AdS/CFT and Hadron Formation in QCD

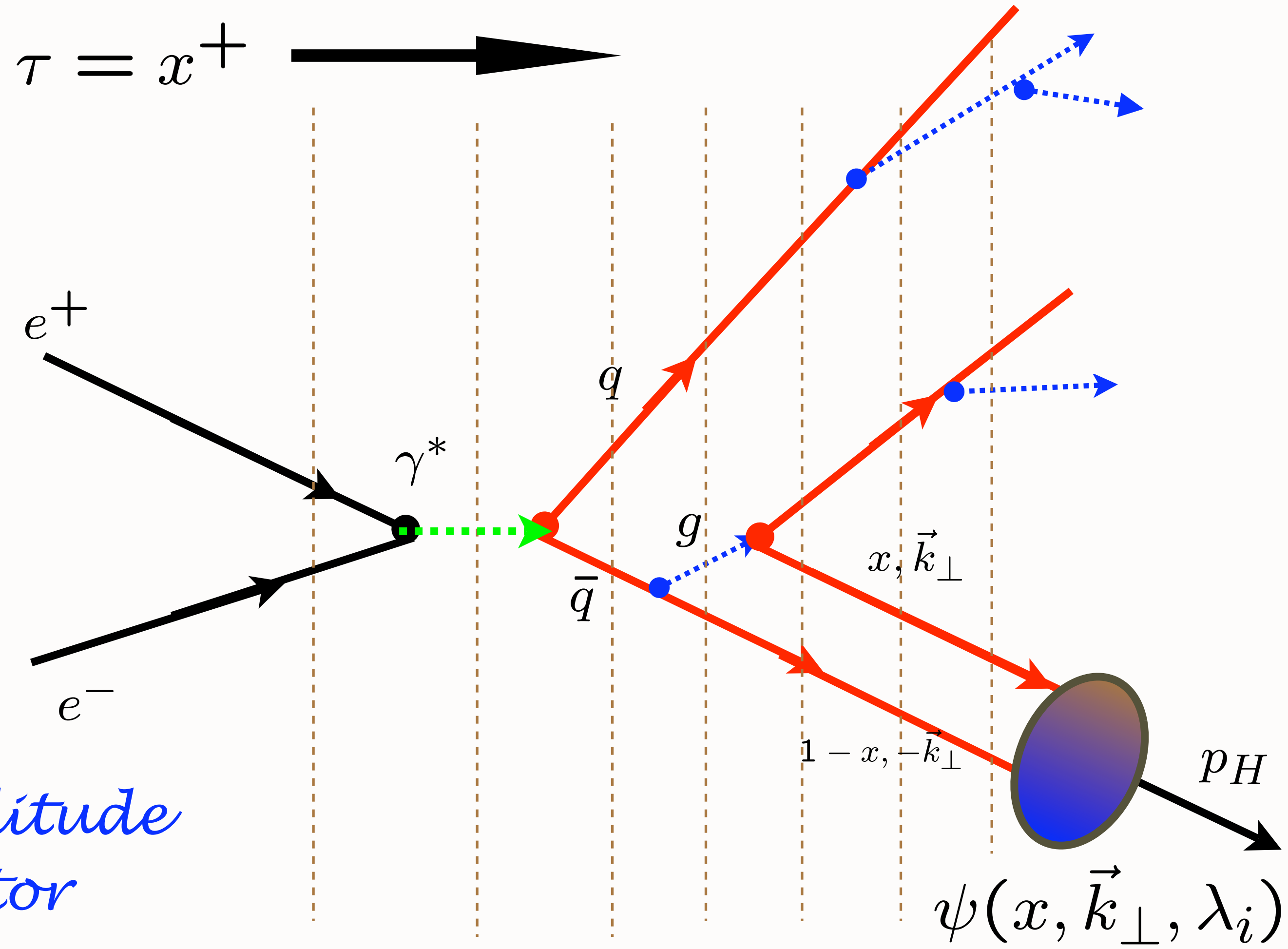
Stan Brodsky, SLAC

Ringberg workshop on non-perturbative QCD of jets

Monday 08 January 2007 - Wednesday 10 January 2007

Schloss Ringberg

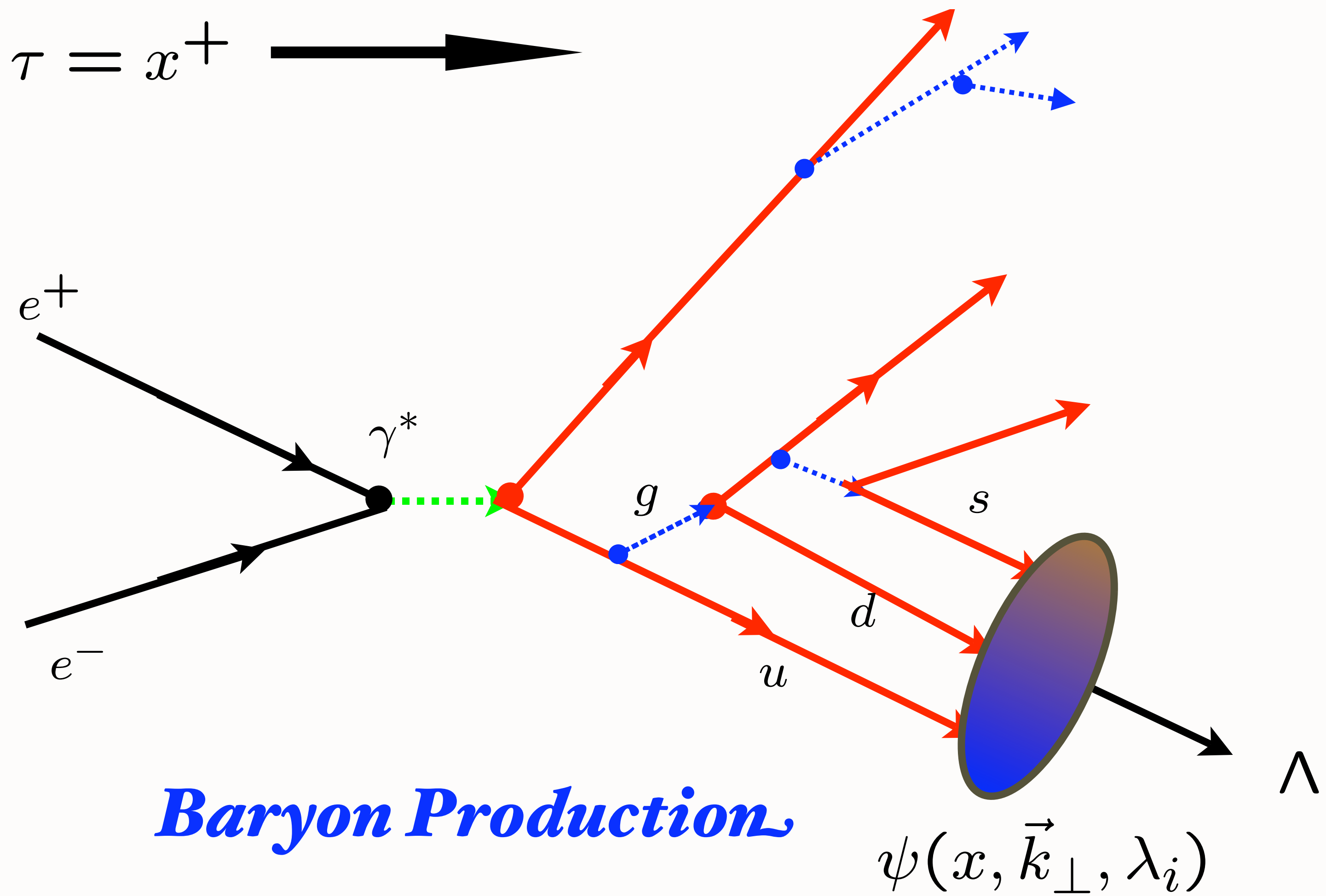
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front Wavefunctions

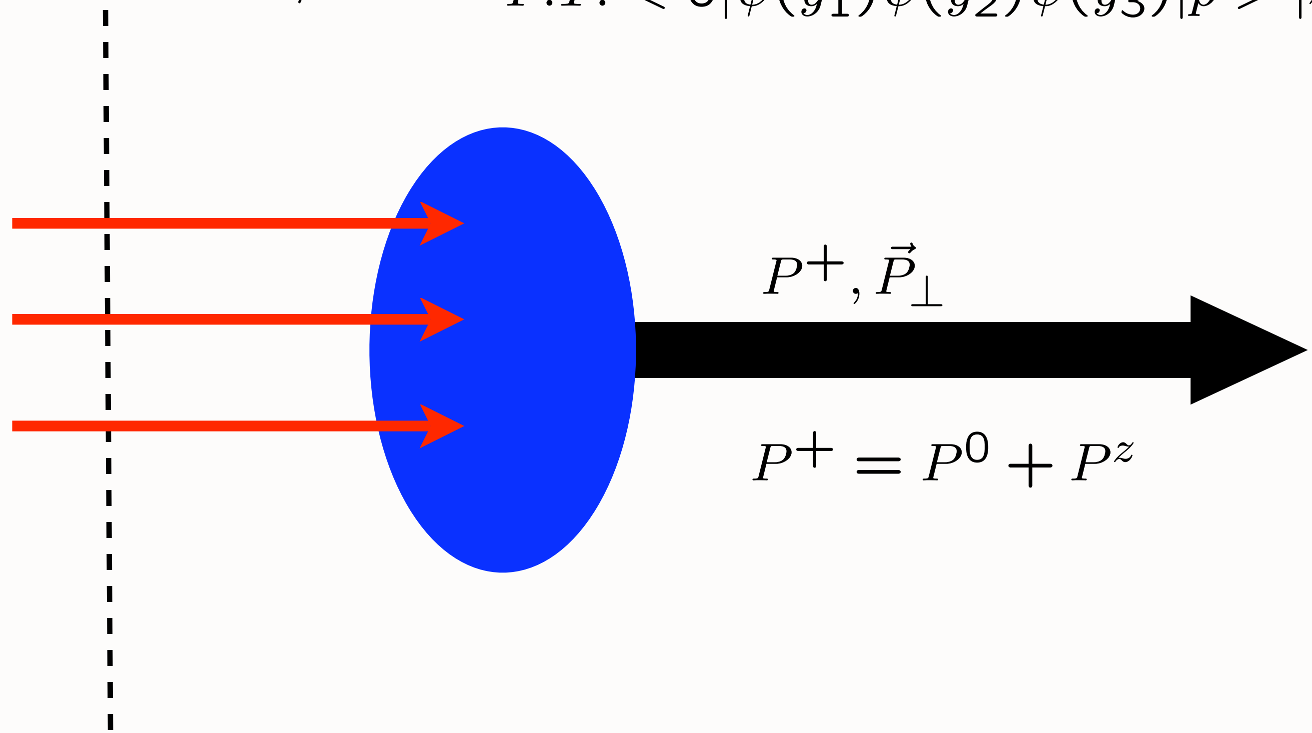
Fixed $\tau = t + z/c$

$F.T. < 0|\psi(y_1)\psi(y_2)\psi(y_3)|p > |_{\tau_i = 0}$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

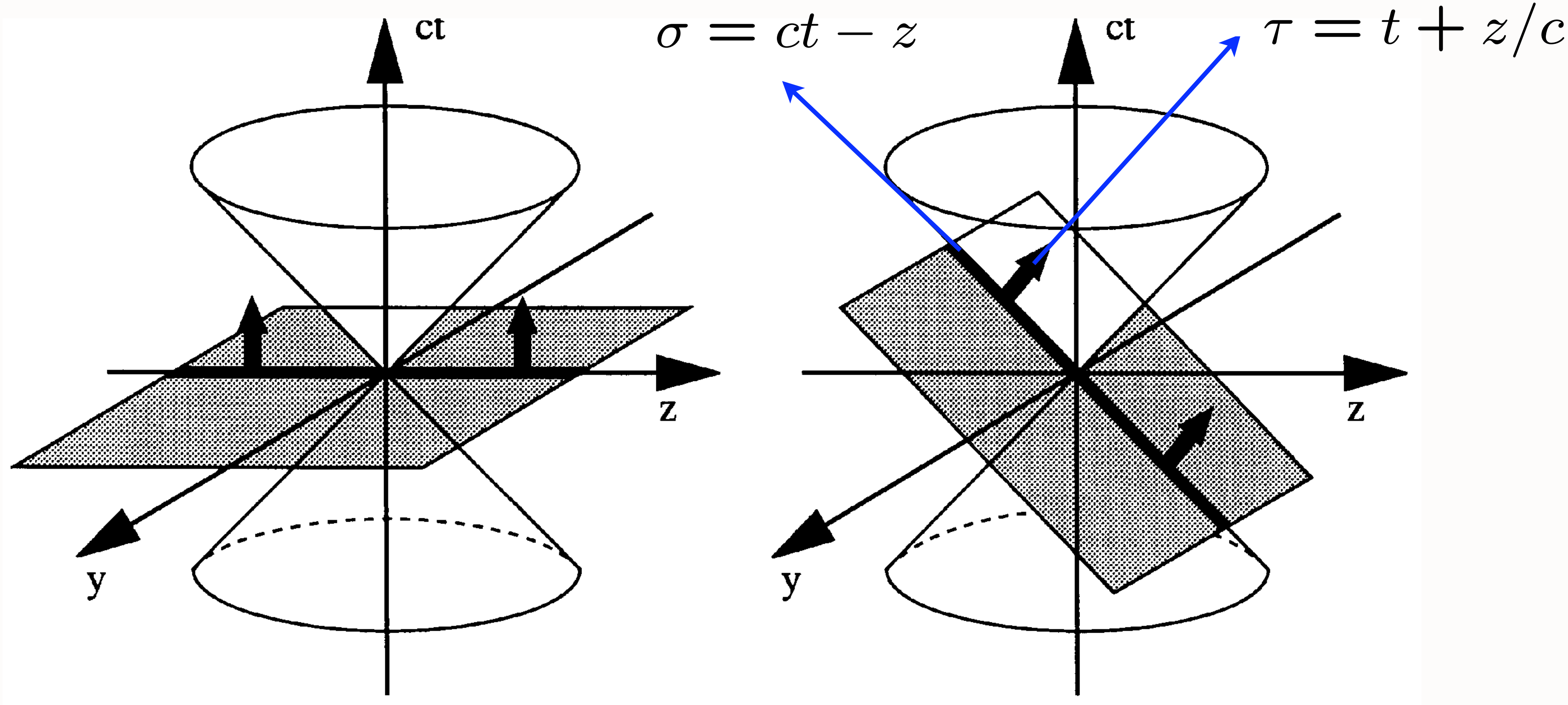


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

Dirac's Amazing Idea: The "Front Form"

Evolve in light-front time!



Instant Form

Front Form

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2 \; |\Psi_h\rangle$$

Compute LFWFS from first principles

$$H_{LC}^{QCD} = P_\mu P^\mu = \quad P^- P^+ - \vec{P}_\perp^2$$

The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\Psi_h(P^+, \vec{P}_\perp)\right\rangle =$$

$$\sum_{n,\lambda_i} \int \; [dx_i \; d^2\vec{k}_{\perp i}] \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

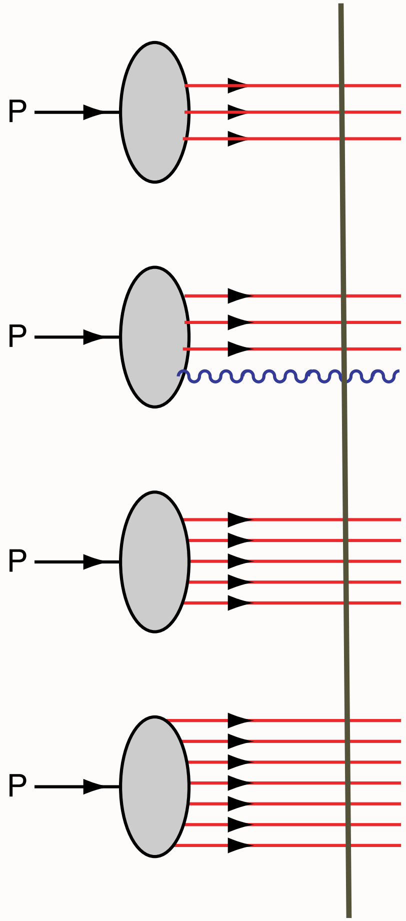
$$\times \; |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

$$\sum_n \int [dx_i \; d^2\vec{k}_{\perp i}] \; |\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 = 1$$

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

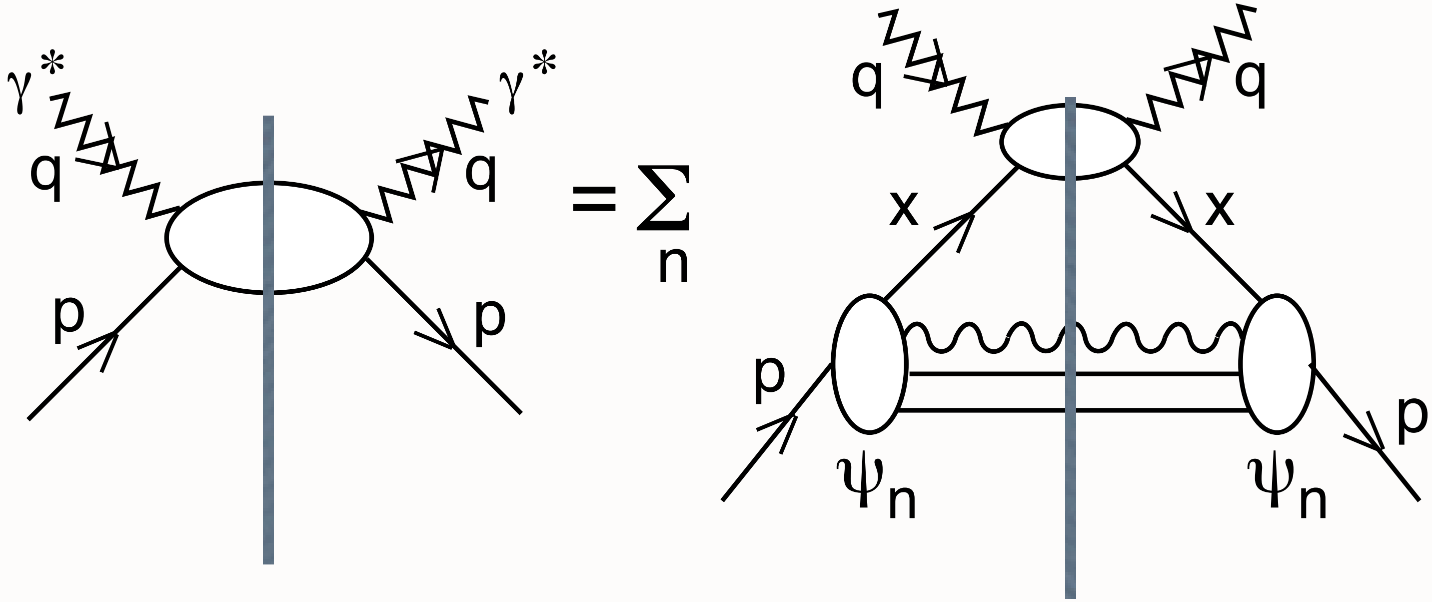
$$\psi_n(x, k_\perp) \qquad x_i = \frac{k_i^+}{P^+}$$



$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Invariant under boosts. Independent of P^μ

Deep Inelastic Lepton Proton Scattering

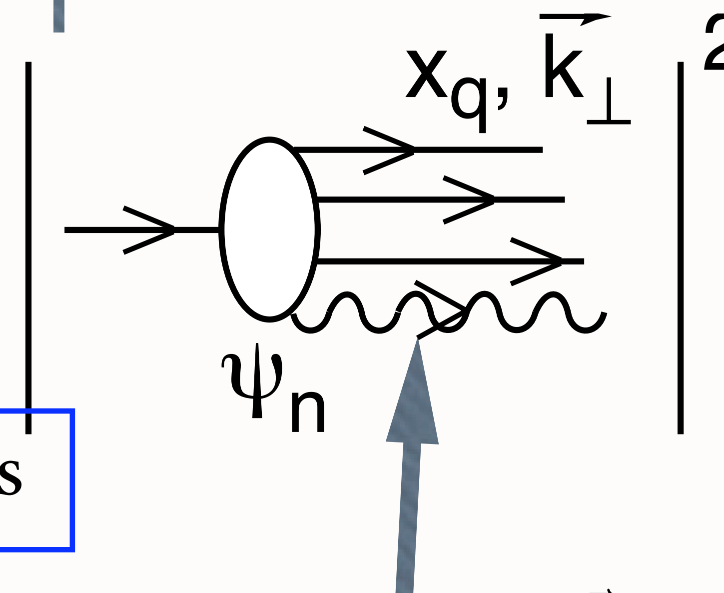


Imaginary Part of
Forward Virtual Compton Amplitude

$q(x,Q^2) = \sum_n \int^{k_\perp^2 \le Q^2 \perp} d^2k_\perp \, |\Psi_n(x,k_\perp)|^2$

$x = x_q$

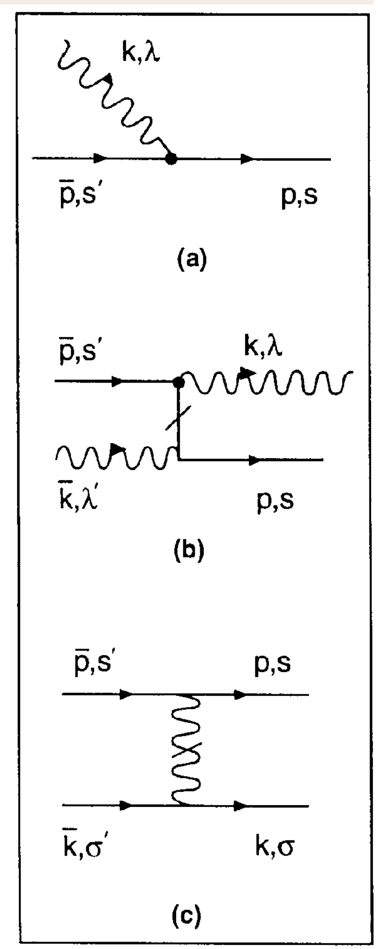
All spin, flavor distributions

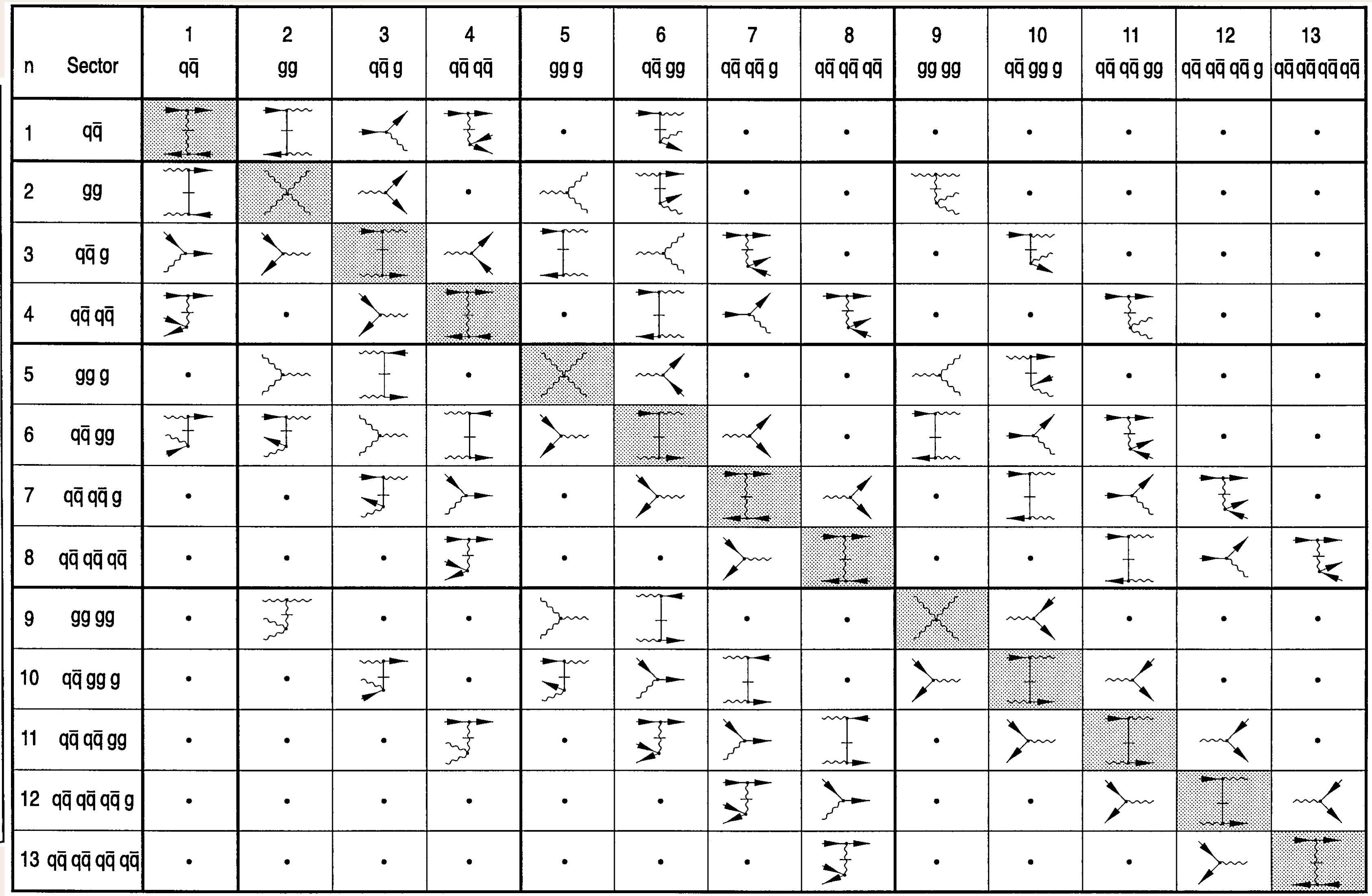


Light-Front Wave Functions $\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Light-Front QCD Heisenberg Equation

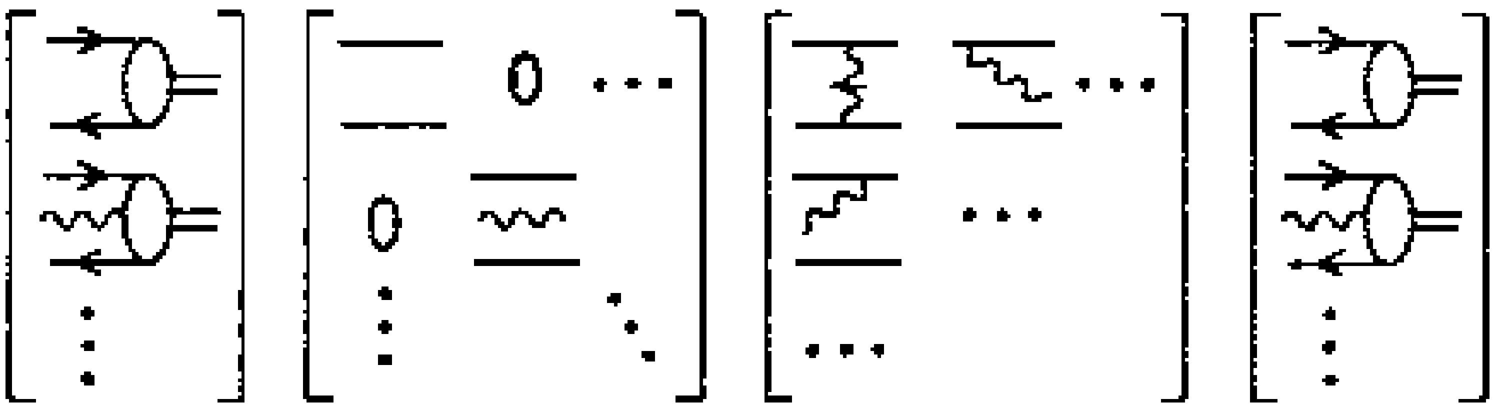
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$





LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



Angular Momentum on the Light-Front

A^+=0 gauge: No unphysical degrees of freedom

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

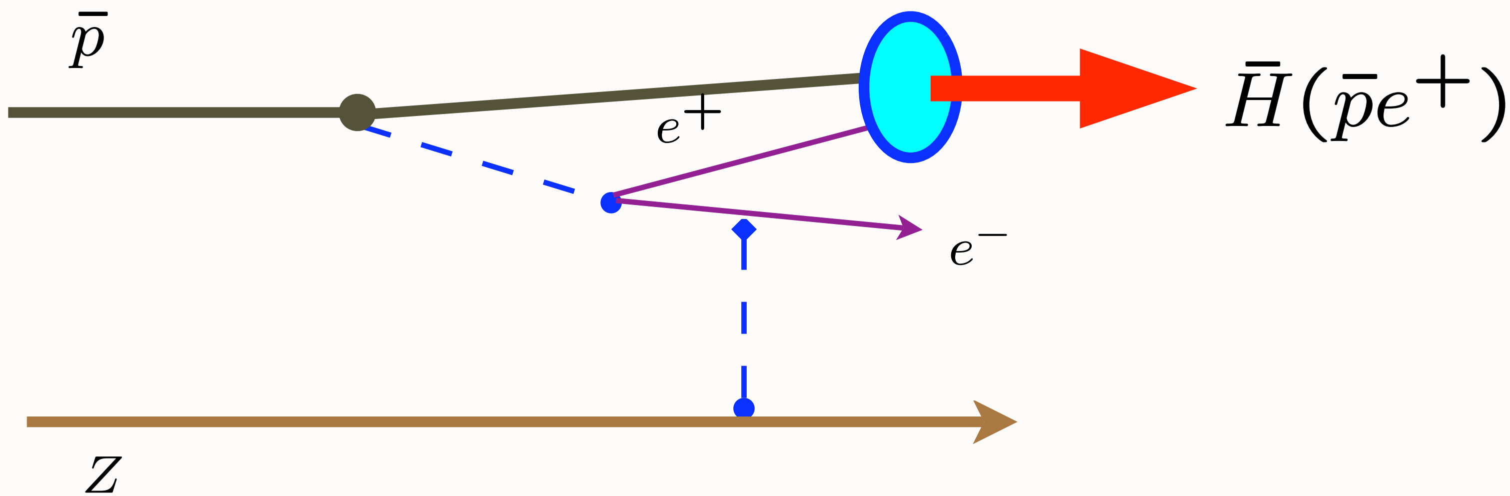
n-1 orbital angular momenta

Creating Hadrons

- Coalescence of co-moving quarks
- Maximal probability at minimum off-shellness
- Hadronization formation at a given light-front time described by light-front wavefunction $\psi_n^H(x_i, k_{\perp i}, \lambda_i)$
- Example in QED: Formation of anti-hydrogen
- Exclusive amplitudes controlled by LFWS
- LFWFs predicted by AdS/CFT

Formation of Relativistic Anti-Hydrogen

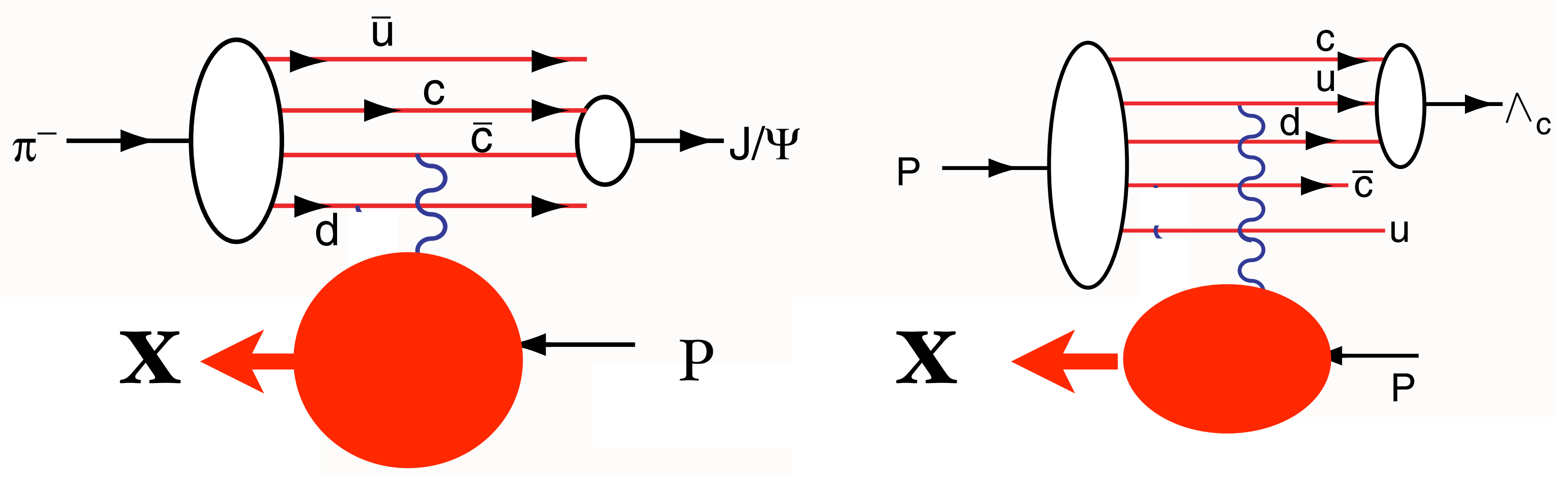
Coalescence of co-moving positron and antiproton



Munger, Schmidt,sjb

Observed at CERN and FermiLab

Leading Hadron Production from Intrinsic Charm



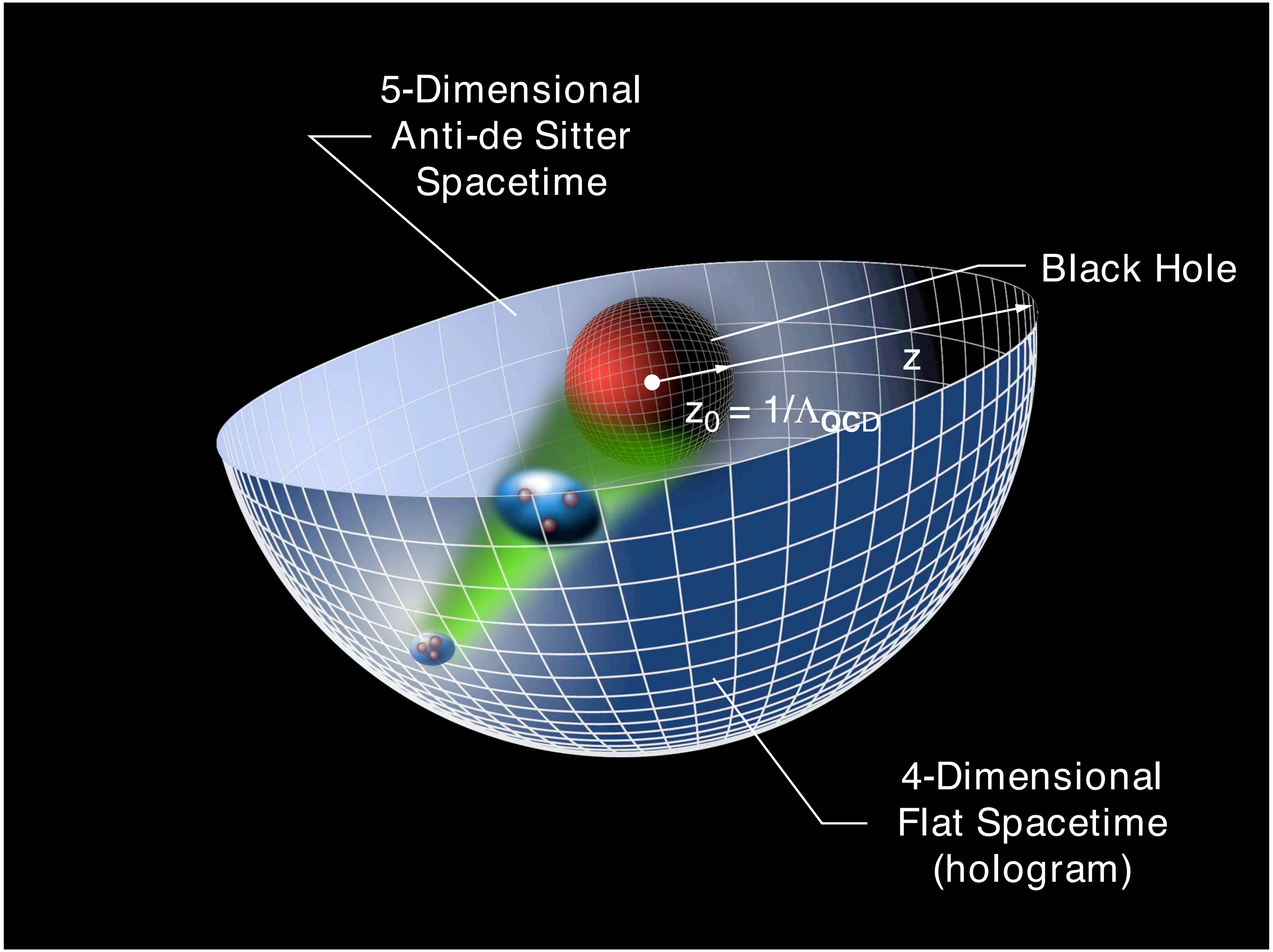
Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ, Λ_c and other Charm Hadrons at High x_F

Parton Correlations contained in LFWFs

$$\frac{d\sigma}{dx_F dQ^2 d\cos\theta}(\pi N \to \ell^+\ell^- X) = A(1-x_F)^2(1+\cos^2\theta)+B(1-x_F)^0\,\frac{\Lambda^2_{QCD}}{Q^2}$$

- Higher Twist contribution to DY, DIS
- Off-shell corrections to DGLAP at x ~ 1, z ~1
- Direct Production of Hadrons in Hard Subprocess
- Proton Decay

The Impact of AdS/CFT on QCD Phenomenology

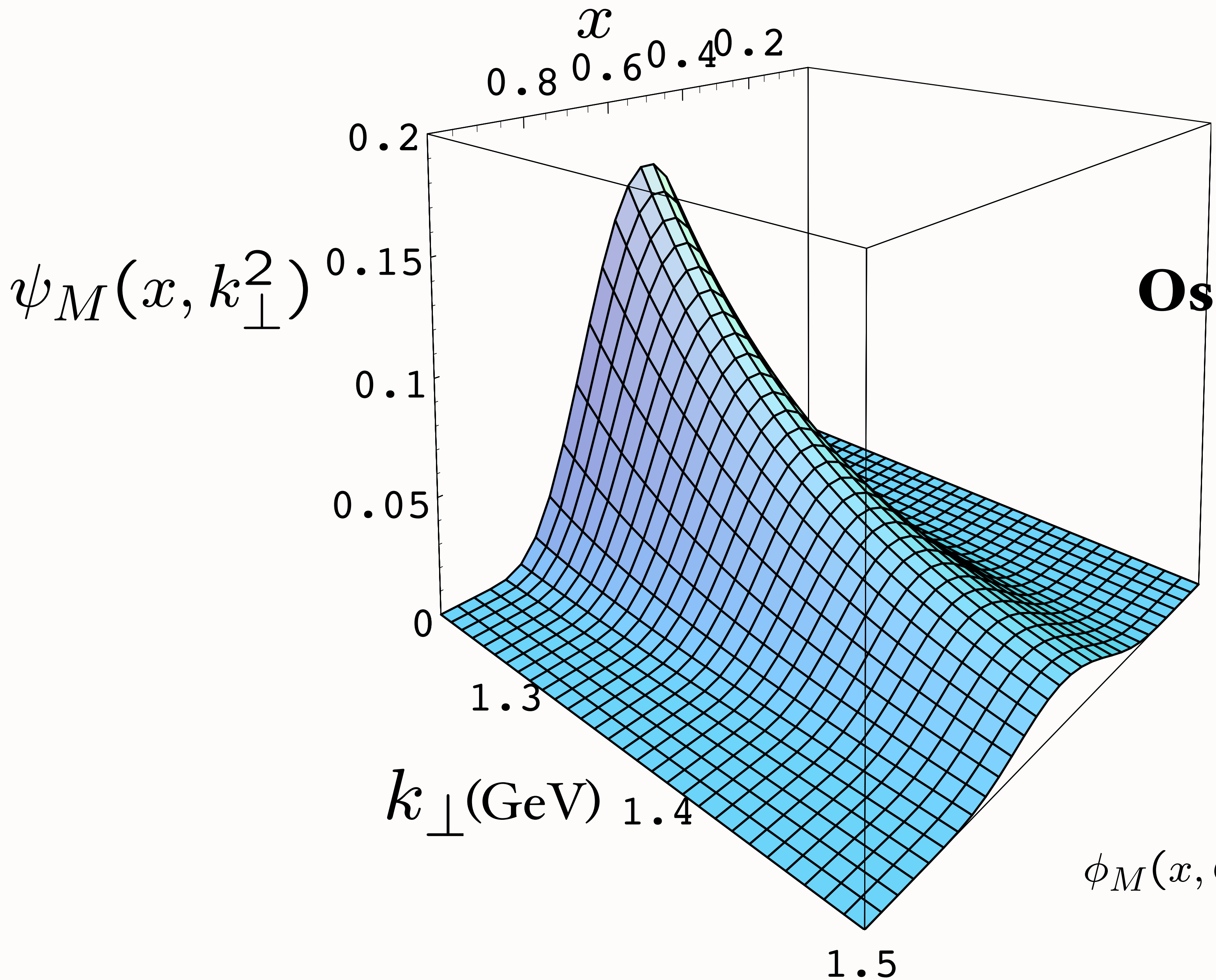


Changes in length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

AdS/CFT and Hadron Formation in QCD

Prediction from AdS/CFT: Meson LFWF



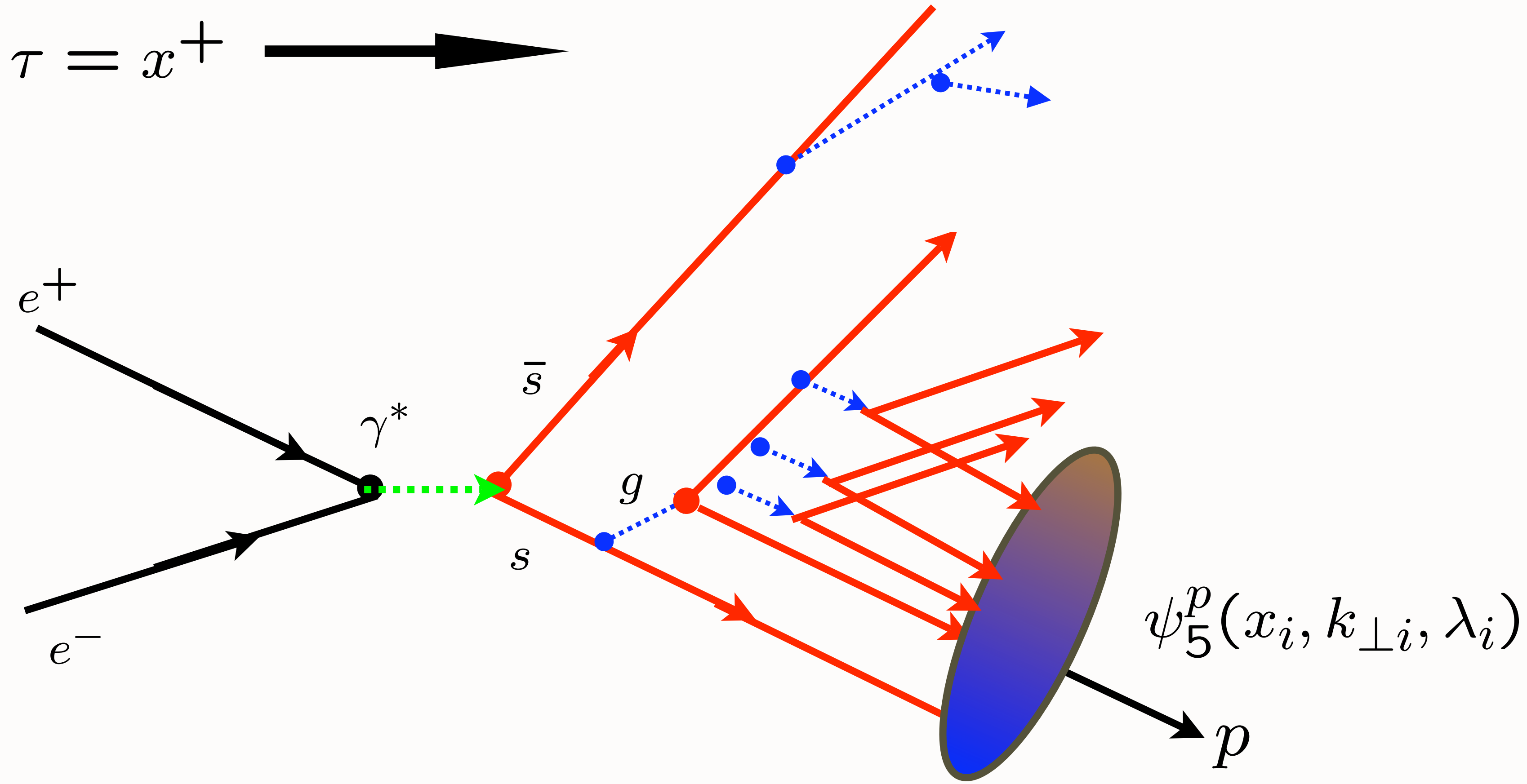
Harmonic Oscillator model

$\kappa = 0.77$GeV

de Teramond, sjb

$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$

Hadronization at the Amplitude Level

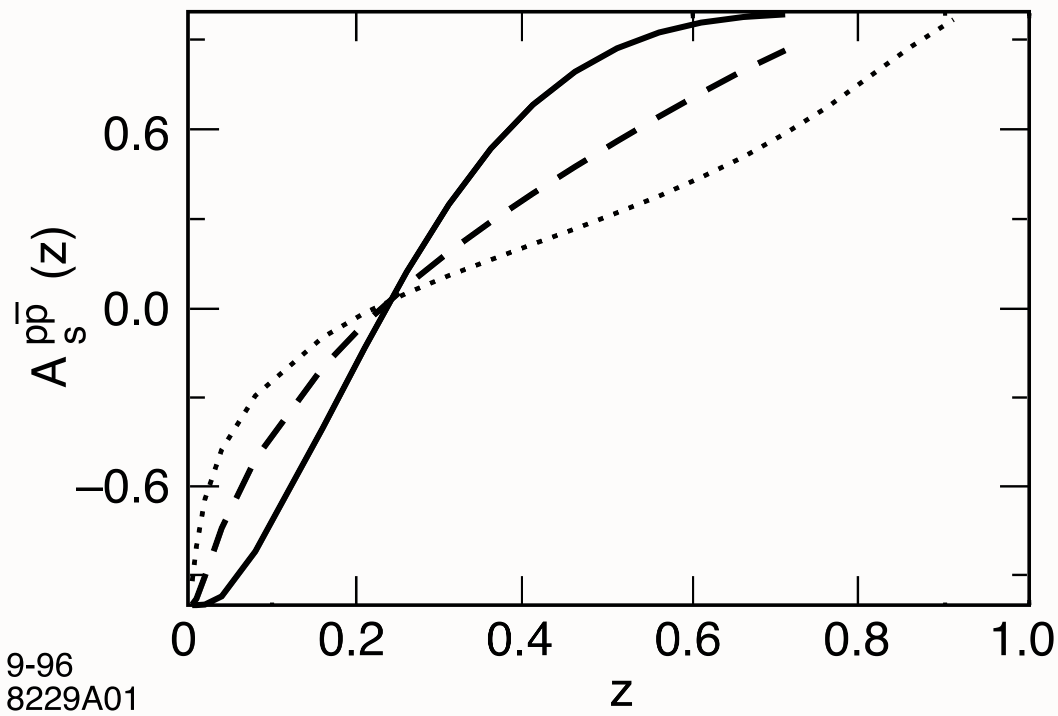


Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$

B-Q Ma, sjb

$$D_{s\to p}(z) \neq D_{s\to\bar{p}}(z)$$



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to\bar{p}}(z)}{D_{s\to p}(z) + D_{s\to\bar{p}}(z)}$$

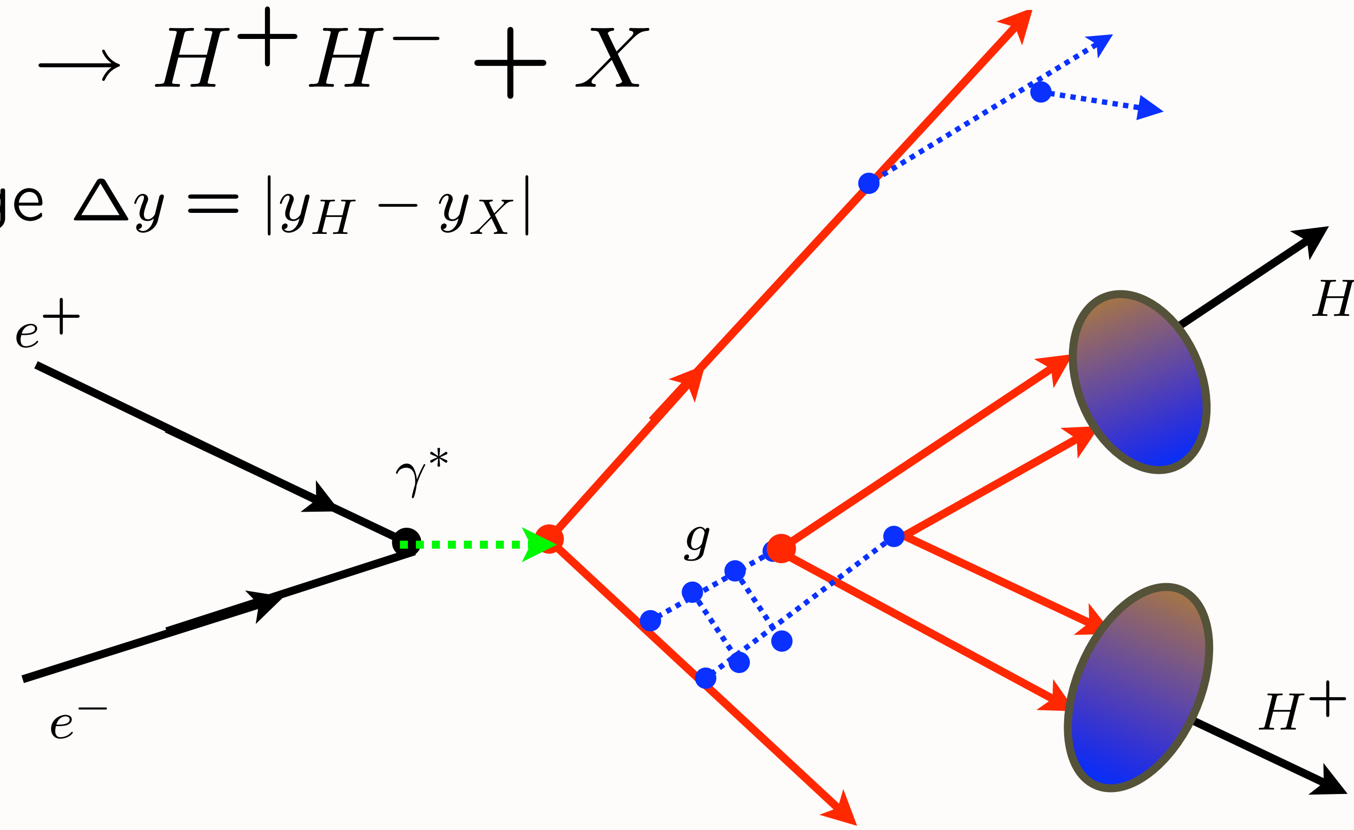
Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s} > \simeq |K^+\Lambda >$

Hadronization at the Amplitude Level

$$e^+e^- \rightarrow H^+H^- + X$$

Large $\Delta y = |y_H - y_X|$

Bjorken, Lu, sjb
Kopeliovich, Schmidt, sjb



Timelike Pomeron ***C= + Gluonium Trajectory***

Large Rapidity Gap Events

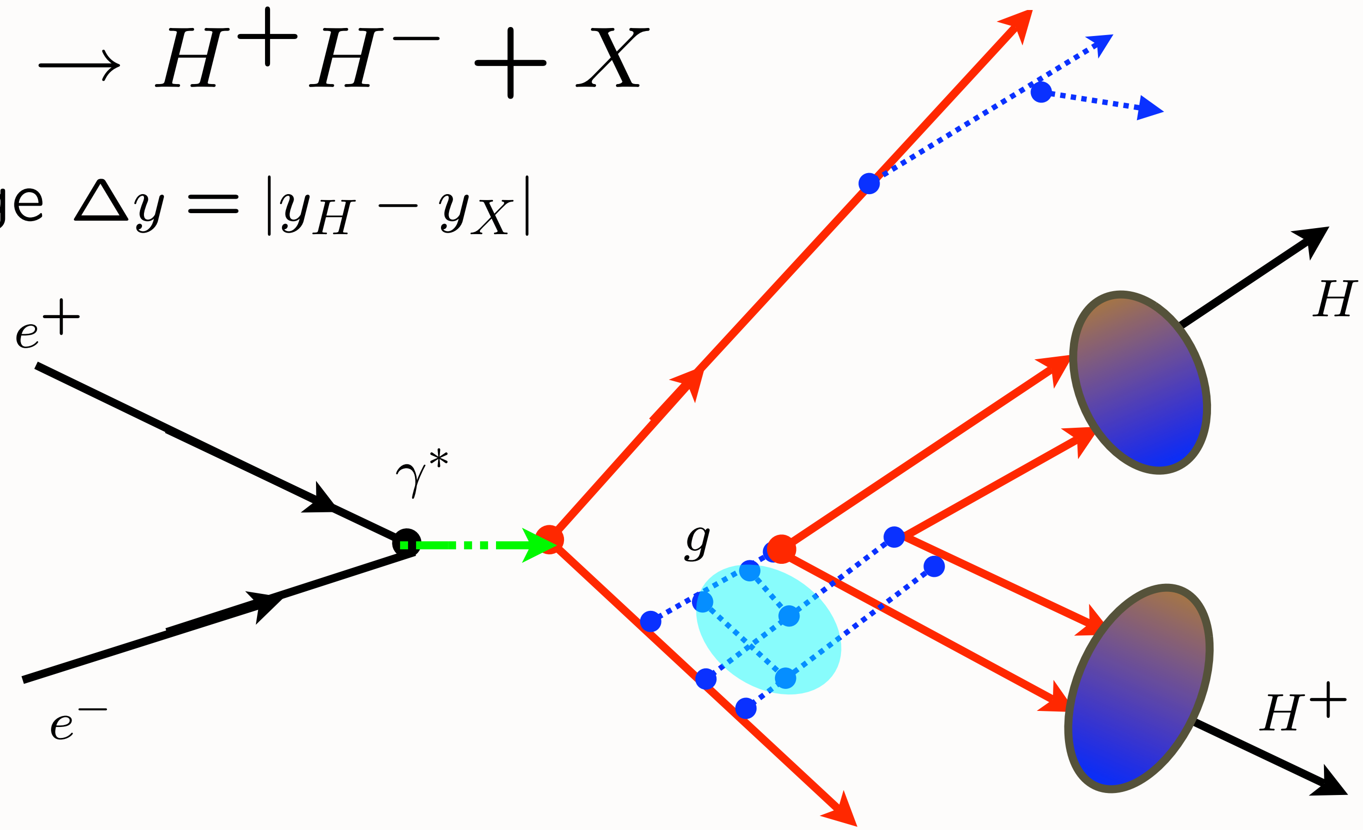
Crossing analog of Diffractive DIS $eH \rightarrow eH + X$

Hadronization at the Amplitude Level

$$e^+e^- \to H^+H^- + X$$

Kopeliovich, Schmidt, sjb

Large $\Delta y = |y_H - y_X|$



Timelike Odderon

Large Rapidity Gap Events

C= - Gluonium Trajectory

H^+H^- asymmetry from Odderon-Pomeron interference

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six-quark wavefunction
- 5 color-singlet combinations of 6 color-triplets -- only one state is | n p
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions x_i ($i=1,2,\ldots,6$) can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, C_d, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum Q, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy]=\delta(1-\sum_{i=1}^{6} y_i)\prod_{i=1}^{6} dy_i$, $C_F=(n_c^2-1)/2n_c=\frac{4}{3}$, $\beta=11-\frac{2}{3}n_f$, and n_f is the effective number of flavors$\}$

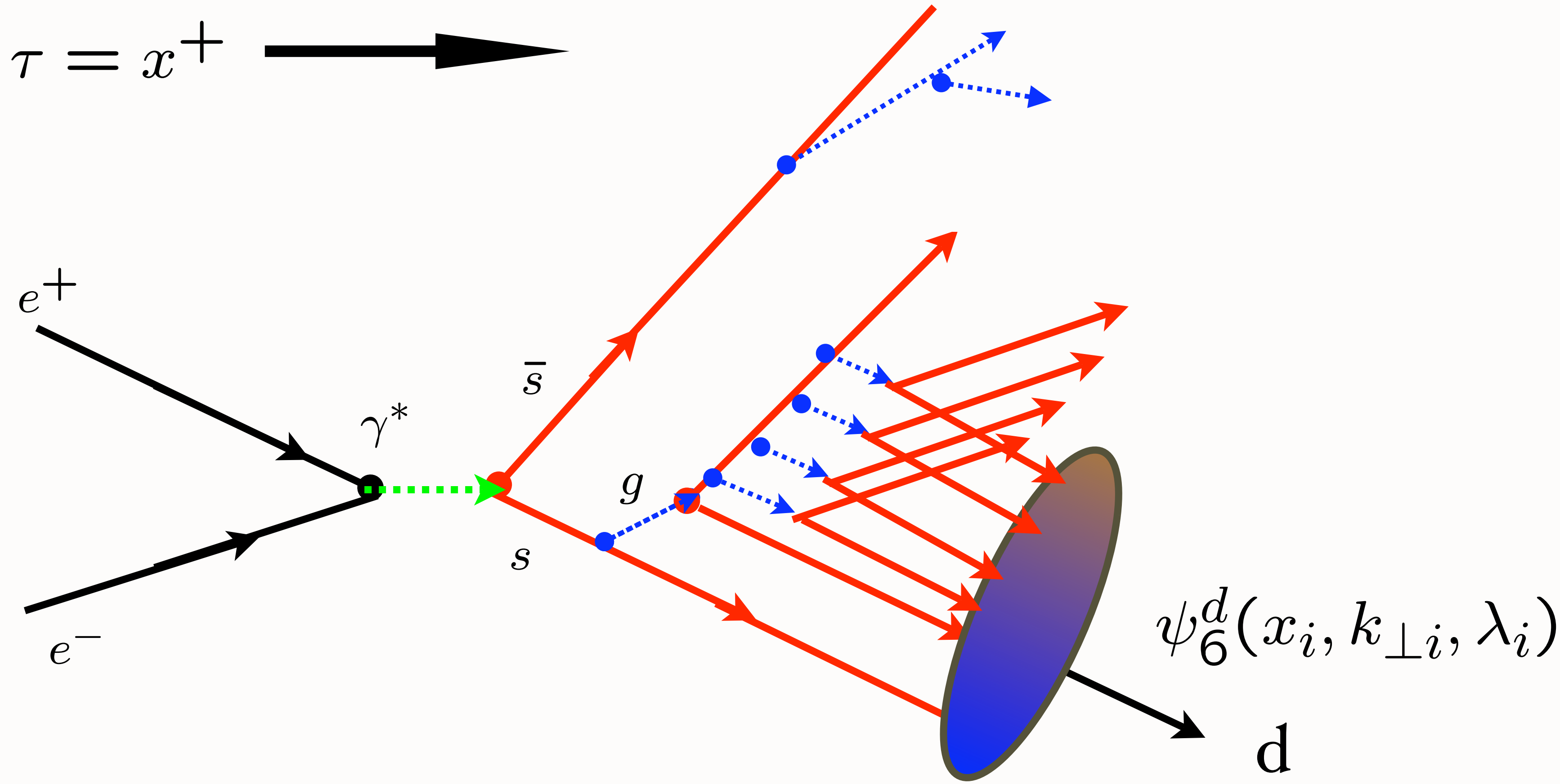
$$\prod_{k=1}^{6} x_k\left[\frac{\partial}{\partial\xi}+\frac{3C_F}{\beta}\right]\tilde{\Phi}(x_i,Q)=-\frac{C_d}{\beta}\int_0^1 [dy]V(x_i,y_i)\,\tilde{\Phi}(y_i,Q),$$

$$\xi(Q^2)=\frac{\beta}{4\pi}\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\alpha_s(k^2)\sim\ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right).$$

$$V(x_i,y_i)=2\prod_{k=1}^{6} x_k\sum_{i\neq j}^{6}\theta(y_i-x_i)\prod_{l\neq i,j}^{6}\delta(x_l-y_l)\frac{y_j}{x_j}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j}+\frac{\Delta}{y_i-x_i}\right)$$

where $\delta_{h_i\bar{h}_j}=1$ (0) when the helicities of the constituents $\{i,j\}$ are antiparallel (parallel). The infrared singularity at $x_i=y_i$ is cancelled by the factor $\Delta\tilde{\Phi}(y_i,Q)=\tilde{\Phi}(y_i,Q)-\tilde{\Phi}(x_i,Q)$ since the deuteron is a color singlet.

Hadronization at the Amplitude Level

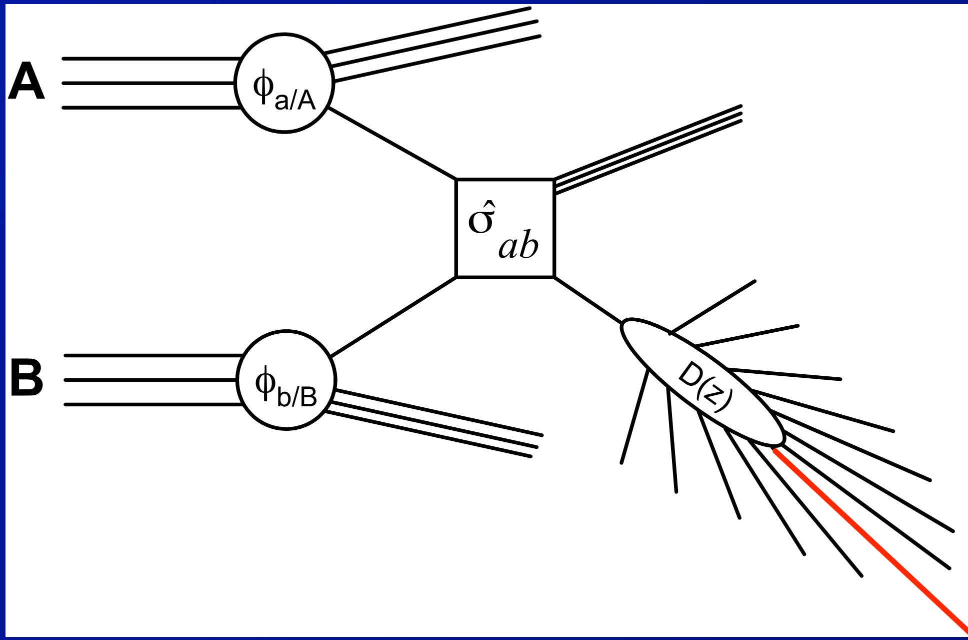


"Hidden-Color" Components $|(uud)_{8C}(ddu)_{8C}>$

New Hadronization Mechanism

Single High-pt Hadron Production

Cole

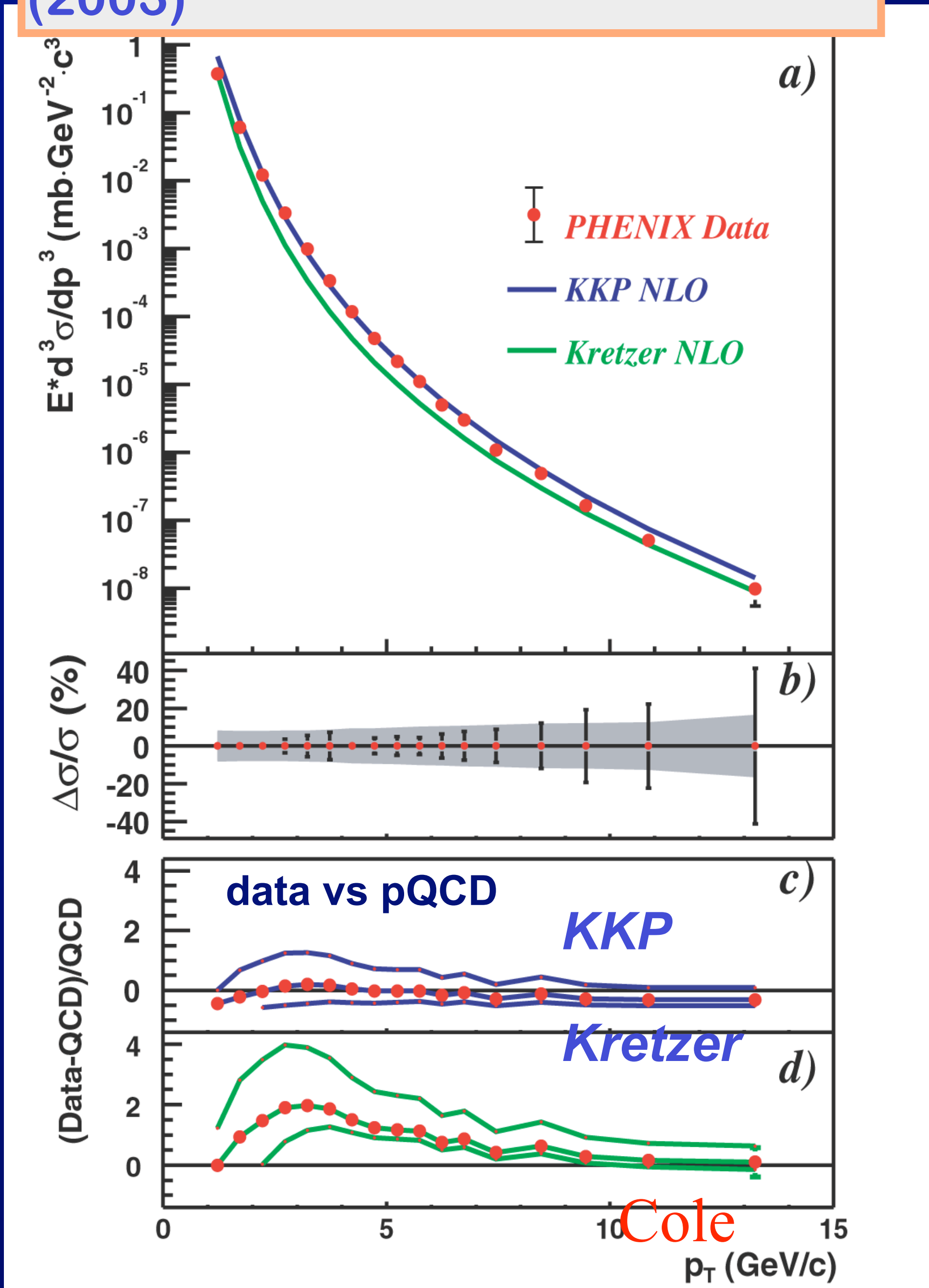


$$E\frac{d^3\sigma}{dp^3} = \sum_{abc}\int dx_a dx_b \phi_{a/A}(x_a,Q^2,\mu)\phi_{b/B}(x_a,Q^2,\mu)$$

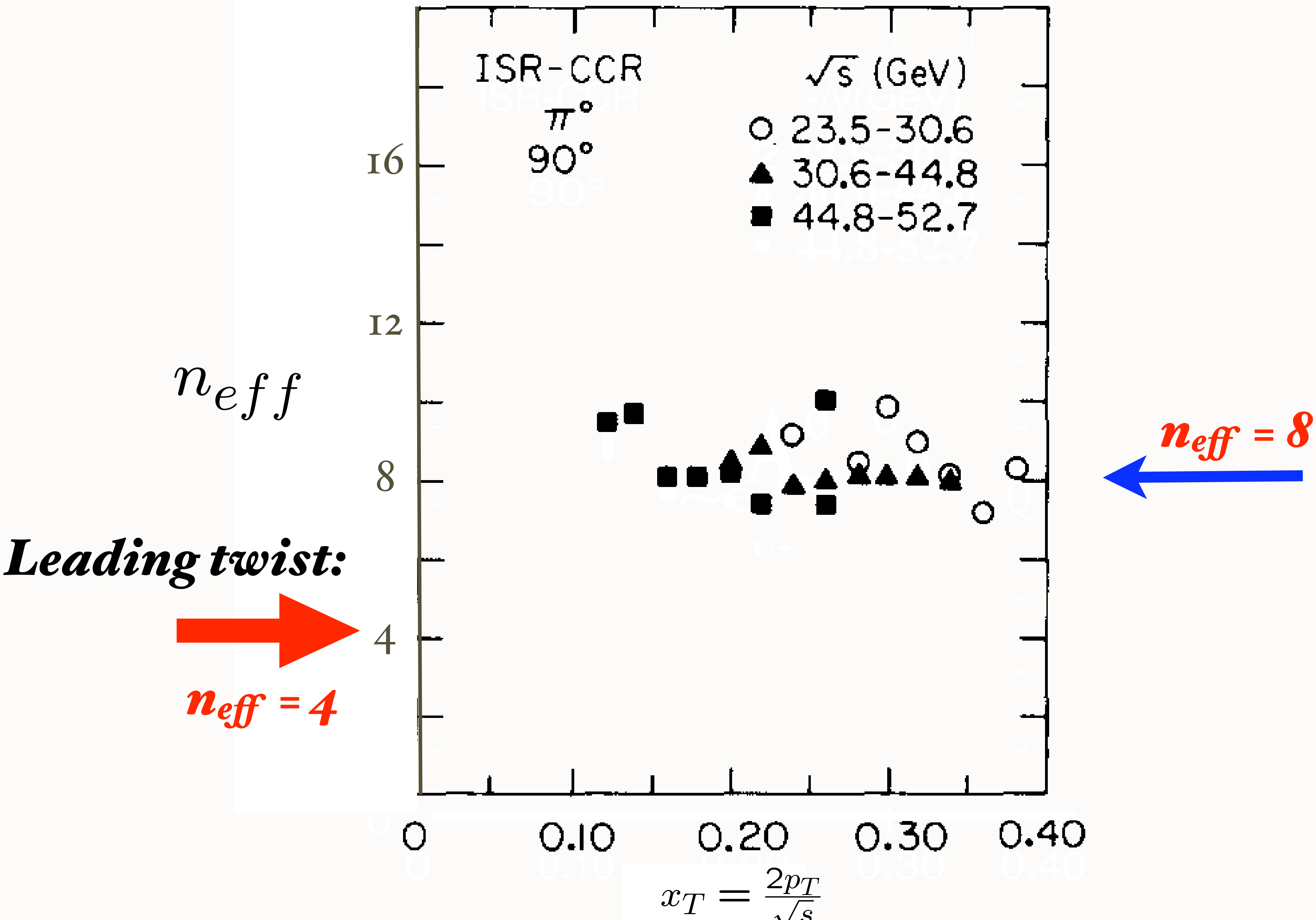
$$\times\frac{D_{\pi^0/c}(z,Q^2,\mu)}{z\pi}\frac{d\hat{\sigma}}{dt}$$

- **NLO calculation agrees well with PHENIX π^0 spectrum (!?)**
  - BUT, FF dependence ?
  - Lore: KKP better for gluons
  - Calc. Includes resummation!

Phys. Rev. Lett. 91, 241803 (2003)

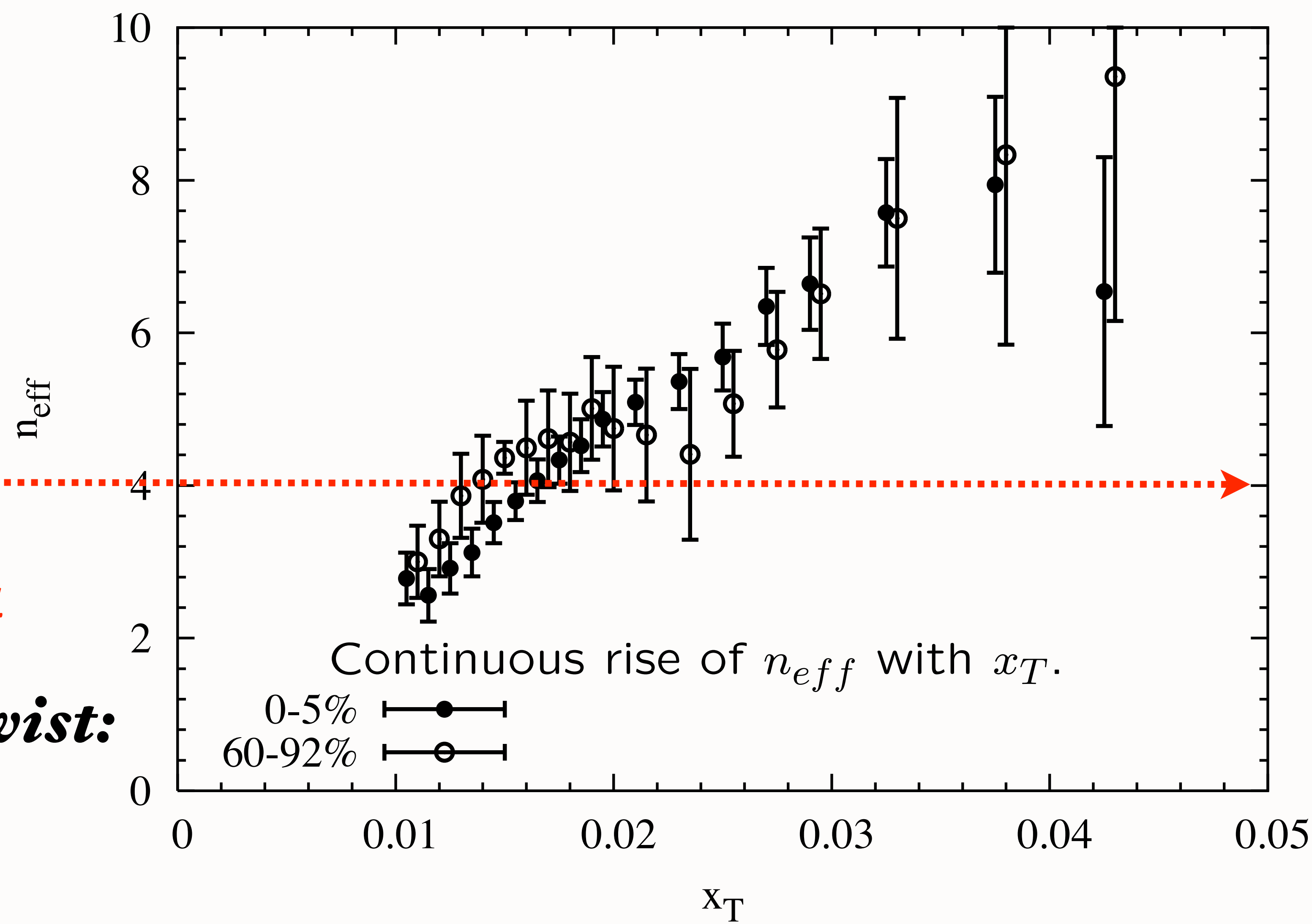


$$E\frac{d\sigma}{d^3p}(pp \to \pi^0 X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$



Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available p_T range. Shown are data for central $(0-5\%)$ and for peripheral $(60-90\%)$ collisions.

STAR

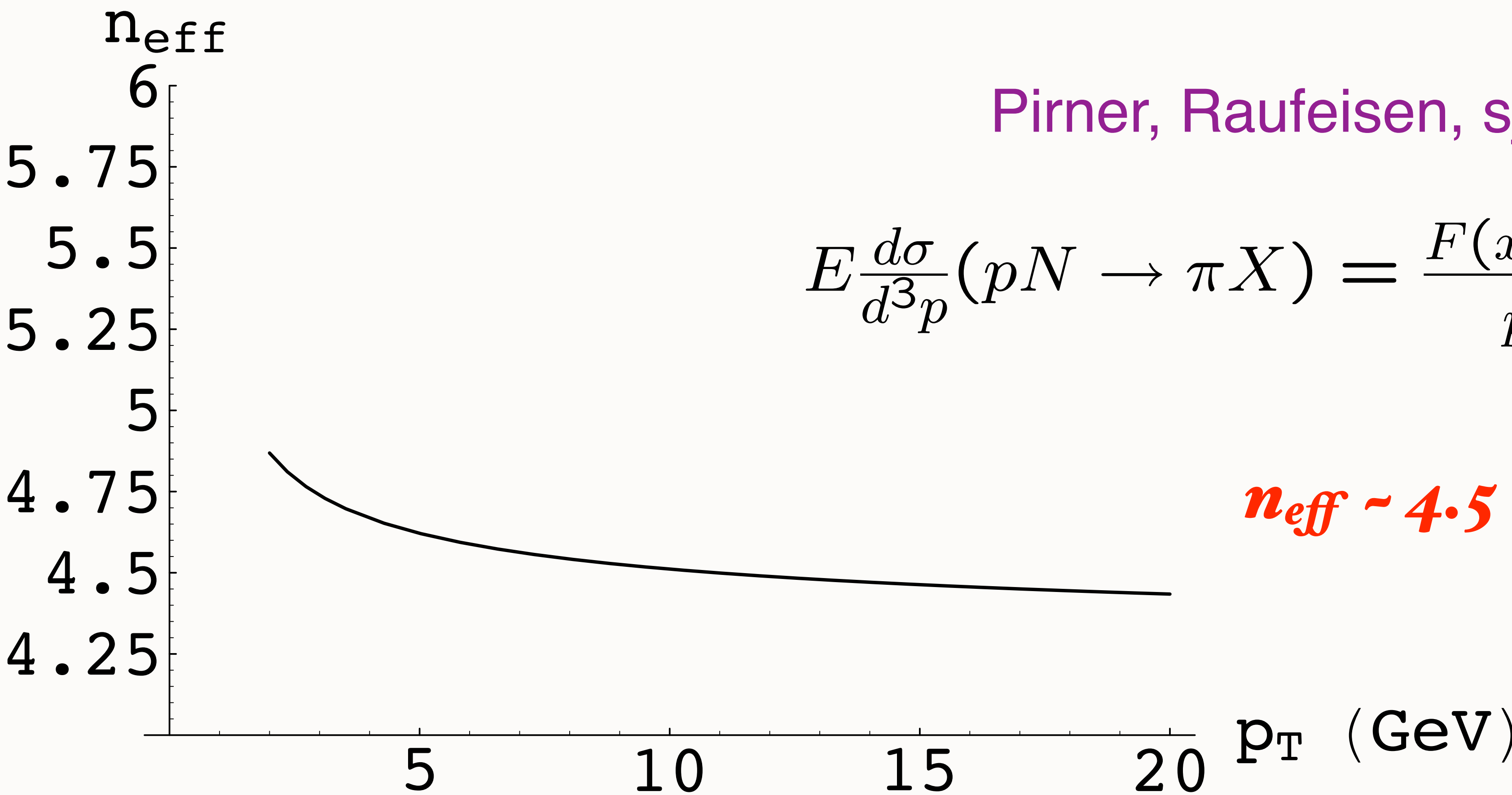


$n_{eff} = 4$

Leading twist:

$$E\frac{d\sigma}{d^3p}(pN \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$

QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling



Pirner, Raufeisen, sjb

$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$

$n_{eff} \sim 4.5$

Key test of PQCD: power fall-off at fixed x_T

$$d\sigma(h_a h_b \to hX) = \sum_{abc} G_{a/h_a}(x_a) G_{b/h_b}(x_b) dx_a dx_b \frac{1}{2\hat{s}} \left|A_{fi}\right|^2 dX_f D_{h/c}(z_c) dz_c.$$

$$E\frac{d^3\sigma(h_a h_b \to hX)}{d^3p} = \frac{F(y, x_R)}{p_T^{n(y,x_R)}}.$$

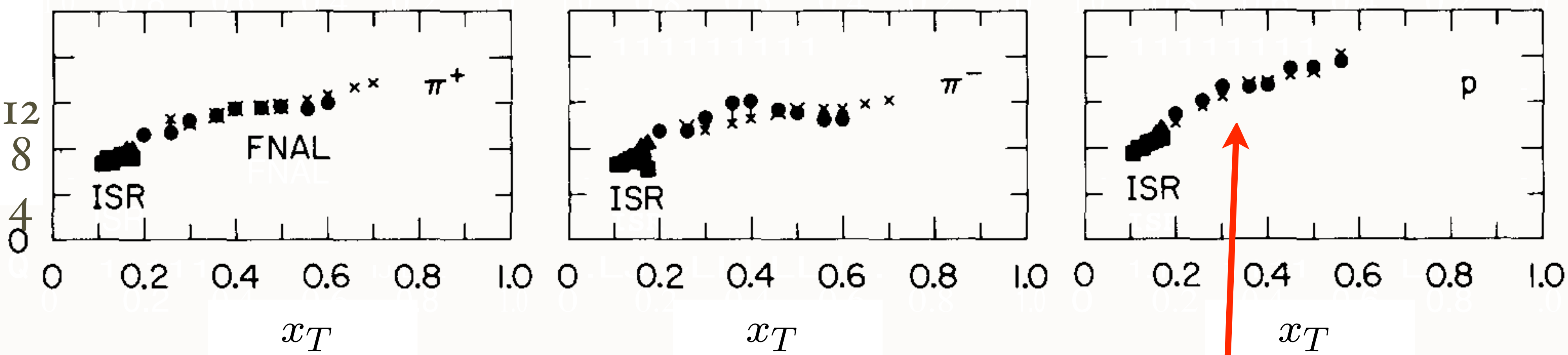
$$n = 2n_{active} - 4,$$

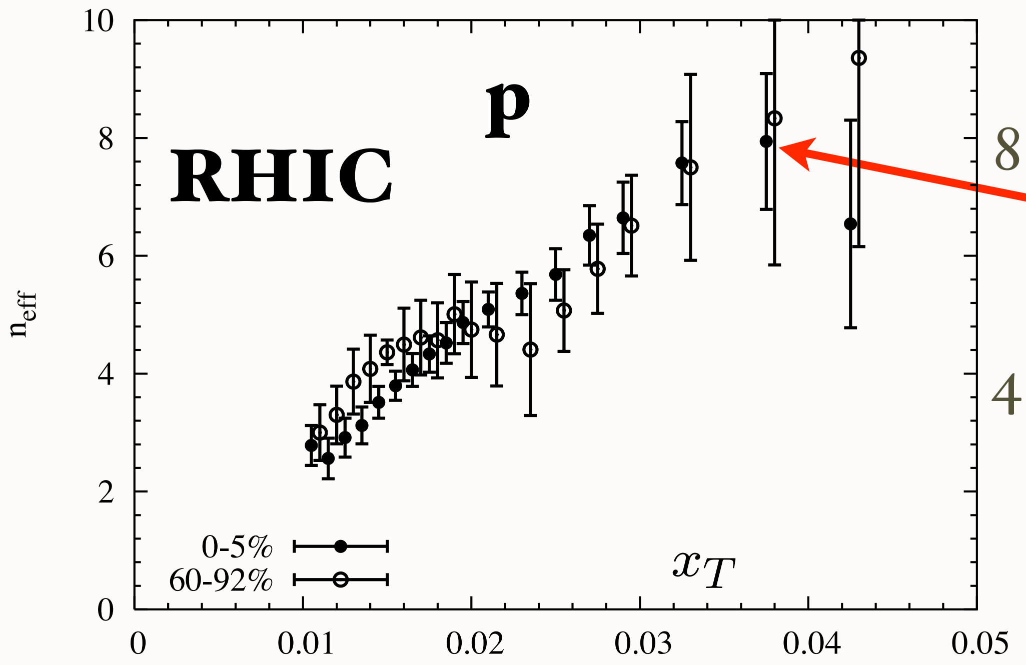
$$n_{eff}(p_T) = -\frac{d\ln E\frac{d^3\sigma(h_a h_b \to hX)}{d^3p}}{d\ln(p_T)}$$

$$E\frac{d^3\sigma(h_a h_b \to hX)}{d^3p} = \left[\frac{\alpha_s(p_T^2)}{p_T^2}\right]^{n_{active}-2} \frac{(1-x_R)^{2n_s-1+3\xi(p_T)}}{x_R^{\lambda(p_T)}} \alpha_s^{2n_s}(k_{x_R}^2) f(y).$$

$$\xi(p_T) = \frac{C_R}{\pi}\int_{k_{x_R}^2}^{p_T^2} \frac{dk_\perp^2}{k_\perp^2}\alpha_s(k_\perp^2) = \frac{4C_R}{\beta_0}\ln\frac{\ln(p_T^2/\Lambda_{QCD}^2)}{\ln(k_{x_R}^2/\Lambda_{QCD}^2)}.$$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$





$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{12}}$$

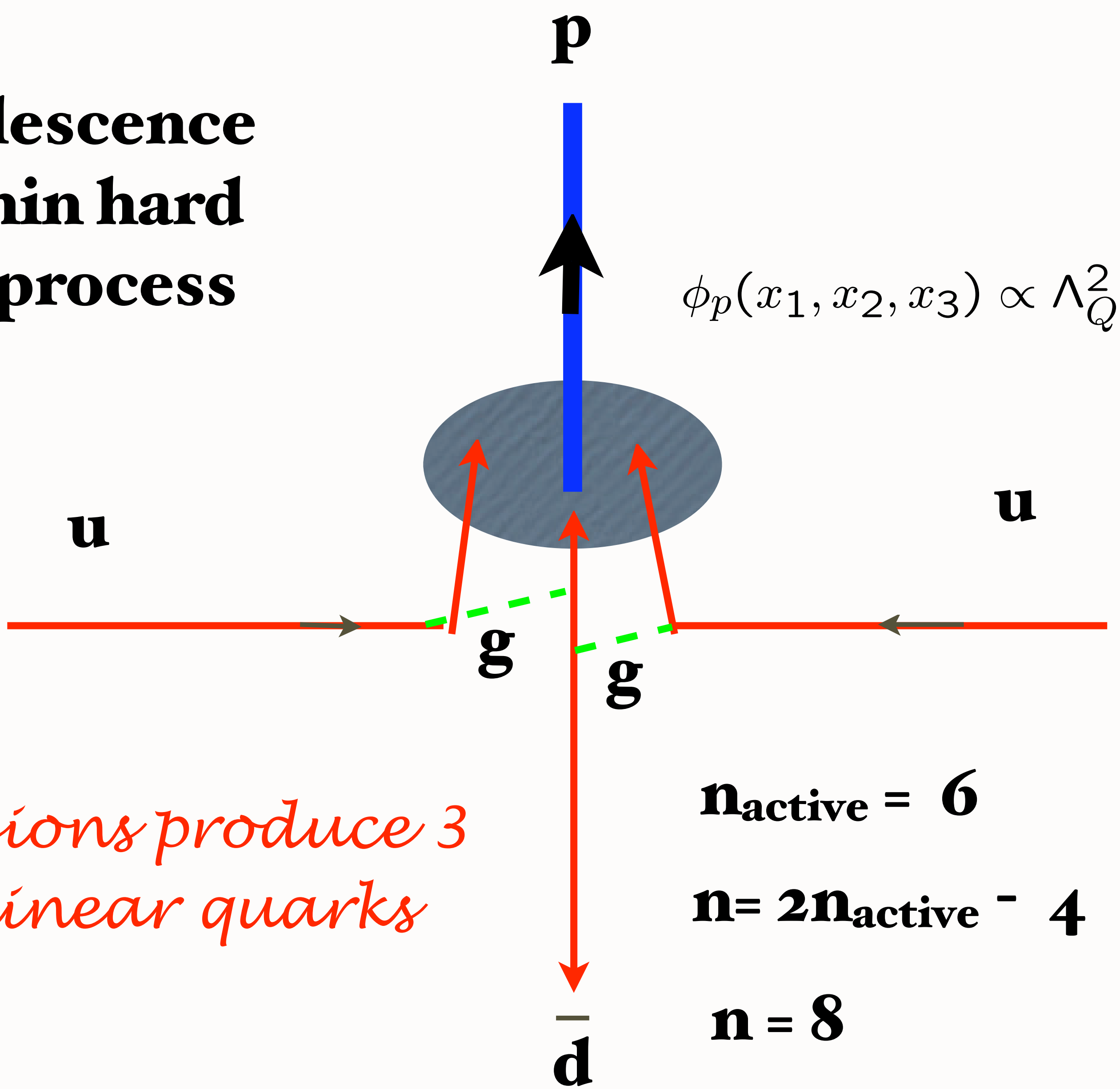
$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{8}}$$

Trend consistent with RHIC at small x_T

Proton made within hard subprocess

Bjorken
Blankenbecler, Gunion, sjb

Coalescence within hard subprocess



$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$

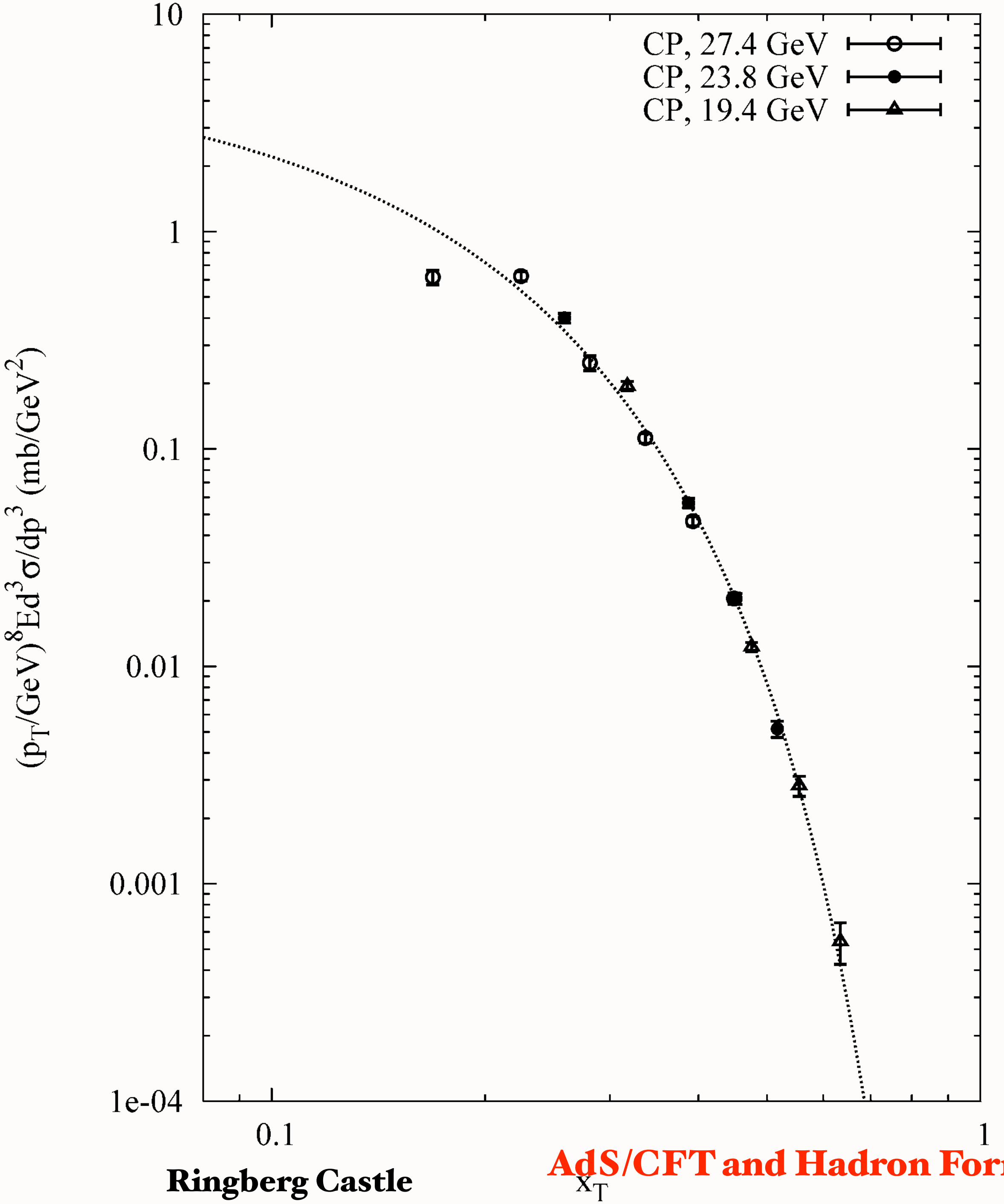
Collisions produce 3 collinear quarks

$n_{active} = 6$

$n = 2n_{active} - 4$

$n = 8$

Invariant cross sections for $pp \to (\pi^+ + \pi^-)/2 + X$



Chicago-Princeton FermiLab Measurements

$$E\frac{d\sigma}{d^3p}(pp \to \pi X) = \frac{F(x_T,\theta_{CM})}{p_T^{2N}}$$

$$n_{eff} = 2N = 8$$

$$qq \to qq\text{:}\ \ n_{eff} = 4$$

$$gq \to \pi q\text{:}\ \ n_{eff} = 6$$

$$\pi q \to \pi q\text{:}\ \ n_{eff} = 8$$

$$F(x_T, \theta_{CM} = \pi/2) = C(1 - x_T)^9$$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

Role of higher twist in hard inclusive reactions

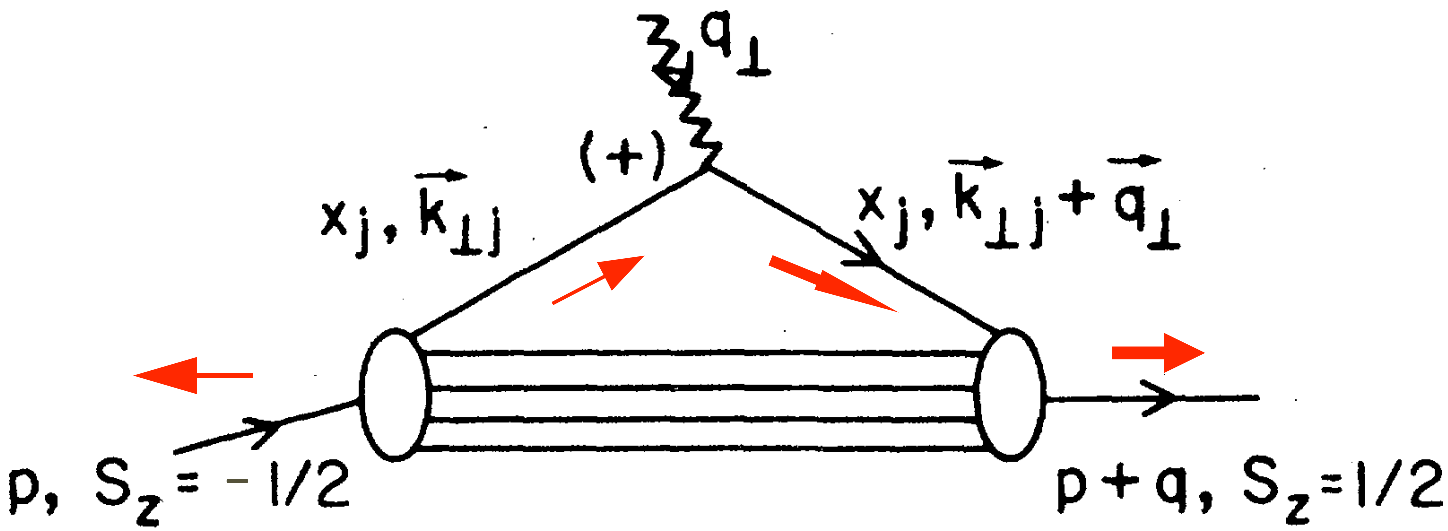
- Hadron can be produced directly in hard subprocess as in exclusive reactions
- Sum over reactions
- Trigger bias: No wasted same-side energy
- Exclusive -inclusive connection important at high x_T
- Possible explanation of $n_{eff} = 8, 12$ observed at ISR, Fermilab: Chicago-Princeton experiments
- Direct Hadron Production -- color transparency and reduced same side absorption
- Critical to plot data at fixed x_T
- Interpretation of RHIC data is modified if higher twist subprocesses play an important role

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



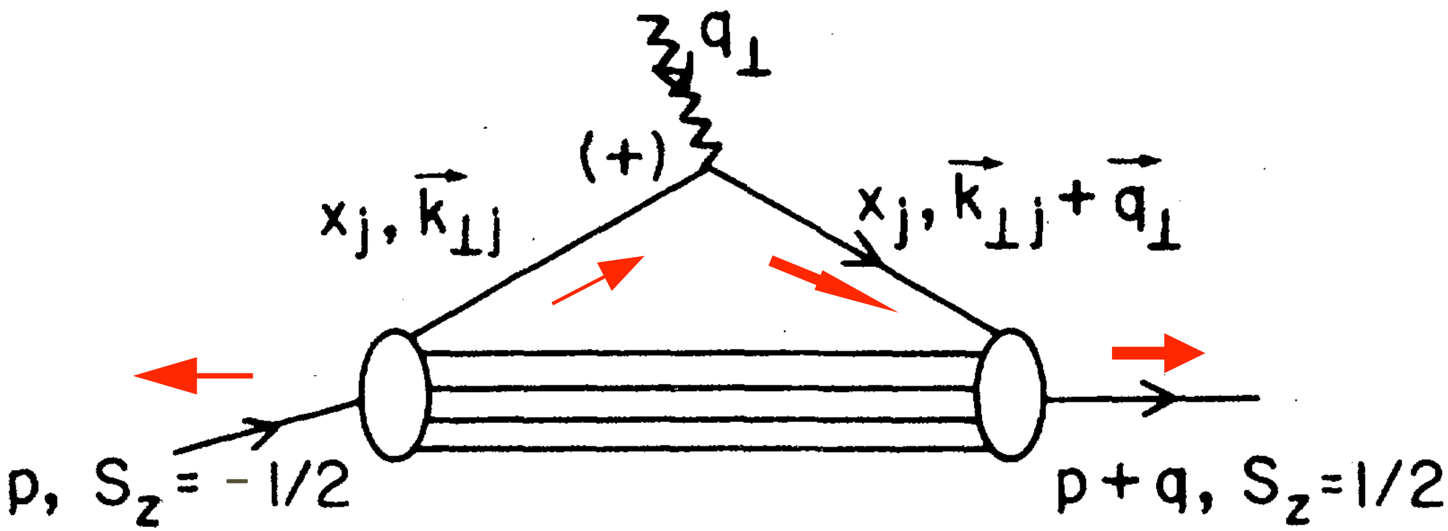
Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Anomalous gravitomagnetic moment B(0)

Equivalence theorem: B(0)=0

graviton

sum over constituents

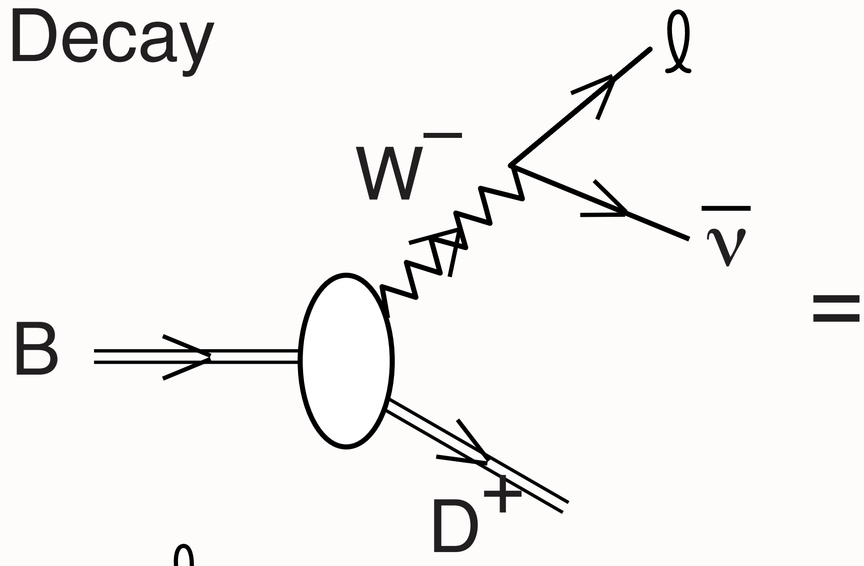


$$B(0) = 0$$

Each Fock State

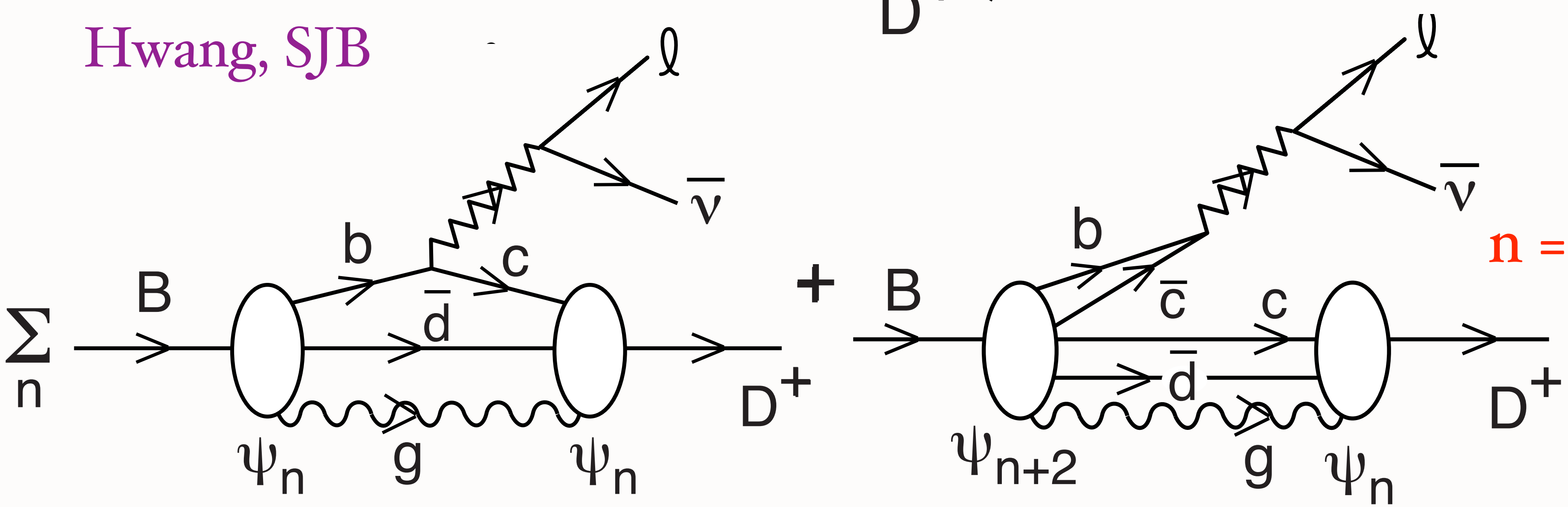
Weak Exclusive Decay

$\langle D \mid J^{+}(0) \mid B \rangle$



Exact Formula

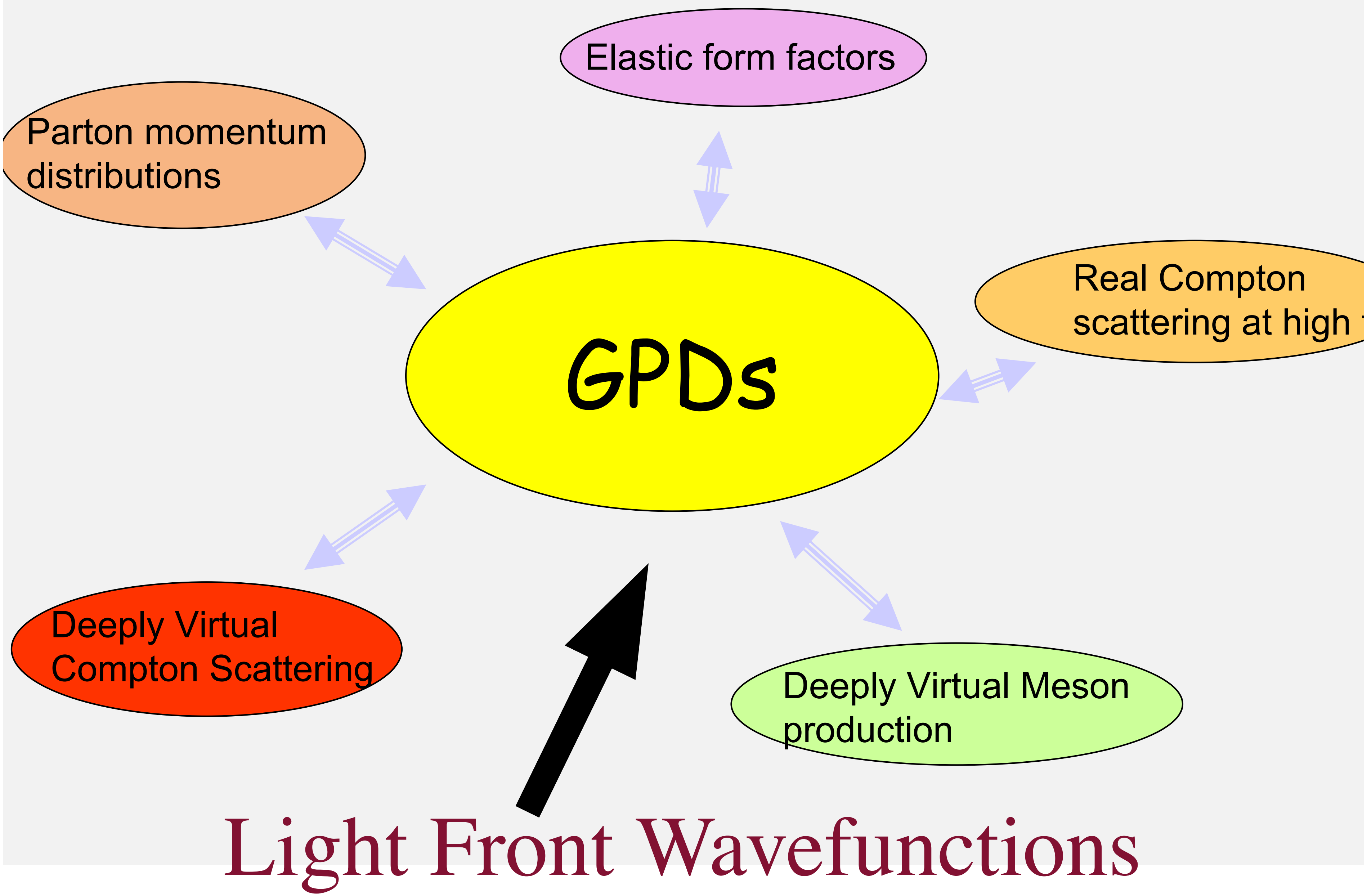
Hwang, SJB



$n = n' + 2$

Annihilation amplitude needed for Lorentz Invariance

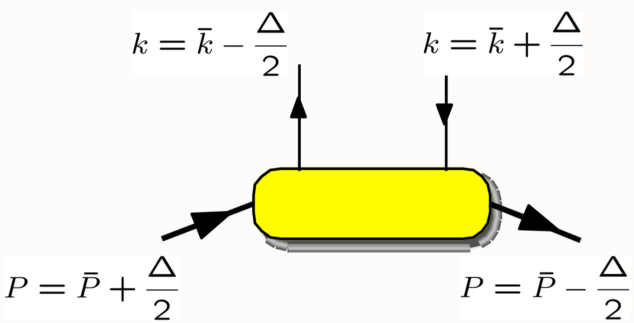
A Unified Description of Hadron Structure

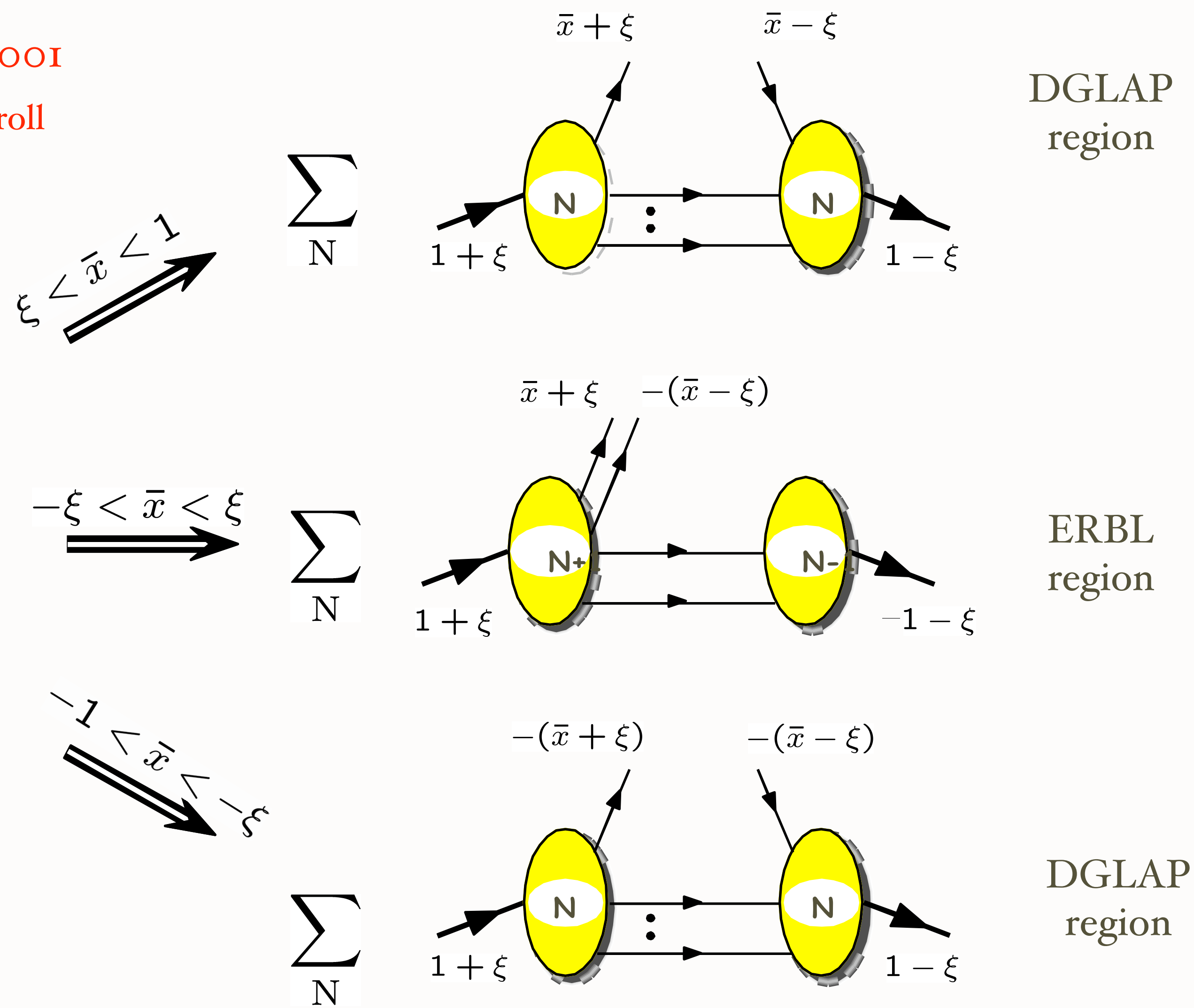


Light-Front Wave Function Overlap Representation

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll





N=3 VALENCE QUARK ⇒ Light-cone Constituent quark model

N=5 VALENCE QUARK + QUARK SEA ⇒ Meson-Cloud model

Pasquini

The Generalized Parton Distribution $E(x, \zeta, t)$

The generalized form factors in virtual Compton scattering $\gamma^*(q) + p(P) \to \gamma^*(q') + p(P')$ with $t = \Delta^2$ and $\Delta = P - P' = (\zeta P^+, \boldsymbol{\Delta}_\perp, (t + \boldsymbol{\Delta}_\perp^2)/\zeta P^+)$, have been constructed in the light-front formalism. [Brodsky, Diehl, Hwang, 2001]

We find, under $\boldsymbol{q}_\perp \to \boldsymbol{\Delta}_\perp$, for $\zeta \le x \le 1$,

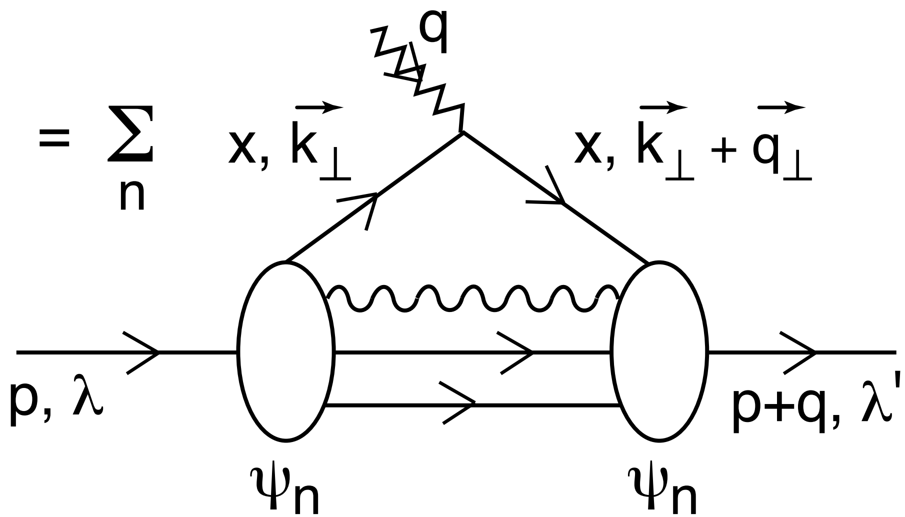
$$\frac{E(x,\zeta,0)}{2M} = \sum_a (\sqrt{1-\zeta})^{1-n} \sum_j \delta(x - x_j) \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \times \psi_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i) \mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}} \psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i)\,,$$

with $x_j' = (x_j - \zeta)/(1-\zeta)$ for the struck parton j and $x_i' = x_i/(1-\zeta)$ for the spectator parton i.

The E distribution function is related to a $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}}$ matrix element at finite ζ as well.

Form Factors $\ell p \to \ell' p'$ $\langle p' \lambda' | J^+ (0) | p \lambda \rangle$

$F_{\lambda\lambda'}(Q^2) = \sum_n$

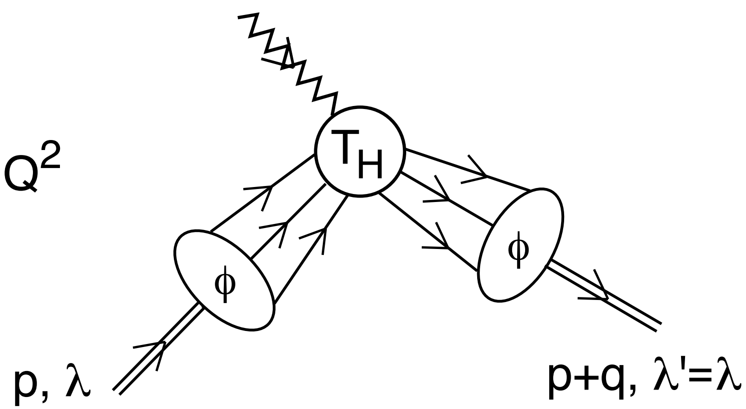


Lepage, Sjb
Efremov
Radyushkin

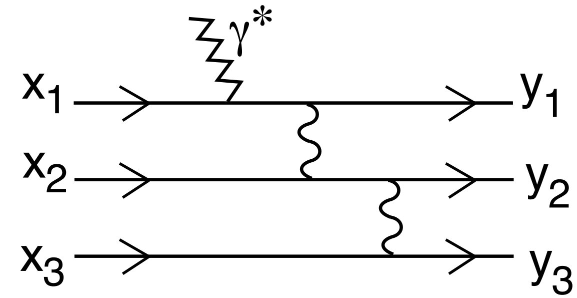
QCD Factorization

Scaling Laws from PQCD or AdS/CFT

⇨ Large Q^2



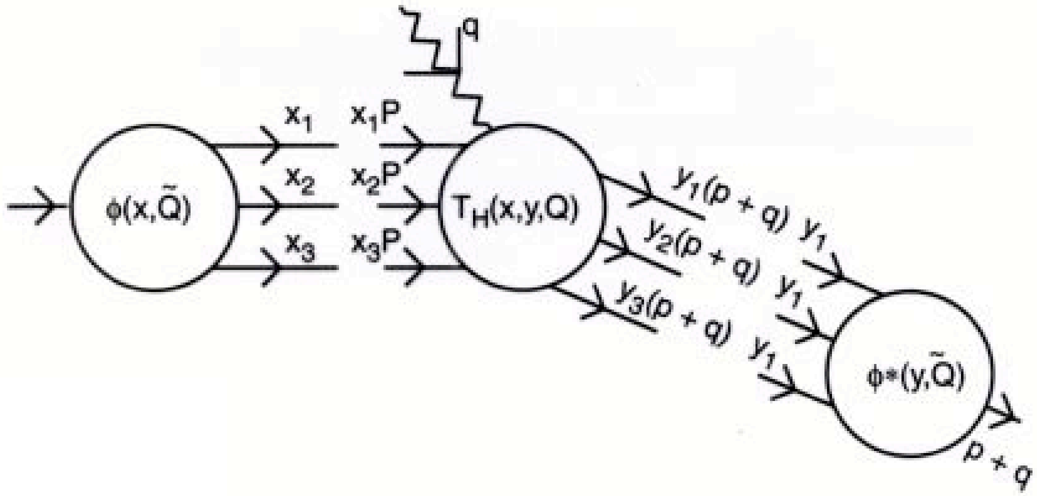
$T_H = \sum$



+ ...

$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i,)$$

Another example of hadronization: Exclusive Amplitudes



$$M = \int \prod dx_i dy_i \phi_F(x,\tilde{Q}) \times T_H(x_i,y_i,\tilde{Q}) \phi_I(y_i,Q)$$

T_H emphasizes short distances at high Q

$$\phi(x_i,Q) \equiv \Pi_{i=1}^{n-1} \int^Q d^2\vec{k}_\perp \ \psi_n(x_i,\vec{k}_{\perp i})$$

Hadron Distribution Amplitude

Hadron Distribution Amplitudes

$$\phi(x_i, Q) \equiv \prod_{i=1}^{n-1} \int^Q d^2\vec{k}_\perp \ \psi_n(x_i, \vec{k}_{\perp i})$$

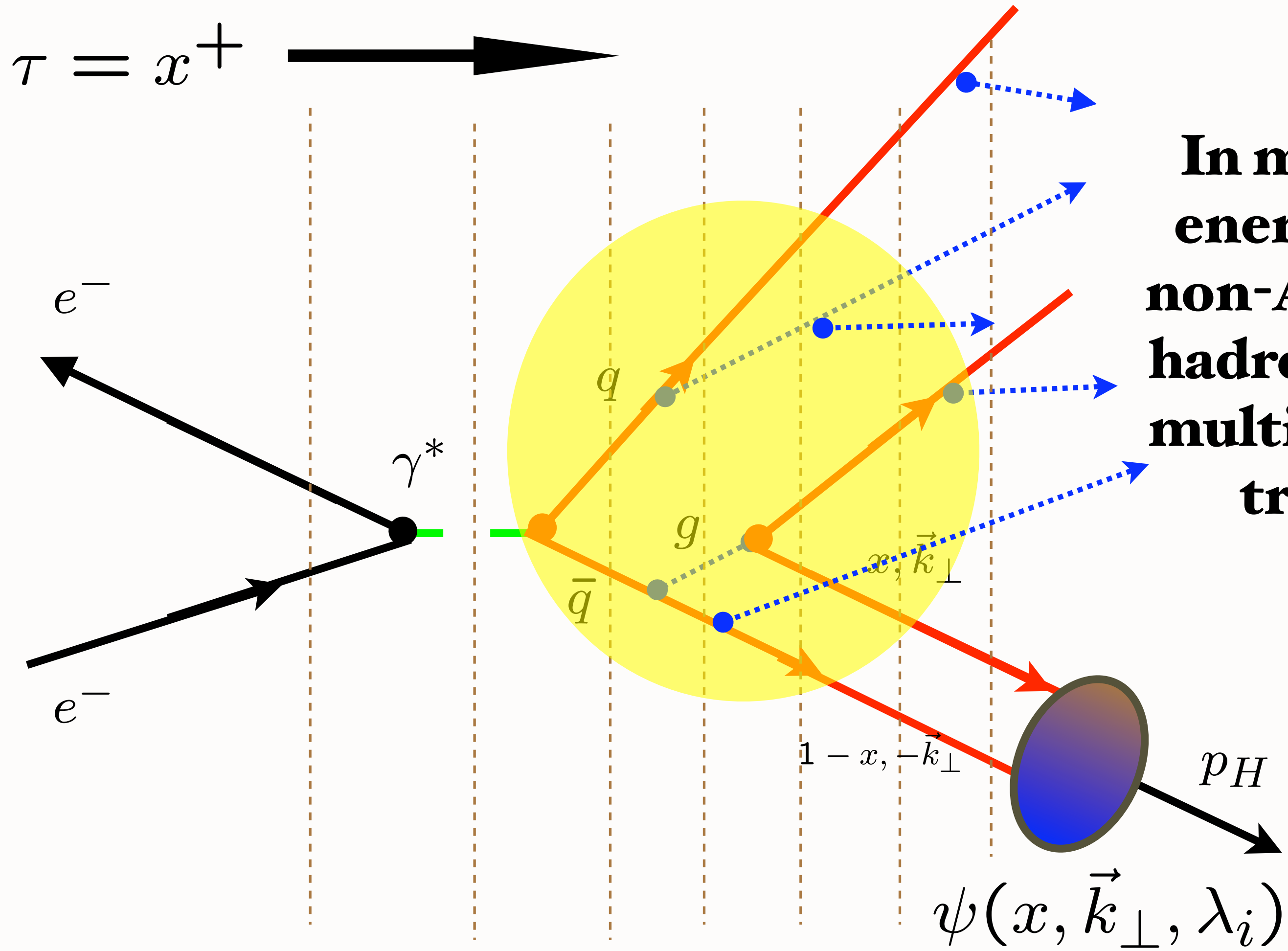
- Fundamental measure of valence wavefunction

Lepage, SJB

- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

$$\phi_\pi(x, Q) = P_\pi^+ \int \frac{dz^-}{4\pi} e^{i\pi P_\pi^+ z^-/2} < 0|\psi(0)\frac{\gamma^+\gamma^5}{2\sqrt{2n_C}}\psi(z)|\pi >^{(Q)} |_{z^+=\vec{z}_\perp=0}$$

Hadronization at the Amplitude Level



In medium: study energy loss, LPM, non-Abelian effects; hadronization time, multiplicities; color transparency

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

QCD Lagrangian

gluon dynamics — quark kinetic energy + quark-gluon dynamics — mass term

$$L_{QCD} = -\frac{1}{4g^2}\,\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{nf} i\,\overline{\psi_f}\,D_\mu\,\gamma^\mu\,\psi_f + \sum_{f=1}^{nf} m_f\,\overline{\psi_f}\,\psi_f$$

QCD color charge — field strength tensor — covariant derivative — quark field

QCD: $N_C = 3$ Quarks: 3_C Gluons: 8_C.

$\alpha_s = \frac{g^2}{4\pi}$ is dimensionless

Classical Lagrangian is scale invariant for massless quarks

If $\beta = \frac{d\alpha_s(Q^2)}{d\log Q^2} = 0$ then QCD is invariant under conformal transformations:

Parisi

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

- Holographic Model: Initial “semi-classical” approximation to QCD: Remarkable agreement with light hadron spectroscopy

- Mapping to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD} ; variational methods

Strongly Coupled Conformal QCD and Holography

- Conformal Theories are invariant under the Poincaré and conformal transformations with $M^{\mu\nu}$, P^μ, D, K^μ, the generators of $SO(4,2)$.

- QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops. For $\beta = d\alpha_s(Q^2)/dQ^2$, QCD is a conformal theory: Parisi, Phys. Lett. B **39**, 643 (1972).

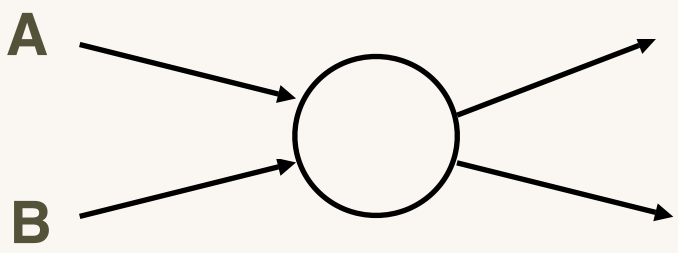
- Growing theoretical and empirical evidence that $\alpha_s(Q^2)$ has an IR fixed point: von Smekal, Alkofer and Hauck, arXiv:hep-ph/9705242; Alkofer, Fischer and Llanes-Estrada, hep-th/0412330; Deur, Burkert, Chen and Korsch, hep-ph/0509113 . . .

- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies Brodsky and Farrar, Phys. Rev. Lett. **31**, 1153 (1973); Matveev *et al.*, Lett. Nuovo Cim. **7**, 719 (1973).

Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H-1} \qquad -t = Q^2$$

Farrar & sjb; Matveev et al

Conformal symmetry and PQCD predicts leading-twist power behavior

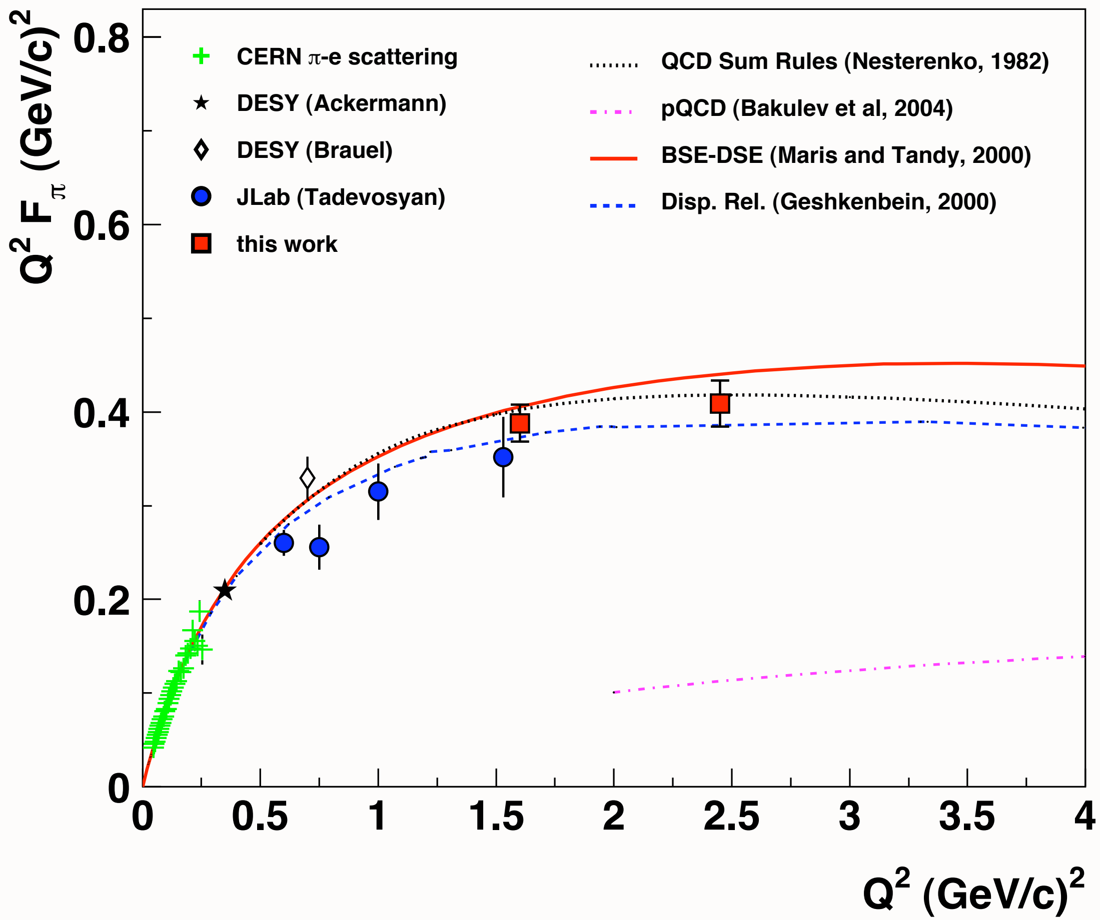
Characteristic scale of QCD: 300 MeV

Leading-Twist Scaling cannot be postponed!

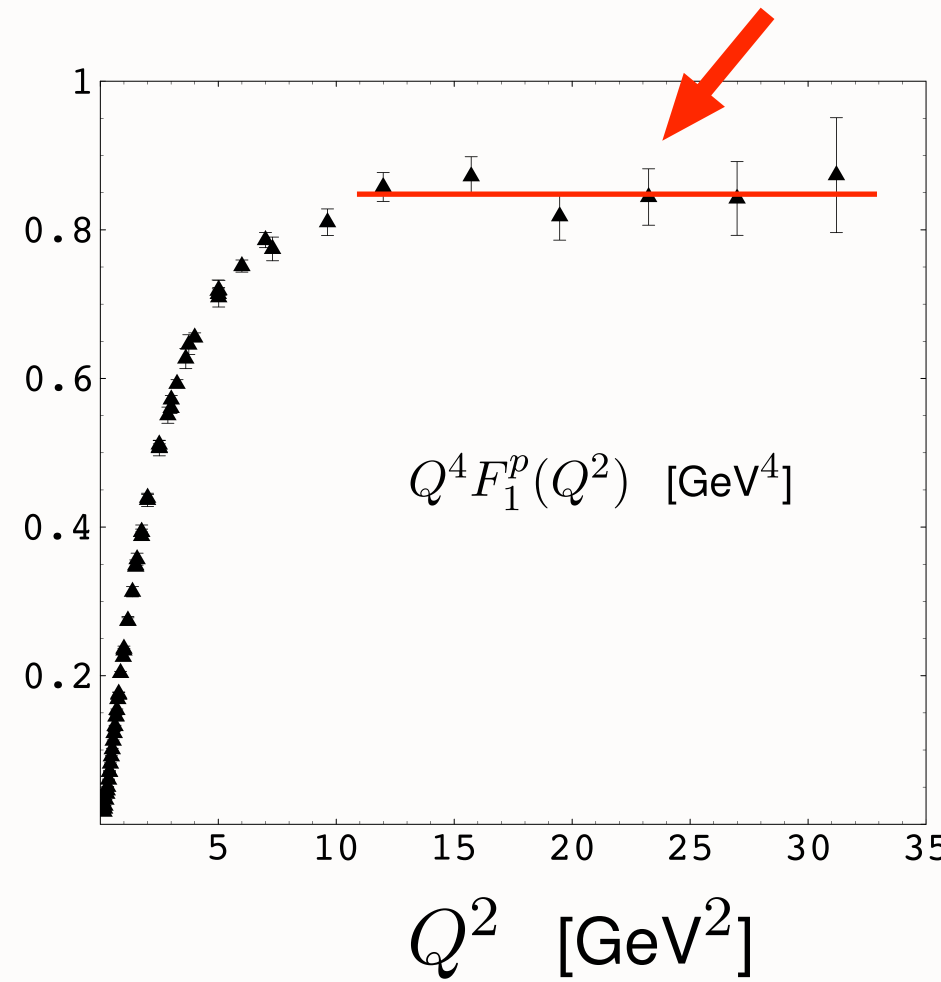
New J-PARC, GSI, J-Lab, Belle, Babar tests

Conformal behavior: $Q^2 F_\pi(Q^2) \to$ const

$Q^4 F_1(Q^2) \to$ const



Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**



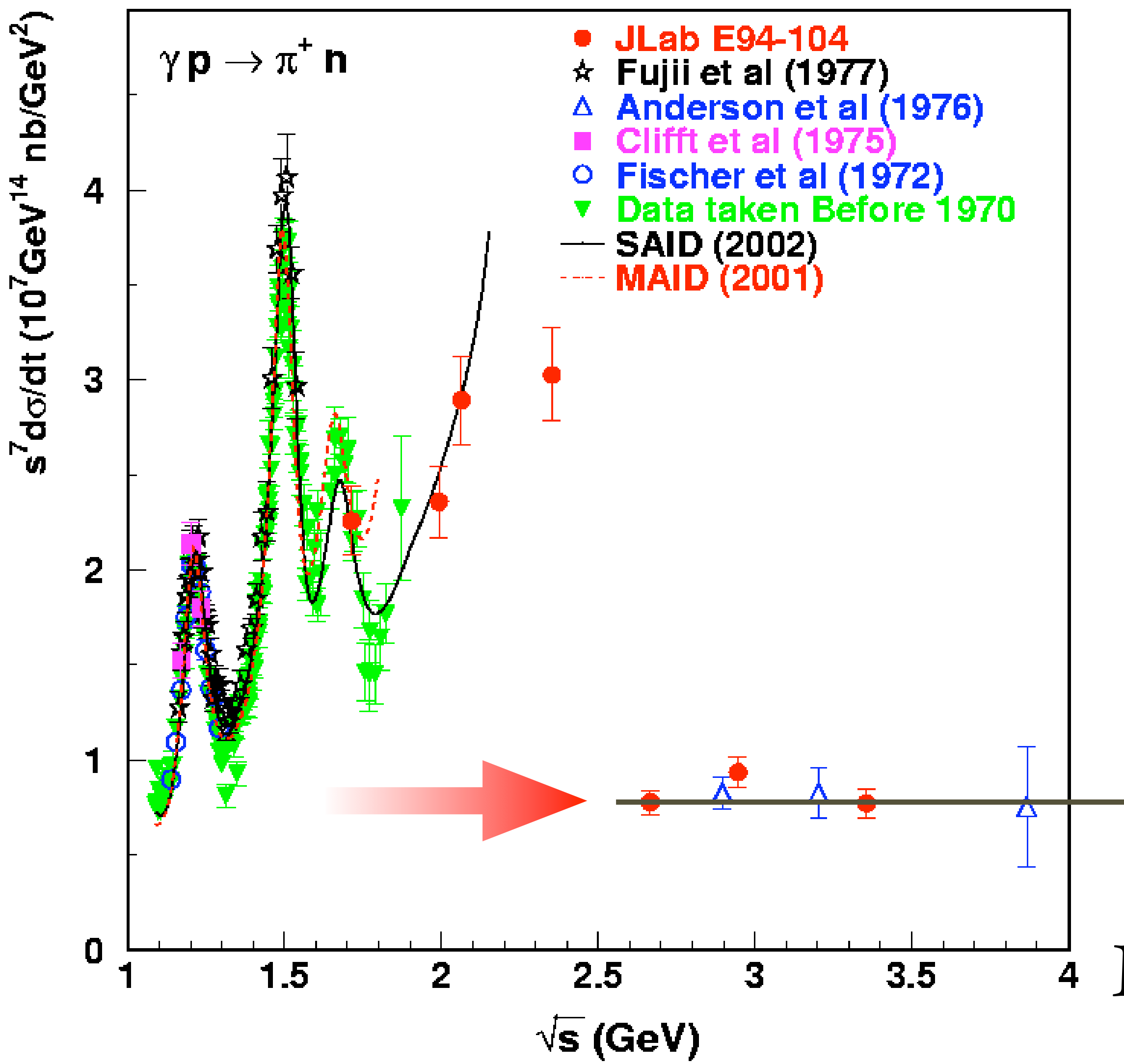
Generalized parton distributions from nucleon form-factor da
M. Diehl (**DESY**) , **Th. Feldmann** (**CERN**) , **R. Jakob**, **P. Kroll** (**W**
DESY-04-146, CERN-PH-04-154, WUB-04-08, Aug 2004. 68pp.
Published in Eur.Phys.J.C39:1-39,2005
e-Print Archive: **hep-ph/0408173**

G. Huber

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Taveklidze



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$

fixed θ_{CM} scaling

PQCD and AdS/CFT:

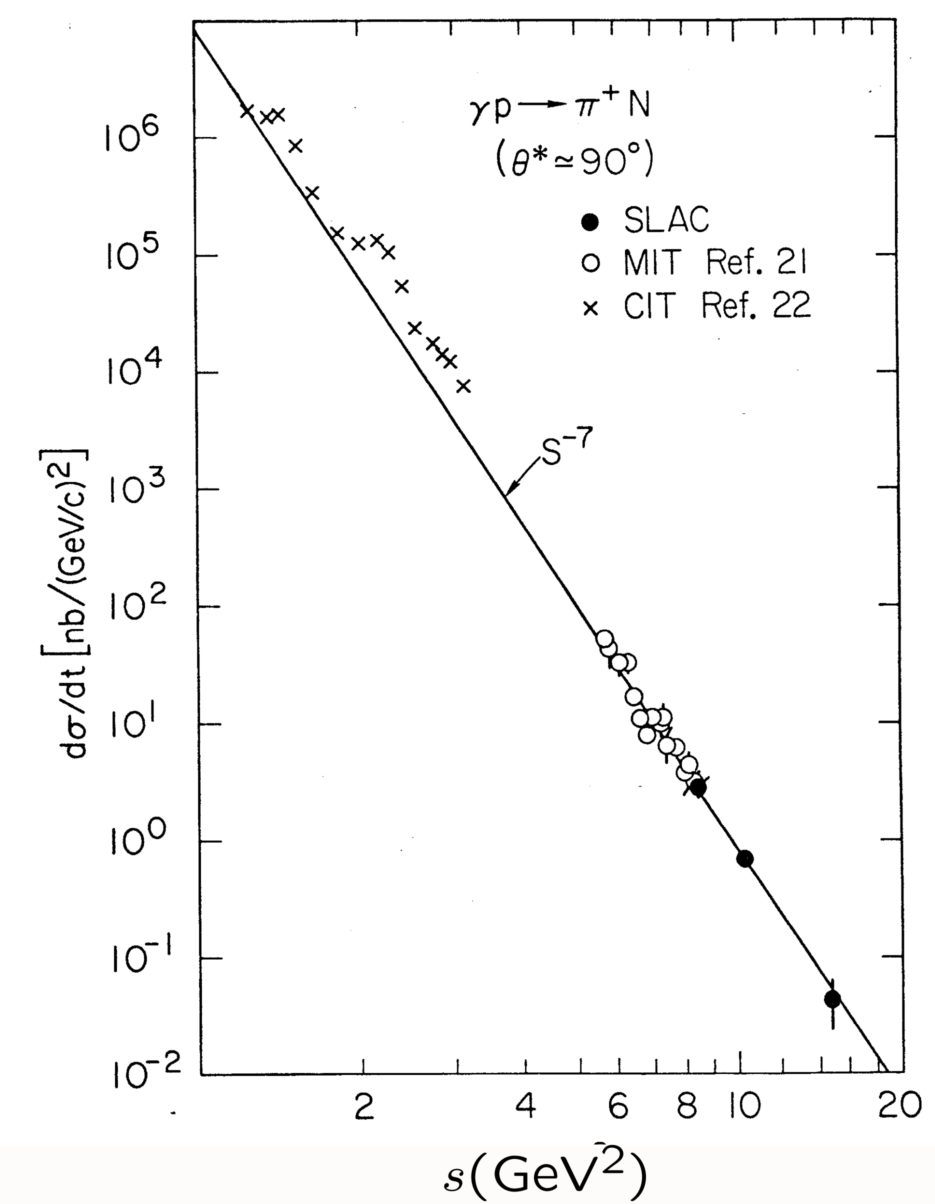
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

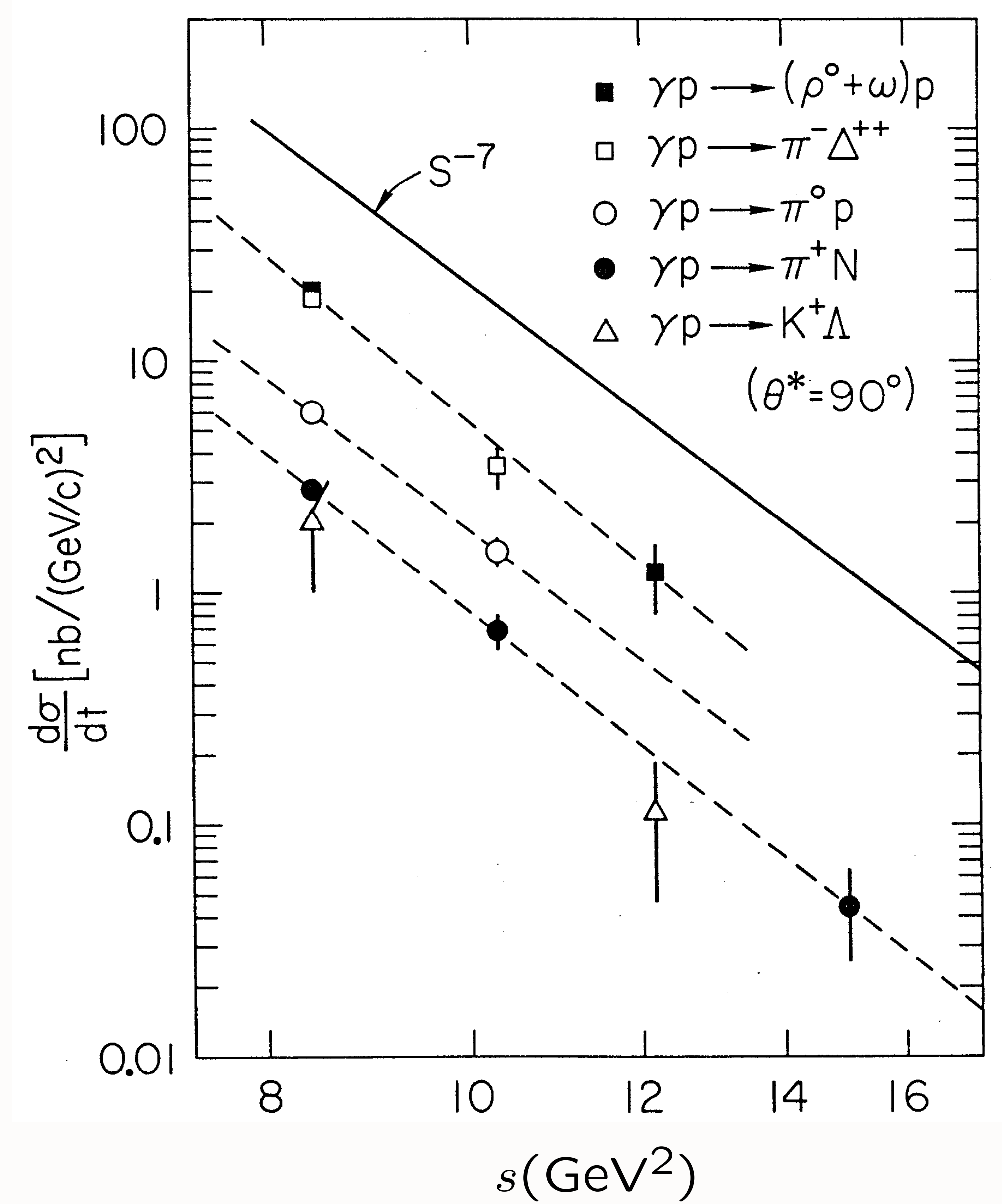
$$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

No sign of running coupling

Conformal invariance at high momentum transfer!

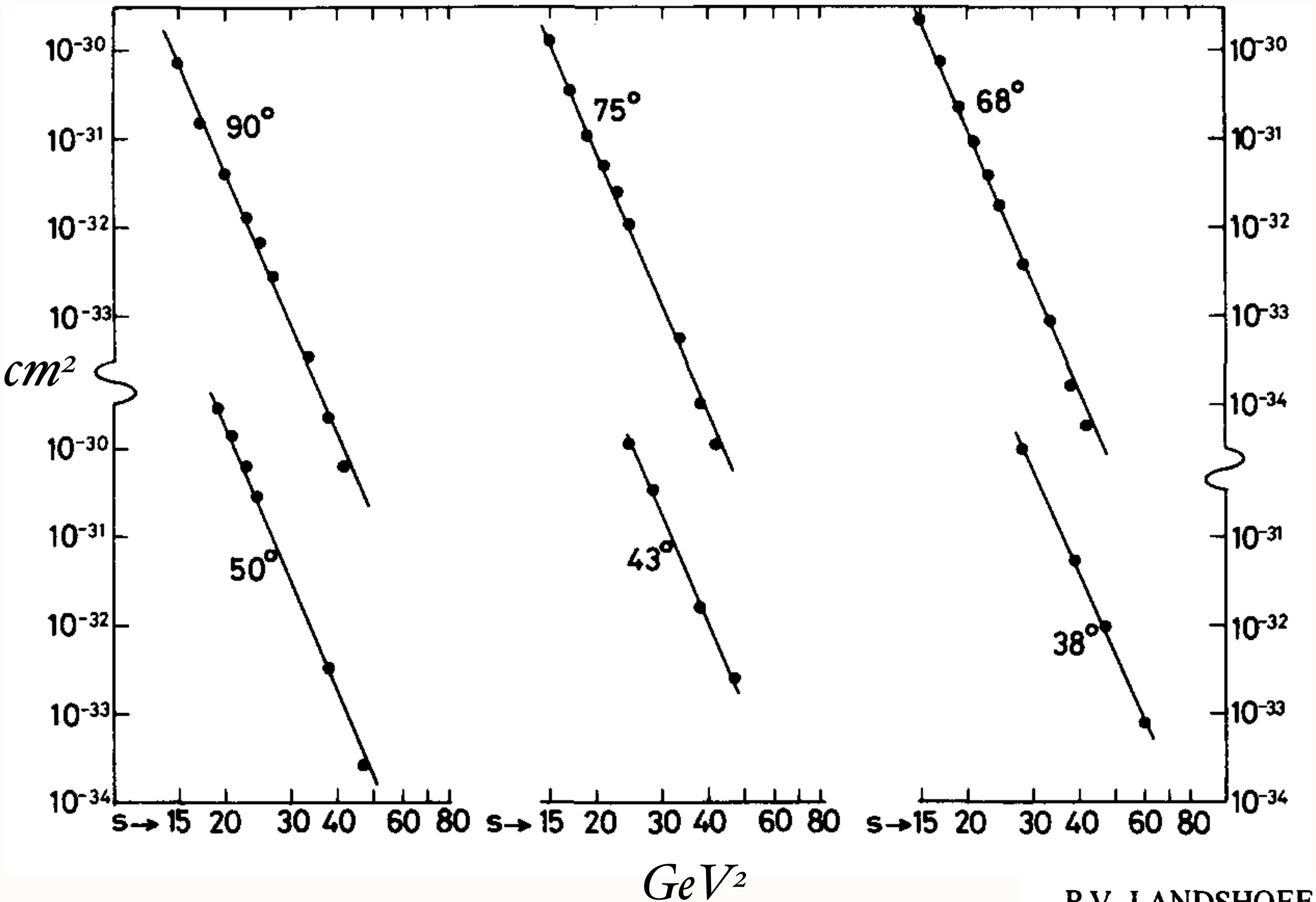




Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $\quad n = 4 \times 3 - 2 = 10$



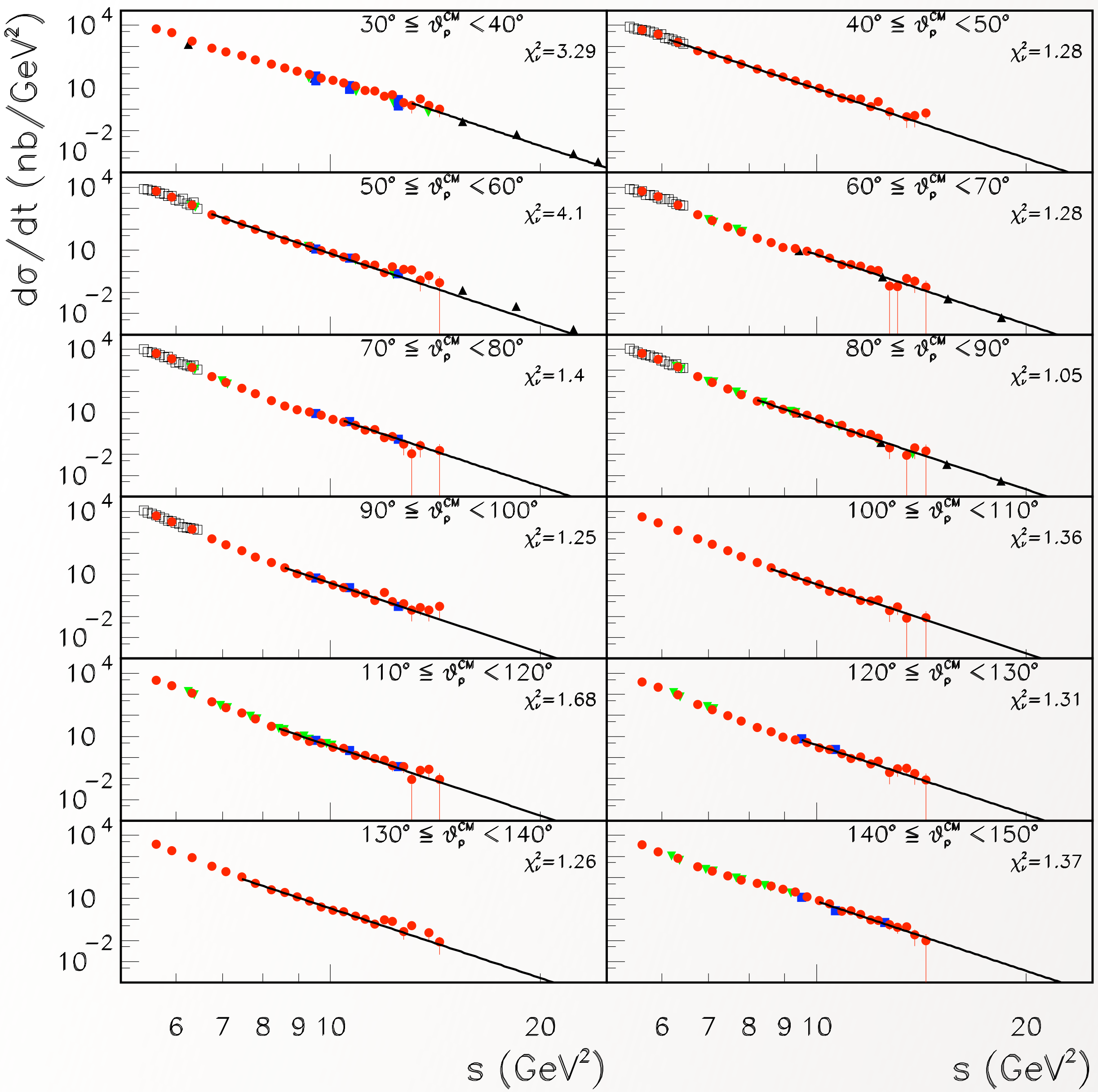
P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

Deuteron Photodisintegration



J-Lab

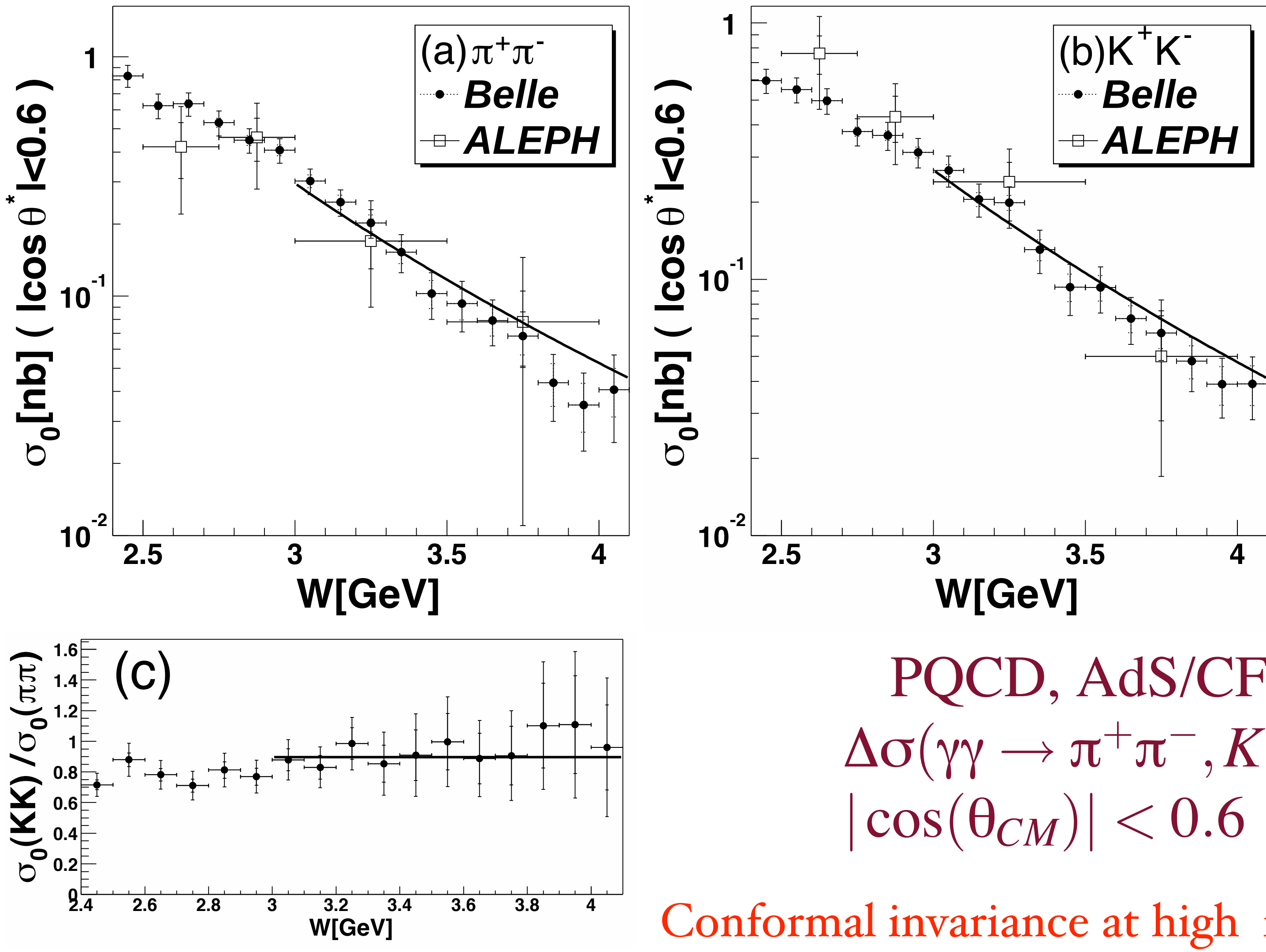
PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$n_{tot}-2 =$
(1 + 6 + 3+ 3) - 2 = 11

Conformal invariance at high momentum transfers!



Two-Photon Reactions

Hard Exclusive Processes: Fixed angle

PQCD, AdS/CFT:

$$\Delta\sigma(\gamma\gamma \to \pi^+\pi^-, K^+, K^-) \sim 1/W^6$$

$$|\cos(\theta_{CM})| < 0.6$$

Conformal invariance at high momentum transfers!

Fig. 5. Cross section for (a) $\gamma\gamma\to\pi^+\pi^-$, (b) $\gamma\gamma\to K^+K^-$ in the c.m. angular region $|\cos\theta^*| < 0.6$ together with a W^{-6} dependence line derived from the fit of $s|R_M|$. (c) shows the cross section ratio. The solid line is the result of the fit for the data above 3 GeV. The errors indicated by short ticks are statistical only.

Why do dimensional counting rules work so well?

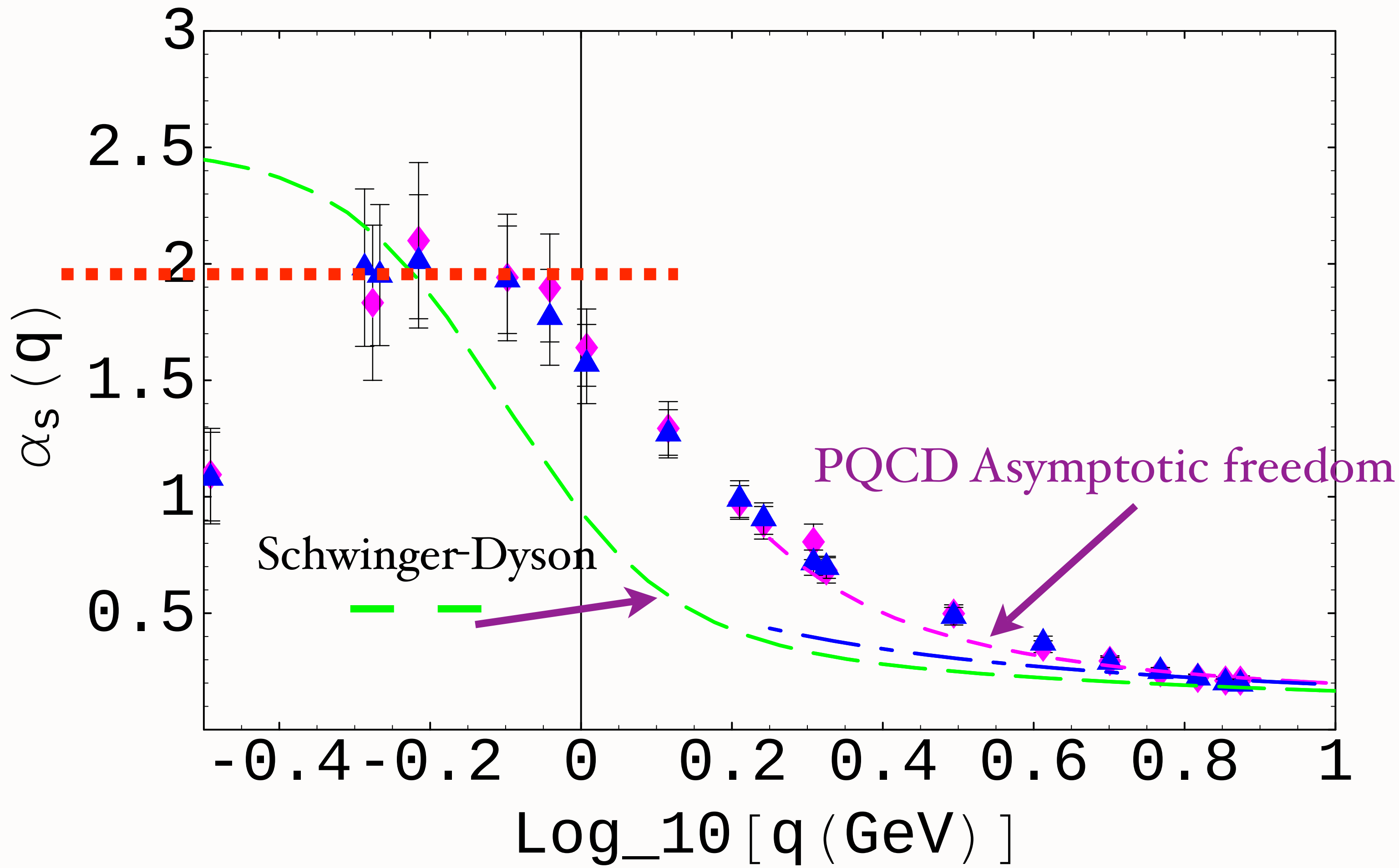
- **PQCD predicts log corrections from powers of α_s, logs, pinch contributions** Lepage, sjb; Efremov, Radyushkin

- **DSE: QCD coupling (mom scheme) has IR Fixed point!** Alkofer, Fischer, von Smekal et al.

- **Lattice results show similar flat behavior** Furui, Nakajima

- **PQCD exclusive amplitudes dominated by integration regime where α_s is large and flat**

Infrared-Finite QCD Coupling?



Lattice simulation (MILC)

Furui, Nakajima

DSE: Alkofer, Fischer, von Smekal et al.

Define QCD Coupling from Observable

Grunberg
Neubert
Maxwell

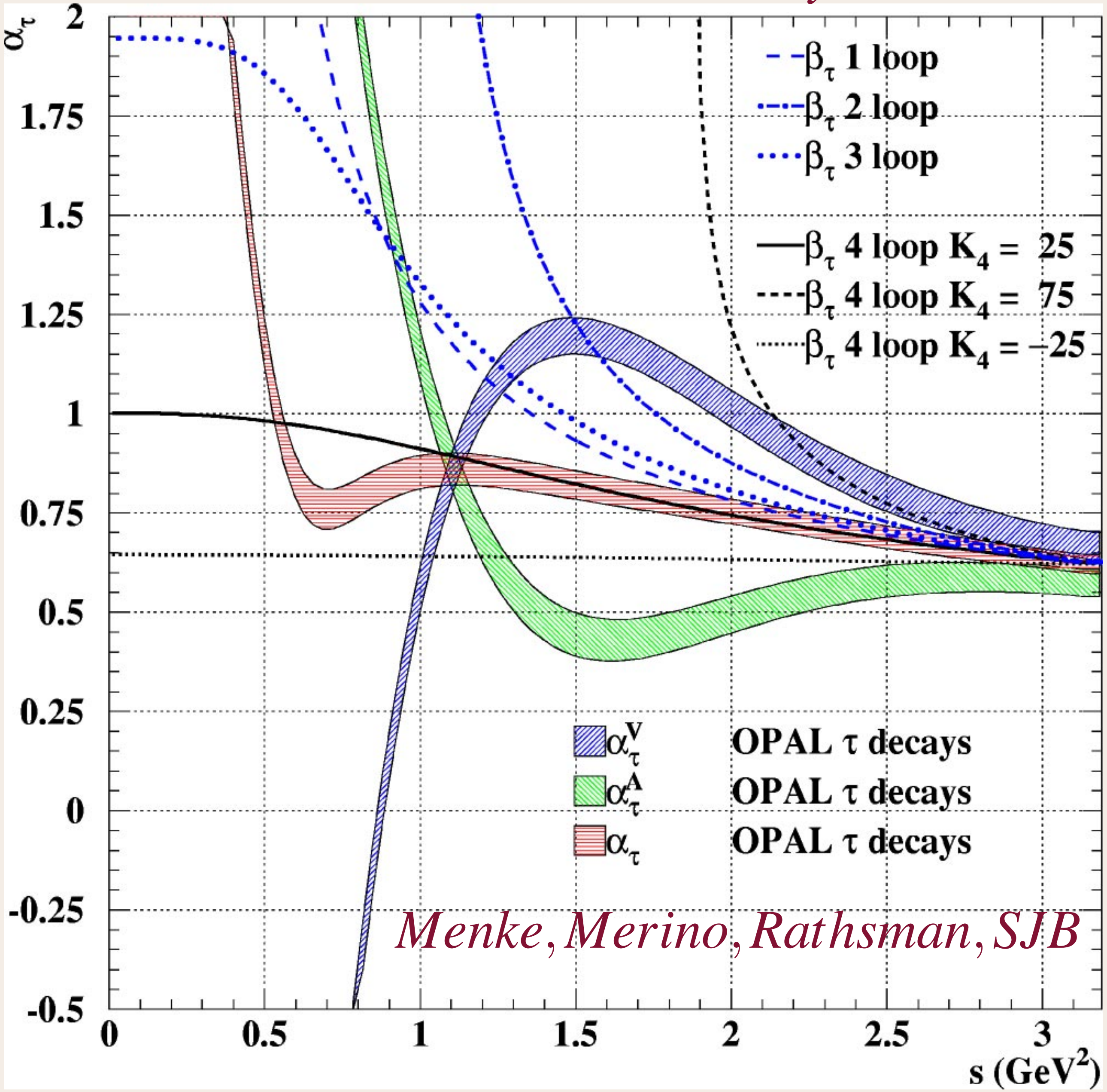
$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2\ [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to X e\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu) \times [1 + \frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Commensurate scale relations: Relate observable to observable at commensurate scales H.Lu, Rathsman, sjb

Effective Charges: analytic at quark mass thresholds, finite at small momenta

QCD Effective Coupling from *hadronic τ decay*



Menke, Merino, Rathsman, SJB

Heuristic Arguments for an IR Fixed Point

$$\alpha_s(Q^2) \simeq \text{const at small } Q^2$$

- Semi-Classical approximation to massless QCD
- No particle creation or absorption $\beta = 0$
- Conformal symmetry broken by confinement
- Effective gluon mass: vacuum polarization vanishes at small momentum transfer
- $\Pi(Q^2) \propto \frac{Q^2}{m_g^2}$ $\quad Q^2 << 4m_g^2$ $\quad \alpha_s(Q^2) \simeq \text{const}$

Analog of Serber-Uehling vacuum polarization in QED:

$$\Pi(Q^2) = \frac{\alpha}{15\pi}\frac{Q^2}{m_e^2} \qquad Q^2 << 4m_e^2$$

Decoupling of long wavelength gluonic interactions

Maldacena

AdS/CFT: mapping of $AdS_5 \times S_5$ to conformal N=4 SUSY

- QCD not conformal; however, it has some manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- IR fixed point? $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- Semi-classical approximation to QCD

Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses

- Eigensolutions of ERBL evolution equation for distribution amplitudes

V. Braun et al;
Frishman, Lepage, Sachrajda, sjb

- Commensurate scale relations: relate observables at corresponding BLM scales:
Generalized Crewther Relation

- Use AdS/CFT

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

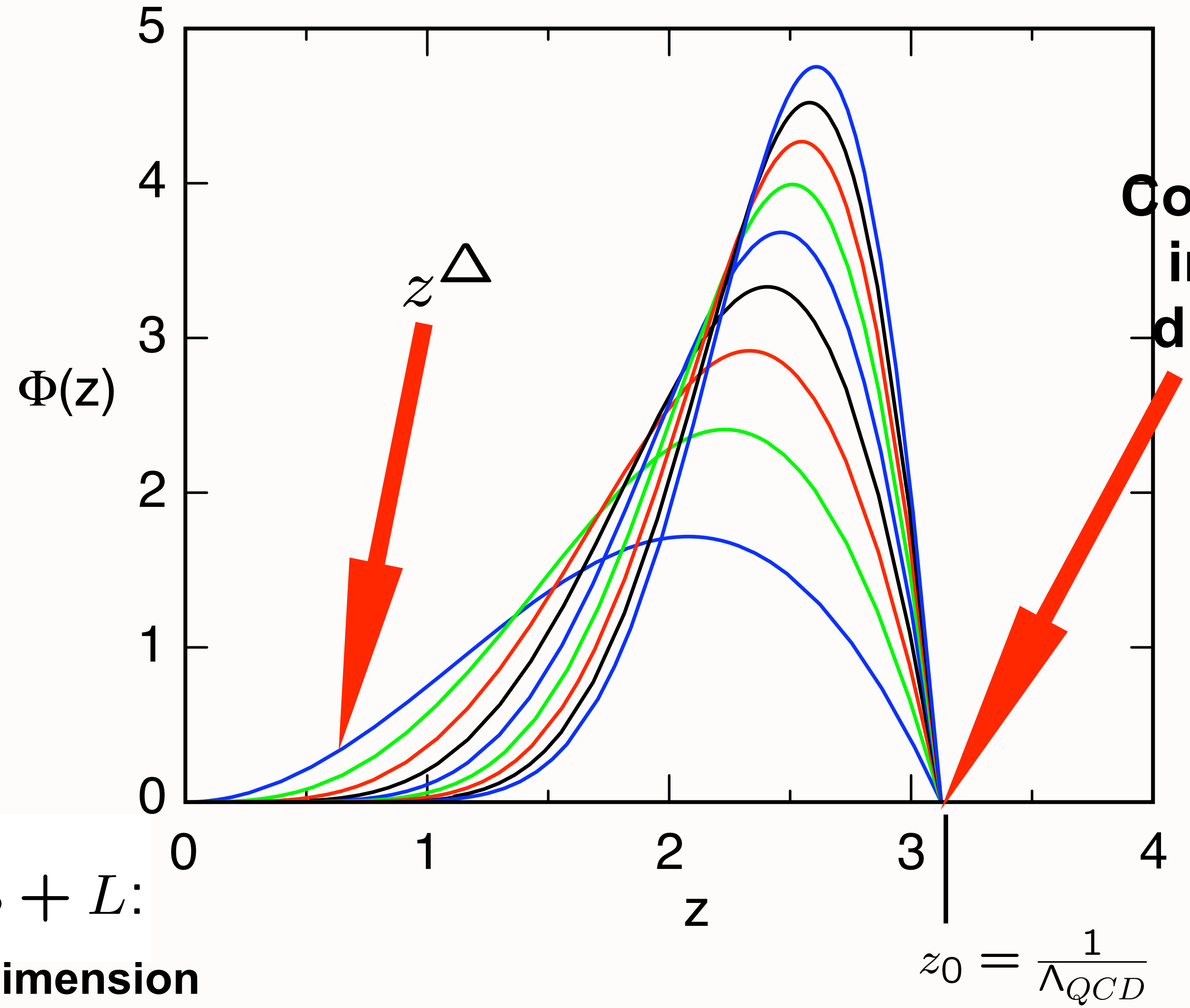
AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Alternative: Add Confining HO Potential

Identify hadron by its interpolating operator at z -- > 0



$\Delta = 3 + L$:

Twist dimension of baryon

de Teramond, sjb

Prediction from AdS/QCD

Only one parameter!

Entire light quark baryon spectrum

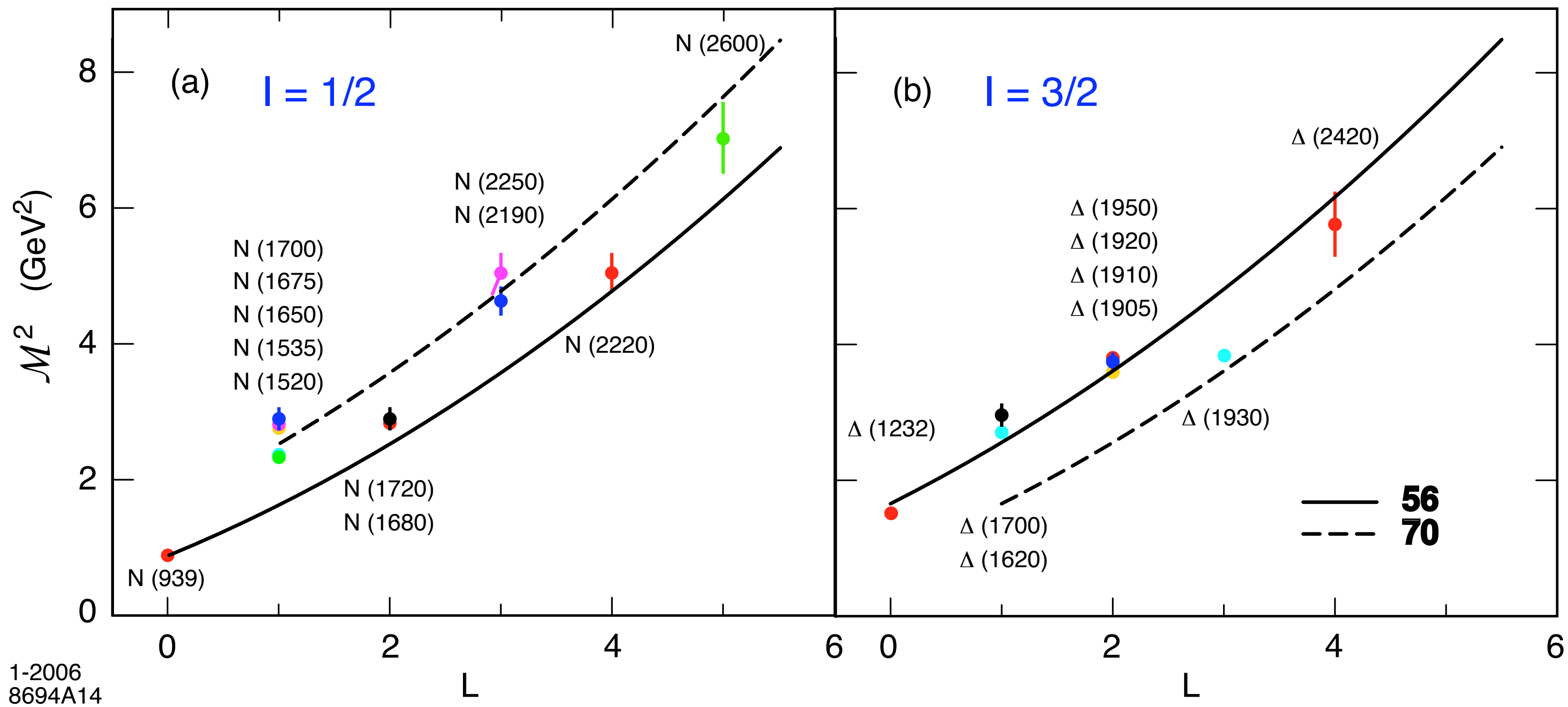


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Guy de Teramond
SJB

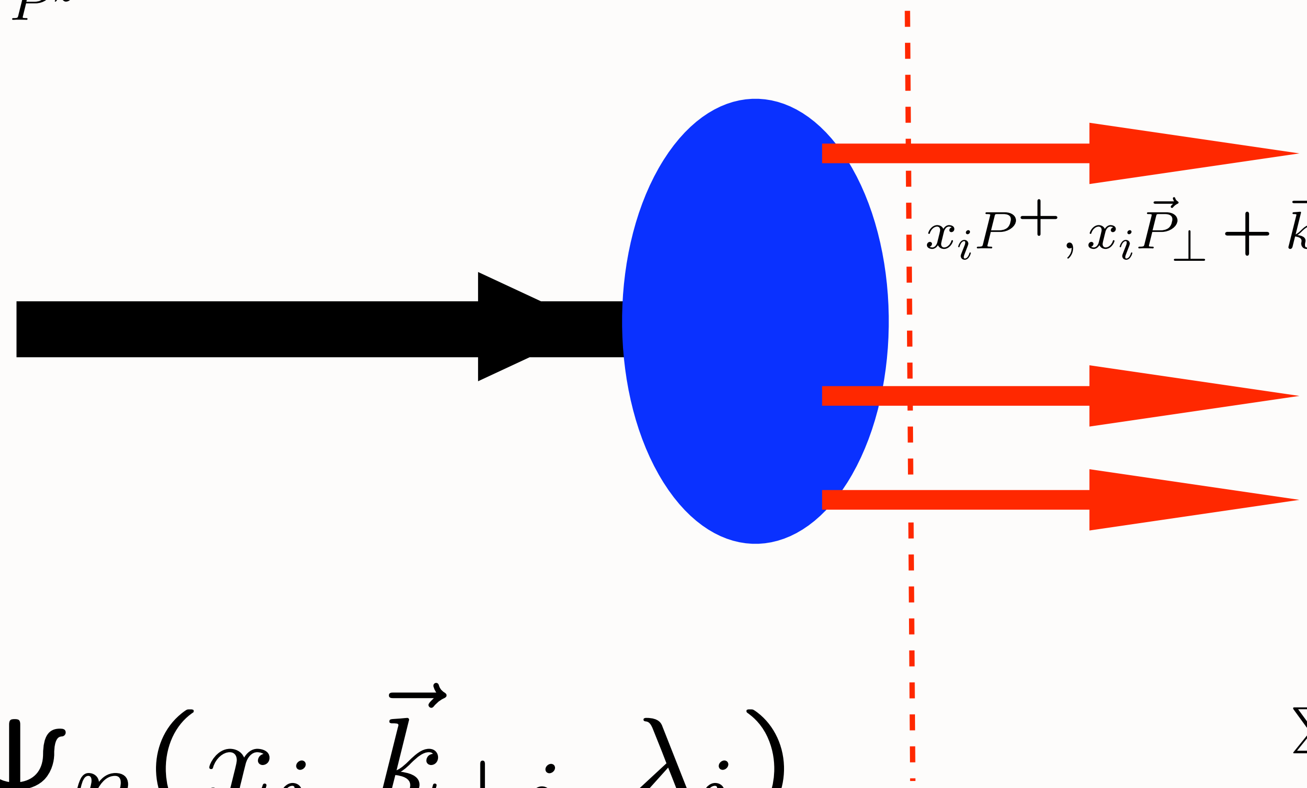
- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

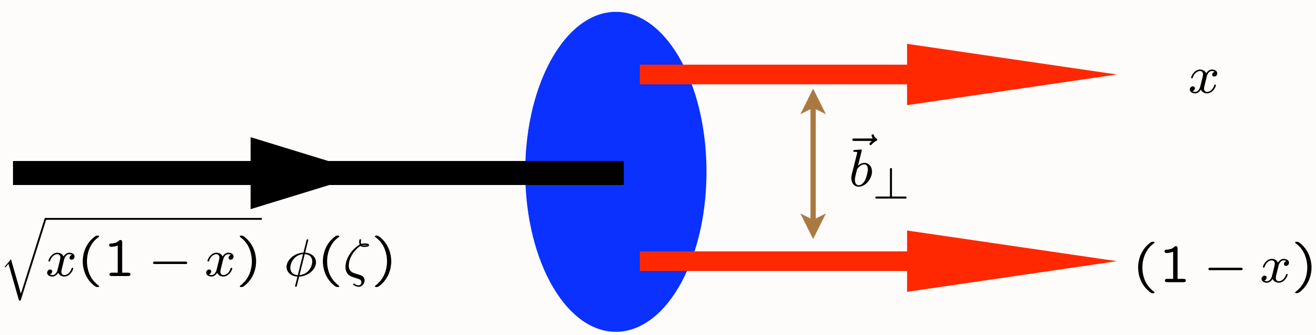
$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

G. de Teramond and sjb

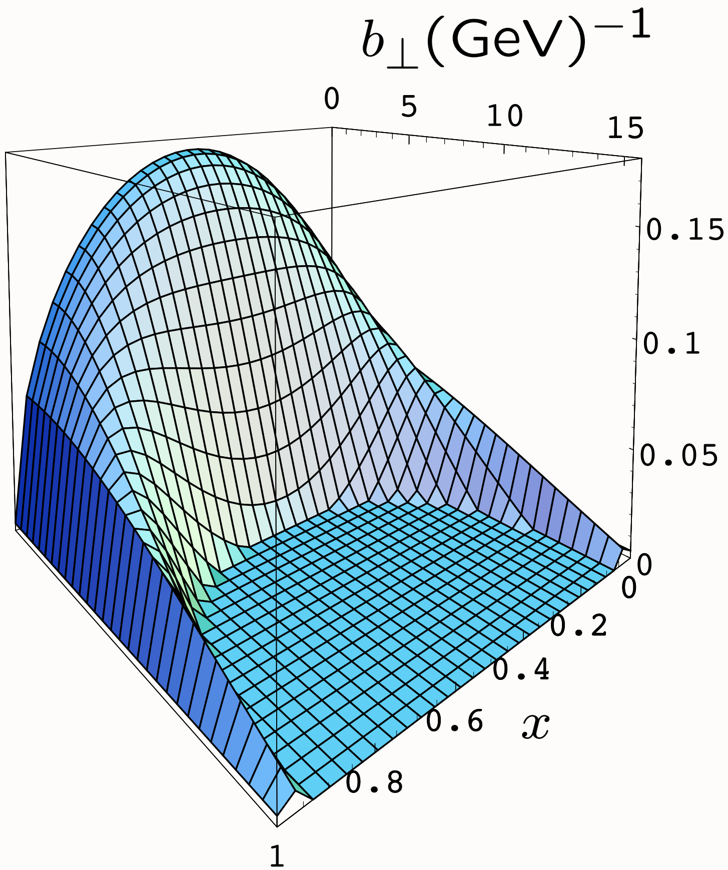
Mapping between LF(3+1) and AdS_5

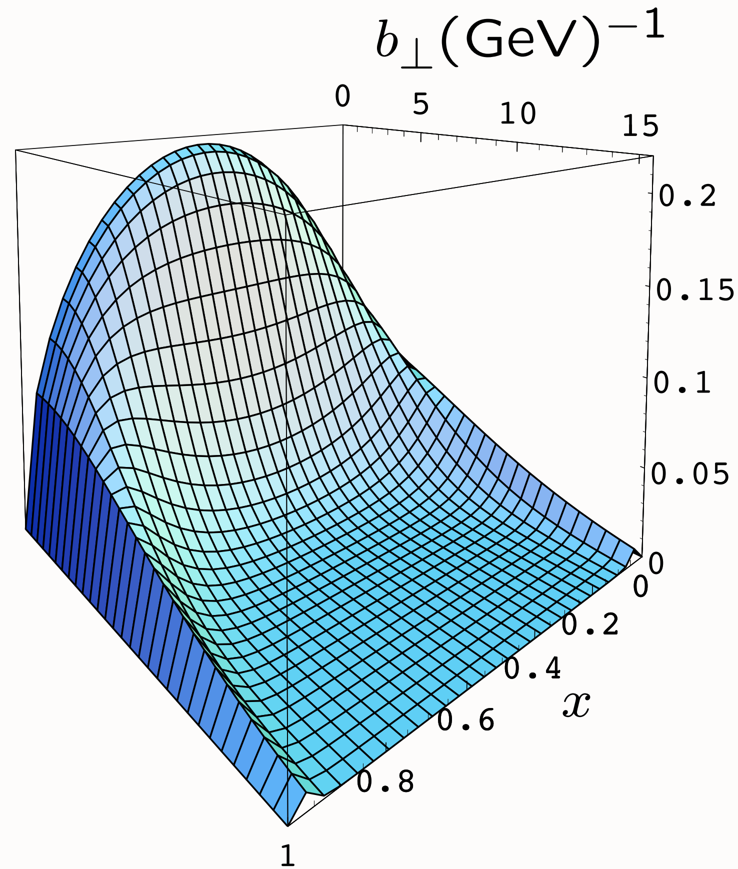
LF(3+1)		AdS_5
$\psi(x, \vec{b}_\perp)$	↔	$\phi(z)$
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$	↔	z



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$





$\Lambda_{QCD} = 0.32$ GeV

$\kappa = 0.76$ GeV.

Truncated Space

Harmonic Oscillator

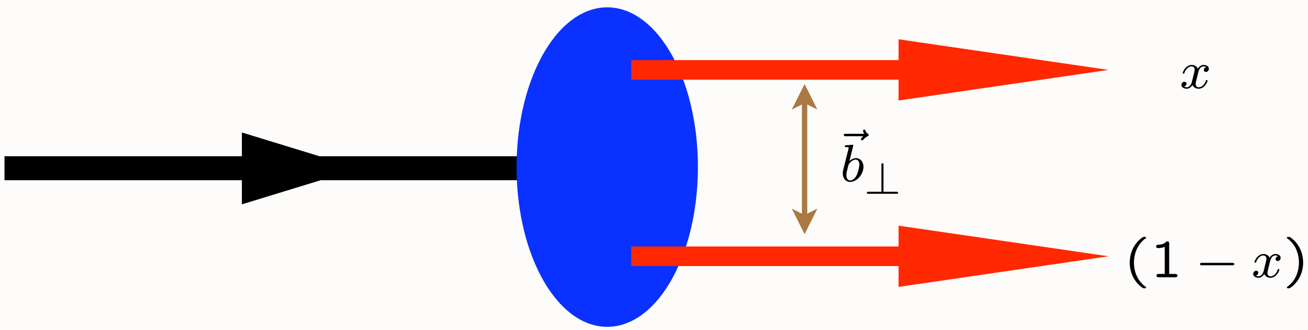
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

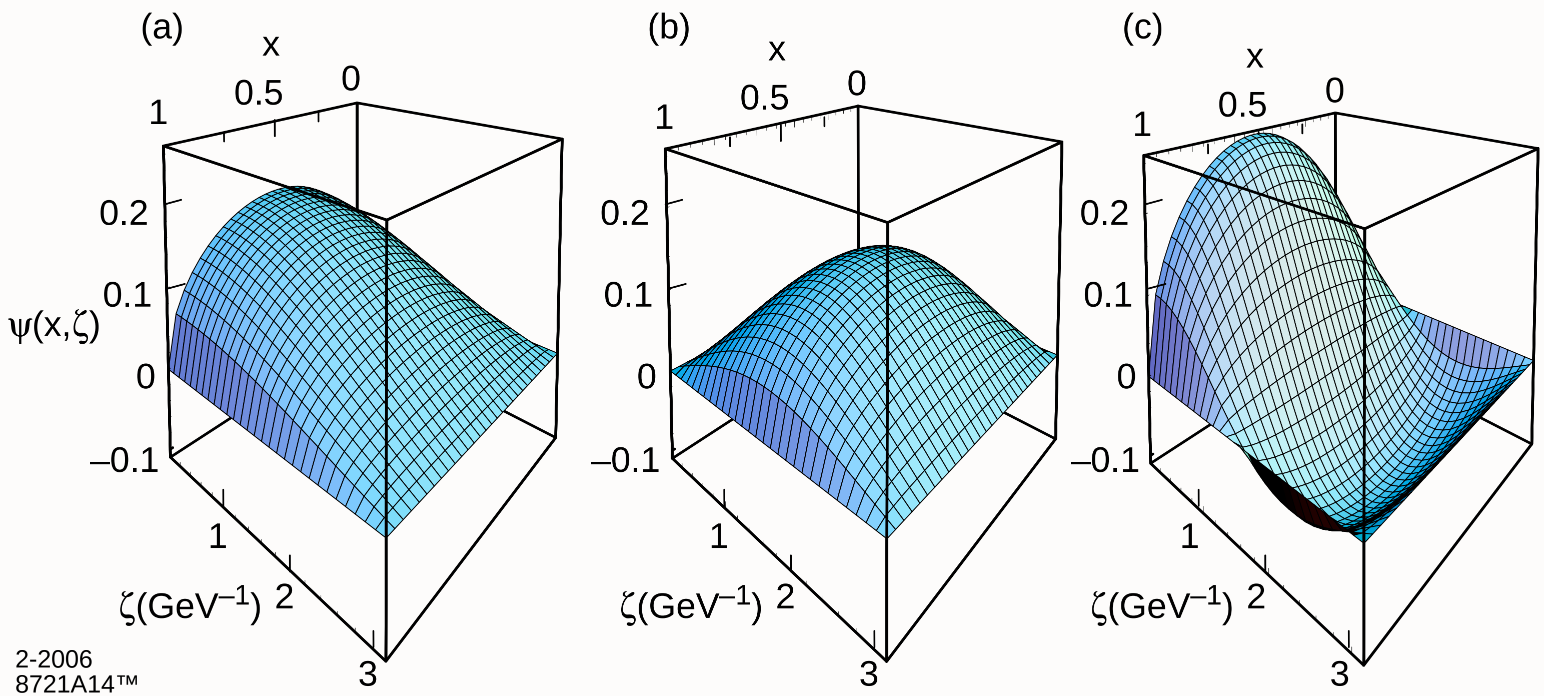


Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

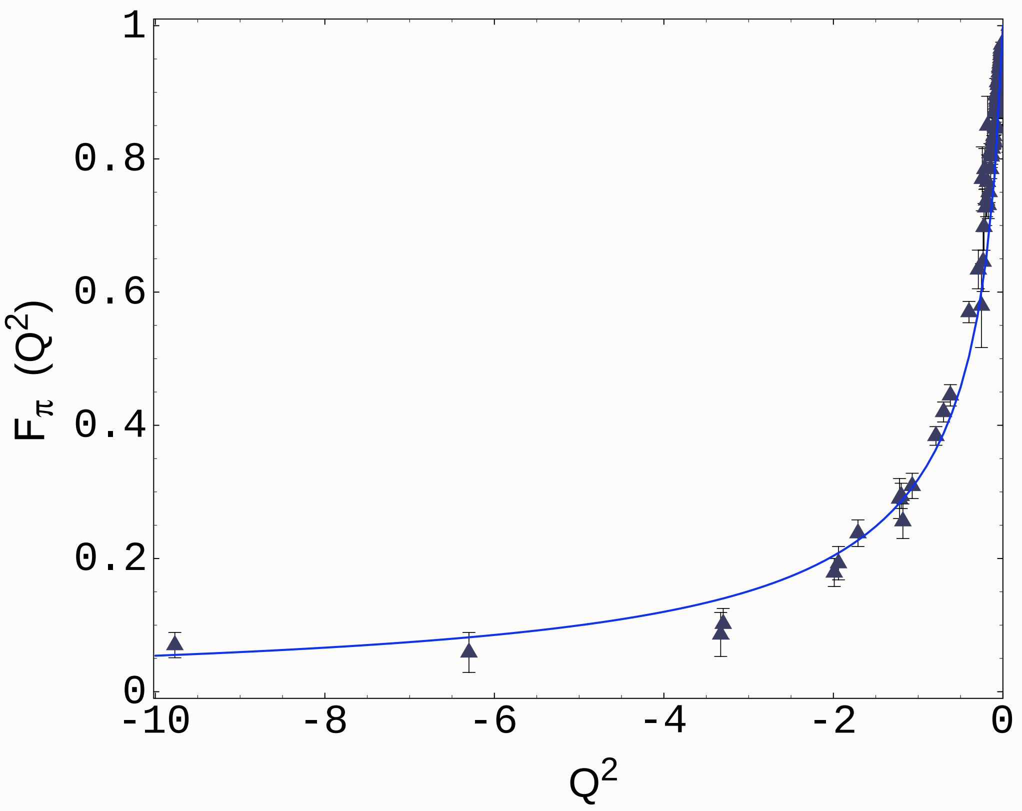
Two parton LFWF bound state:

$$\widetilde{\psi}_{\overline{q}q/\pi}(x,\zeta) = B_{L,k}\sqrt{x(1-x)}J_L\left(\zeta\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\big(z \le \Lambda^{-1}_{\rm QCD}\big),$$



(a) ground state $L=0$, $k=1$, (b) first orbital $L=1$, $k=1$, (c) first radial $L=0$, $k=2$.

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$



Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

Data Compilation from Baldini, Kloe and Volmer

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Action for scalar field in AdS_5

$$S[\Phi] = \kappa' \int d^4x dz \sqrt{g} \left[g^{\ell m} \partial_\ell \Phi^* \partial_m \Phi - \mu^2 \Phi^* \Phi \right]$$

where $[\kappa'] = L^{-2}$ $\quad g^{\ell m} = \frac{z^2}{R^2} \eta^{\ell m} \quad \sqrt{g} = R^5 / z^5$

Action is invariant under scale transformations

$$x^\mu \to \lambda x^\mu, \quad z \to \lambda z.$$

$$\Phi(x^\ell) = \Phi(\lambda x^\ell)$$

Variation wrt Φ

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0$$

Solutions of form: $\Phi(x,z) = e^{-iP\cdot x} f(z)$ $P_\mu P^\mu = \mathcal{M}^2$

$$S = -\kappa R^3 \int \frac{dz}{z^3} \left[(\partial_z f)^2 - \mathcal{M}^2 f^2 + \frac{(\mu R)^2}{z^2} f^2 \right]$$

Variation of S wrt f :

$$z^5 \partial_z \left(\frac{1}{z^3} \partial_z f \right) + z^2 \mathcal{M}^2 f - (\mu R)^2 f = 0.$$

$$\left[z^2 \partial_z^2 - 3z\partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] f = 0,$$

Introduce confinement, break conformal invariance

P-S Boundary Condition $f(z = \frac{1}{\Lambda_{QCD}}) = 0$

Normalization in truncated space $R^3 \int_0^{\Lambda_{\mathrm{QCD}}^{-1}} \frac{dz}{z^3}\, f^2(z) = 1$

Identify Orbital Angular Momentum $(\mu R)^2 = -4 + L^2$

- Wave equation in AdS for bound state of two scalar partons with conformal dimension $\Delta = 2 + L$

$$\left[z^2\partial_z^2 - 3z\,\partial_z + z^2\,\mathcal{M}^2 - L^2 + 4\right]\Phi(z) = 0,$$

with solution

$$\Phi(z) = Ce^{-iP\cdot x}z^2 J_L(z\mathcal{M}).$$

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^n \sigma_i.$
- The twist τ is equal to the number of partons $\tau = n$.

Introduce confinement, break conformal invariance

$$f(z = \frac{1}{\Lambda_{QCD}}) = 0$$

Match fall-off at small z to Conformal Dimension of hadron state at short distances

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$

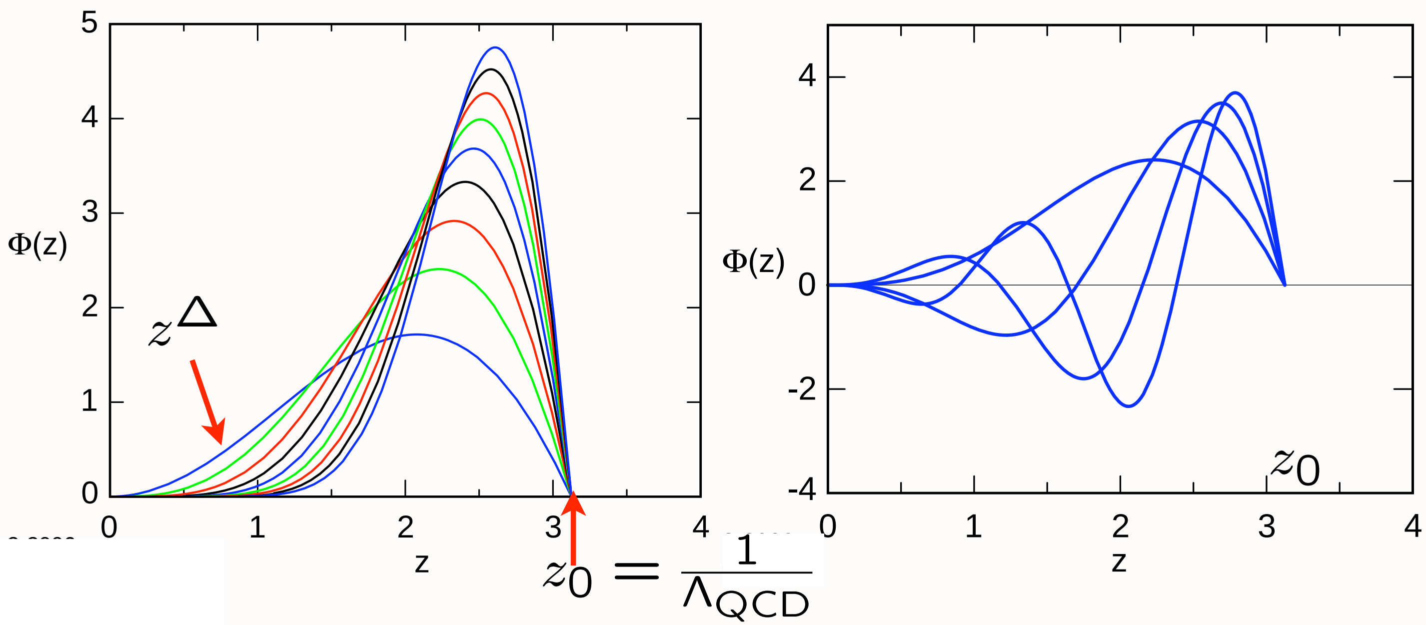
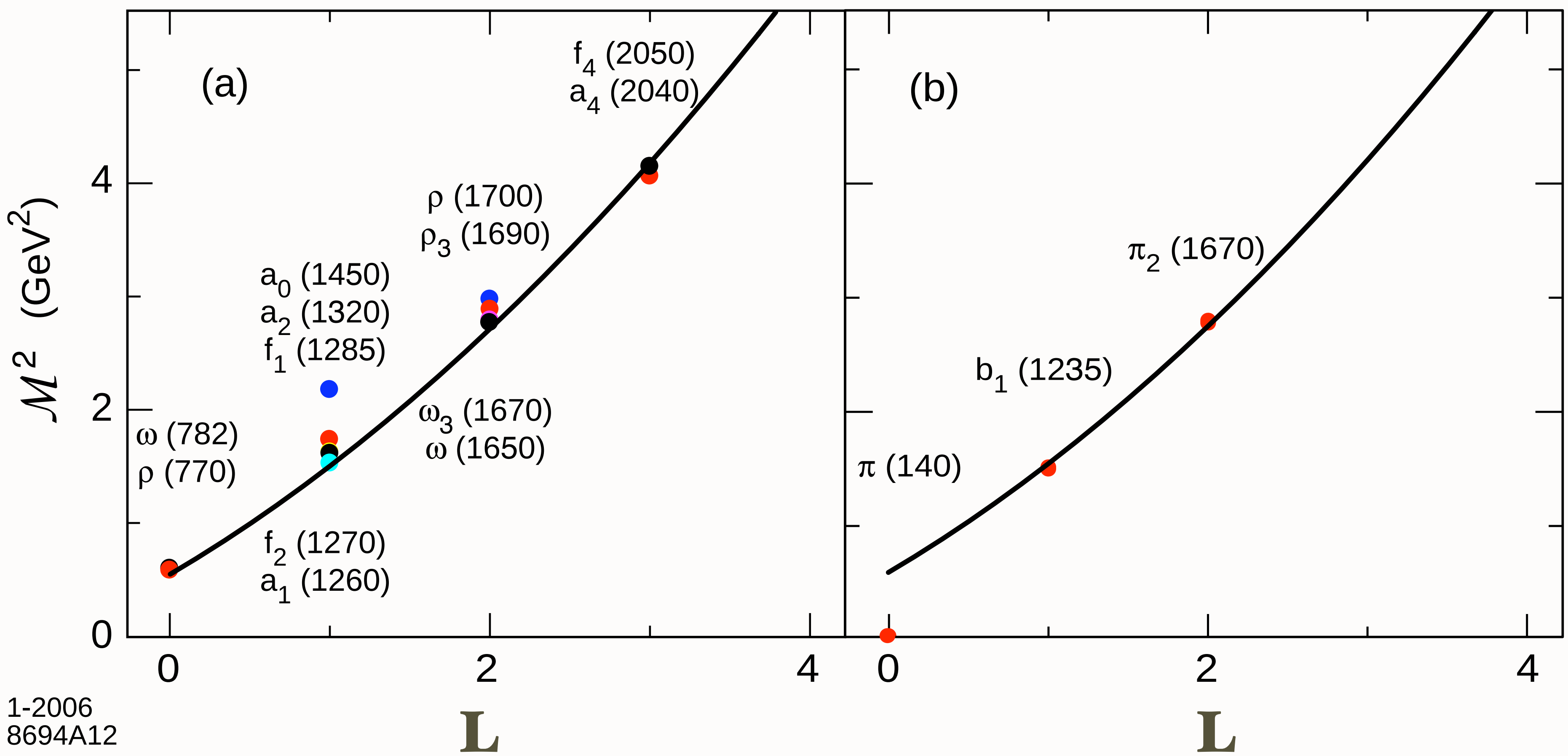


Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.



Light meson orbital spectrum $\Lambda_{QCD} = 0.32 \text{ GeV}$

Guy de Teramond
SJB

Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

Wave Equation:

$$\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P)\Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow \quad$ parallel Regge trajectories for baryons !

$$\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k} \Lambda_{QCD}, \qquad \mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Predictions of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum

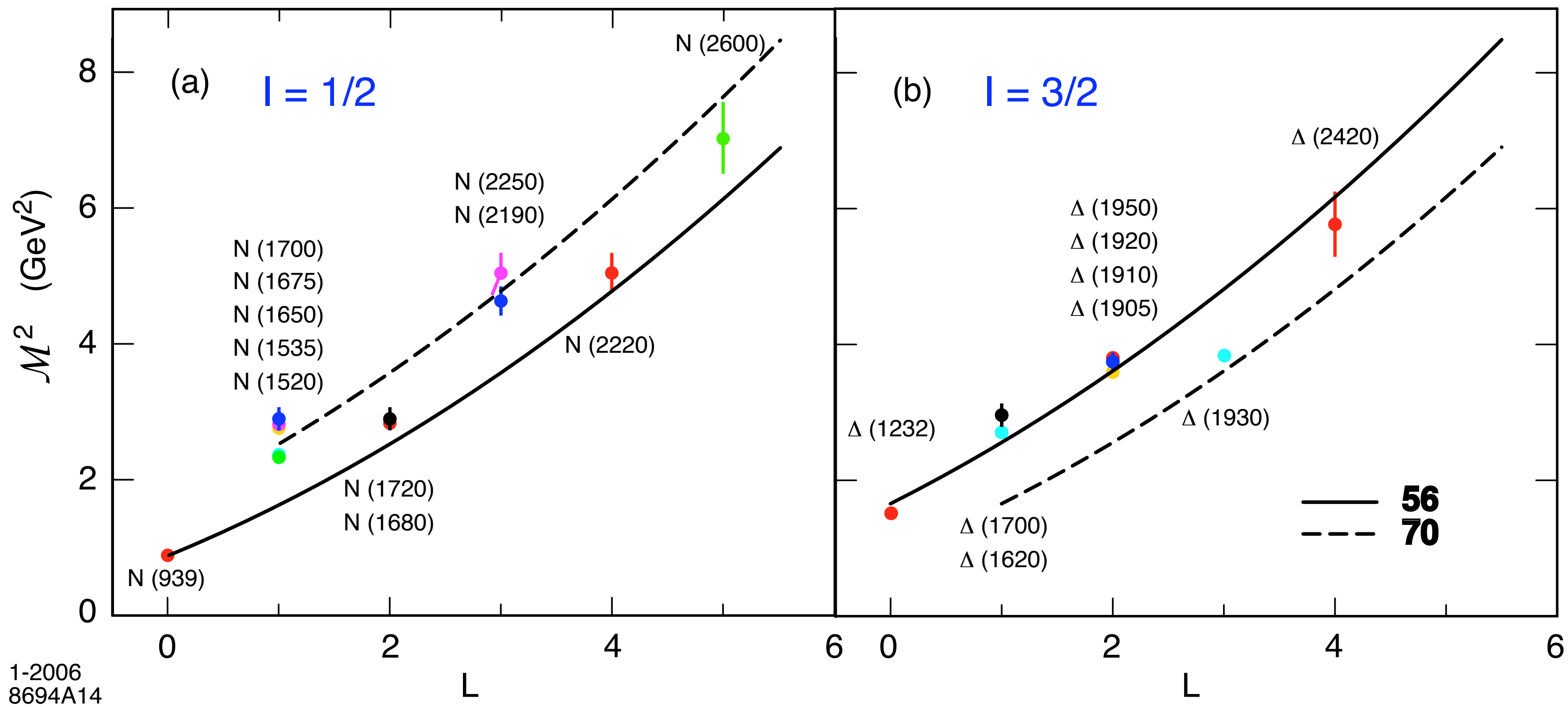


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Guy de Teramond
SJB

AdS/CFT and Integrability

- Conformal Symmetry plus Confinement: Reduce AdS/QCD Equations to Linear Form
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations
- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).

AdS/CFT LF Equation for Mesons with HO Confinement

Karch, et al.

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

LF Hamiltonian

$$H^\nu_{LF}\phi_\nu = \mathcal{M}^2_\nu\phi_\nu$$ **Bilinear** $$H^\nu_{LF} = \Pi^\dagger_\nu \Pi_\nu,$$

where

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

de Teramond, sjb

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)\right] = \frac{2\nu+1}{\zeta^2} - 2\kappa^2.$$

AdS/CFT LF Equation for Mesons with HO Confinement

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

Define $$b_\nu^\dagger = -i\Pi_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta$$

$$b_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \qquad b_\nu^\dagger b_\nu = b_{\nu+1} b_{\nu+1}^\dagger$$

Ladder Operator $$b_\nu^\dagger|\nu\rangle = c_\nu|\nu+1\rangle$$

$$\left(-\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right)\phi_\nu(\zeta) = c_\nu\phi_{\nu+1}(\zeta)$$

$$\phi_\nu(z) = C z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} G_\nu(\zeta),$$

$$2xG_\nu(x) - G'(x) = xG_{\nu+1}(x)$$

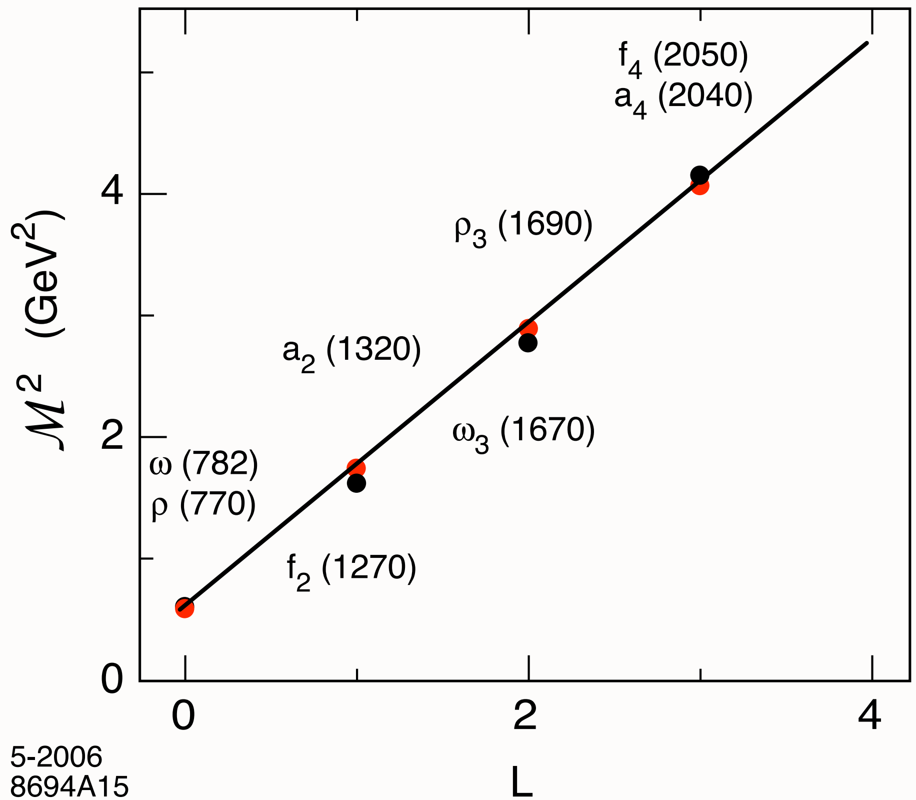
defines the associated Laguerre function $L_n^{\nu+1}(x^2)$

$$\phi_\nu(z) = C_\nu z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2).$$

Subtract Vacuum Energy

$$\mathcal{M}^2 \to \mathcal{M}^2 - 2\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + \frac{1}{2}).$$



$\mathcal{M}^2(\mathrm{GeV}^2)$

L

$J = L + 1$ vector meson Regge trajectory for $\kappa \simeq 0.54$ GeV

Holographic Harmonic Oscillator Model: Baryons

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

Frame-Independent LF Dirac Equation

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right)$$

$$\Pi_\nu^\dagger(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 + \kappa^2\zeta\gamma_5\right)$$

Coupled Equations

$$\begin{pmatrix} 0 & -\frac{d}{d\zeta} \\ \frac{d}{d\zeta} & 0 \end{pmatrix}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} - \begin{pmatrix} 0 & \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \\ \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta & 0 \end{pmatrix}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \mathcal{M}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$-\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - \kappa^2\zeta\psi_- = \mathcal{M}\psi_+,$$

$$\frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - \kappa^2\zeta\psi_+ = \mathcal{M}\psi_-.$$

HO due to Linear Potential!

$$V = -\beta\kappa^2\zeta$$

Holographic Harmonic Oscillator Model: Baryons

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right)$$

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 + \kappa^2\zeta\gamma_5\right)$$

$$\left(H_{LF} - \mathcal{M}^2\right)\psi(\zeta) = 0, \qquad H_{LF} = \Pi^\dagger\Pi$$

Uncoupled Schrodinger Equations **Harmonic Oscillator Potential!**

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2(\nu+1)\kappa^2 + \mathcal{M}^2\right)\psi_+(\zeta) = 0,$$

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4(\nu+1)^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\nu\kappa^2 + \mathcal{M}^2\right)\psi_-(\zeta) = 0,$$

Solution

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$

$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2),$$

Same eigenvalue! $\mathcal{M}^2 = 4\kappa^2(n+\nu+1)$

Holographic Baryon Spectrum

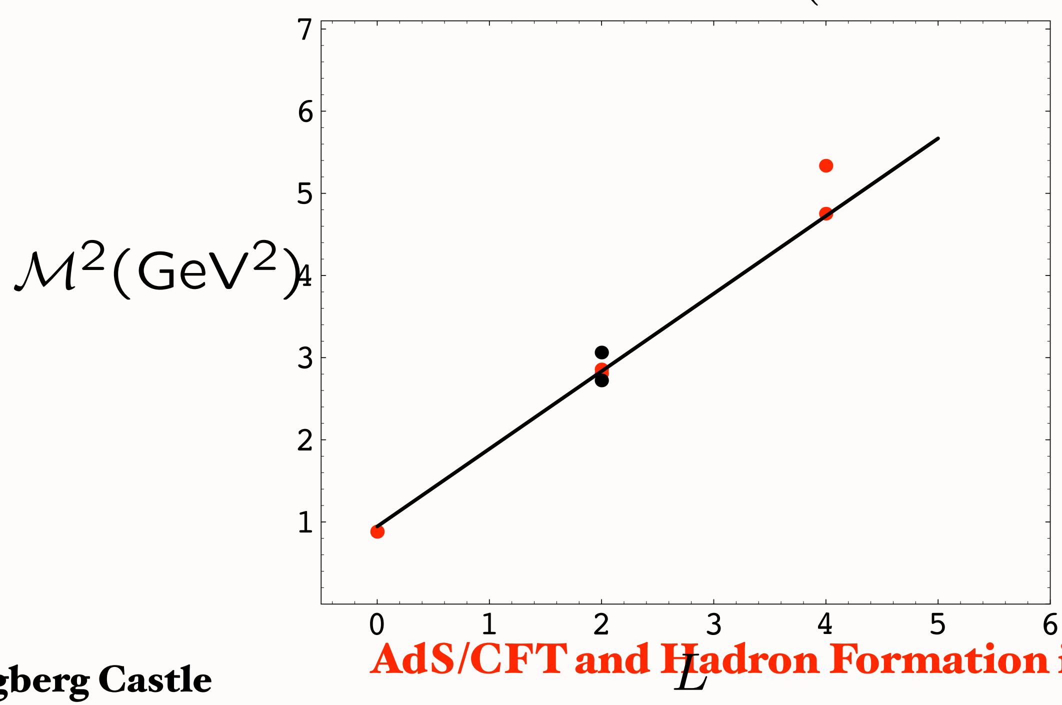
$$\psi(\zeta) = \kappa^{2+L}\sqrt{\frac{n!}{(n+L+2)!}}\zeta^{\frac{3}{2}+L}e^{-\kappa^2\zeta^2/2}\left[L_n^{L+1}\left(\kappa^2\zeta^2\right)u_+ + \frac{\kappa\zeta}{\sqrt{n+L+2}}L_n^{L+2}\left(\kappa^2\zeta^2\right)u_-\right.$$

$$\mathcal{M}^2 = 4\kappa^2(n+L+2).$$

Vacuum Energy Shift?

$$\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n+L+1).$$



$J = L + 1/2$ Regge trajectory

$\kappa \simeq 0.49$ GeV

Same slope in L and n

Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$\begin{aligned} F(Q^2)_{I\to F} &= R^{3+2\sigma} \int_0^\infty \frac{dz}{z^{3+2\sigma}}\, e^{(3+2\sigma)A(z)} \Phi_F(z)\, J(Q,z)\, \Phi_I(z) \\ &\simeq R^{3+2\sigma} \int_0^{z_o} \frac{dz}{z^{3+2\sigma}}\, \Phi_F(z)\, J(Q,z)\, \Phi_I(z), \end{aligned}$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.$$

- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z\to 0$:

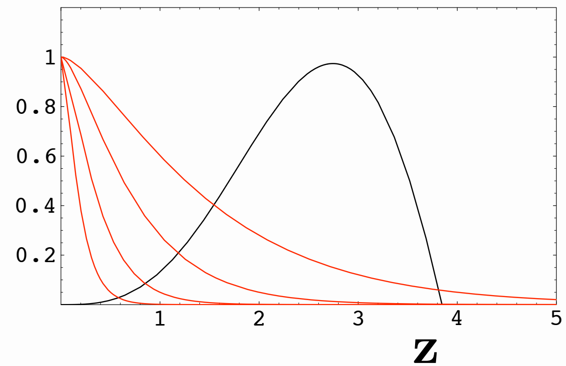
$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

- Propagation of external perturbation suppressed inside AdS.

- At large enough $Q \sim r/R^2$, the interaction occurs in the large-r conformal region. Important contribution to the FF integral from the boundary near $z \sim 1/Q$.

$\mathbf{J}(\mathbf{Q}, \mathbf{z}), \quad \mathbf{\Phi}(\mathbf{z})$

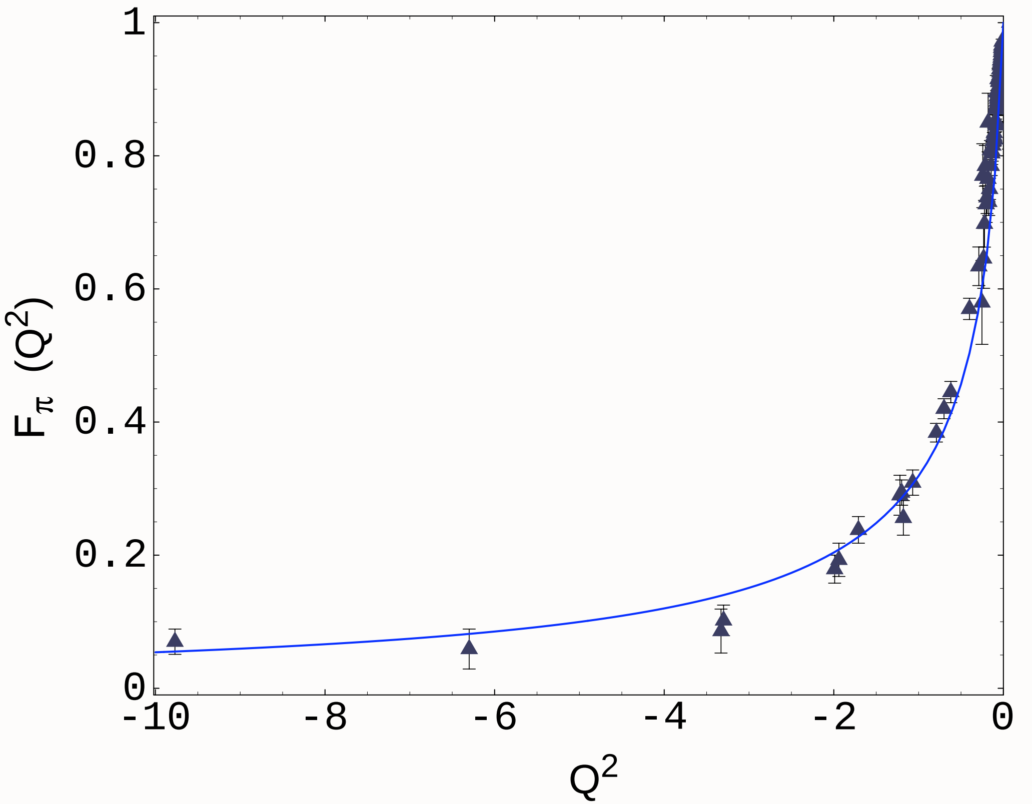


- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau - 1},$$

General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.



Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

Data Compilation from Baldini, Kloe and Volmer

Example: Evaluation of QCD Matrix Elements

Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ (1-\gamma_5) \psi_d \right| \pi^- \right\rangle = i\sqrt{2} P^+ f_\pi,$$

with

$$\left| \pi^- \right\rangle = \left| d\overline{u} \right\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\,d\downarrow} d^\dagger_{c\,u\uparrow} - b^\dagger_{c\,d\uparrow} d^\dagger_{c\,u\downarrow} \right) \left| 0 \right\rangle.$$

Use light-cone expression:

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

Lepage and Brodsky '80

Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{\rm QCD}}{8 J_1(\beta_{0,1})} = 83.4 \text{ Mev},$$

for $\Lambda_{\rm QCD} = 0.2$ GeV (fixed from the pion FF).

Experiment: $f_\pi = 92.4$ Mev.

Pion Decay Constant in HO Model

$$f_\pi = 2\sqrt{N_C}\int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\,\psi_{\overline{q}q/\pi}(x,\vec{k}_\perp)$$

$$= 2\sqrt{N_C}\int_0^1 dx\,\phi(x, Q^2 \to \infty),$$

$$\phi(x,Q^2) = \int^{Q^2} \frac{d^2\vec{k}_\perp}{16\pi^3}\psi(x,\vec{k}_\perp)$$

$$\psi_{\overline{q}q/\pi}(x,\vec{k}_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}}\, e^{-\frac{\vec{k}_\perp^2}{2\kappa^2 x(1-x)}}$$

$$f_\pi = \frac{\sqrt{3}\kappa}{8} = 86.6 \text{ MeV} \qquad \kappa = 0.4 \text{ GeV}.$$

G. de Teramond and sjb

$$f_\pi = 92.4 \quad \text{MeV}$$ Exp.

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} \Phi_{P'}(z) J(Q,z) \Phi_P(z).$$

$$\Phi(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2 e^{-\kappa^2 z^2/2}.$$

$$J(Q,z) = zQK_1(zQ).$$

$$F(Q^2) = 1 + \frac{Q^2}{4\kappa^2} \exp\left(\frac{Q^2}{4\kappa^2}\right) Ei\left(-\frac{Q^2}{4\kappa^2}\right)$$

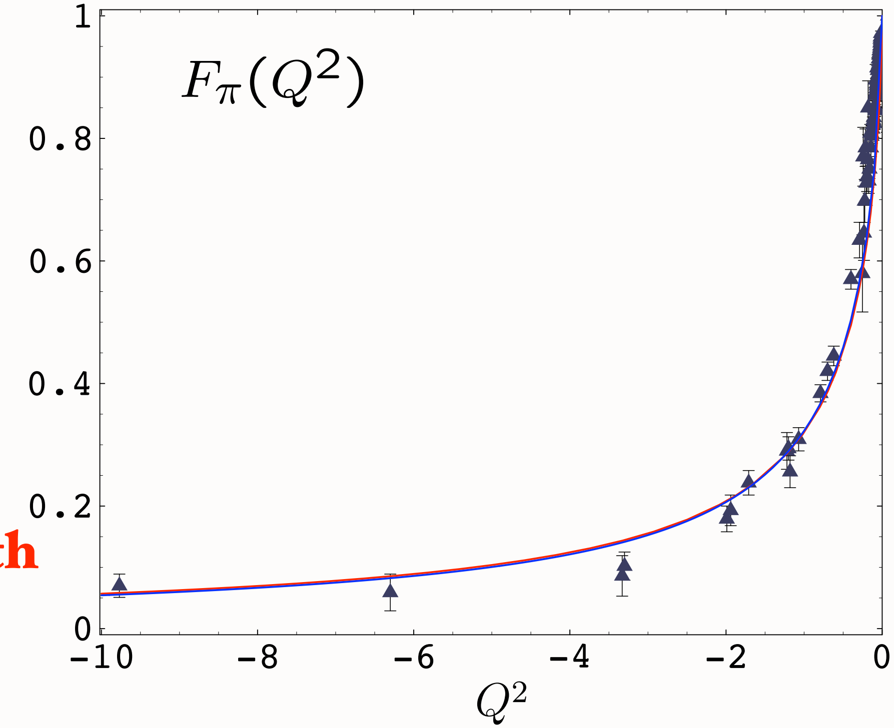
$$Ei(-x) = \int_\infty^x e^{-t} \frac{dt}{t}.$$

Space-like Pion Form Factor

$\kappa = 0.4$ GeV

$\Lambda_{QCD} = 0.2$ GeV.

Identical Results for both confinement models



$$F(Q^2) \to \frac{4\kappa^2}{Q^2}$$

$\kappa = 2\Lambda_{QCD}$

High Q^2 from short distances

$$z^2 = \zeta^2 = b_\perp^2 x(1-x)$$
$$= \mathcal{O}\left(\frac{1}{Q^2}\right)$$

Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu \Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \qquad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1 \pm \gamma_5}{2}\, u(P).$$

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}.$$

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_-(z)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

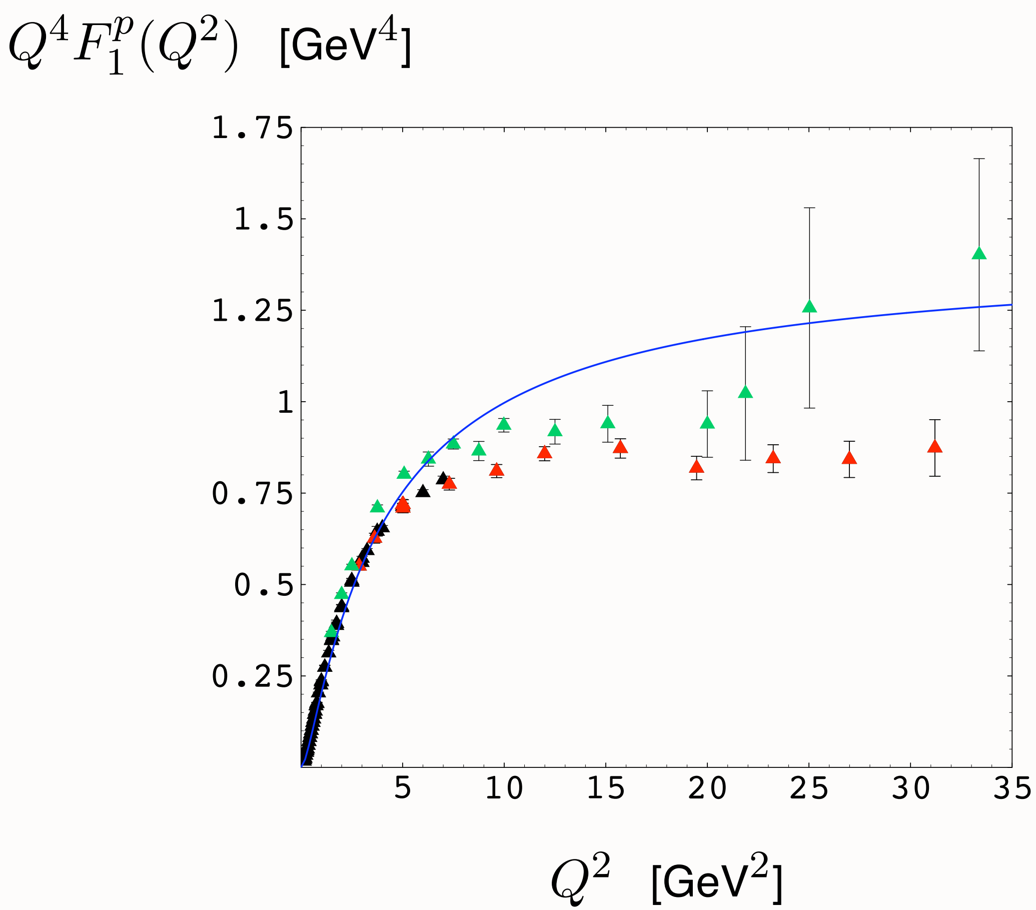
$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right],$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

Dirac Proton Form Factor

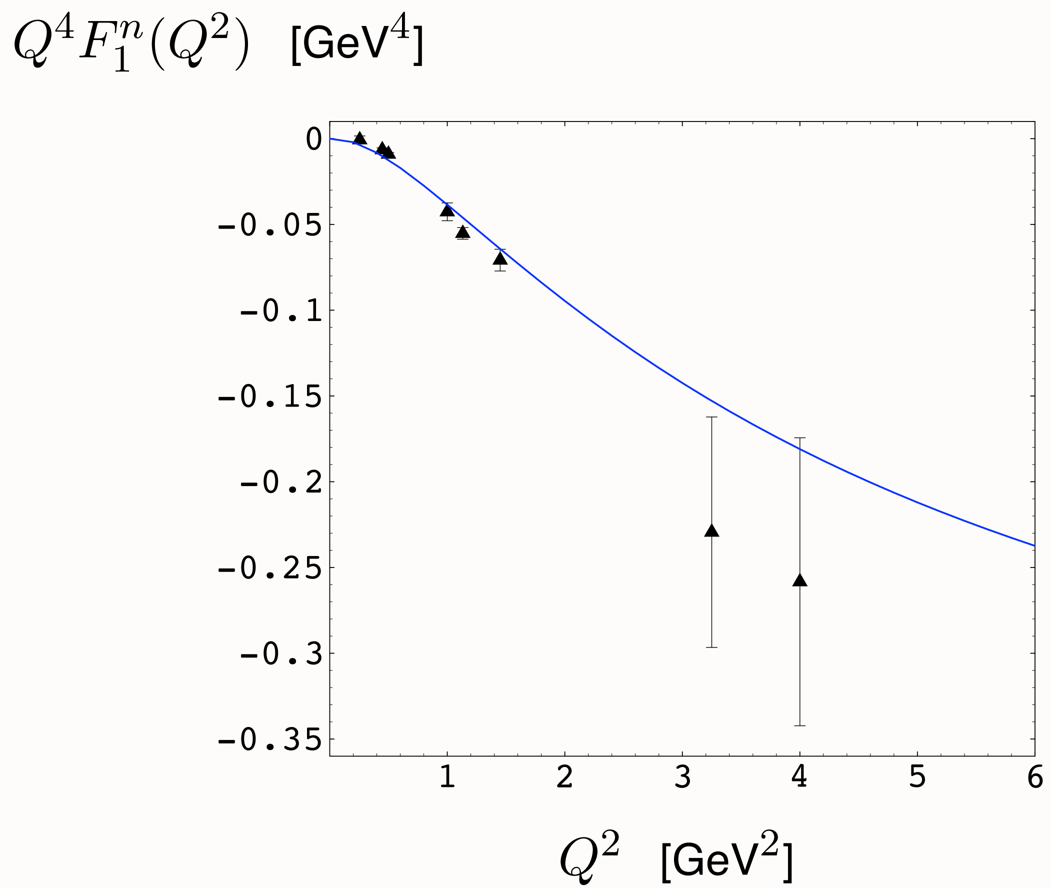
(Valence Approximation)



Prediction for $Q^4 F_1^p(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Analysis of the data is from Diehl (2005). Red points are from Sill (1993). Superimposed Green points are from Kirk (1973).

Dirac Neutron Form Factor

(Valence Approximation)



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Holographic Model for QCD Light-Front Wavefunctions

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\left(bqx\right)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Identical DYW and AdS5 Formulae: Two parton case

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

  the solution for the electromagnetic potential in AdS space, and

$$\boxed{\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right) \theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)}$$

  the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$\boxed{z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|}$$

General n-parton case

- Form factor in AdS is the overlap of normalizable modes dual to the incoming and outgoing hadrons Φ_P and $\Phi_{P'}$ with the non-normalizable mode $J(Q,z)$ dual to the external source

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) J(Q,z) \Phi_P(z).$$

Polchinski and Strassler, hep-th/0209211

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta).$$

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

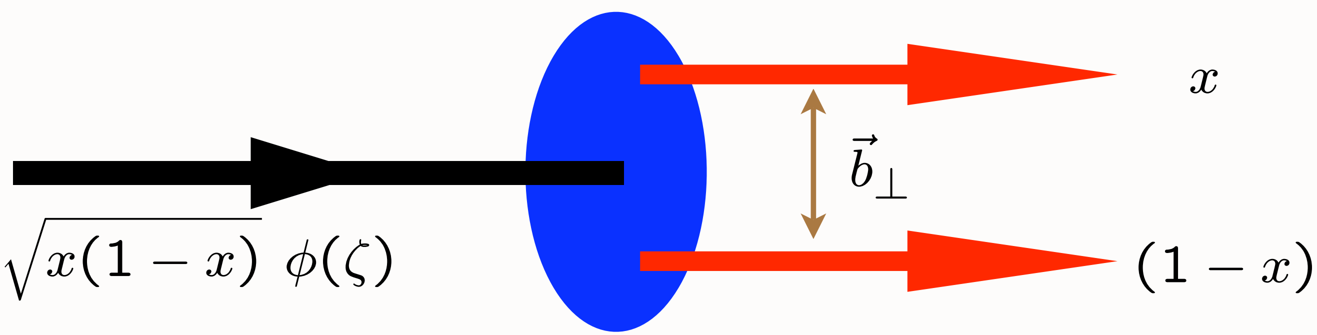
the solution for $J(Q,\zeta)$!

Mapping between LF(3+1) and AdS_5

LF(3+1) ↔ AdS_5

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \quad \longleftrightarrow \quad z$$



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

Holography: Map AdS/CFT to 3+1 LF Theory

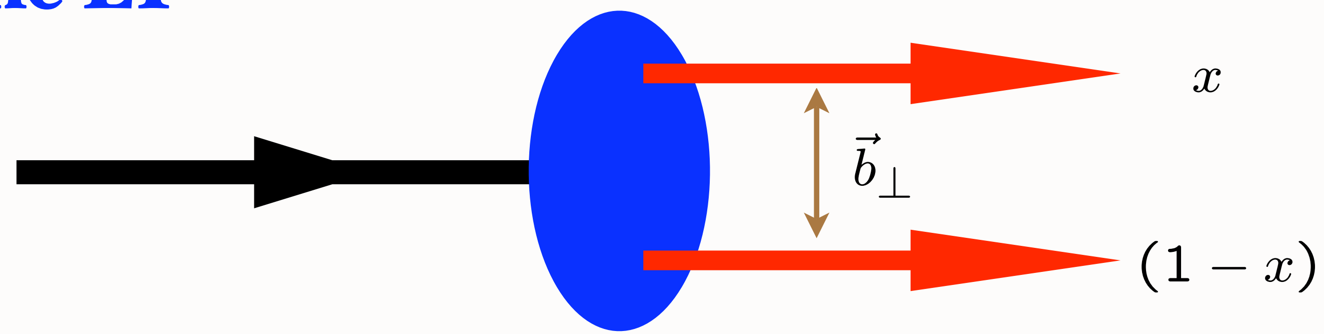
Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

radial variable on the LF

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$

Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

General solution:

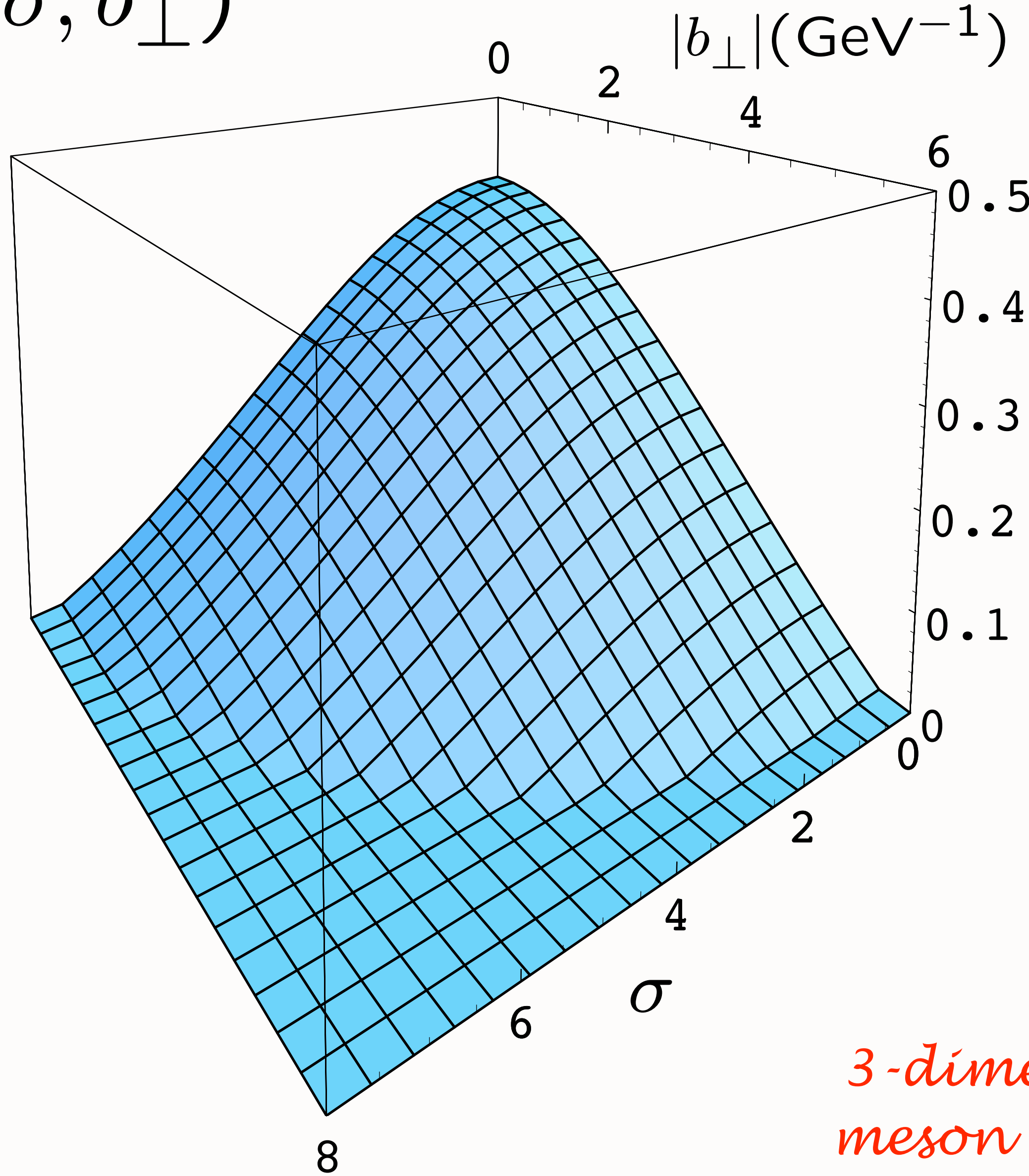
$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

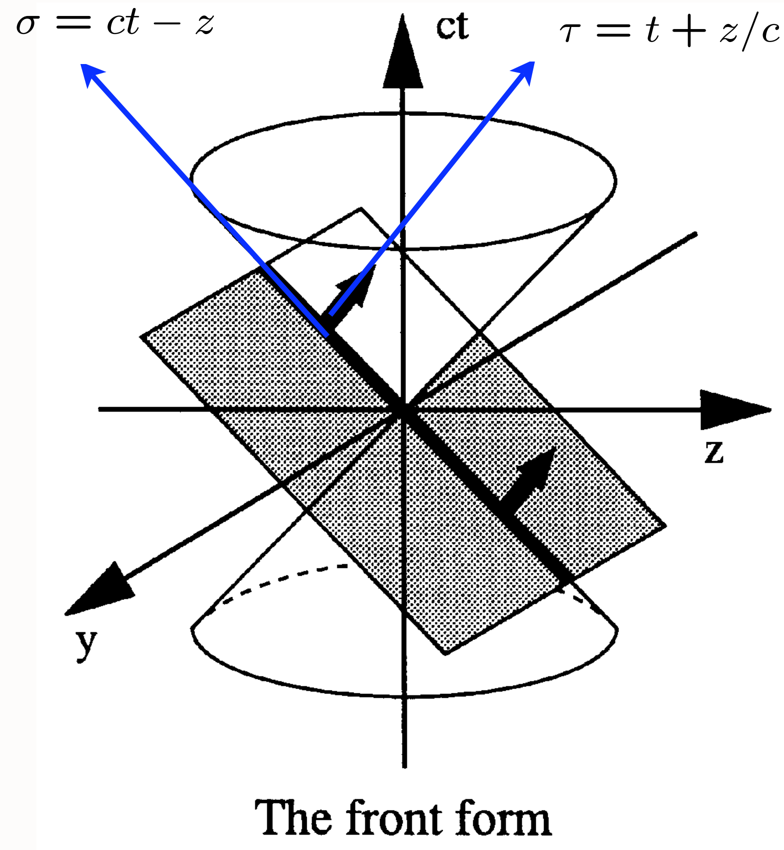
$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda_{QCD}^{-2}}{x(1-x)}\right),$$

AdS/CFT Holographic Model

G. de Teramond
SJB

$\psi(\sigma, b_\perp)$



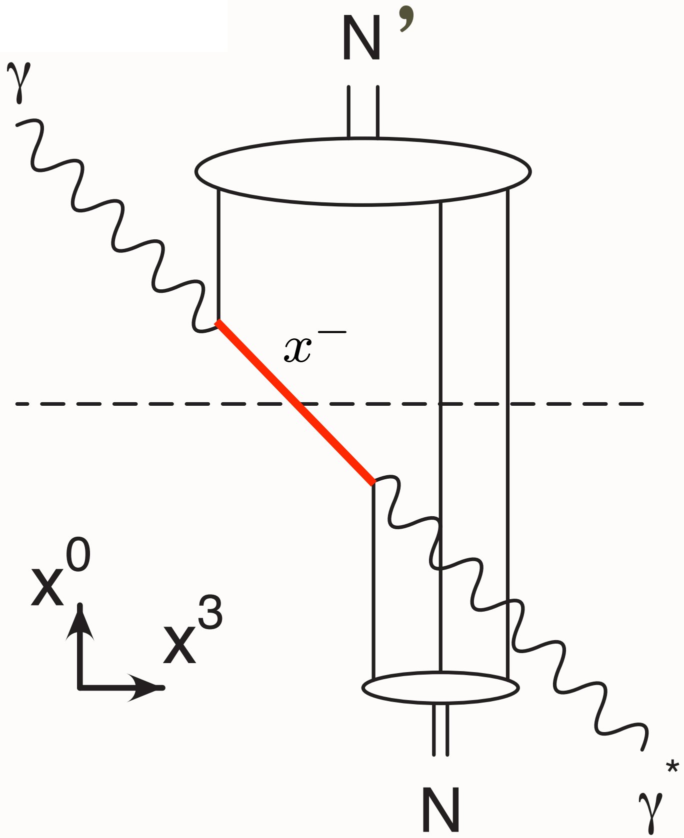


3-dimensional photograph: meson LFWF at fixed LF Time

Space-time picture of DVCS

P. Hoyer

$\sigma = \frac{1}{2} x^- P^+$



$x^+ = \mathbf{x}_\perp = 0$)

The position of the struck quark differs by x^- in the two wave functions

Measure x- distribution from DVCS:
Use Fourier transform of skewness,
the longitudinal momentum transfer

$$\zeta = \frac{Q^2}{2p \cdot q}$$

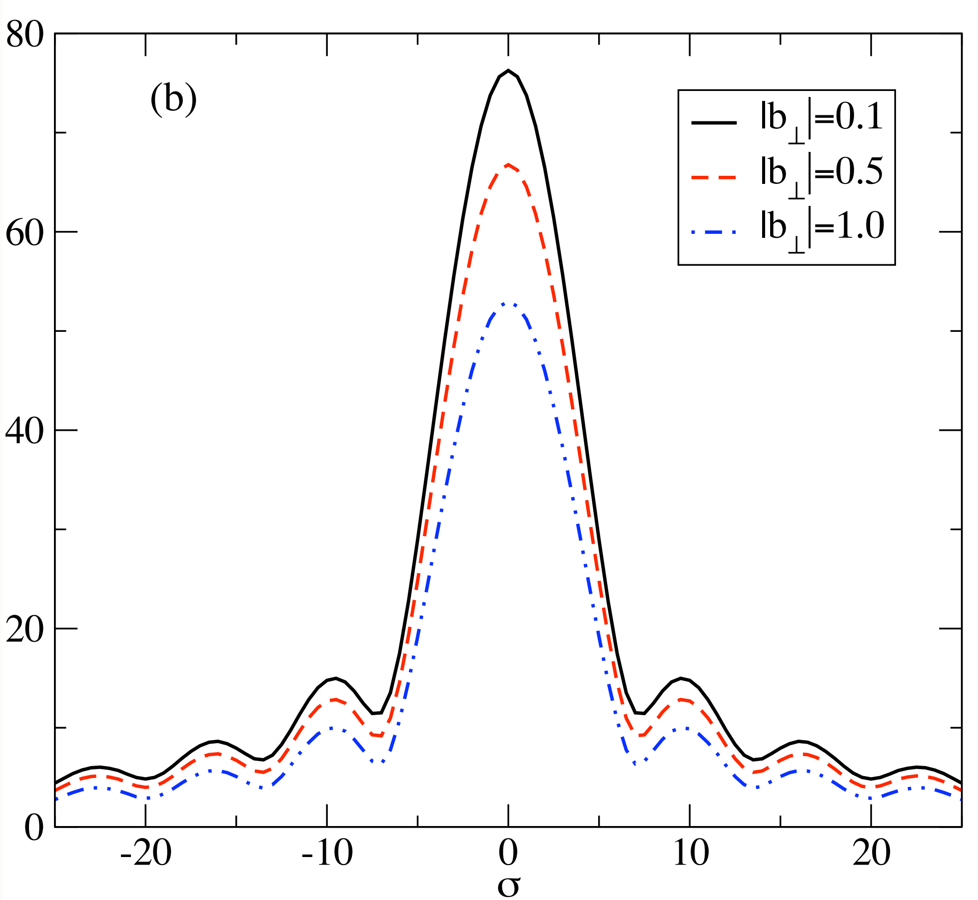
S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, b_\perp) = \frac{1}{2\pi}\int d\zeta e^{i\sigma\zeta}\tilde{A}(b_\perp,\zeta)$$

$\sigma = \frac{1}{2}x^- P^+$ $\zeta = \frac{Q^2}{2p\cdot q}$

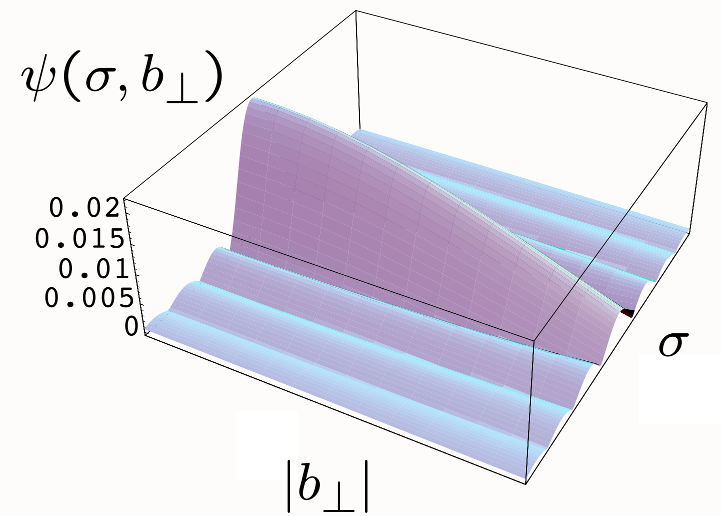


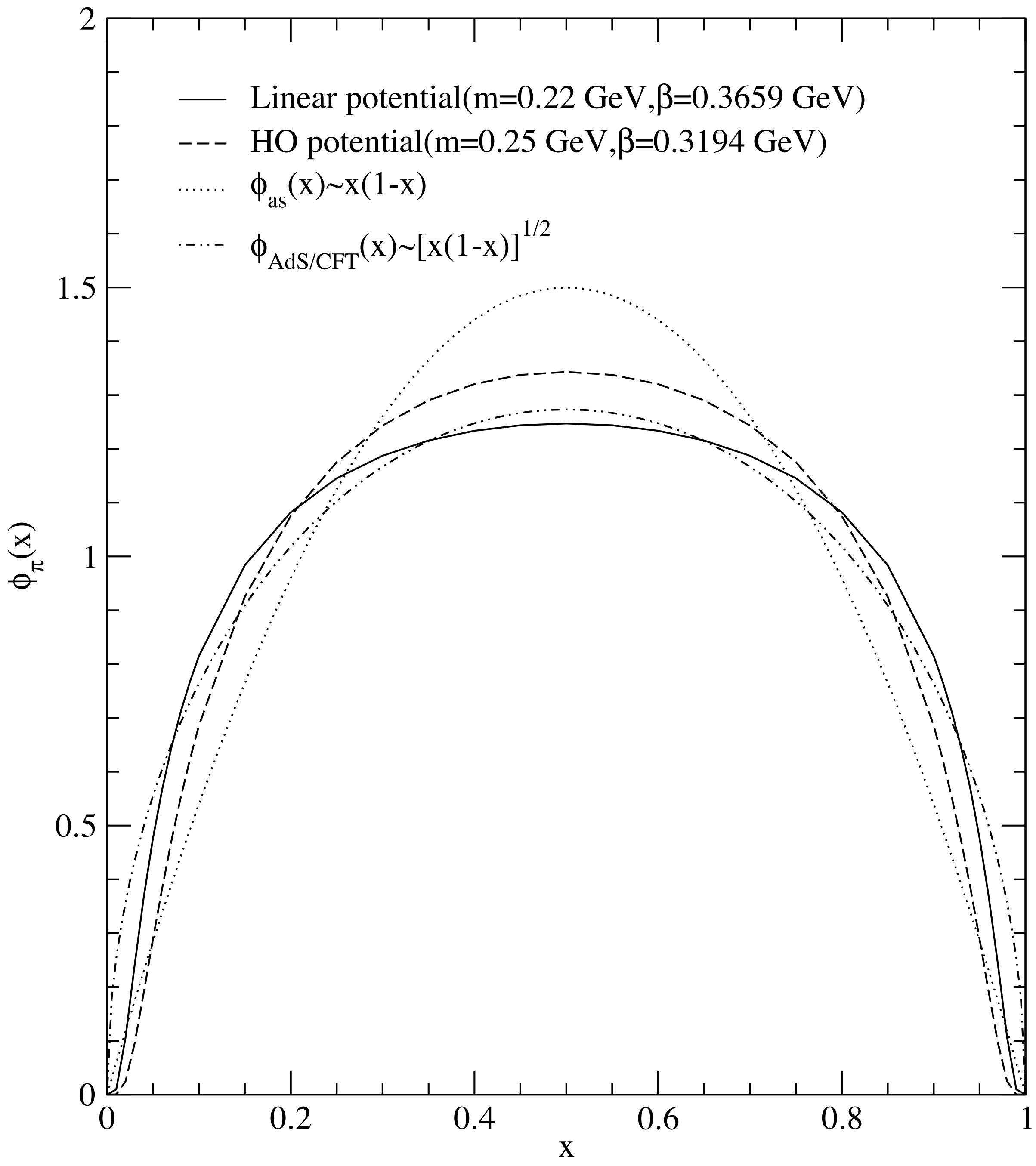
The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

$\Lambda_{QCD} = 0.32$





AdS/CFT:

$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

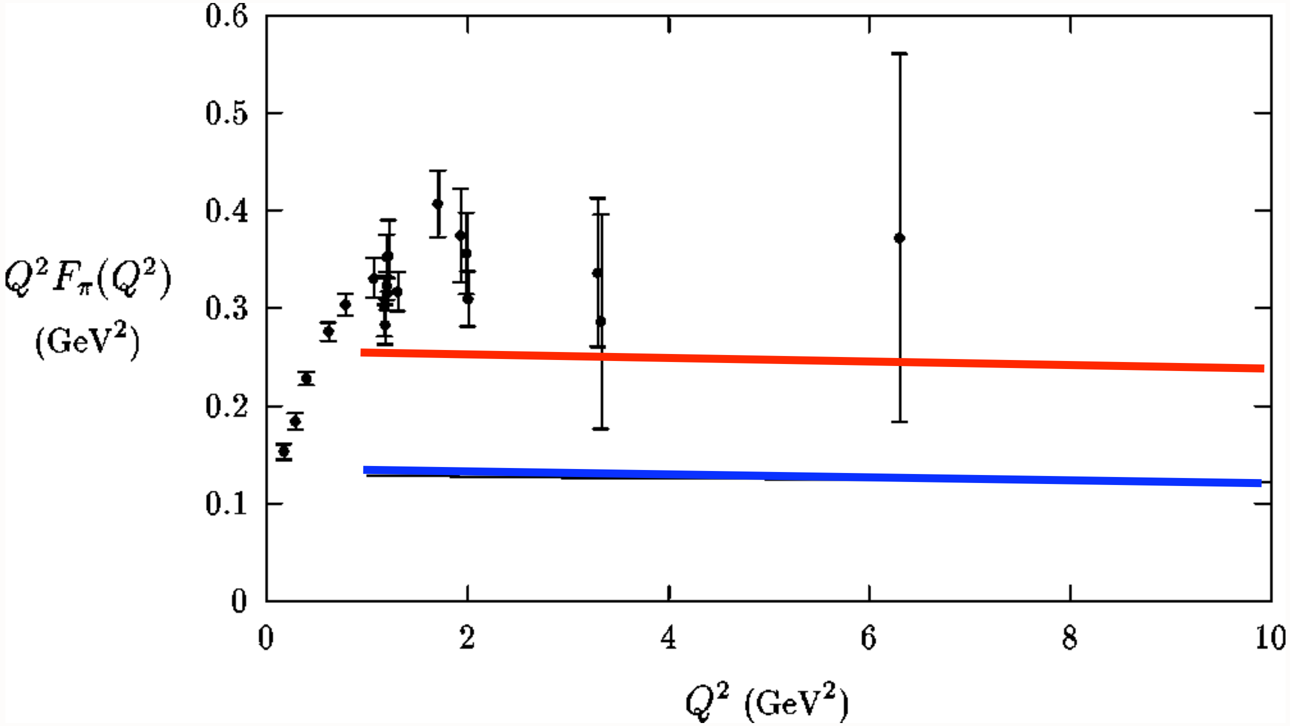
Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

n QCD

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb

$$F_\pi(Q^2) = \int_0^1 dx\, \phi_\pi(x) \int_0^1 dy\, \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

$\phi_{asymptotic} \propto x(1-x)$

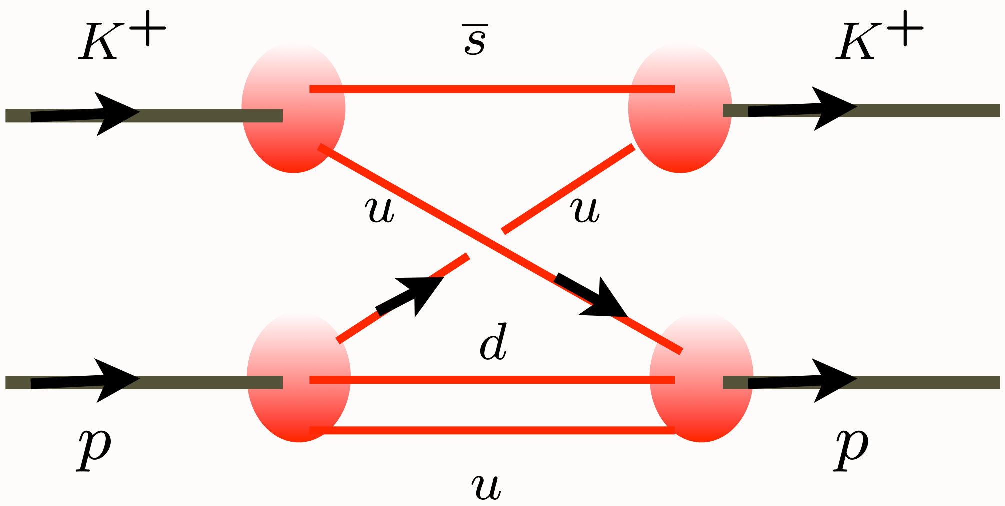
Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

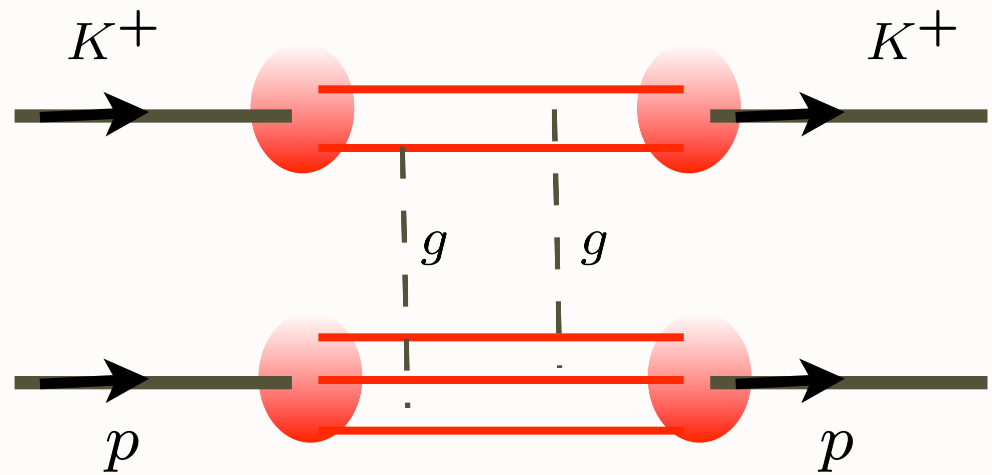
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb



Quark Interchange
(Spin exchange in atom-atom scattering)



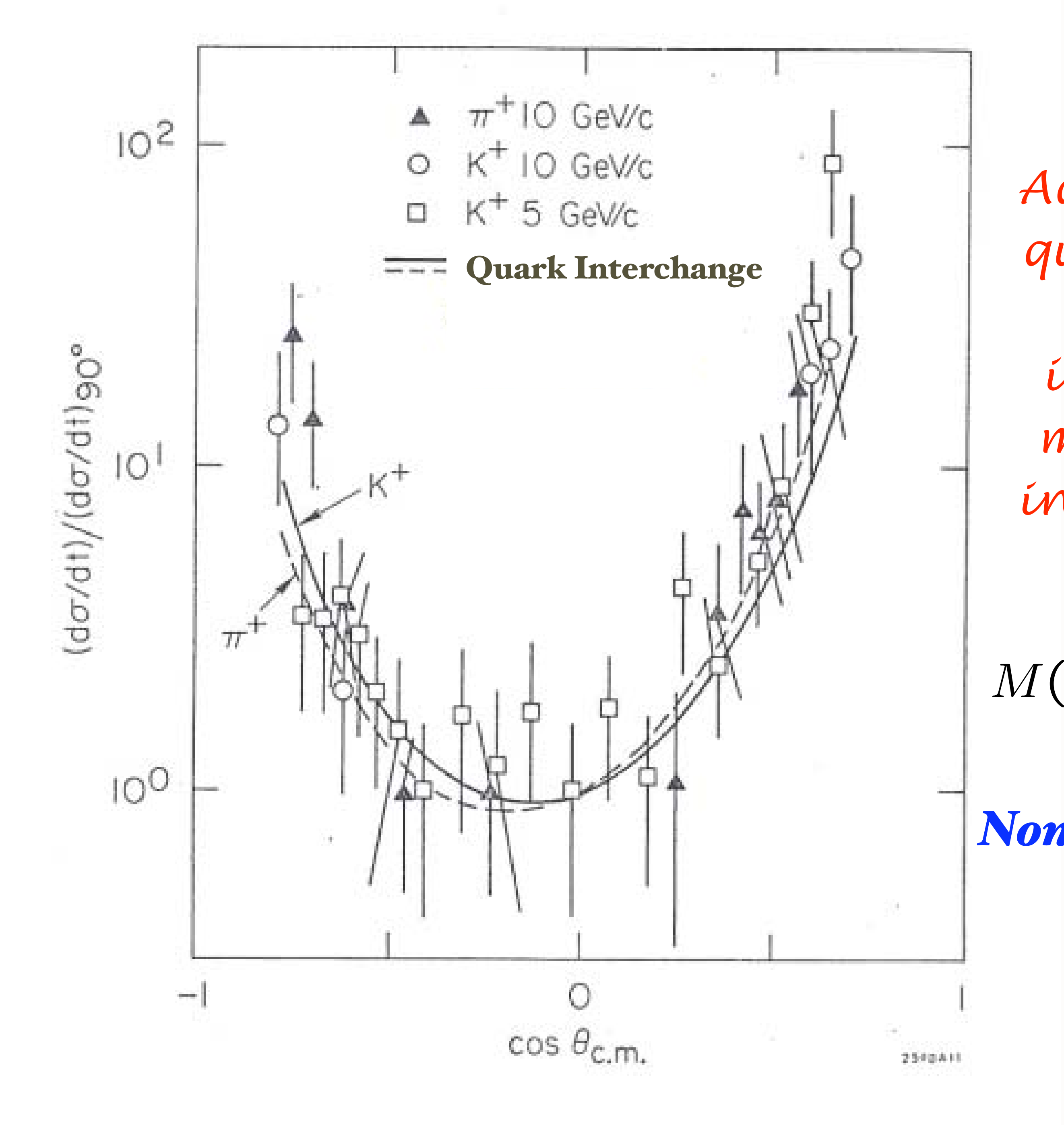
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \ \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
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and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^{0}/\Sigma^{0})K^{0}$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\pm}$,

$K^{\pm}p \to pK^{\pm}$,

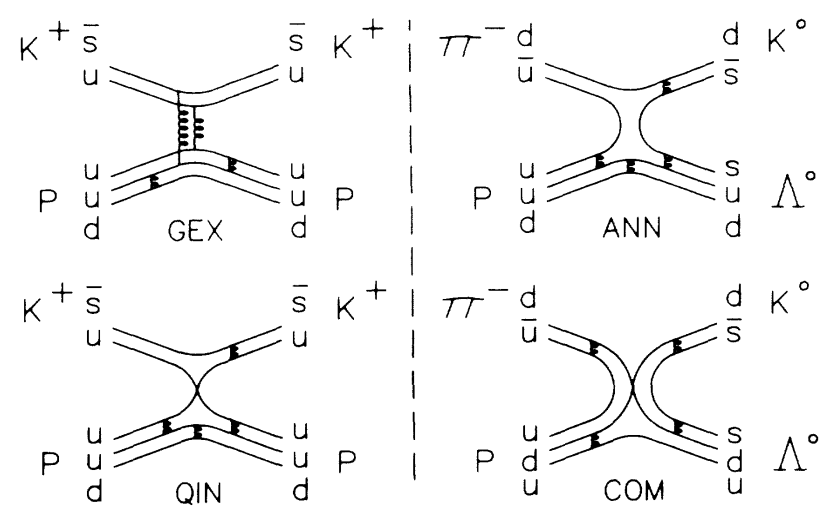
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^{0}K^{0}, \Sigma^{0}K^{0}$,

$p^{\pm}p \to pp^{\pm}$.



Features of Light-Front Formalism

- *Hidden Color* Of Nuclear Wavefunction
- *Color Transparency, Opaqueness*
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)
- *Direct mapping to AdS/CFT* (de Teramond, sjb)
- New Effective LF Equations (de Teramond, sjb)
- Light-Front Amplitude Generator

Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role if ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

AdS/CFT and QCD

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

- Representation of Semi-Classical QCD
- Confinement at Long Distances and Conformal Behavior at short distances
- Non-Perturbative Derivation of Dimensional Counting Rules
- Hadron Spectra, Regge Trajectories, Light-Front Wavefunctions; QCD at the amplitude level
- Goal: A first approximant to physical QCD

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

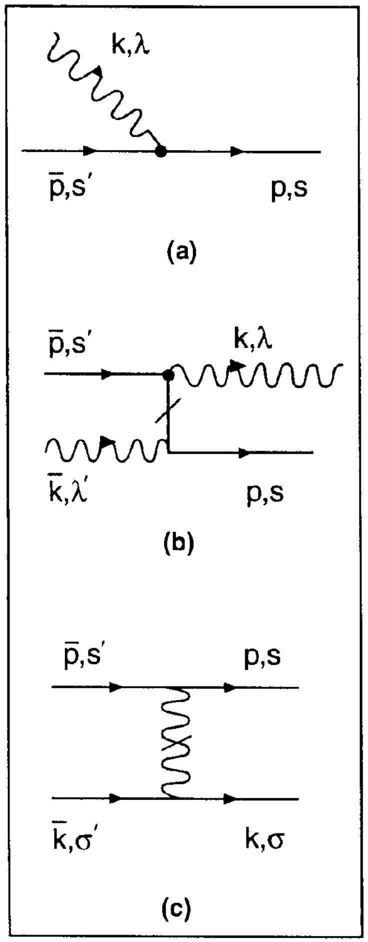
- Good initial approximant
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

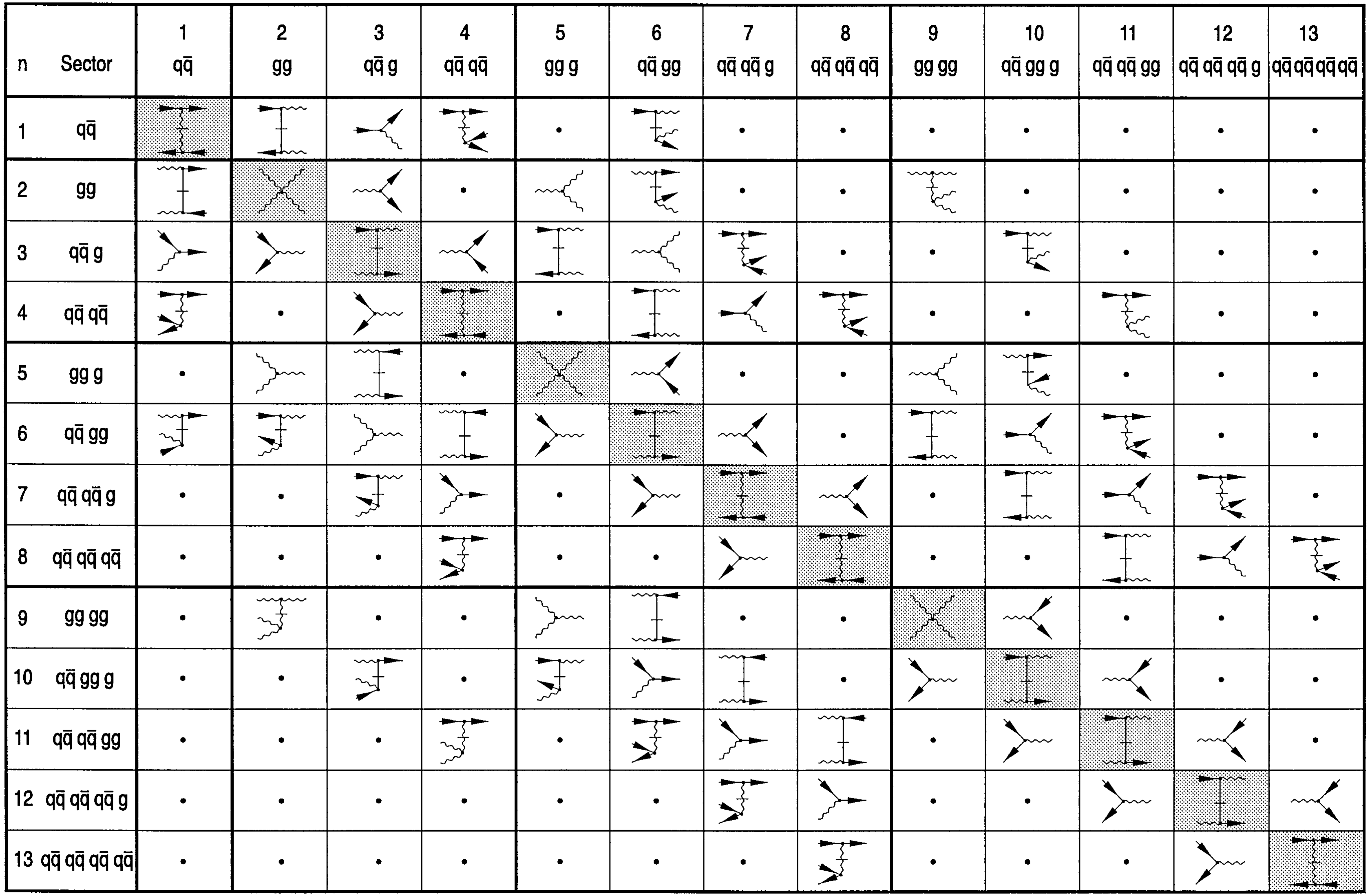
Vary, Harinandrath, sjb

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

DLCQ





Use AdS/QCD basis functions

Pauli, Pinsky, sjb

AdS/QCD

- New initial approximation to QCD based on conformal invariance, and confinement

- Underlying principle: Semi-Classical QCD

- AdS5: Mathematical representation of conformal gauge theory

- Chiral symmetry, heavy quark masses

  R. Apreda, J. Erdmenger, D. Lust, C. Seig, hep-th/0610276

- Systematically improve using DLCQ

- Successes: Hadron spectra, LFWFs, dynamics

- QCD at the Amplitude Level

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension $3, \frac{9}{2}$ and 4 states $\overline{q}q$, qqq, and gg appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small x:

  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:

  Brodsky and de Téramond, hep-th/0310227. E. van Beveren and Rupp, hep-th 0610199

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:

  Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

  R. Apreda, J. Erdmenger, D. Lust, C. Seig, hep-th/0610276

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 . . .

SCIENCE VOL 265 15 SEPTEMBER 1995

A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

Frank and Ernest

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