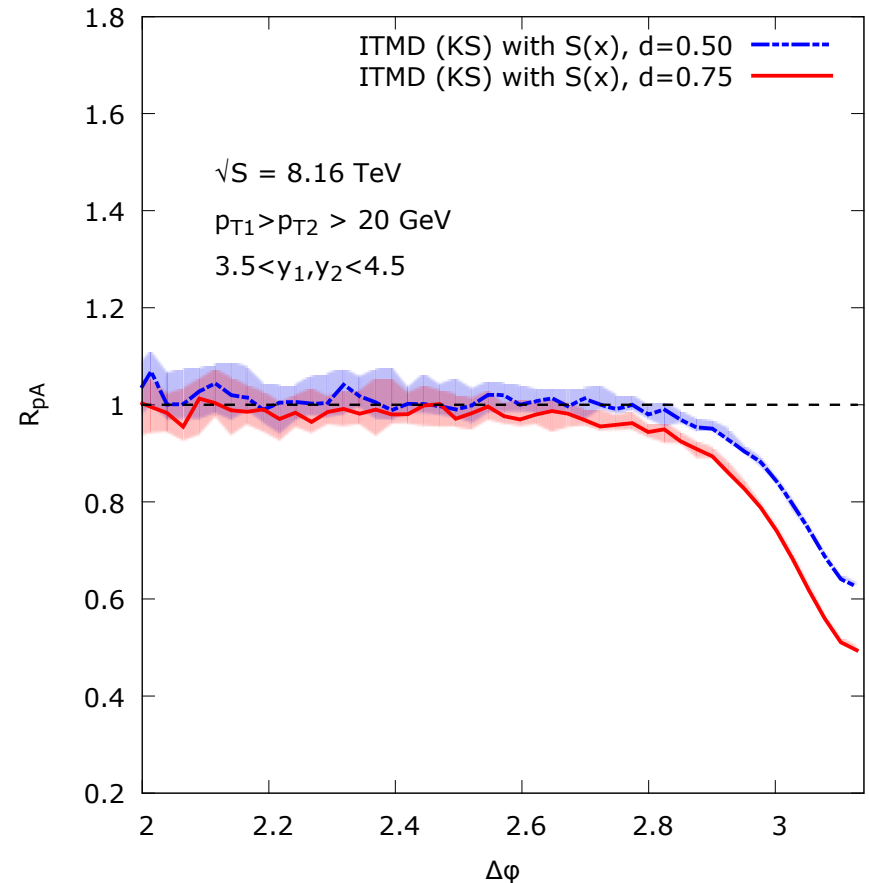
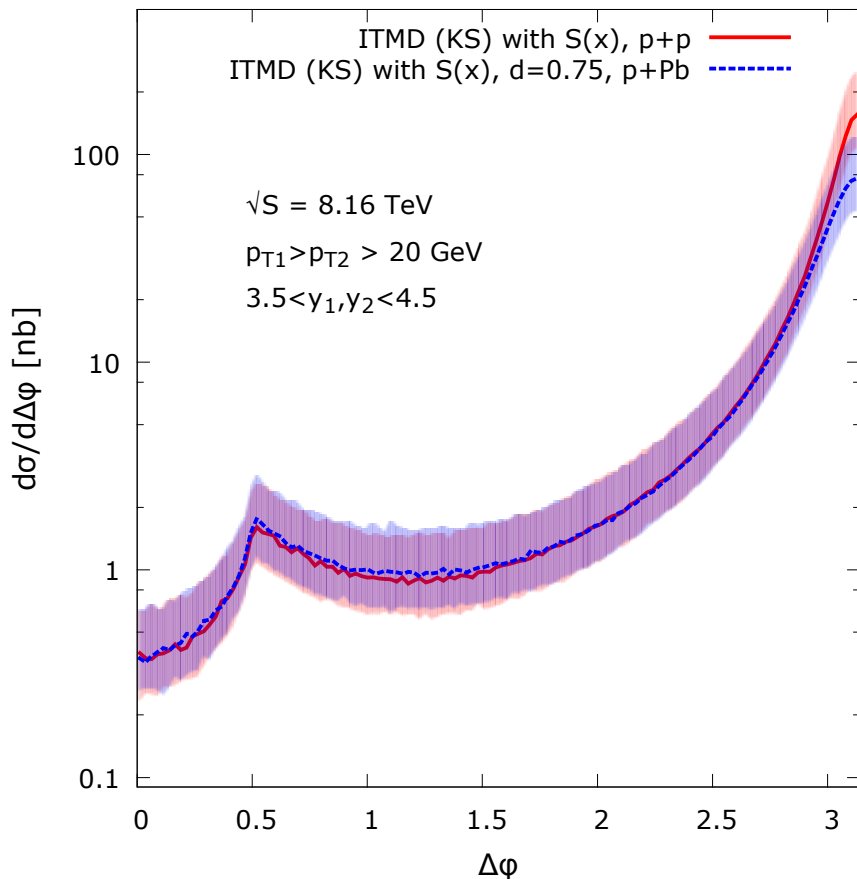
A. van Hameren, P. Kotko, K. Kutak, CM and S. Sapeta
Phys.Rev. D89 (2014) 9, 094014, arXiv:1402.5065

A. van Hameren, P. Kotko, K. Kutak, CM, E. Petreska and S. Sapeta
JHEP 12 (2016) 034, arXiv:1607.03121

Transverse-momentum imbalance

- with a free parameter to vary the nuclear saturation scale

$$Q_{sA}^2 = d A^{1/3} Q_{sp}^2$$

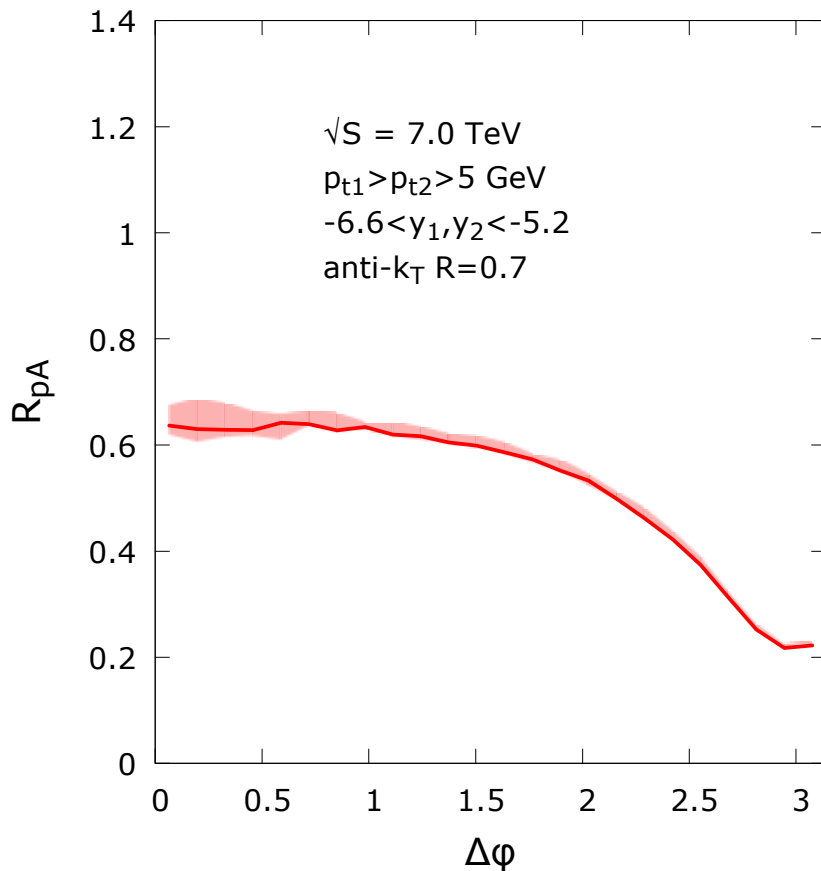


non-linear effects become sizeable near $\Delta\phi = \pi$, as expected

R_{pA} of forward-forward di-jets

- strong nuclear modification predicted

due to strong non-linear effects



the kinematics for this plot are chosen assuming that the di-jets are detected in an upgrade of the CASTOR detector

for detection in the central part of CMS (i.e. $3 < y < 5$), R_{pA} goes from 1 to 0.6 with increasing $\Delta\phi$

- prospects for a measurement

CMS: analysis started but now on hold
ATLAS: expressed interest to do it
ALICE: forward jets seems impossible
LHC-b: forward jets possible?

Conclusions

- for forward di-jet production, TMD factorization and CGC calculations are consistent with each other in the overlapping domain of validity

small x and leading power of the hard scale

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

- saturation physics is relevant if the di-jet transverse momentum imbalance $|k_t|$ is of the order of the saturation scale Q_s

- at small- x , the "soft" factorization breaking is expected, understood, and is not a issue in saturation calculations:

the more appropriate description of the parton content in terms of classical fields allows to use information extracted from a process to predict another

- given an initial condition, all the gluon TMDs can be obtained at smaller values of x , from the JIMWLK equation

the scale dependence of the TMDs, which at small x boils down to Sudakov logarithms, can also be implemented (future work)