Double-parton scattering and Poisson statistics

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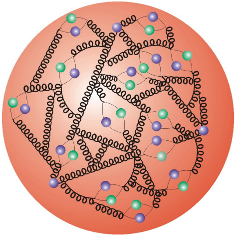
(supported in part by Polish National Science Center
grant no. 2015/19/B/ST2/00989)

QCD challenges in pp, pA and AA collisions at high energies
27 February – 3 March 2017, ETC*, Trento, Italy

Contents

Double Parton Scattering



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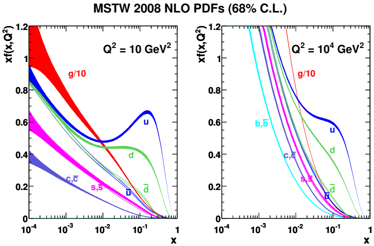
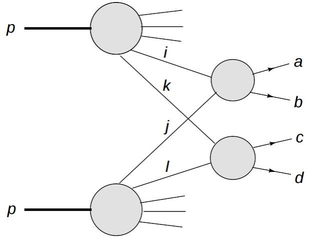




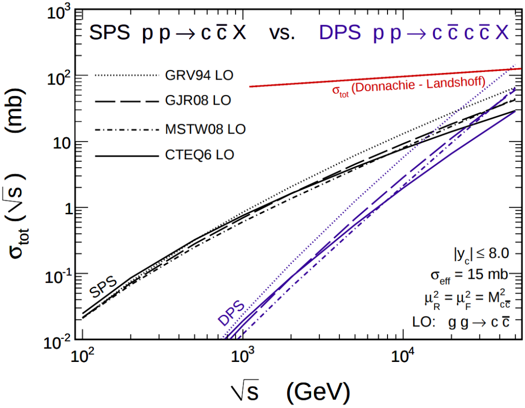
Factorized formula

$$\sigma_{\text{DPS}} = \frac{1}{2\sigma_{\text{eff}}}\sigma_{\text{SPS}}^2$$

- Well known fact: neglects correlations between partons
- Argued in this presentation: applies only to processes with small cross sections

Motivation: DPS Charm

Famous result by Łuszczak, Maciuła, Szczurek: Phys. Rev. D79, 094034 (2012)



Puzzle

$c\bar{c}c\bar{c}X$ is a subset of $c\bar{c}X \implies \sigma_{\text{SPS}} \geq \sigma_{\text{DPS}}$

Solution

proper interpretation of inclusive cross section + Poisson statistics

Double-parton scattering and Poisson statistics

Rafał Staszewski

Introduction

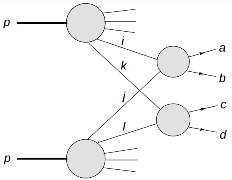
Toy model

Realistic model

Summary and conclusions

Definitions

- **process**: *e.g.* $pp \to c\bar{c}X$
- **subprocess**: *e.g.* $gg \to c\bar{c}$



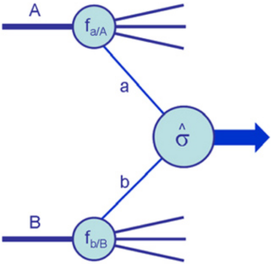
- **inclusive SPS**: process containing **at least one** subprocess
- **exclusive SPS**: process containing **exactly one** subprocess
- **inclusive DPS**: process containing **at least two** subprocesses
- **exclusive DPS**: process containing **exactly two** subprocesses
- and so on (TPS, QPS, ...)

Cross sections for all processes must, **by definition**, be smaller than the total inelastic cross section.

Inclusive cross section

Inclusive cross section:

$$\sigma_{\text{inc}} = \int f(x_1, \mu^2) f(x_2, \mu^2) \hat{\sigma}(x_1, x_2, \mu^2)$$



- σ_{inc} is the cross section for the **subprocess**
- Processes containing several subprocesses are "counted" several times

$$\sigma_{\text{inc}} = \sigma_{\text{excSPS}} + 2\sigma_{\text{excDPS}} + 3\sigma_{\text{excTPS}} + 4\sigma_{\text{excQPS}} + \ldots$$

- Inclusive cross section may exceed total inelastic cross section

Contents

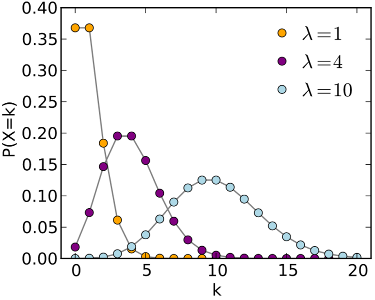
Poisson statistics

- Average number of subprocesses per process:

$$\sigma_{\text{inc}} = \bar{n}\sigma_{\text{inel}}.$$

- Poisson distribution

$$P\left(n\right) = e^{-\bar{n}}\frac{\bar{n}^n}{n!}$$



- exclusive SPS

$$\sigma_{\text{excSPS}} = P(n=1)\cdot\sigma_{\text{inel}}$$

- exclusive DPS

$$\sigma_{\text{incDPS}} = P(n=2)\cdot\sigma_{\text{inel}}$$

- inclusive SPS

$$\sigma_{\text{incSPS}} = P(n\geq 1)\cdot\sigma_{\text{inel}}$$

- inclusive DPS

$$\sigma_{\text{excDPS}} = P(n\geq 2)\cdot\sigma_{\text{inel}}$$

Small cross section limits

- Example calculation for inclusive SPS

$$\sigma_{\text{incSPS}} = P(n \geq 1) \cdot \sigma_{\text{inel}} = [1 - P(0)] \cdot \sigma_{\text{inel}} = (1 - e^{-\bar{n}})\sigma_{\text{inel}}$$

$$\sigma_{\text{incSPS}} \xrightarrow{\bar{n}\to 0} \bar{n}\sigma_{\text{inel}} = \sigma_{\text{inc}}$$

- At $\bar{n} \to 0$:

$$\sigma_{\text{incSPS}} = \sigma_{\text{excSPS}} = \sigma_{\text{inc}}$$

- Example calculation for inclusive SPS

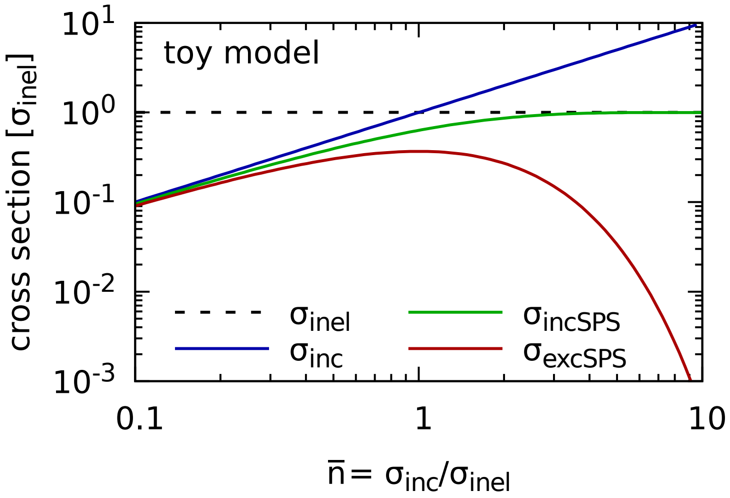
$$\sigma_{\text{excDPS}} = P(n = 2) \cdot \sigma_{\text{inel}} = \frac{1}{2} e^{-\bar{n}} \bar{n}^2 \sigma_{\text{inel}} \xrightarrow{\bar{n}\to 0} \frac{1}{2\sigma_{\text{inel}}} \sigma_{\text{inc}}^2$$

- At $\bar{n} \to 0$:

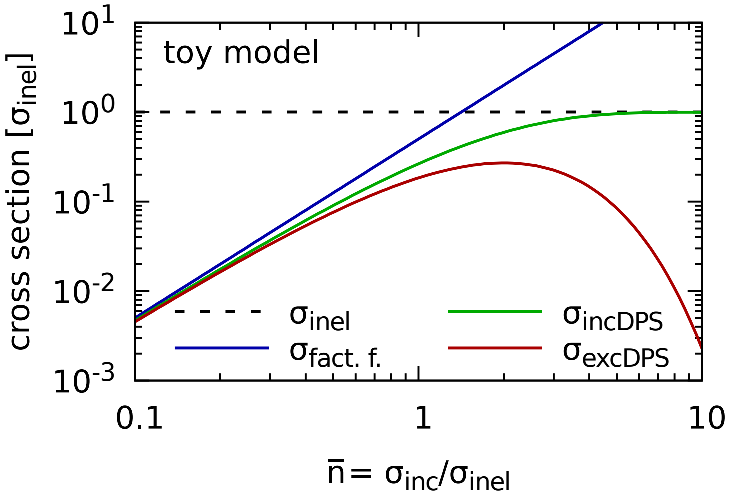
$$\sigma_{\text{incDPS}} = \sigma_{\text{excDPS}} = \frac{1}{2\sigma_{\text{eff}}} \sigma_{\text{SPS}}^2$$

with $\sigma_{\text{eff}} = \sigma_{\text{inel}}$.

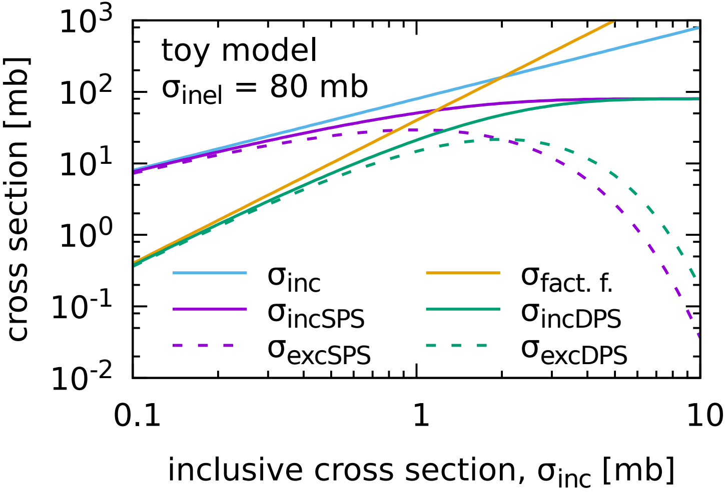
Results


toy model
cross section [σinel]
10^1
10^0
10^-1
10^-2
10^-3
0.1
1
10
n̄ = σinc/σinel
σinel
σinc
σincSPS
σexcSPS

Results


toy model
cross section [σ_{inel}]
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0.1
1
10
$\bar{n} = \sigma_{inc}/\sigma_{inel}$
σ_{inel}
$\sigma_{fact.\ f.}$
σ_{incDPS}
σ_{excDPS}

Results


toy model
σinel = 80 mb
cross section [mb]
inclusive cross section, σinc [mb]
σinc
σincSPS
σexcSPS
σfact. f.
σincDPS
σexcDPS
10^3
10^2
10^1
10^0
10^-1
10^-2
0.1
1
10

Contents

Impact parameter dependence

- Average number of interactions

$$\bar{n} \to \bar{n}(b)$$

- Inclusive cross section

$$\sigma_{\text{inc}} = \int \bar{n}(b) d^2\boldsymbol{b}.$$

- Probability

$$P(n) \to P(n;b) = e^{-\bar{n}(b)} \frac{(\bar{n})^n}{n!}.$$

- Cross sections for various processes

$$\sigma_{\text{incSPS}} = \int P(n > 0; b)\, d^2\boldsymbol{b},$$

Similar in spirit to what is done for MPI modeling in MC event generators, *e.g.* T. Sjostrand and M. van Zijl, Phys. Rev. D **36**, 2019 (1987).

Overlap function

Since

$$\sigma_{\text{inc}} = \int \bar{n}(b) d^2\boldsymbol{b}.$$

it is possible to define the **overlap function** $F(b)$ such that

$$\bar{n}(b) = \sigma_{\text{inc}} F(b)$$

$F(b)$ is normalised to unity:

$$\int F(b) d^2\boldsymbol{b} = 1.$$

A practical (but not necessary for the model) assumption is the universality of $F(b)$.

Limit of $\sigma_{\text{inc}} \to 0$

In the limit of $\sigma_{\text{inc}} \to 0$

$$\sigma_{\text{inc}} = \sigma_{\text{incSPS}} = \sigma_{\text{excSPS}}$$

$$\sigma_{\text{incDPS}} = \sigma_{\text{excDPS}} = \frac{1}{2}\sigma_{\text{inc}}^2 \int F^2(b) d^2\boldsymbol{b}.$$

In this limit the factorised formula

$$\sigma_{\text{DPS}} = \frac{1}{2\sigma_{\text{eff}}}\sigma_{\text{SPS}}^2$$

is recovered with effective cross section given by $F^2(b)$:

$$\frac{1}{\sigma_{\text{eff}}} = \int F^2(b) d^2\boldsymbol{b}$$

Overlap function

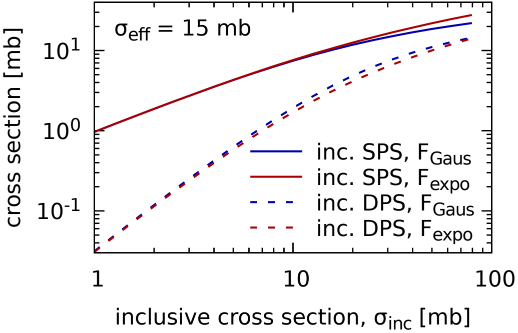
- Gaussian form

$$F_{\text{Gaus}}(b) = \frac{2}{\sigma_{\text{eff}}} \exp\left(-\frac{2\pi b^2}{\sigma_{\text{eff}}}\right)$$

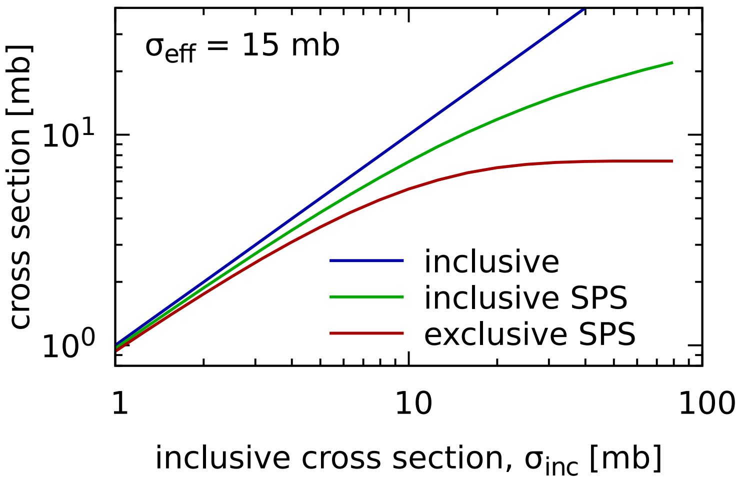
- Exponential form

$$F_{\text{expo}}(b) = \frac{4}{\sigma_{\text{eff}}} \exp\left(-b\sqrt{\frac{8\pi}{\sigma_{\text{eff}}}}\right)$$

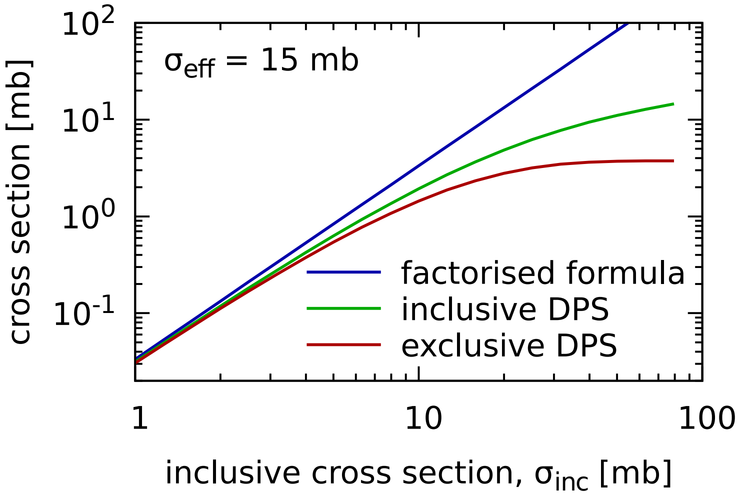
- Parameters chosen to reproduce σ_{eff}



Results


σeff = 15 mb
cross section [mb]
10^1
10^0
1
10
100
inclusive cross section, σinc [mb]
inclusive
inclusive SPS
exclusive SPS

Results


σ_{eff} = 15 mb
cross section [mb]
10^2
10^1
10^0
10^{-1}
1
10
100
inclusive cross section, σ_{inc} [mb]
factorised formula
inclusive DPS
exclusive DPS

Contents

Summary and conclusions

- For processes with cross sections comparable to total cross sections proper statistical treatment is important for calculations of DPS processes
- One needs to distinguish between inclusive and exclusive SPS, DPS, TPS, ...
- $\sigma_{\text{inc}} = \int f_1 f_2 \hat{\sigma}$ should be interpreted as cross section for a given subprocess and it can exceed total inelastic cross section
- Factorised formula for σ_{DPS} is valid only for processes with small cross sections
- The proposed formalism relies only on a proper counting of parton–parton processes, it does not introduce any new parameters