Seminar at ECT*, Trento, Italy, 2 March 2017

# The challenge of understanding QCD

Stanisław D. Głazek

*University of Warsaw and Yale University*

Precise description of processes that involve quarks and gluons requires a mathematically well-defined concept of a particle. The talk addresses this issue starting with the concept of quantum field operator. I describe the renormalization group procedure that makes the concept of effective particles sufficiently precise for studying the QCD Hamiltonian and finding out if the theory can explain the notions of quarks and gluons as constituents of hadrons.

References:

S. D. Głazek and A. P. Trawiński, Few-Body Syst. **58**, 49 (2017)
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stglazek@fuw.edu.pl

# Outline

Renormalization group procedure for effective particles (**RGPEP**)

helps in describing hadrons as bound states of their constituents

- quantum fields and the size of quanta
- asymptotic freedom, bound state equations, scattering

Insigts to be mentioned also concern:
ridge effect in pp collisions, proton radius in QED and CQM potential in AdS/QCD.

stglazek@fuw.edu.pl

# The concept of a quantum field operator

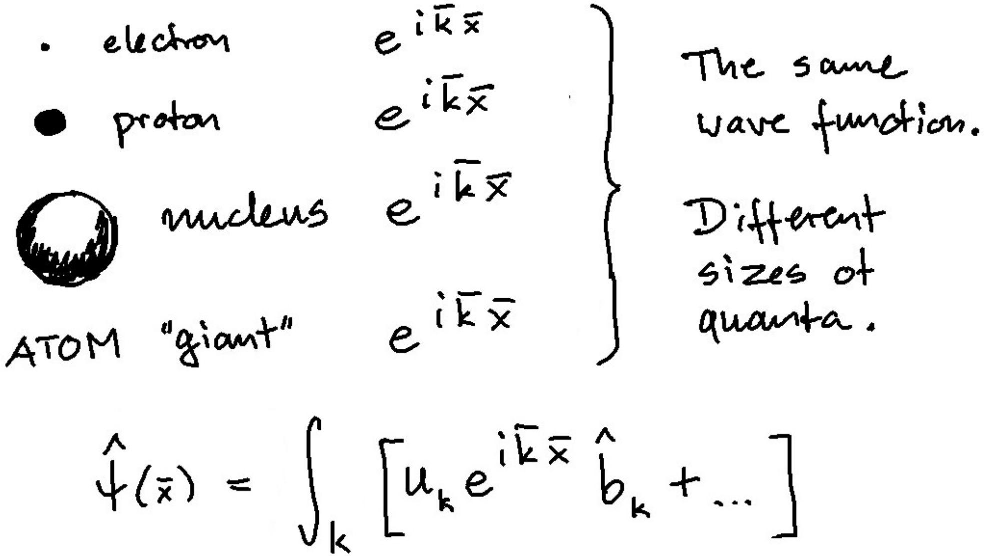


FIG. 1: **Quantum field operators are blind to the size of individual quanta.**

stglazek@fuw.edu.pl

# Illustration: Canonical quantization of the YM field

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\,F^{\mu\nu}F_{\mu\nu}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

$$\mathcal{T}^{\mu\nu} = -F^{a\mu\alpha}\partial^\nu A^a_\alpha + g^{\mu\nu}F^{a\alpha\beta}F^a_{\alpha\beta}/4$$

$$P^\nu = \int d\sigma_\mu\, \mathcal{T}^{\mu\nu}$$

$$\mathcal{H} = \mathcal{T}^{00}$$

.

$$\mathcal{H}_{YM} = \mathcal{H}_{A^2} + \mathcal{H}_{A^3} + \mathcal{H}_{A^4}$$

quantization $\quad \hat{A}^\mu = \sum_{\sigma c} \int_k \left[ t^c \, \varepsilon^\mu_{k\sigma} \, a_{k\sigma c} \, e^{-ikx} + t^c \, \varepsilon^{\mu *}_{k\sigma} \, a^\dagger_{k\sigma c} \, e^{ikx} \right]_{on \ \Sigma}$

$$\hat{H}_{YM} = \int_\Sigma \, : \mathcal{H}_{YM}(\hat{A}) :$$

stglazek@fuw.edu.pl

.

$$\mathcal{H}_{A^3} = g\ i\partial_\alpha A^a_\beta\ [A^\alpha, A^\beta]^a$$

stglazek@fuw.edu.pl

$$\hat{H}_{A^3} = \int_{x \in \Sigma} : g \; i\partial_\alpha \hat{A}^a_\beta(x) \; [\hat{A}^\alpha(x), \hat{A}^\beta(x)]^a :$$

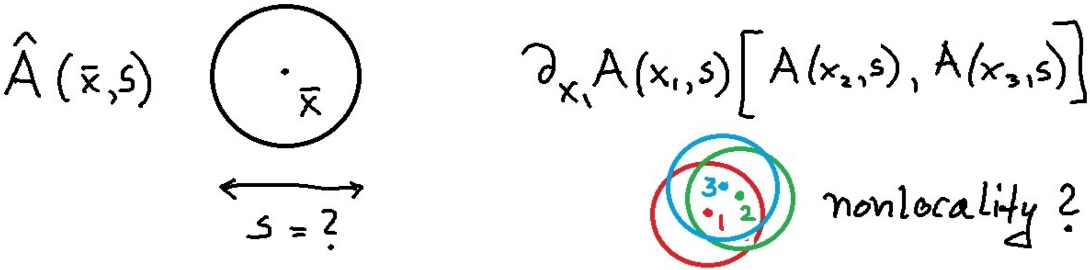


FIG. 2: Nonlocality is approximated by a local interaction for wavelengths greater than $s$. The particle size $s$ is the scale parameter in the RGPEP.

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$$\hat{H}_{A^3,s} = \int_{x_i \in \Sigma} g_s(x_1, x_2, x_3) \ : i\partial_\alpha \hat{A}^a_\beta(x_1, s) \ [\hat{A}^\alpha(x_2, s), \hat{A}^\beta(x_3, s)]^a :$$

$$g_s(x_1, x_2, x_3) = ?$$

SDG, K. G. Wilson, Phys. Rev. D **48**, 5863 (1993)

K. G. Wilson *et al.*, Phys. Rev. D **49**, 6720 (1994)

F. J. Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

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SDG, Acta Phys. Pol. B **43**, 1843 (2012)

M. Gomez-Rocha, SDG, Phys. Rev. D **92**, 065005 (2015)

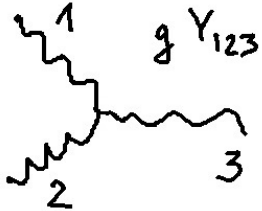
stglazek@fuw.edu.pl

# RGPEP initial condition at $s = 0$ for local QCD

$$\hat{H}_{A^3,s=0} = \int_{x \in \Sigma} : g \, i\partial_\alpha \hat{A}^a_\beta(x,0) \, [\hat{A}^\alpha(x,0), \hat{A}^\beta(x,0)]^a :$$

$$\hat{A}^\mu(x, s=0) = \sum_{\sigma c} \int_k \left[ t^c \, \varepsilon^\mu_{k\sigma} \, a_{k\sigma c, s=0} \, e^{-ikx} + t^c \, \varepsilon^{\mu *}_{k\sigma} \, a^\dagger_{k\sigma c, s=0} \, e^{ikx} \right]_{on \ \Sigma}$$

$$\hat{H}_{A^3,s=0} = \sum_{123} \int_{123} \delta_{12.3} \ \left[ g \, Y_{123} \ a^\dagger_{1,0} \, a^\dagger_{2,0} \, a_{3,0} + g \, Y^*_{123} \ a^\dagger_{3,0} \, a_{2,0} \, a_{1,0} \right]$$




stglazek@fuw.edu.pl

# Regularization

**Field** or **Interaction**

Kinematical symmetries of Hamiltonians in the Minkowski space-time

stglazek@fuw.edu.pl

# RGPEP

$$H_0(a_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, ..., i_n} c_0(i_1, ..., i_n)\ a^\dagger_{i_1,0} \cdots a_{i_n,0}$$

$$\boxed{c_0 = \text{the initial condition} = \text{canonical QCD with counterterms}}$$

$$H_s(a_s) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, ..., i_n} c_s(i_1, ..., i_n)\ a^\dagger_{i_1,s} \cdots a_{i_n,s}$$

$$\boxed{c_s = \ ?}$$

One of the functions $c_s$ is $g_s(i_1, i_2, i_3)$.

# RGPEP generator and non-perturbative QCD

$$H_s(a_s) = H_0(a_0)$$

$$a_s = U_s \, a_0 \, U_s^\dagger$$

$$H_s = H_f + H_I$$

$$H_s' = [G_s, H_s]$$

$$G_s = [H_f, \tilde{H}_s]$$

stglazek@fuw.edu.pl

$$\textbf{NP} \quad \textbf{QCD} \;\rightarrow\; H'_s = \left[[H_f, \tilde{H}_s], H_s\right]$$

$$\textbf{AF} \;\; \textbf{pQCD} \;\rightarrow\; H_s = H_f + gH_{1s} + g^2 H_{2s} + g^3 H_{3s} + ...$$

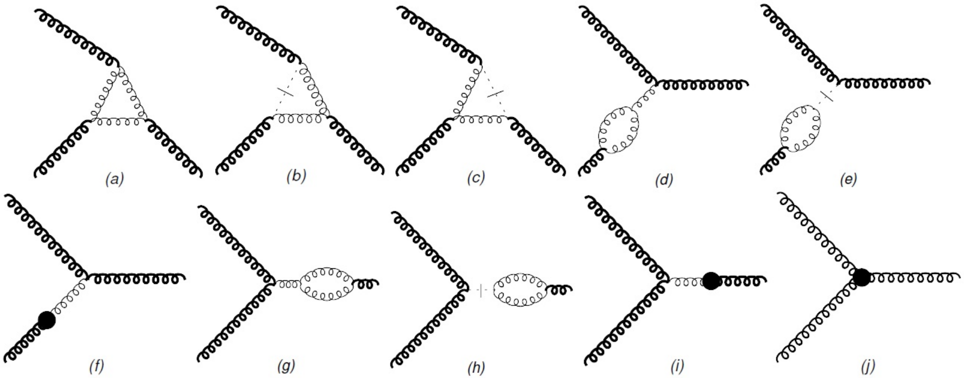
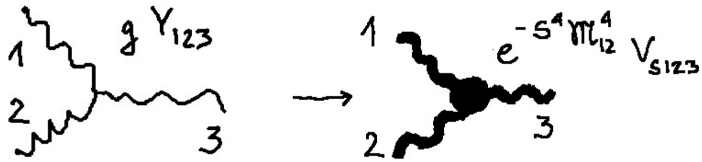



FIG. 3: Third-order contributions to the three-gluon vertex - these are not Feynman diagrams

$$H_{A^3\,(1+3)s} = \sum_{123} \int_{123} \delta_{12.3}\; e^{-s^4 \mathcal{M}_{12}^4} \left[ V_{s123}\, a_{1,s}^\dagger a_{2,s}^\dagger a_{3,s} + V_{s123}^*\, a_{3,s}^\dagger a_{2,s} a_{1,s} \right]$$




$$\lim_{\kappa^\perp \to 0} V_{s123} = g_s\, Y_{123} \qquad\qquad g_s = g_0 + \frac{g_0^3}{48\pi^2} N_c\, 11\, \ln \frac{s}{s_0}$$

RGPEP Hamiltonian $\beta(s)$ $\leftrightarrow$ Gross-Wilczek-Politzer $\beta(\lambda)$

Minkowskian $s$ $\leftrightarrow$ $1/\lambda$ Euclidean

three-dimensional length $\leftrightarrow$ four-dimensional length

M. Gomez-Rocha, SDG, Phys. Rev. D **92**, 065005 (2015)

stglazek@fuw.edu.pl

# Effective-particle picture of nucleons

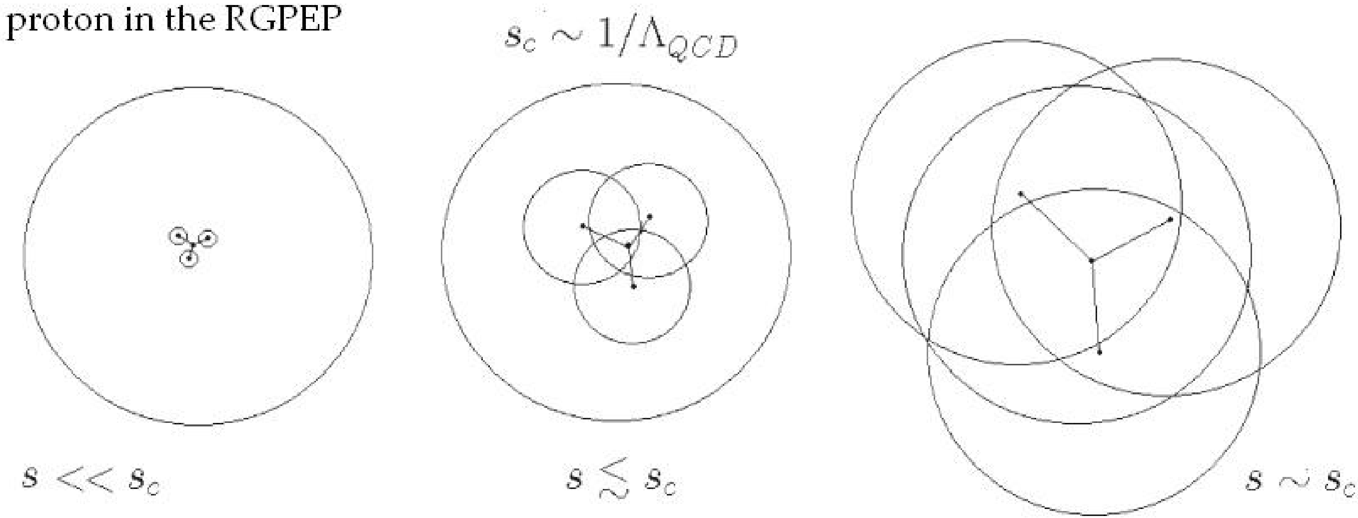



FIG. 4: The RGPEP scale-dependent proton picture

# Effective-particle color arrangement

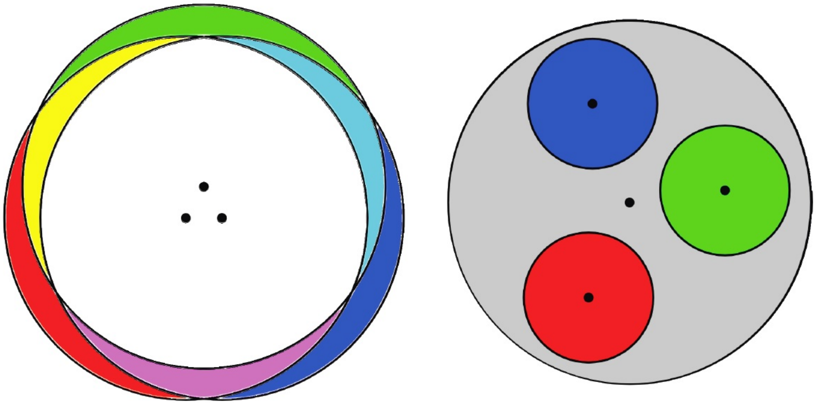
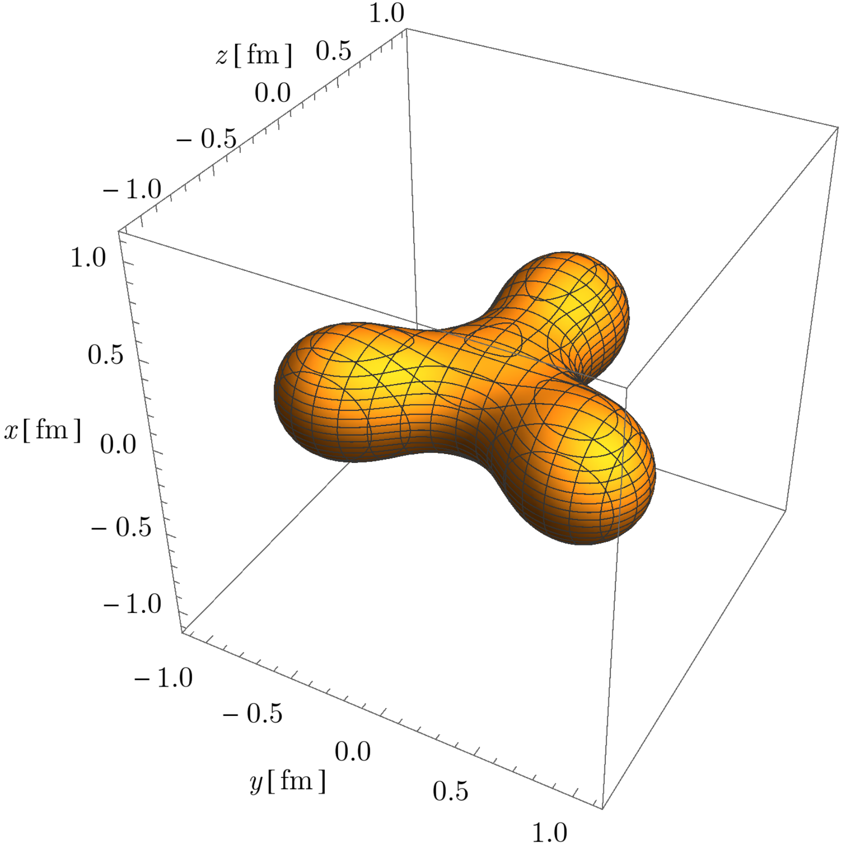


FIG. 5: Nucleon white interior with colored boundary coupling to mesons

**Ridge effect in *pp* collisions** from 7 TeV to 13 TeV

P. Kubiczek, SDG, Lith. J. Phys. **55**, 155 (2015) : YY vs. II

# YY



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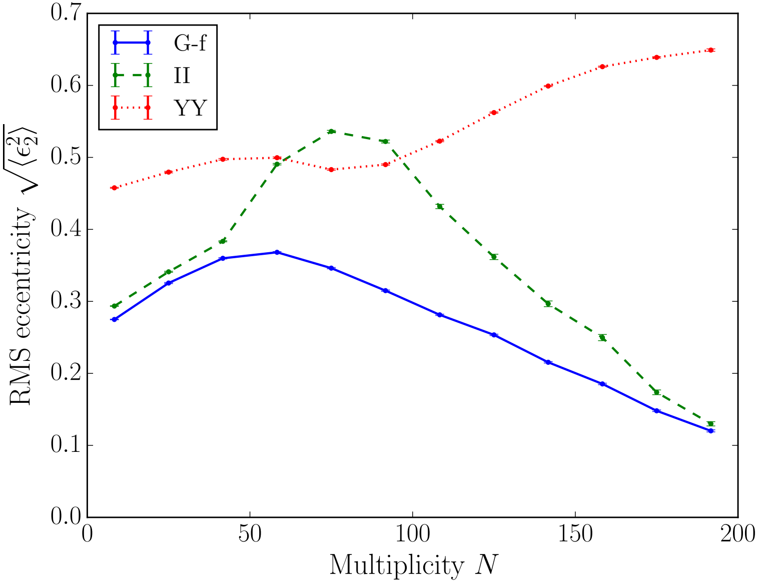
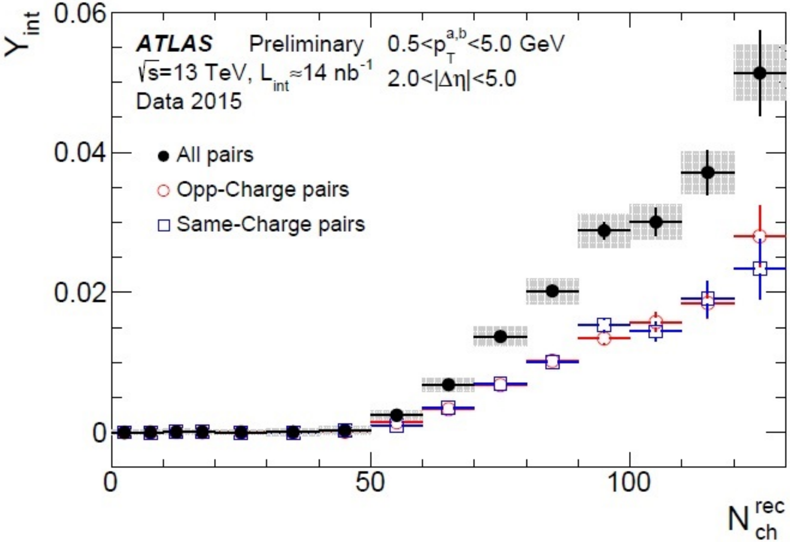



stglazek@fuw.edu.pl




**Miguel Arratia** for the ATLAS

**EPS HEP2015 talk in Vienna, 23 July 2015**

stglazek@fuw.edu.pl

# The issue of proton radius in QED

RGPEP derivation of the Schrödinger equation from QED:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi$$

| Family of effective Hamiltonians: $\hat{H}_{QEDs}$ |

$s = 0$ in canonical theory

| $\hat{H}_{QEDs}|\psi_s\rangle = E|\psi_s\rangle \quad \rightarrow \quad \hat{H}_{Sch} = ?$ |

How does the parameter $s$ enter in $\hat{H}_{Sch}$?

stglazek@fuw.edu.pl

# The RGPEP interpretation of atomic valence constituents

$$\begin{bmatrix} \cdots \\ \cdot\cdot \\ e\gamma e\bar{e}p\gamma \\ e\gamma p\gamma \\ e\gamma p \\ \boxed{ep} \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} \cdots \\ \cdot\cdot \\ e\gamma e\bar{e} \\ e\gamma\gamma \\ e\gamma \\ e \end{bmatrix} \times \begin{bmatrix} \cdots \\ \cdot\cdot \\ pe\bar{e} \\ p\gamma\gamma \\ p\gamma \\ p \end{bmatrix} + \ldots \quad \sim \quad |e_s p_s\rangle + \ldots$$

$$\psi_0(e_0, p_0) + \infty \quad \rightarrow \quad \psi_s(e_s, p_s) + corrections$$

SDG, Phys.Rev. D **90**, 045020 (2014).

stglazek@fuw.edu.pl

# "True" Schrödinger equation in QED $\hat{H}_{QEDs}|\psi_s\rangle = E|\psi_s\rangle$

$$\frac{\vec{p}^{\,2}}{2\mu}\,\psi_s(\vec{p}) + \int \frac{d^3k}{(2\pi)^3}\, V_s(\vec{p},\vec{k}\,)\,\psi_s(\vec{k}\,) = -E_B\,\psi_s(\vec{p})$$

$$V_s(\vec{p},\vec{k}\,) = e^{-s^4(\vec{p}^{\,2}-\vec{k}^{\,2})^2/c^4} \; \frac{-4\pi}{(\vec{p}-\vec{k}\,)^2} \; G_E(\vec{q}^{\,2}) \qquad \mathbf{4}\%$$

definition of relative electron-proton momentum $\vec{p}$ in atoms

matches the definition of quark momentum in **AdS/QCD** through

the Brodsky - de Teramond light-front holography

SDG, Acta Phys. Pol. B **42**, 1933 (2011)

$$c = \sqrt{m_l m_p}/(m_l + m_p)$$

# Jet production in pion-nucleus collisions

A. P. Trawiński, PhD Thesis, UW 2016

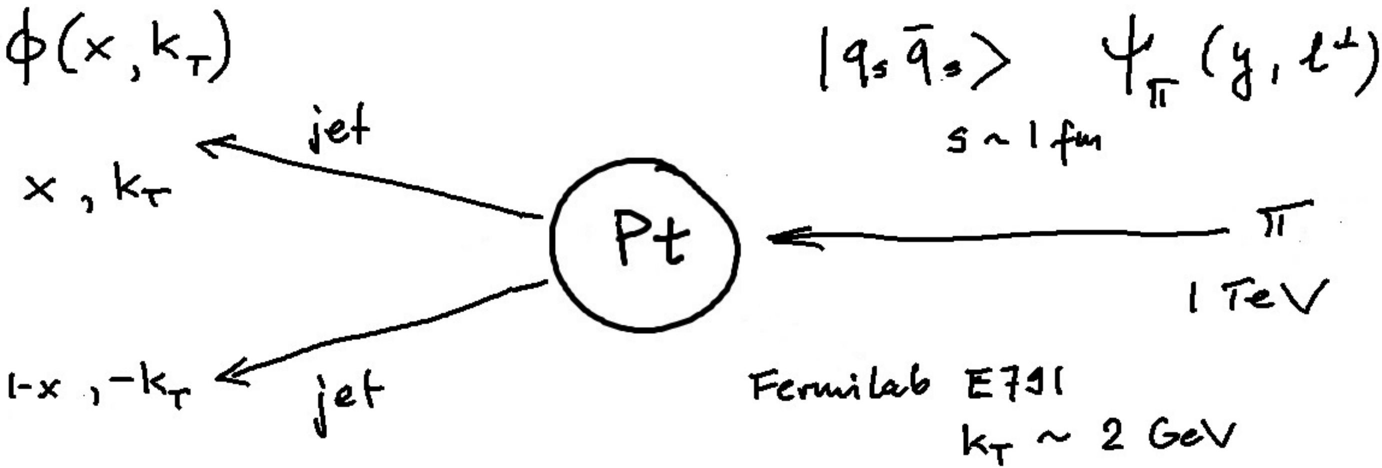


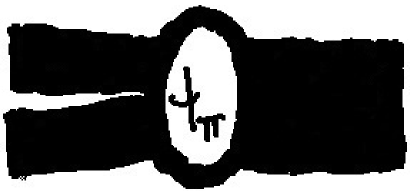

FIG. 6: Pion is cut into two quark jets by a gluon in a nucleus.

# The RGPEP pion with $U_{\rm eff}$ from AdS/QCD

the RGPEP scale $s_c \sim 1/\Lambda_{QCD}$

$q_{s_c}$

$\bar{q}_{s_c}$



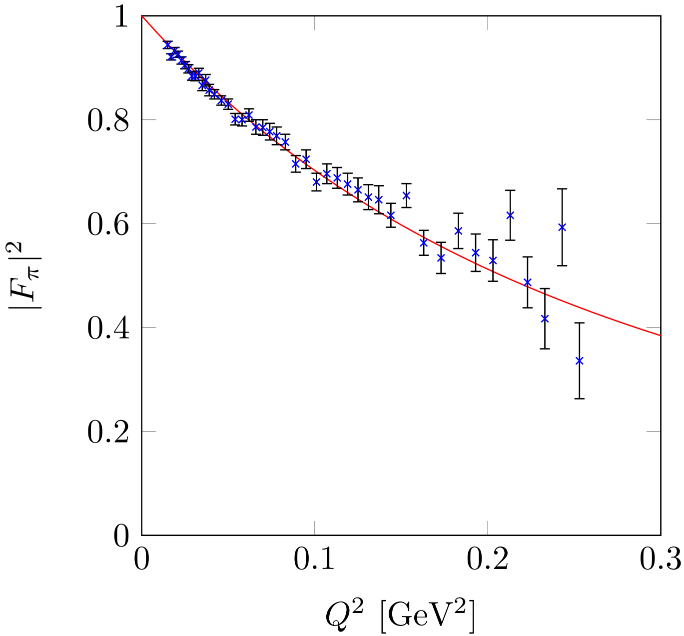

$\pi$

$$\psi_\pi \sim e^{-s_c^2 \mathcal{M}^2_{q\bar{q}}}$$

$$\mathcal{M}^2_{q\bar{q}} = \frac{k^{\perp 2} + m^2_{s_c}}{x(1-x)}$$

stglazek@fuw.edu.pl

# The RGPEP pion of $\sim 1$ fm quarks with $U_{\rm eff}$ from AdS/QCD




Data: A. R. Amendolia, Nucl. Phys. B **277**, 168 (1986)    Theory: A. P. Trawiński, PhD Thesis, UW 2016

# $|\pi\rangle = |q_{s_c}\bar{q}_{s_c}\rangle$ in terms of quarks and gluons with $1/s \sim 100$ GeV

$$q_s = U_s\, q_0\, U_s^\dagger$$

$$q_{s_1} = U_{s_1} U_{s_2}^\dagger\, q_{s_2}\, U_{s_2} U_{s_1}^\dagger$$

$$W = U_{s_1} U_{s_2}^\dagger$$

$$|\pi\rangle = |q_{s_c}\bar{q}_{s_c}\rangle = W\, |q_s \bar{q}_s\rangle$$

$$s \sim 1/100 \text{ GeV}$$

stglazek@fuw.edu.pl

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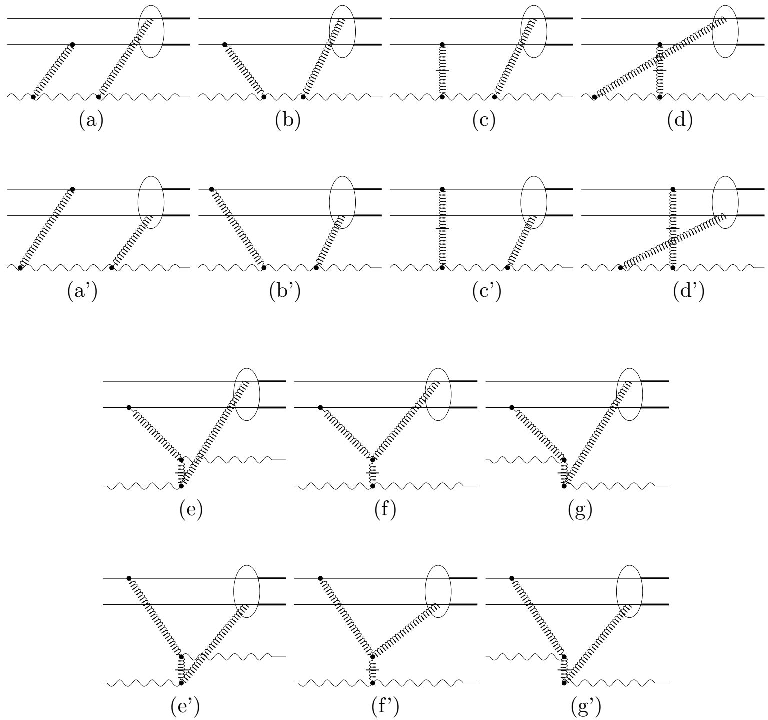


FIG. 7: Examples of pion splitting on gluons in a nucleus, described using $H_{QCDs}$ with $1/s \sim 100$ GeV.



stglazek@fuw.edu.pl

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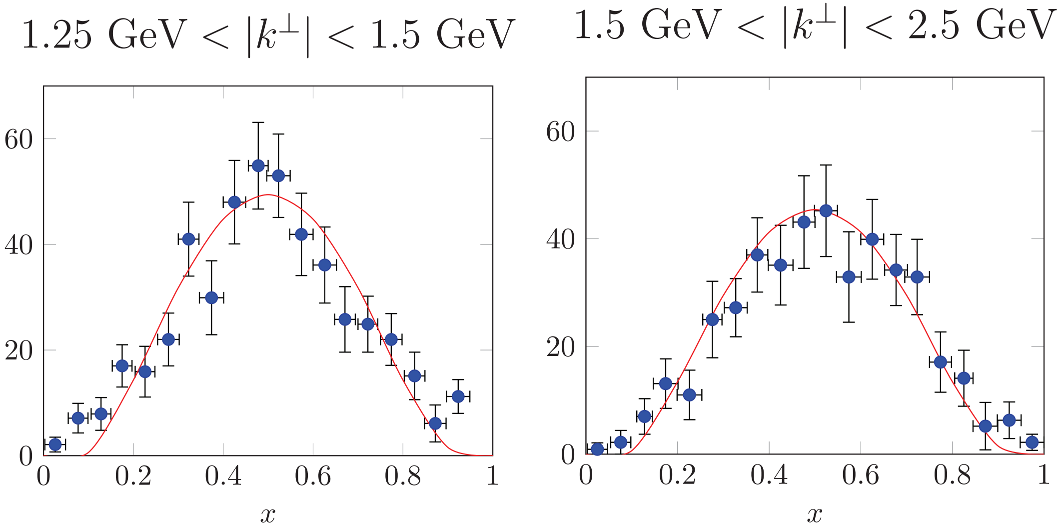


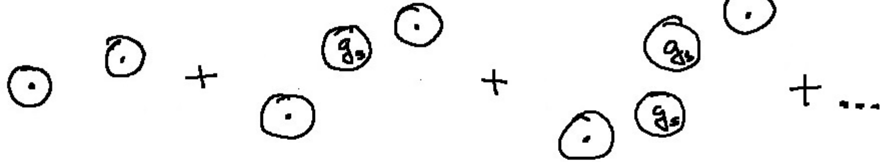

Jet counts distribution $\phi(x, k_T)$, for jets induced by pions impinging on Pt, in two jet-$k_T$ bins.

Data from E791, Phys. Rev. Lett. **86**, 4768 (2001); theory from A. P. Trawiński, PhD Thesis, UW 2016.

stglazek@fuw.edu.pl

# Example of a current challenge for $H_{QCDS}$

$$|\Upsilon\rangle = |Q_s \bar{Q}_s\rangle + |Q_s \bar{Q}_s g_s\rangle + |Q_s \bar{Q}_s g_s g_s\rangle + \ldots$$




What are these functions?

$$\psi(\bar{x}_1, \bar{x}_2), \ \psi(\bar{x}_1, \bar{x}_2, \bar{x}_3), \ \psi(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) \ \ldots$$

Maria Gómez-Rocha (ECT*), Kamil Serafin (Warsaw), Jai More (Mumbai)

→ QCD of decay, production and properties of heavy quarkonia

including their **gluon components**

stglazek@fuw.edu.pl

# Conclusion:

**Effective particle approach opens new options**

**for theoretical and phenomenological studies.**

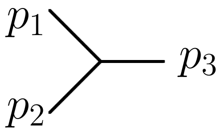
stglazek@fuw.edu.pl

# Regularization

example $\hat{H}_{A^3} \to \hat{H}_{A^3\,R}$

$$\hat{H}_{A^3} = \sum_{123} \int_{123} \delta_{12.3} \left[ g\, Y_{123}\, a_1^\dagger a_2^\dagger a_3 + g\, Y_{123}^*\, a_3^\dagger a_2 a_1 \right]$$

$$\hat{H}_{A^3\,R} = \sum_{123} \int_{123} \delta_{12.3}\ R \left[ g\, Y_{123}\, a_1^\dagger a_2^\dagger a_3 + g\, Y_{123}^*\, a_3^\dagger a_2 a_1 \right]$$




$$x_1 = p_1^+/p_3^+ = x \qquad k_1^\perp = p_1^\perp - x_1 p_3^\perp = \kappa^\perp$$

$$x_2 = p_2^+/p_3^+ = 1-x \qquad k_2^\perp = p_2^\perp - x_2 p_3^\perp = -\kappa^\perp$$

$$r_i = x_i^\delta \, e^{-k_i^{\perp 2}/\Delta^2} \qquad i = 1,2,3 \qquad x_3 = 1 \qquad k_3^\perp = 0^\perp$$

$$\boxed{R = \Pi_{i=1}^3 \, r_i}$$

$$Y_{123} = i f^{c_1 c_2 c_3} \left[ \varepsilon_1^* \varepsilon_2^* \cdot \varepsilon_3 \kappa - \varepsilon_1^* \varepsilon_3 \cdot \varepsilon_2^* \kappa \frac{1}{x_2} - \varepsilon_2^* \varepsilon_3 \cdot \varepsilon_1^* \kappa \frac{1}{x_1} \right]$$

stglazek@fuw.edu.pl

# Suggestion concerning glueballs

$$\begin{array}{ccccc} \text{bare gluons of size } 0 & & \text{gluons of size } s & & \\ \begin{bmatrix} \dots \\ |gggggg\rangle \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \end{bmatrix} & = & \begin{bmatrix} \dots \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \\ |g\rangle \end{bmatrix} & \otimes & \begin{bmatrix} \dots \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \\ |g\rangle \end{bmatrix} + \dots \end{array}$$

$$|gg\rangle + |ggg\rangle + \dots = |g_s g_s\rangle + |g_s g_s g_s\rangle + \dots$$

stglazek@fuw.edu.pl

# QCD bound states and QED Hydrogen atom analogy

$$V_c = -\frac{\alpha_{\rm atom}}{r} \quad \sim \alpha_{\rm atom} = \alpha(s_{\rm atom})$$

$$\psi(\vec{r}\,) \sim e^{-\alpha_{\rm atom}\mu|\vec{r}\,|} \qquad \tilde{\psi}(\vec{k}\,) \sim \frac{1}{[\vec{k}^{\,2} + \alpha^2_{\rm atom}\mu^2]^2}$$

$$\left[P^+ \hat{H}_{QED}(s_{atom}) - P^{\perp\,2}\right] |\psi_{\rm atom}\rangle = M^2_{\rm atom}|\psi_{\rm atom}\rangle$$

$H_{QCD} \neq H_{QED}$ in the RGPEP $\quad m_g(s) =? \quad \rightarrow \quad$ hadrons

**Eigenvalue problem ↔ Operators creating hadrons**

stglazek@fuw.edu.pl

$$\mathbf{NP} \;\rightarrow\; H_t' = \Big[[H_f, \tilde{H}_t], H_t\Big]$$

$$\mathbf{AF} \;\rightarrow\; H_t = H_f + gH_{1t} + g^2 H_{2t} + g^3 H_{3t} + \ldots$$

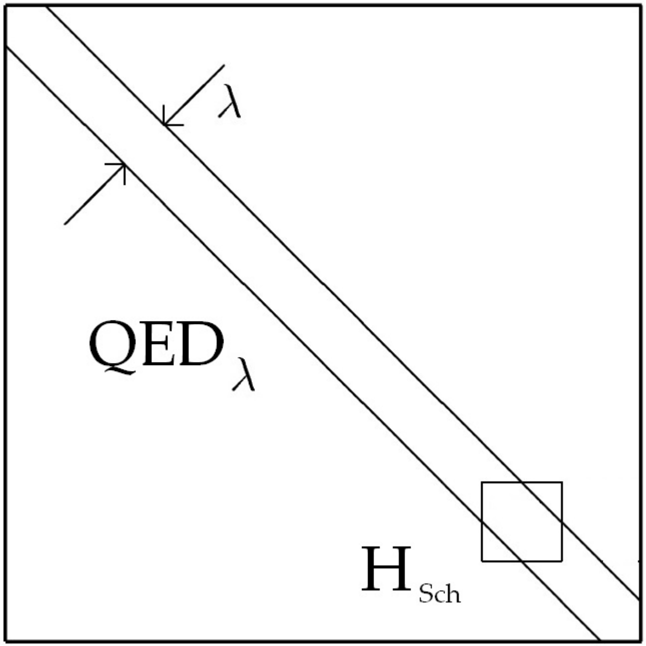
$$H_{1t}' = \Big[[H_f, \tilde{H}_{1t}], H_f\Big]$$

$$H_{1t\,mn}' = -(\mathcal{M}_m^2 - \mathcal{M}_n^2)^2 H_{10\,mn}$$

$$H_{1t} = f_t \; H_{10}$$

$$f_t = e^{-t(\mathcal{M}_c^2 - \mathcal{M}_a^2)^2}$$

$$H_{A^3\,1t} = \sum_{123} \int_{123} \delta_{12.3}\; e^{-t\,\mathcal{M}_{12}^4} \; \Big[ Y_{123}\, a_{t1}^\dagger a_{t2}^\dagger a_{t3} + Y_{123}^*\, a_{t3}^\dagger a_{t2} a_{t1} \Big]$$




stglazek@fuw.edu.pl