New Insights into non-perturbative QCD from String Theory

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Motivation

A new description of strongly coupled large N gauge theories from string theory

Starting point: AdS/CFT correspondence

Duality: Anti-de Sitter supergravity in 5d / supersymmetric conformal field theory in 4d

Generalization: 5d gravity description of

4d non-supersymmetric $SU(N)$ gauge theory ($N \to \infty$) with quark degrees of freedom (conjecture)

AdS/CFT Correspondence

(Maldacena 1997, AdS: Anti de Sitter space, CFT: conformal field theory)

- Arises from String Theory in a particular low-energy limit:

  ’t Hooft coupling $\lambda = g^2 N$ large and fixed , $N \rightarrow \infty$

- Duality Quantum Field Theory $\Leftrightarrow$ Gravity Theory

- Duality: Quantum field theory at strong coupling $\Leftrightarrow$ Gravity theory at weak coupling

- Conformal field theory in four dimensions $\Leftrightarrow$ Supergravity Theory on $AdS_5 \times S^5$

New results and applications

- Deep inelastic scattering

  Pomeron — Polchinski, Strassler et al

- Gravity dual descriptions of confining gauge theories

- Adding flavour to AdS/CFT — Karch/Katz

- Spontaneous chiral symmetry breaking

  $U(1)_{\mathrm{A}}$ symmetry — Evans, J.E., Guralnik et al

  Non-perturbative calculation of meson spectra by solving 2nd order gravity equations of motion.

- Finite-temperature field theories

  Quark-Gluon Plasma — Son, Starinets et al

- AdS/QCD (‘bottom-up approach’) — Brodsky, ...

Outline

I. Introduction to the AdS/CFT correspondence

II. Hard scattering and the pomeron in AdS/CFT

AdS/CFT correspondence

- Anti-de Sitter space is a curved space with constant negative curvature. It has a boundary.

  Metric: $ds^2 = e^{2r/L}\,\eta_{\mu\nu}dx^\mu dx^\nu - dr^2$

- The isometry group of $(d+1)$-dimensional AdS space coincides with the conformal group in d dimensions ($SO(d,2)$).

- The AdS/CFT correspondence provides a dictionary between field theory operators and supergravity fields.

$$\mathcal{O}_\Delta \leftrightarrow \phi_m \ , \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + L^2 m^2}$$

- Items in the same dictionary entry have the same quantum numbers under superconformal symmetry.

- $L^4 = 4\pi g^2 N \alpha'^2$

AdS/CFT correspondence

- field-operator correspondence:

$$\langle e^{\int d^d x \, \phi_0(\mathbf{x}) \mathcal{O}(\mathbf{x})} \rangle_{\text{CFT}} = Z_{\text{string}} \Big|_{\phi(0,\mathbf{x}) = \phi_0(\mathbf{x})}$$

Generating functional for correlation functions of particular composite operators in the quantum field theory

coincides with

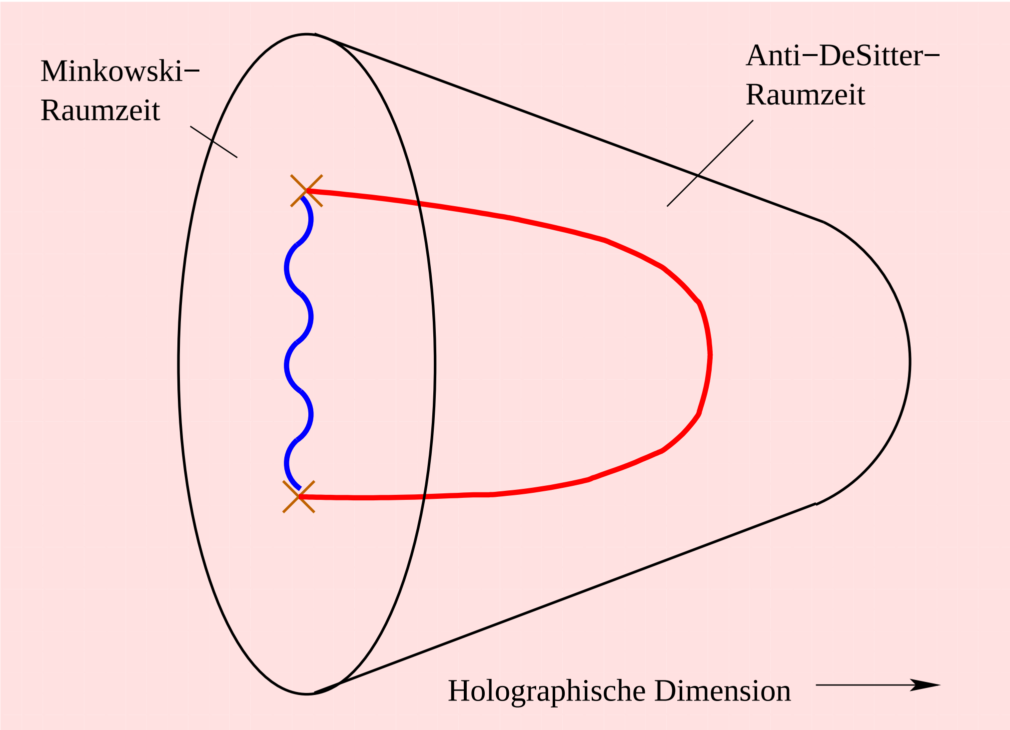
classical tree diagram generating functional in supergravity

AdS/CFT for $\mathcal{N} = 4$ super Yang-Mills

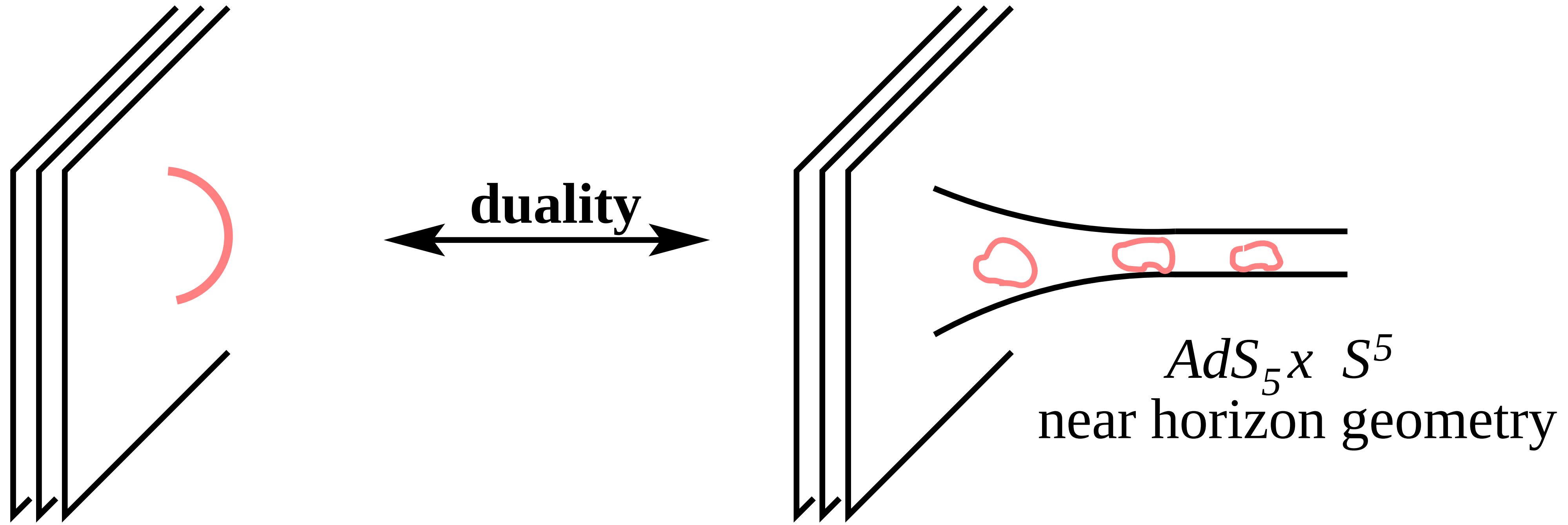
IIB Supergravity on $AdS_5 \times S^5 \Leftrightarrow$ large N limit of $\mathcal{N} = 4$ Super Yang-Mills

isometries: $SO(4,2) \times SU(4)$, $\mathcal{N} = 4$ SYM: $\beta \equiv 0$ $\Leftrightarrow$ conformal field theory

- 1 vector field A_μ
- 4 complex Weyl fermions $\lambda_{\alpha A}$ ($\bar{4}$ of $SU(4)_R$)
- 6 real scalars ϕ_i (6 of $SU(4)_R$)



D3 branes in 10d



$\Downarrow$ low-energy limit (λ large, $N \to \infty$)

$d = 4\ \mathcal{N} = 4\ SU(N)$ Super-Yang-Mills theory at large N
all fields in adjoint representation of gauge group

IIB Supergravity theory on $AdS_5 \times S^5$

Extensions of AdS/CFT

Aim: Extend gravity/field theory correspondence to other (more realistic) field theories

$\mathcal{N} = 4\ SU(N)$ Super-Yang-Mills theory

- $N \to \infty$
- Supersymmetry
- Conformal Symmetry
- all fields in adjoint representation of the gauge group

QCD:

- $N = 3$
- no supersymmetry
- Confinement
- quarks are in the fundamental representation of the gauge group

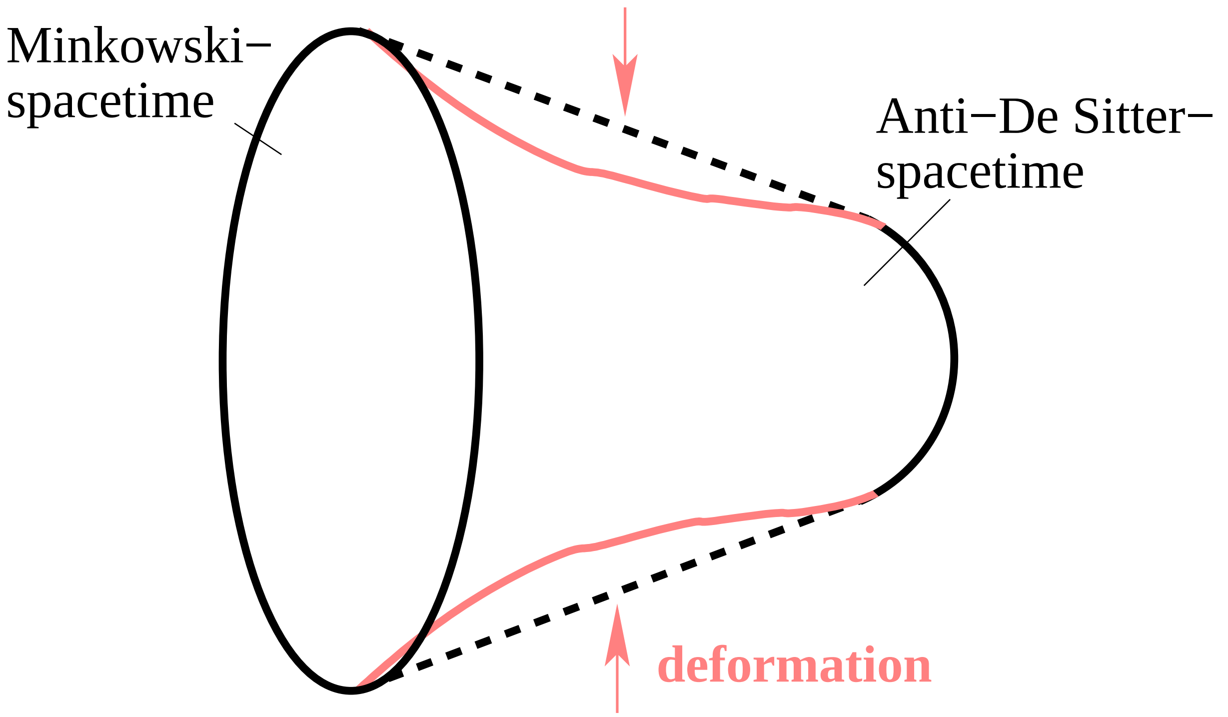
Possible extensions of AdS/CFT:

- relax $N \to \infty$ low energy limit $\Leftrightarrow$ consider string theory instead of supergravity
- break supersymmetry and conformal symmetry $\Leftrightarrow$ deform AdS space
- add fields in the fundamental representation of the gauge group — Kaminski's talk

Deformations of AdS/CFT

metric of deformed AdS space: $ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2$

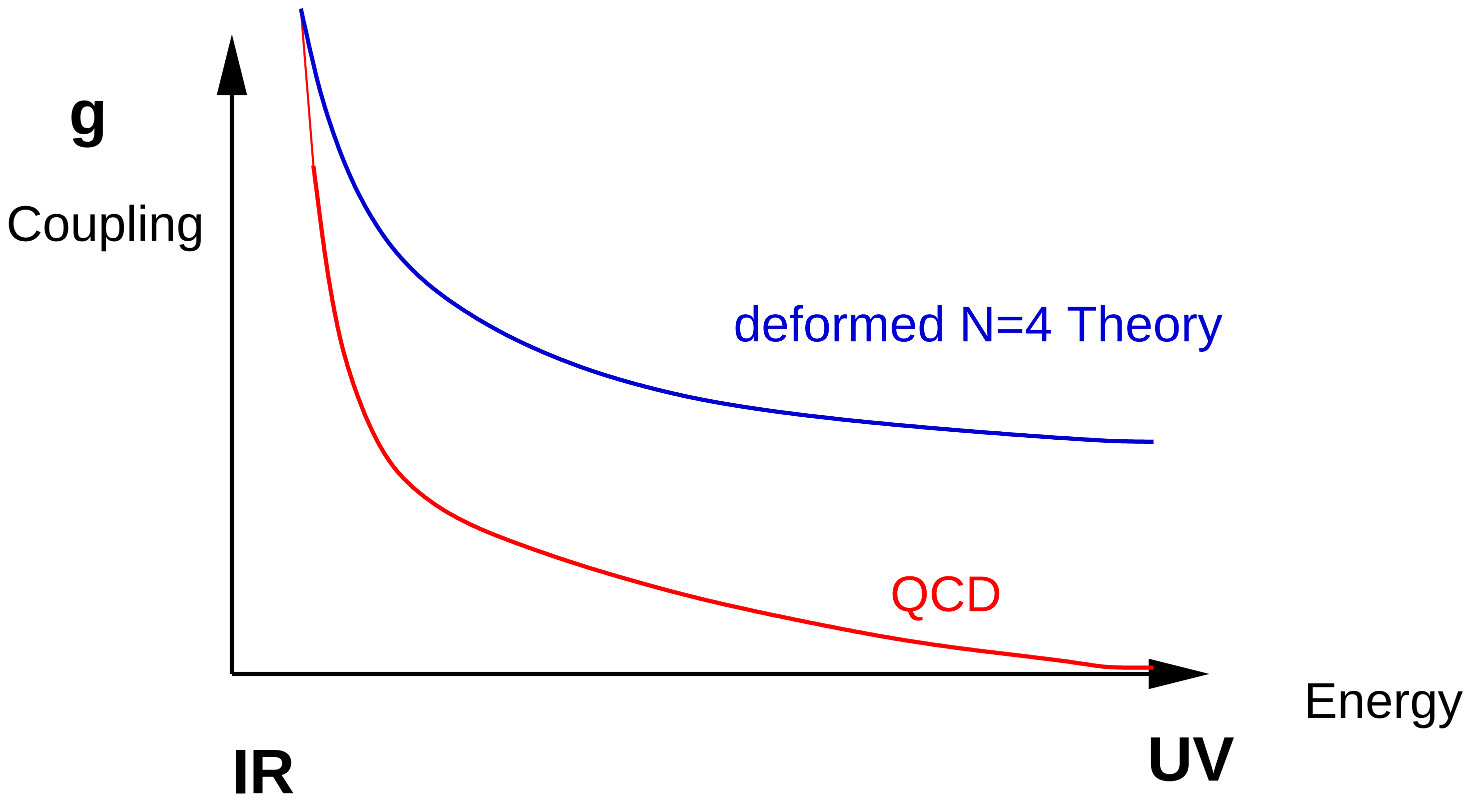
AdS: $A(r) = \frac{r}{L}$



Holographic RG flows: Fifth dimension $\Leftrightarrow$ energy scale

Renormalization group equation from supergravity gradient flow

Running gauge coupling



II. Hard scattering ad the pomeron in AdS/CFT

Polchinski, Strassler '01, '02

Pomeron: Polchinski, Strassler, Brower, Tan '06

Simplest example:

Fixed-angle scattering of glueballs - hard scattering (power law)

String theory in flat space: Soft scattering

Here: Warped space !

Metric (conformal case):

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + R^2 ds_X^2$$

Hard scattering

$$p_\mu = \frac{r}{L}\tilde{p}_\mu$$

p_μ conserved momentum, corresponding to invariance under translation of x^μ

$\tilde{p}_\mu$ momentum in local inertial coordinates for momenta localized at r

Holographic encoding of gauge theory physics:

Low energy states at small r, high energy states at large r (near boundary)

Simplest way to obtain dual of a non-conformal theory:

Introduce cut-off on the geometry

(For instance $r_{\min} = \Lambda L^2$, Λ mass of lightest glueball)

Hard scattering in AdS/CFT

$2 \longrightarrow m$ scattering of closed strings

Exclusive glueball scattering at large energy $\sqrt{s}$ and fixed angles

Dilaton

$$\Phi = e^{ip\cdot x}\psi(r,\Omega)$$

Dilaton dual to field theory operator $\mathrm{tr} F^{\mu\nu} F_{\mu\nu}$

Amplitude:

$$\mathcal{A}(p) = \int dr\, d^5\Omega \sqrt{-g}\, \mathcal{A}_{\mathrm{string}}(\tilde{p}) \prod_{i=1}^{m+2} \psi_i(r,\Omega)$$

For λ large, Φ is slowly varying in transverse directions

$\Rightarrow$ Ten-dimensional scattering takes place at a point in transverse directions

Hard scattering

Amplitude:

$$\mathcal{A}(p) = \int dr\, d^5\Omega \sqrt{-g}\, \mathcal{A}_{\rm string}(\tilde{p}) \prod_{i=1}^{m+2} \psi_i(r,\Omega)$$

For string amplitude $\mathcal{A}_{\rm string}(\tilde{p})$: Gaussian approximation robust for λ large, $N \to \infty$

$$\mathcal{A}_{\rm string}(\tilde{p}) = g^{2m} \alpha'^{2m-1} F(\tilde{p}\sqrt{\alpha'})$$

At large r:

$$\psi_i(r,\Omega) = C f_i(r/r_{\rm min}) g(\Omega)\,, \qquad f_i \to (r/r_{\rm min})^{-\Delta_i}$$

Δ_i conformal dimension, $\Delta = \sum_{i=1}^{m+2} \Delta_i$

Result:

$$\mathcal{A}(p) \sim \frac{(gN)^{\frac{1}{4}(\Delta-2)}}{N^m \Lambda^{m-2}} \left(\frac{\Lambda}{p}\right)^{\Delta-4}$$

Hard scattering

Discussion of Amplitude

$$\mathcal{A}(p) \sim \frac{(gN)^{\frac{1}{4}(\Delta-2)}}{N^m \Lambda^{m-2}} \left(\frac{\Lambda}{p}\right)^{\Delta-4} \quad :$$

Energy scaling: Same as QCD result, with identification $\Delta_i = n_i$

n_i number of hard constituents in i^{th} hadron

QCD result:

$$\mathcal{A}(p) \sim s^{2-\frac{1}{2}n} \,, \quad n = \sum_{i=1}^{m+2} n_i$$

For states with spin: twist $\tau_i \equiv \Delta_i - \sigma_i$

Coupling dependence: $g^2 N$ has to be replaced by $gN^{1/2}$ to obtain agreement with QCD

Pomeron in AdS/CFT

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Brower, Polchinski, Strassler, Tan '06

Pomeron in field theory:

Universal, colorless, flavorless coherent excitation that dominates hadronic elastic scattering

at large s, small t , large N

contributes the leading singularity in the angular momentum plane

Pomeron in AdS/CFT:

Calculation of field theory amplitude from string amplitude in AdS space with cut-off

Approximation: Ten-dimensional string theory amplitude local

Four-dimensional scattering given by coherent sum over scattering in the six transverse dimensions

Pomeron in AdS/CFT: Field theory from string theory amplitude

Ten-dimensional string theory S matrix:

$$\mathcal{S} = i \int d^4x \, d^6y \sqrt{-G} \mathcal{A}_{\rm local}(x,y) \ ,$$

$$\mathcal{A}_{\rm local}(x,y) \to \mathcal{T}_{10}(\tilde{p}) \prod_{\substack{\rm ext.\\ \rm states}} e^{ip_i\cdot x}\psi_i(y)$$

$\mathcal{T}_{10}(\tilde{p})$ flat spacetime string theory amplitude

$\tilde{p}^\mu = \frac{R}{r}p^\mu$: momenta seen by local inertial observer

Four-dimensional amplitude:

$$\mathcal{T}_4 = \int d^6y \sqrt{-G}\, \mathcal{T}_{10}(\tilde{p}) \prod_{\substack{\rm ext.\\ \rm states}} \psi_i(y)$$

$$\mathcal{S} = i(2\pi)^4\delta^4(\Sigma p)\mathcal{T}_4$$

Pomeron in AdS/CFT

Consider Regge scattering: s large with t fixed

Local inertial quantities:

$$\tilde{s} = \frac{R^2}{r^2} s \,, \quad \tilde{t} = \frac{R^2}{r^2} t$$

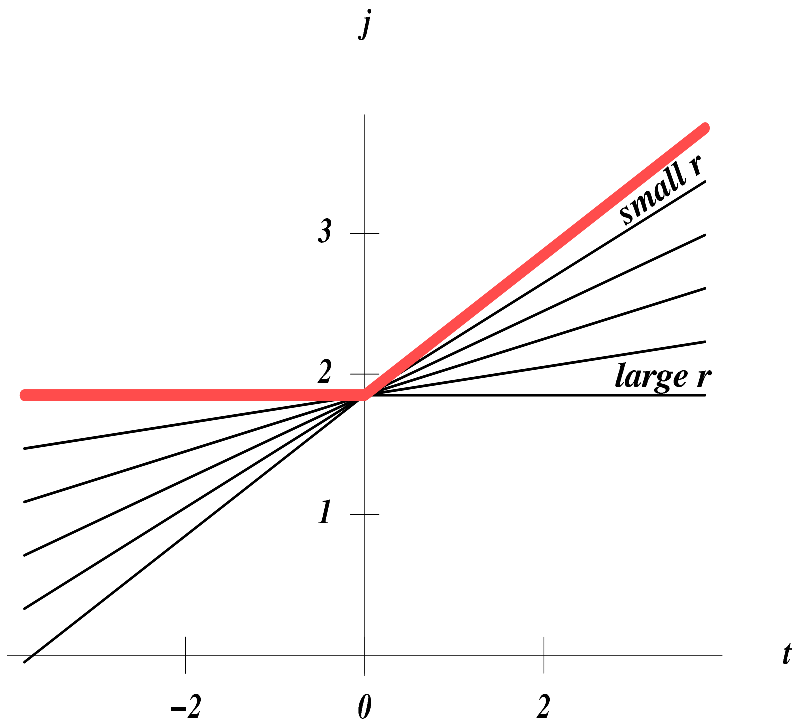
At fixed r: $\mathcal{T}_{10}(\tilde{s}, \tilde{t}) \to f(\alpha' \tilde{t})(\alpha' \tilde{s})^{2+\alpha' \tilde{t}/2}$

$$\mathcal{T}_4(s,t) = \int d^6 y \sqrt{-G}\, f(\alpha' R^2 t/r^2)(\alpha' R^2 s/r^2)^{2+\alpha' R^2 t/2r^2} \prod_{i=1}^{4} \psi_i(y)$$

Regge behaviour may be read off from exponent:

Gives unified description of behaviour expected from QCD!

Pomeron in AdS/CFT



At large s, highest trajectory will dominate:

t positive: r small: soft (Regge) pomeron, properties determined by confining dynamics: glueball

t negative: r large: hard (BFKL) pomeron, two-gluon perturbative small object

Pomeron in AdS/CFT

String theory gives unified description of BFKL and Regge behaviour

(Hard and soft pomeron)

Full analysis beyond local approximation: s large compared to λ

$$\lambda, s \to \infty\,, \quad \frac{\ln s}{\sqrt{\lambda}} \quad \text{fixed}$$

Conclusion

The AdS/CFT correspondence and its generalizations offer new possibilities for a string-theory based description of large N gauge theories.

Four-dimensional scattering amplitudes are obtained from ten-dimensional string amplitudes in a warped space.

A unified description of hard and soft pomeron is obtained.