Top Quark Mass: Fitting, Threshold and Reconstruction

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Outline

- Quark masses in QCD
- Why we want m_t to very high precision
- Top Threshold Scan at ILC
- Fitting Methods at Tevatron
- Reconstruction at LHC and ILC
- Conclusions

Remarks on Quark Masses

- Important QCD input parameters for SM predictions
- Confinement $\Longrightarrow$ quark masses not physical observables

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G_{\mu\nu}^2 + \sum_i \bar{\psi}_i \, (\not{D} - M_i) \, \psi$$

  - ▷ defined as formal parameters in QCD action
  - ▷ (renormalization) scheme dependent
  - ▷ to be well defined: $m_q^{\rm schemeA} = m_q^{\rm schemeB} (1 + \alpha_s + \alpha_s^2 + \ldots)$
  - ▷ some schemes more appropriate than others

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▷ defined as formal parameters in QCD action

pole scheme:

▷ correlations to input parameters, order of p.th.

▷ degrades convergence of p.th. ("$\mathcal{O}(\Lambda_{\rm QCD})$ renormalon") due to linear sensitivity to IR momenta

$\Rightarrow$ Not used any more today when high precision required

Remarks on Quark Masses

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$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G^2_{\mu\nu} + \sum_i \bar{\psi}_i \, (\not{D} - M_i) \, \psi$$

▷ defined as formal parameters in QCD action

$\overline{\rm MS}$ scheme:

▷ short-distance mass

▷ "running" parameter, $\overline{m}_q(\mu)$

▷ for processes where quark is very energetic $(E \gg m_q)$ or far off-shell $(q^2 \neq m_q^2)$

▷ standard mass for comparison: $\overline{m}_q(\overline{m}_q)$

Remarks on Quark Masses

- Important QCD input parameters for SM predictions
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$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G^2_{\mu\nu} + \sum_i \bar{\psi}_i \, (\not{D} - M_i) \, \psi$$

▷ defined as formal parameters in QCD action

threshold schemes:

▷ short-distance mass

▷ processes where quark is close to mass-shell $(q^2 \sim m_q^2)$

non-relativistic sum rules

B physics

heavy quarkonia

top-antitop threshold

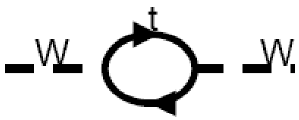
kinetic mass

PS mass

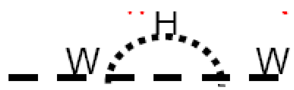
1S mass

Need for a precise Top mass

Electroweak precision observables



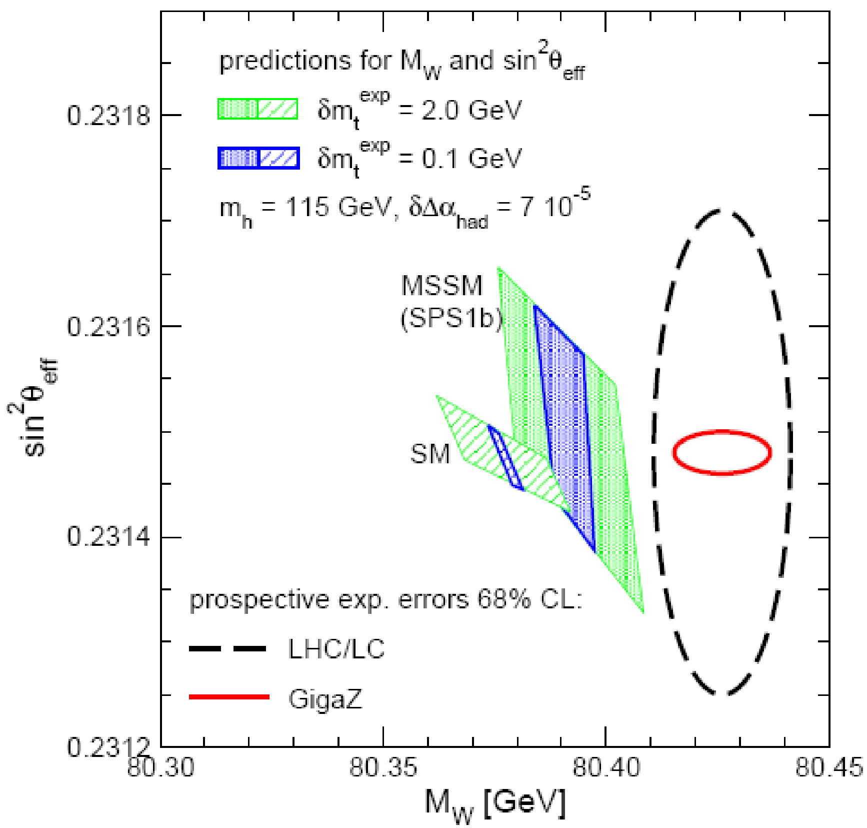
$\delta M_W \propto m_t^2$



$\delta M_W \propto \ln(M_H)$

$$\sin\theta_W \times \Big(1 + \delta(m_t, m_H, \ldots)\Big)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$

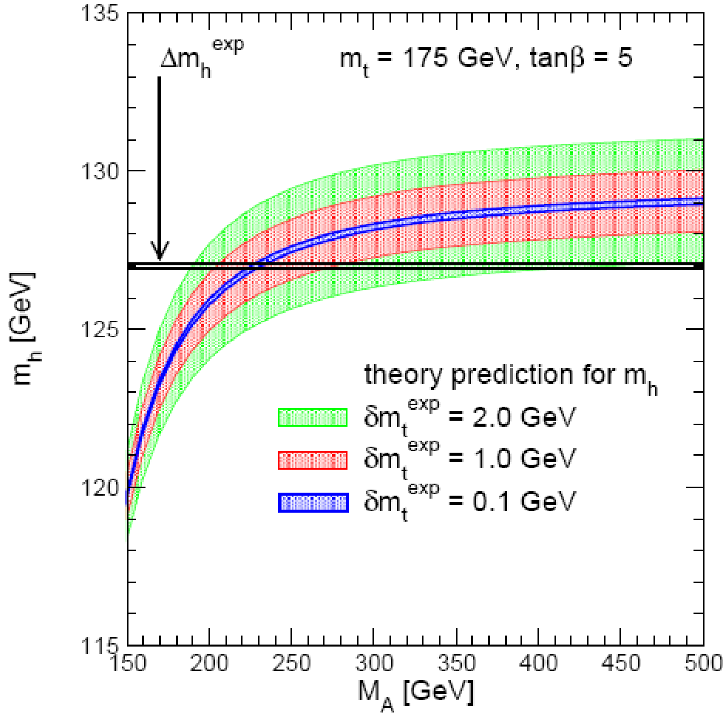


Need for a precise Top mass

Mass of Lightest MSSM Higgs Boson

$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

	LHC	LC
δm_h	1 GeV	50 MeV
needed δm_t	4 GeV	0.2 GeV
expected δm_t	1-2 GeV	~ 0.1 GeV



Need for a precise Top mass

Dark Matter Constraints in $MSSM_{mSUGRA}$

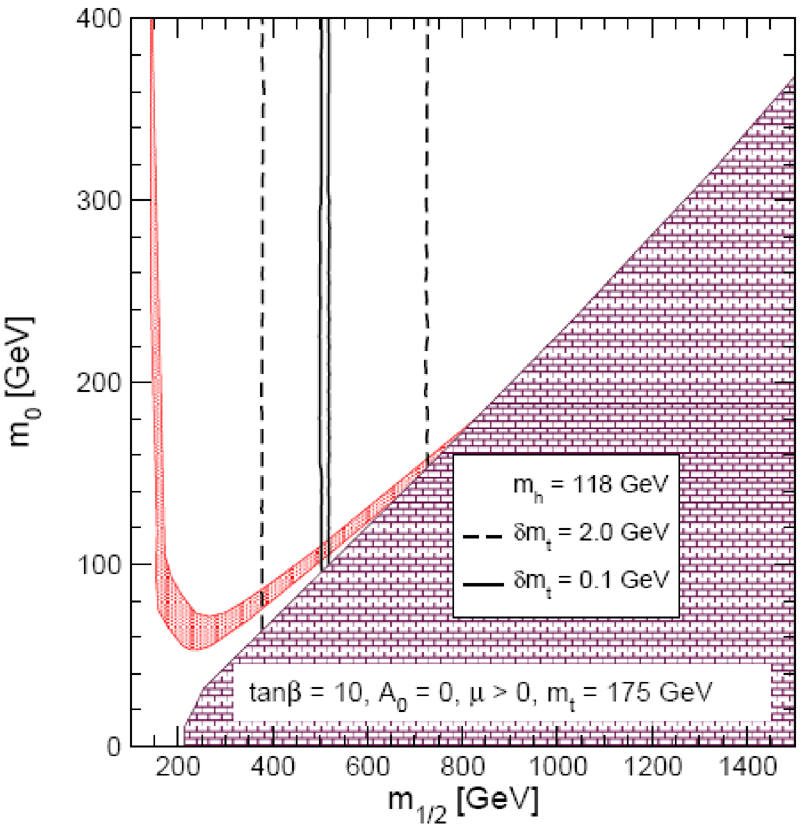
scenario:

$\star$ LSP = neutralino

$\star$ $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$

WMAP $\Rightarrow 0.094 \leq \Omega_\chi h^2 \leq 0.129$

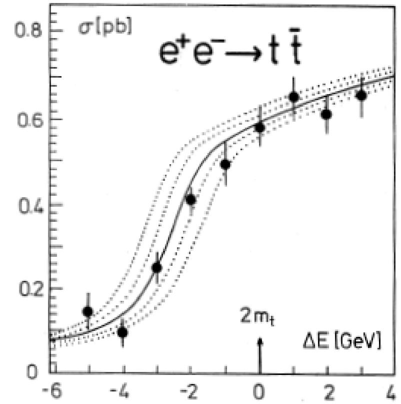
$m_h = 118 \pm 0.05$ GeV



Top Threshold Scan

Threshold Scan: $\sqrt{s} \simeq 350$ GeV (Phase I)

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood
  (renormalons, summations)



$\rightarrow \delta m_t^{\rm exp} \simeq 50$ MeV

$\rightarrow \delta m_t^{\rm th} \simeq 100$ MeV

What mass?

$\sqrt{s}_{\rm rise} \sim 2m_t^{\rm thr} +$ pert.series

(short distance mass: 1S $\leftrightarrow \overline{\rm MS}$)

$$m_t \gg p \sim m_t v \gg E \sim m_t v^2 \sim \Gamma_t > \Lambda_{\rm QCD}$$

- Effective Theory: NRQCD
  - Two correlated renormalization scales

    $E_{\rm kin} \sim \frac{p^2}{m_t} \Rightarrow \mu_{\rm usoft} = \frac{\mu^2_{\rm soft}}{m_t}$
  - Theory unstable top quark

    Γ_t **cuts off nonperturvative effects**

vNRQCD

Luke, Manohar, Rothstein; Stewart, Hoang

vNRQCD

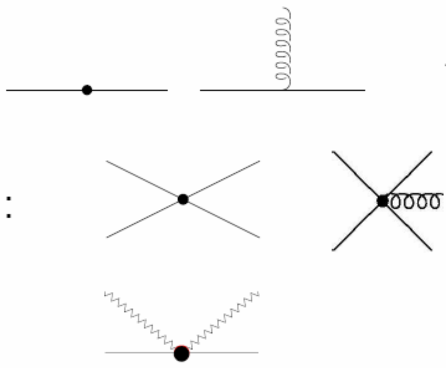
$$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm potential} + \mathcal{L}_{\rm soft}$$

$$D^\mu = \partial^\mu + i g_s(m\nu^2) A^\mu$$

$\mathcal{L}_{\rm usoft}$:

$\mathcal{L}_{\rm potential}$:

$\mathcal{L}_{\rm soft}$:



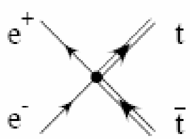
$$\psi_{\mathbf{p}}^\dagger(x)\left\{ iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} - \delta m_t + i\frac{\Gamma_t}{2}\right\}\psi_{\mathbf{p}}(x)$$

$$V(\nu)\,\psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}}$$

$$U_{\mu\nu}(\nu)\;\psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu \psi_{\mathbf{p}}$$

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2}\mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2}\Lambda + \frac{\mathcal{V}_t(\nu)}{m^2}T\right]$$

external currents: (production & annihilation)



$$\mathbf{O}_{\mathbf{p}} = C_{V,A}(\nu)\cdot[\bar{e}\gamma^i(\gamma_5)e][\psi_{\mathbf{p}}^\dagger \boldsymbol{\sigma}^i \tilde{\chi}_{-\mathbf{p}}^*] + \cdots \qquad t\bar{t}\,(^3S_1)$$

Threshold Cross Section

$$\sigma_{\rm tot} \;\propto\; {\rm Im}\left[\int d^4x\, e^{-i\hat{q}x} \left\langle 0 \left| T\, \mathbf{O}_{\mathbf{p}}^\dagger(0)\, \mathbf{O}_{\mathbf{p}'}(x) \right| 0 \right\rangle\right]$$

$$\propto\; {\rm Im}\left[\left(C_A(\nu)^2 + C_V(\nu)^2\right) G(0,0,\sqrt{s})\right]$$

$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

fully known

at NNLL order ✓

Manohar,Stewart; AH '99-'03
Pineda,Soto '00-'01
Peter '94, Schröder '98

NLL ✓

NNLL (matching) ✓ Benke etal; Czarnecki etal '99

NNLL (non-mixing) ✓ AH '03

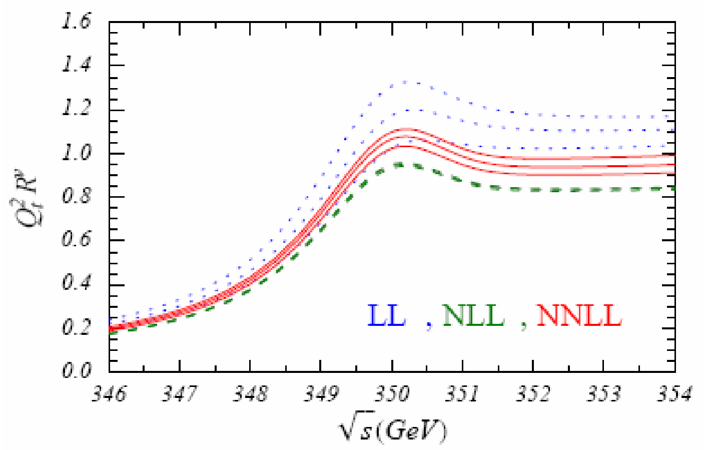
NNLL (mixing) not completed

spin-dependent (soft) Penin etal. '04

usoft $\mathcal{V}_{2,r}$ Stahlhofen, AH 06

NNLL Cross Section

Manohar,Stewart,Teubner,AH 03



- top 1S mass
- NNLL renormalization group improved NRQCD
- fully analytic results

$$\Rightarrow \quad \delta m_t^{\mathrm{1S,th}} \approx 100\ \mathrm{MeV}$$

$$M_t^{\mathrm{1S}} = m_t \Big\{ 1 - [\,\Delta^{\mathrm{LL}}\,] - [\,\Delta^{\mathrm{NLL}}\,] - [\,\Delta_c^{\mathrm{NNLL}} + \Delta_m^{\mathrm{NNLL}}\,] \Big\}$$

$$\Delta^{\mathrm{LL}}(\nu) = \frac{a^2}{8}\,,$$

$$\Delta^{\mathrm{NLL}}(\nu) = \frac{a^3}{8\pi\, C_F}\Big[\beta_0\Big(L+1\Big) + \frac{a_1}{2}\Big]\,,$$

$$\Delta_c^{\mathrm{NNLL}}(\nu) = \frac{a^4}{8\pi^2 C_F^2}\Big[\beta_0^2\Big(\frac{3}{4}L^2 + L + \frac{\zeta_3}{2} + \frac{\pi^2}{24} + \frac{1}{4}\Big) + \beta_0\,\frac{a_1}{2}\Big(\frac{3}{2}L + \cdots\Big)$$
$$+\frac{\beta_1}{4}\Big(L+1\Big) + \frac{a_1^2}{16} + \frac{a_2}{8}\Big]\,,$$

$$\Delta_m^{\mathrm{NNLL}}(\nu) = -\frac{a^2}{8}\,\mathcal{V}_k^{(s)}(\nu) - \frac{a^3}{8\pi}\Big[\frac{\mathcal{V}_2^{(s)}(\nu)}{2} + \mathcal{V}_s^{(s)}(\nu) + \frac{3\mathcal{V}_r^{(s)}(\nu)}{8}\Big] + \frac{5}{128}\,a^4\,,$$

$$L \equiv \ln\Big(\frac{\nu}{a}\Big)\,.$$

$$\bar{m}_t(\bar{m}_t) \;=\; \Big[175 - 7.58\,\epsilon(\mathrm{LO}) - 0.96\,\epsilon^2(\mathrm{NLO}) - 0.23\,\epsilon^3(\mathrm{NNLO})$$
$$\pm 0.2(\delta M_{1S}) \pm x\,0.07(\delta\alpha_s)\Big]\ \mathrm{GeV}\,.$$

$$\alpha_s(M_Z) = 0.118 \pm x\,0.001$$

Fitting Methods at Tevatron

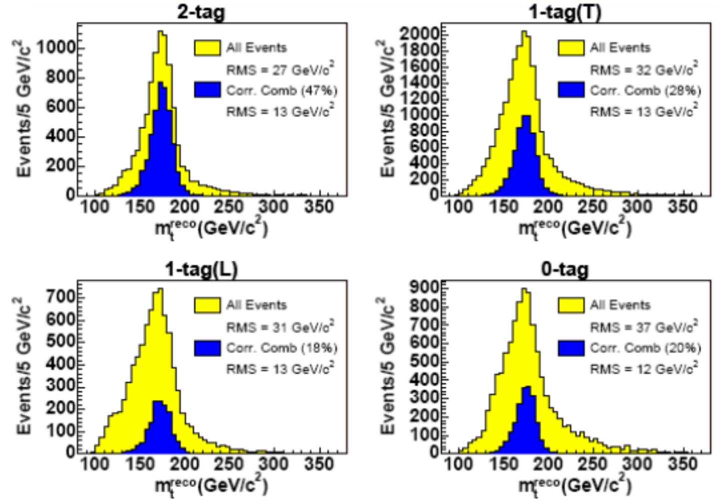
Template Method (CDF II)

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2} + \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{b\ell\nu} - m_t^{reco})^2}{\Gamma_t^2} + \frac{(M_{bjj} - m_t^{reco})^2}{\Gamma_t^2}$$

Usually pick solution with lowest χ^2.

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamics Method (D0 II)

- Principle: compute event-by-event probability as a function of m_t making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

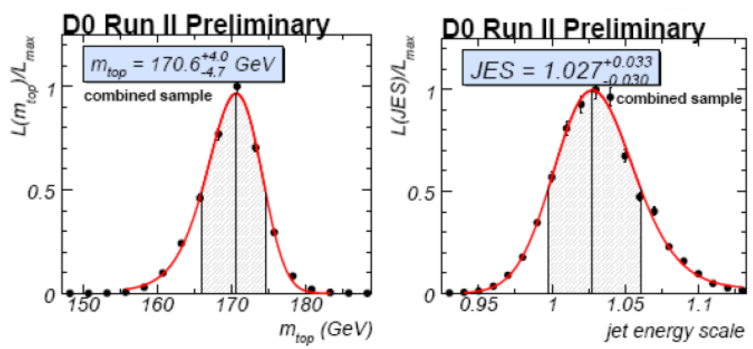
$$P(x;m_t) = \frac{1}{\sigma} \int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1) f(q_2)\, W(x\,|\,y)$$

parton distribution functions

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

Lepton+jets (370 pb^{-1})



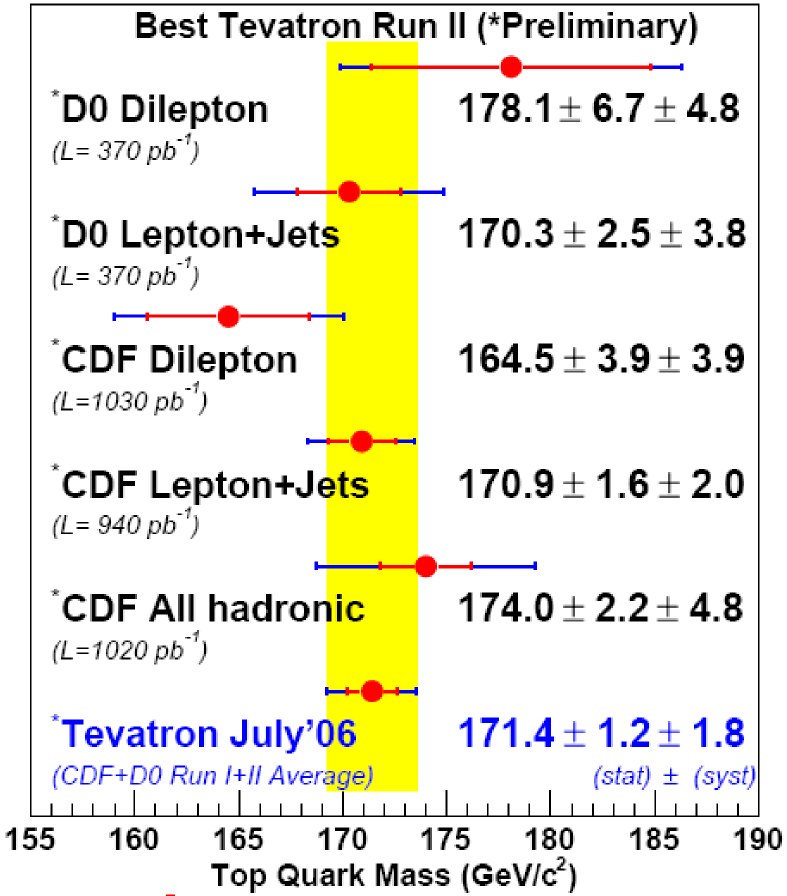
Fitting Methods at Tevatron

Experimental Issues

- Jet Energy scale
- underlying events
- b-tagging
- detector effects

What mass is measured?

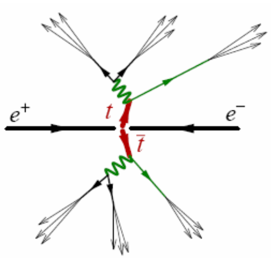
- Top mass used in the MC ?
- Stable under higher order corrections ?
- A problem hard to address

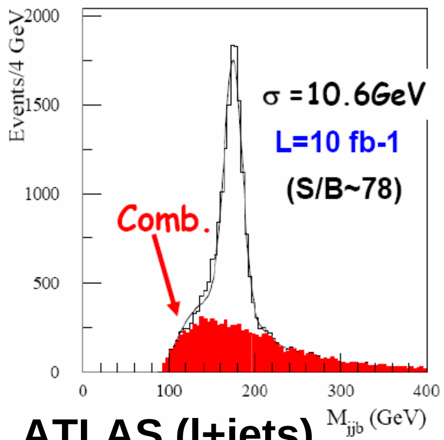


Problem becomes relevant now !

Reconstruction at LHC and ILC

- Invariant Mass Reconstruction
  - suffi cient events for reconstruction (lepton+jets)
  - $\sigma_{\text{tot}}(\text{LHC}) = 850pb \to 10^8\ t\bar{t}$ pairs
  - $\sigma_{\text{tot}}(\text{ILC}) = 1pb \to 10^5\ t\bar{t}$ pairs





ATLAS (l+jets)

$\rightarrow$ things to worry about:

- JES
- jet-jet interconnection
- background + underlying events
- how good is MC?

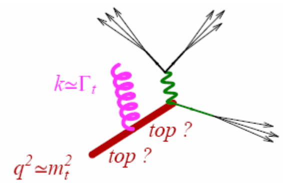
$\delta m_t^{\text{exp ex}} \simeq 1$ GeV (LHC)

$\delta m_t^{\text{exp ex}} \simeq 0.2$ GeV (ILC)

Source of uncertainty	Hadronic δM$_{top}$ (GeV)	Fitted δM$_{top}$ (GeV)
Light jet scale	0.9	0.2
b-jet scale	0.7	0.7
b-quark fragm	0.1	0.1
ISR	0.1	0.1
FSR	1.9	0.5
Comb bkg	0.4	0.1
Total	2.3	0.9

Systematics uncertainties:

Jet energy calibration
'Out of cone' showering
B-fragmentation



What mass?

Pole Mass ?

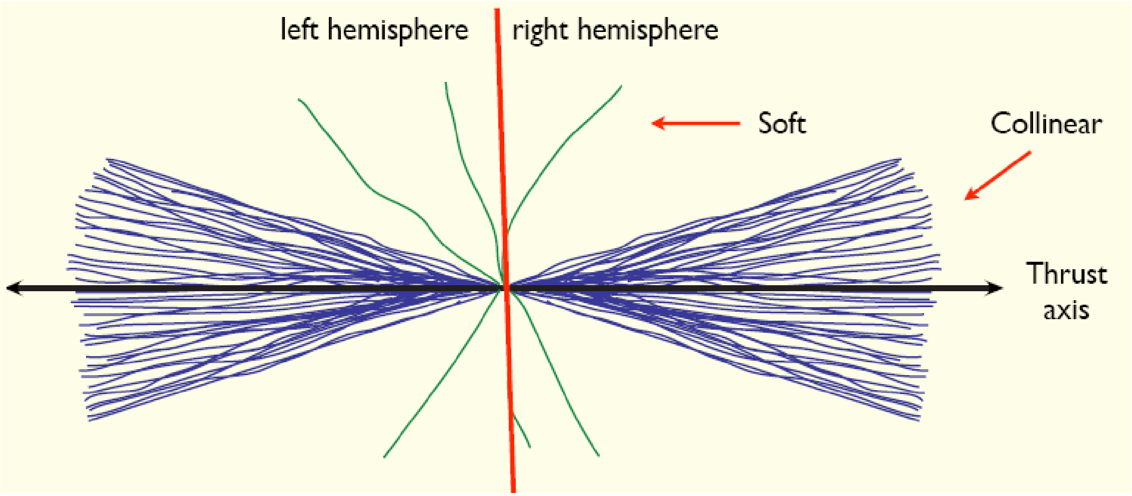
ambiguity: $\Delta m_t \sim \Lambda_{\text{QCD}}$

$\Delta m_t \sim \alpha_s(\Gamma_t)\,\Gamma_t$

Analytic Approach

Fleming,Hoang, Mantry,Stewart

e.g. Double differential hemisphere top mass distribution



$$\frac{d^2\sigma}{ds_t\, ds_{\bar t}}$$

$$s_t - m_t \sim \Gamma_t$$
$$s_{\bar t} - m_t \sim \Gamma_t$$

$$Q \gg m_t \gg \Gamma_t > \Lambda_{QCD}$$

Dijet Event

$1 - T \approx 0$

SCET

Jets with

$s_t - m_t \sim m_t$

Integrate out m

boosted HQET

Jets with

$s_t - m_t \sim \Gamma_t$

Factorization Formula

In SCET:

$$\left(\frac{d^2\sigma}{ds_t\,ds_{\bar{t}}}\right)_{\rm hemi} = \sigma_0\, H(Q,\mu)\int d\ell^+ d\ell^-\, J_n(s_t - Q\ell^+,\mu)\, J_{\bar{n}}(s_{\bar{t}} - Q\ell^-,\mu)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

- Jet functions (generalized "Breit-Wigners")
- Perturbative
- Top width acts as IR-cutoff.
- Depends on a <u>short-distance mass</u> !!

- Soft cross talk between jets
- Non-perturbative
- Does not see top width due to time dilatation.
- Agrees with soft function for massless dijet

See Sonny's talk for details !

Conclusion

- m_t needs to be known as precise as possible.
- Only measurements of Lagrangian masses are meaningful. Pole mass is irrelevant, i.e. not (never!) a physical observable.
- Threshold scan at the ILC the best understood (and probably most precise) method to determine m_t. Not yet clear whether it whether/when ILC will be built.
- Factorization approach to reconstruction feasible. Methods are are totally free of non-perturbative effects at leading order can be explored.
- Important Lesson: The top mass you measure depends on the method you use.