Power Corrections to Event Shapes from Factorized Soft Gluons and SCET

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09 January 2007
Ringberg Workshop on non-perturbative QCD of jets

Outline

- Event Shapes and Nonperturbative Power Corrections

- Factorization in Soft-Collinear Effective Theory (SCET) and QCD

- Power Corrections from Soft Radiation and Universality

C. Bauer, CL, A. Manohar, M. Wise, PRD **70**, 034014 (2004).
CL, G. Sterman, hep-ph/0611061 (PRD).

Event Shapes and Power Corrections

Two-Jet Event Shapes

- **Thrust:** $$T = \frac{1}{Q} \max_{\hat{\mathbf{t}}} \sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|. \quad \text{or } \tau = 1 - T$$
- **Broadening:** $$B = \frac{1}{Q} \sum_i |\mathbf{p}_i \times \hat{\mathbf{t}}|.$$
- ***C* Parameter:** $$C = \frac{3}{2Q^2} \sum_{i,j} |\mathbf{p}_i||\mathbf{p}_j| \sin^2 \theta_{ij}$$
- **Angularities:** $$\tau_a = \frac{1}{Q} \sum_i E_i \sin^a \theta_i (1 - \cos \theta_i)^{1-a}$$

θ_i: angle with thrust axis

$a = 0$: thrust

$a = 1$: broadening

Berger, Kucs, Sterman (2003); Berger, Magnea (2004)

- Infrared-safe for $a < 2$, our analysis of NP effects valid for $a < 1$.

General Class of Event Shapes

- Make use of rapidities:

$$\eta_i = \frac{1}{2}\ln\left(\frac{E_i + p_z^i}{E_i - p_z^i}\right) = \ln\cot\left(\frac{\theta_i}{2}\right)$$

for massless particles

- Many event shapes can be expressed as:

$$e = \frac{1}{Q}\sum_i |\mathbf{k}_i^{\perp}| f_e(\eta_i)$$

$$f_\tau(\eta) = e^{-|\eta|} \qquad f_C(\eta) = \frac{3}{\cosh\eta} \qquad f_{\tau_a}(\eta) = e^{-(1-a)|\eta|}$$

- Other choices for *f* which are sufficiently smooth are also possible

Sources of Nonperturbative Corrections

- Nonperturbative corrections come from radiation with transverse momenta $k_\perp \sim \Lambda_{\rm QCD}$
- Soft emission gives nonperturbative power corrections of order $\sim 1/Q$ in event shape distributions or mean values
- Collinear emission gives subleading corrections of order $\sim 1/Q^{1+b}$, where *b* depends on the event shape

Proposals for Universality of NP corrections

- Shift of event shape mean values:

$$\langle e \rangle = \langle e \rangle_{\mathrm{PT}} + c_e \frac{\mathcal{A}}{Q}$$

$$c_{1-T} = 2 \qquad c_C = 3\pi \qquad c_{\tau_a} = \tfrac{2}{1-a}$$

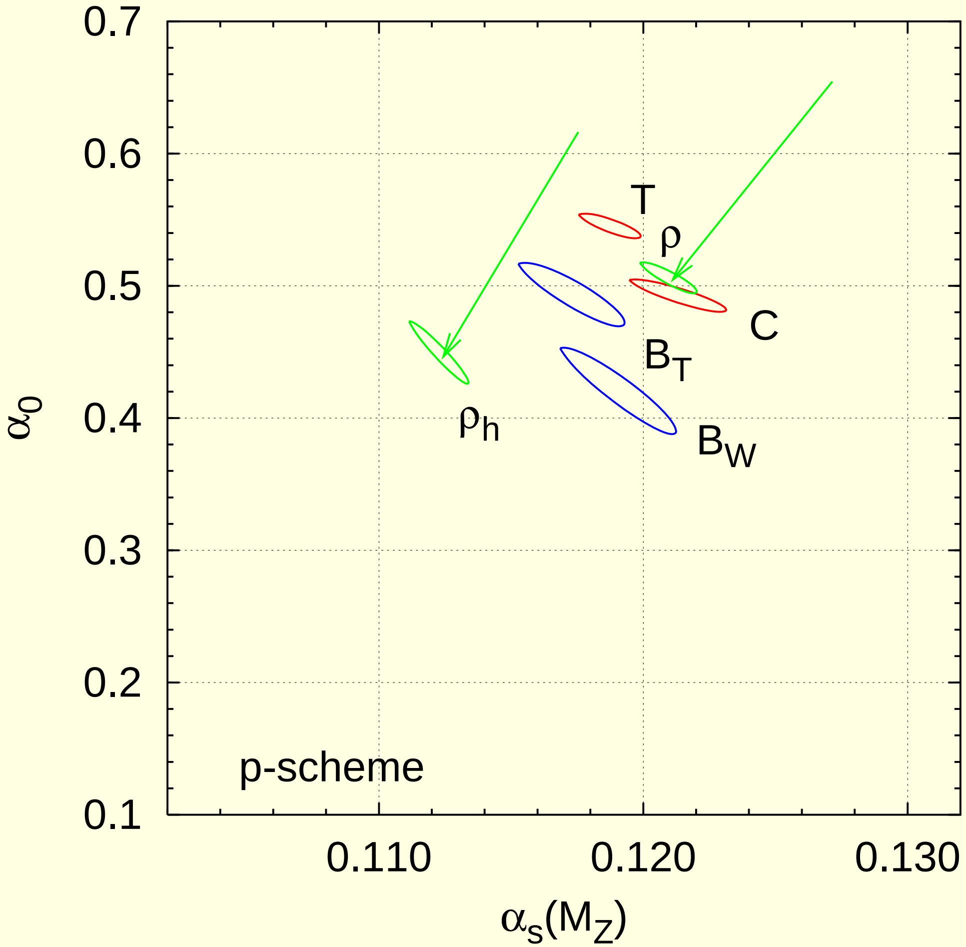
$$\mathcal{A} = \mathcal{A}\big(\alpha_s(Q), \bar{\alpha}_0(\mu_I)\big)$$

- Shift of event shape *distributions:*

$$\frac{d\sigma}{de}(e) = \frac{d\sigma}{de}\left(e - c_e \frac{\mathcal{A}}{Q}\right)\Bigg|_{\mathrm{PT}}$$

Dokshitzer, Webber (1995, 1997)

Experimental Tests



Fit $\alpha_s(\mu), \alpha_0(\mu_I)$ as free parameters to observed 1/Q shifts (typically at $\mu_I \approx 2$ GeV)

Correct value of $\alpha_s(\mu)$ and a universal α_0

From
M. Dasgupta
G. Salam
(2003)

Approaches to Prove Universality

- Numerous approaches based on analysis of low-scale behavior of perturbation theory predict universal coefficient for 1/Q corrections

- Data validate this prediction fairly well

- But models incorporate, at some level, information only of single gluon emission
  - Extended to two gluons, universality maintained (Milan factor)

    Y. Dokshitzer, A. Lucenti, G. Marchesini, G. Salam (1998)

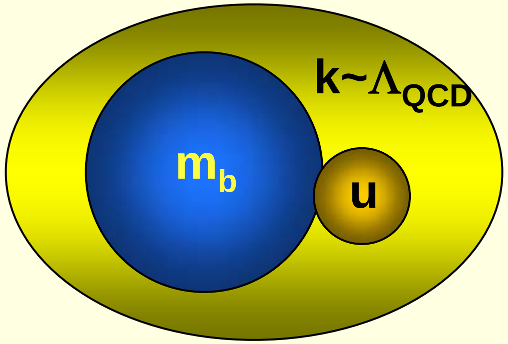
- Can universality of shifts be proved beyond this level?

Factorization in SCET and QCD

Effective Field Theories

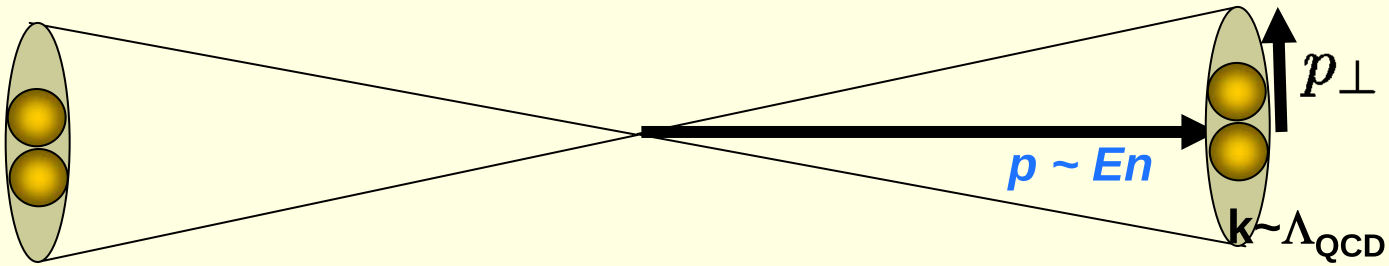
- HQET: effective field theory for fluctuations about heavy quark momentum *mv* = *m*(1,0,0,0)

$$p_b = m_b v + k \\ \sim \mathcal{O}(m_b) + \mathcal{O}(\Lambda_{\mathrm{QCD}})$$



- SCET: effective field theory for fluctuations about light-like trajectory *n* = (1,0,0,1)

$$p_n = En + p_\perp + k \sim \mathcal{O}(Q) + \mathcal{O}(\sqrt{Q\Lambda_{\mathrm{QCD}}}) + \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

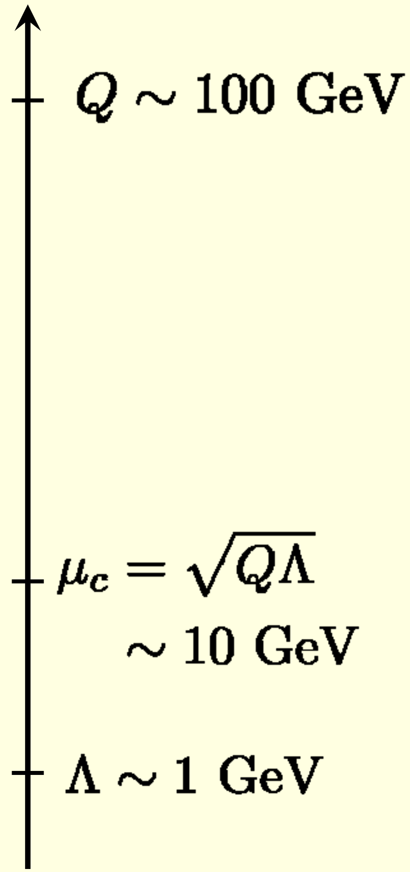


Degrees of Freedom

$n, \bar{n} = (1,0,0,\pm 1)$ $\quad$ $p = (n \cdot p, \bar{n} \cdot p, p_\perp)$

Mode	Scaling
Collinear Quarks ξ_n Gluons A_n	$p_c \sim Q(\lambda^2, 1, \lambda)$ $\Rightarrow p_c^2 \sim Q\Lambda$
Soft Gluons A_s	$p_s \sim Q(\lambda^2, \lambda^2, \lambda^2)$ $\Rightarrow p_s^2 \sim \Lambda^2$

$$\lambda = \sqrt{\frac{\Lambda_{\rm QCD}}{Q}}$$



QCD → SCET

Bauer, Fleming, Pirjol, Stewart (2001)

- Full QCD: $$\mathcal{L}_{\mathrm{QCD}} = \sum_{a=1}^{N_C} \bar{q}^a i\gamma^\mu D_\mu q^a - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$
- Divide momenta into "label" and "residual" components:

$$p = \tilde{p} + \underset{\mathcal{O}(Q\lambda^2)}{k}, \quad \tilde{p}^\mu = \underset{\mathcal{O}(Q)}{\bar{n}\cdot p}\,\frac{n^\mu}{2} + \underset{\mathcal{O}(Q\lambda)}{p_\perp^\mu}$$

- Factor out label momenta from collinear fields:

$$\psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\psi_{n,p}(x)$$

- Project out large & small components of quark spinors:

$$\xi_{n,p} = \frac{\not{n}\bar{\not{n}}}{4}\psi_{n,p}\,, \quad \Xi_{n,p} = \frac{\bar{\not{n}}\not{n}}{4}\psi_{n,p}$$

massless, dynamical

integrate out, effective mass ~Q

SCET Lagrangian

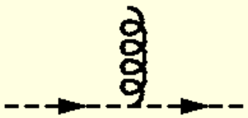
Bauer, Fleming, Pirjol, Stewart (2001)

- Leading order in λ:

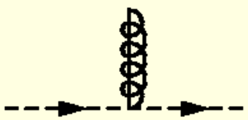
$$\mathcal{L}_{\xi\xi} = \sum_{p,p'\neq 0} e^{-i(\tilde{p}-\tilde{p}')\cdot x}\,\bar{\xi}_{n,p'}\left(in\cdot D_{us} + \sum_{q\neq 0} e^{-i\tilde{q}\cdot x}\, g n\cdot A_{n,q} + i\not{D}_c^{\perp} W \frac{1}{\bar{\mathcal{P}}} W^{\dagger} i\not{D}_c^{\perp}\right)\frac{\bar{\not{n}}}{2}\xi_{n,p}$$

- Feynman rules:

$$= i\frac{\not{n}}{2}\frac{\bar{n}\cdot p}{n\cdot k\,\bar{n}\cdot p + p_{\perp}^2 + i\epsilon}$$



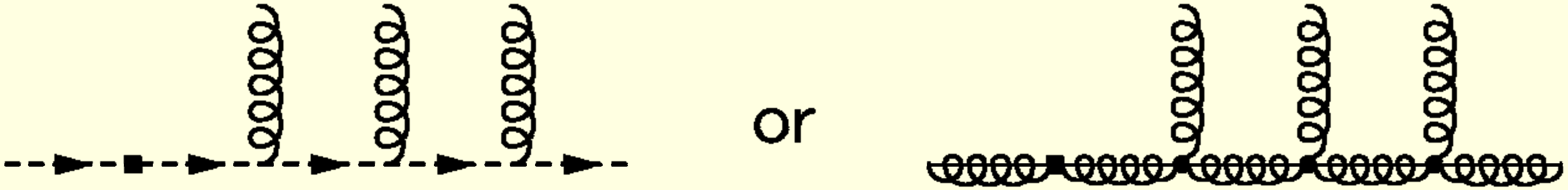
$$= igT^A n_{\mu}\frac{\not{n}}{2}$$



$$= igT^A\left(n_{\mu} + \frac{\gamma_{\mu}^{\perp}\not{p}_{\perp}}{\bar{n}\cdot p} + \frac{\not{p}'_{\perp}\gamma_{\mu}^{\perp}}{\bar{n}\cdot p'} - \frac{\not{p}'_{\perp}\not{p}_{\perp}}{\bar{n}\cdot p\,\bar{n}\cdot p'}\bar{n}_{\mu}\right)\frac{\not{n}}{2}$$

Wilson Lines in SCET

- Collinear Wilson Lines: $W_n(z) = P\exp\left[\int ds\, \bar{n}\cdot A_n(\bar{n}s + z)\right]$
  - Required by gauge invariance
  - Arbitrary number of $\mathcal{O}(1)\ \bar{n}\cdot A_n$ gluons required by power counting
- Soft Wilson Lines: $Y_n(z) = P\exp\left[\int ds\, n\cdot A_s(ns + z)\right]$
  - Sum up couplings of soft gluons:



- only $n\cdot A_s$ component in leading-order SCET Lagrangian

Decoupling of Soft Gluons

Bauer, Pirjol, Stewart (2001)

- Field redefinition: $\xi_n \to Y_n^\dagger \xi_n\,, \quad A_n \to Y_n^\dagger A_n Y_n$
- Removes collinear-soft couplings in leading-order SCET$_{\rm I}$ Lagrangian, for example:

$$\bar{\xi}_n in\cdot D_s \frac{\bar{n}\!\!\!/}{2}\xi_n \to \bar{\xi}_n Y_n in\cdot D_s Y_n^\dagger \frac{\bar{n}\!\!\!/}{2}\xi_n,$$

Now, $Y_n^\dagger$ satisfies: $n\cdot D_s\, Y_n^\dagger(z) = 0$

so the above term in $\mathcal{L}$ reduces to: $\bar{\xi}_n in\cdot\partial \frac{\bar{n}\!\!\!/}{2}\xi_n,$

- At leading order in λ, $\mathcal{L}$ becomes free of couplings to soft gluons, but Wilson lines Y_n must appear in operators containing collinear fields.

Event Shapes in SCET

- Differential $e^+e^- \to \text{hadrons}$ cross-section with respect to event shape *e*:

$$\frac{d\sigma}{de}(Q) = \frac{1}{2Q^2}\sum_{N} |\langle N|\, j^\mu(0)\, |0\rangle\, L_\mu|^2 (2\pi)^4 \delta^4(Q - p_N)\delta(e - e(N))$$

- Match QCD current onto SCET operator:

$$j^\mu_{\text{QCD}} = \bar{q}\gamma^\mu q \quad \longrightarrow \quad j^\mu_{\text{SCET}} = C(\mu)[\bar{\xi}_{\bar{n}} W_{\bar{n}} Y_{\bar{n}}]\gamma^\mu_\perp [Y_n^\dagger W_n^\dagger \xi_n]$$

- Split final state

$$|N\rangle = |J_n\rangle \otimes |J_{\bar{n}}\rangle \otimes |X_s\rangle$$

  - Caveat: partonic statement; one step before hadronization… possible to eliminate this step
- No interactions between sectors in leading-order SCET Lagrangian

Factorization

- Convolution of jet and soft functions:

$$\frac{d\sigma}{de} = \int de_J \, \sigma_J(e_J;\mu) S(e - e_J, \mu)$$

- Collinear jet function calculable perturbatively
- Leading nonperturbative effects in soft function:

$$S(e) = \frac{1}{N_C} \mathrm{Tr} \sum_{X_s} \left| \langle X_s | \, T[Y_n Y_{\bar{n}}^\dagger] \, |0\rangle \right|^2 \delta(e - e(X_s))$$

- Soft function of the form $S(e) = \delta(e - c_e \mathcal{A}/Q)$ would reproduce usual prediction for universal shift of full distribution

Factorization in QCD

- Factorized distribution in QCD:

$$\frac{d\sigma}{de} = \sigma_0(Q) \int de_n \, de_{\bar{n}} \, de_S \, \delta(e - e_n - e_{\bar{n}} - e_S) J_n(Q, e_n) J_{\bar{n}}(Q, e_{\bar{n}}) S_{n\bar{n}}(e_s Q)$$

- Jet functions:

$$J_c^{\mu}(Q, e_{J_c}) = \frac{2}{Q^2} \frac{(2\pi)^6}{N_C} \sum_{N_{J_c}} \mathrm{Tr}\left[\gamma^{\mu} \langle 0| \Phi_{\xi_c}^{(q)\dagger}(0) q(0) |N_{J_c}\rangle \langle N_{J_c}| \bar{q}(0) \Phi_{\xi_c}^{(q)}(0) |0\rangle\right]$$
$$\times \, \delta(e_{J_c} - e(N_{J_c})) \delta(Q - \omega(N_{J_c})) \delta^2(\hat{n}_{J_c} - \hat{n}(N_{J_c}))$$

Wilson lines in direction ξ_c away from light-cone jet direction:

$$\Phi_{\xi_c}^{(f)}(z) = P \exp\left[ig \int_{-\infty}^{0} d\lambda \, \xi_c \cdot \mathcal{A}^{(f)}(\lambda \xi_c + z)\right]$$ enforce gauge invariance

Berger, Kucs, Sterman (2003)

Factorization in QCD

- Soft function constructed from “eikonal” cross-section:

$$\sigma^{(\mathrm{eik})}(e) = \frac{1}{N_C} \sum_{N_{\mathrm{eik}}} \langle 0 | \, \Phi_{\bar{n}}^{(\bar{q})}(0) \Phi_{n}^{(q)\dagger}(0) \, | N_{\mathrm{eik}} \rangle \, \langle N_{\mathrm{eik}} | \, \Phi_{n}^{(q)}(0) \Phi_{\bar{n}}^{(\bar{q})}(0) \, | 0 \rangle \, \delta(e - e(N_{\mathrm{eik}}))$$

  - Models soft radiation away from jets, double-counts collinear radiation along jets

- Remedy: subtract out “eikonal” jet functions

$$J_c^{(\mathrm{eik})}(e_c) = \frac{1}{N_C} \sum_{N_c^{\mathrm{eik}}} \langle 0 | \, \Phi_{\xi_c}^{(f_c)\dagger}(0) \Phi_{\beta_c}^{(f_c)\dagger}(0) \, \big| N_c^{\mathrm{eik}} \big\rangle \, \big\langle N_c^{\mathrm{eik}} \big| \, \Phi_{\beta_c}^{(f_c)}(0) \Phi_{\xi_c}^{(f_c)}(0) \, | 0 \rangle \, \delta(e_c - e(N_c^{\mathrm{eik}}))$$

- Clearer in terms of Laplace transform:

$$\tilde{\sigma}(\nu) = \int_0 de \, \mathrm{e}^{-\nu e} \frac{d\sigma}{de} = \sigma_0 \tilde{J}_n(\nu) \tilde{J}_{\bar{n}}(\nu) \tilde{S}(\nu)$$

- Soft function is constructed out of eikonal functions:

$$\tilde{S}(\nu) = \frac{\tilde{\sigma}^{(\mathrm{eik})}(\nu)}{\tilde{J}_n^{(\mathrm{eik})}(\nu) \tilde{J}_{\bar{n}}^{(\mathrm{eik})}(\nu)}$$

  - Eliminates double-counting of radiation collinear to jets in jet and soft functions

Equivalence to SCET Factorization

- Rearrange eikonal factors: $\tilde{\sigma}(\nu) = \sigma_0 \tilde{\mathcal{J}}_n(\nu) \tilde{\mathcal{J}}_{\bar{n}}(\nu) \tilde{\sigma}^{(\mathrm{eik})}(\nu)$

  where: $\tilde{\mathcal{J}}_c(\nu) = \dfrac{\tilde{J}_c(\nu)}{\tilde{J}_c^{(\mathrm{eik})}(\nu)}$

- Now soft radiation along jet directions subtracted out of collinear jet functions

- Same as "zero-bin subtraction" for collinear fields in SCET Lagrangian,
  - e.g.

$$\mathcal{L}_{\xi\xi} = \sum_{p,p',q\neq 0} e^{-i(\tilde{p}-\tilde{p}'+\tilde{q})\cdot x} \bar{\xi}_{n,p'} g n \cdot A_{n,q} \frac{\bar{\not{n}}}{2} \xi_{n,p}$$

Manohar, Stewart (2006)

- $\tilde{\mathcal{J}}_{n,\bar{n}}$ are SCET jet functions with zero-bins subtracted

- $\tilde{\sigma}^{(\mathrm{eik})}$ is (Laplace-transformed) SCET soft function:

$$S(e) = \frac{1}{N_C} \operatorname{Tr} \sum_{X_s} \left| \langle X_s | T[Y_n Y_{\bar{n}}^\dagger] | 0 \rangle \right|^2 \delta(e - e(X_s))$$

Power Corrections from Soft Radiation, Universality, and Violations Thereof

Energy Flow

cf. Belitsky, Korchemsky, Sterman (2001)

- Consider event shapes expressed in terms of rapidities:

$$e(X) = \sum_{i \in X} |\mathbf{k}_i^\perp| \, f(\eta_i)$$

- Want to eliminate sum over states in soft function:

$$S(e) = \frac{1}{N_C} \mathrm{Tr} \sum_{X_s} \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \, |X_s\rangle \langle X_s | \, Y_n \overline{Y}_{\bar{n}} \, |0\rangle \, \delta\left(e - \frac{1}{Q} \sum_{i \in X_s} |\mathbf{k}_i^\perp| \, f_e(\eta_i) \right)$$

- Using “transverse energy flow” operator:

$$\mathcal{E}_T(\eta) \, |X\rangle = \sum_{i \in X} |\mathbf{k}_i^\perp| \, \delta(\eta - \eta_i) \, |X\rangle$$

- Perform sum over states $|X_s\rangle$:

$$S(e) = \frac{1}{N_C} \mathrm{Tr} \, \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \delta\left(e - \frac{1}{Q} \int_{-\infty}^{\infty} d\eta \, \mathcal{E}_T(\eta) f_e(\eta) \right) Y_n \overline{Y}_{\bar{n}} \, |0\rangle$$

$\overline{Y}_{n,\bar{n}}$: in anti-fundamental representation

Shift in Event Shape Mean Values

- First moment (mean value):

$$\langle e \rangle = \frac{1}{\sigma_{\mathrm{tot}}} \int de\, e\, \frac{d\sigma}{de}$$
$$= \frac{1}{\sigma_{\mathrm{tot}}} \int de\, e \int de_J\, \sigma_J(e_J) S(e - e_J)$$

$$S(e) = \delta(e) - \frac{1}{Q}\delta'(e) \int d\eta\, f_e(\eta) \frac{1}{N_C} \mathrm{Tr}\, \langle 0 | \overline{Y}^\dagger_{\bar{n}} Y^\dagger_n \mathcal{E}_T(\eta) Y_n \overline{Y}_{\bar{n}} | 0 \rangle + \cdots$$

- Shift in first moment:

$$\langle e \rangle = \langle e \rangle_{\mathrm{PT}} + \frac{1}{Q} \int d\eta\, f_e(\eta) \Big\langle \mathcal{E}_T(\eta) \Big\rangle$$

$$\Big\langle \mathcal{E}_T(\eta) \Big\rangle = \frac{1}{N_C} \mathrm{Tr}\, \langle 0 | \overline{Y}^\dagger_{\bar{n}} Y^\dagger_n \mathcal{E}_T(\eta) Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

Boost Invariance in Soft Sector

- Insert Lorentz boosts in soft matrix element:

$$\left\langle \mathcal{E}_T(\eta) \right\rangle = \frac{1}{N_C} \operatorname{Tr} \langle 0 | \overline{Y}^{\dagger}_{\bar{n}} Y^{\dagger}_{n} \mathcal{E}_T(\eta) Y_n \overline{Y}_{\bar{n}} |0\rangle$$

- Boost properties:

$$U(\Lambda(\eta'))^{-1} U(\Lambda(\eta'))$$

$$\begin{aligned}
|0\rangle &\to |0\rangle \\
Y_n(0) &= P \exp\left[ig \int_0^\infty ds\, n \cdot A_s(ns) \right] \\
&\to P \exp\left[ig \int_0^\infty ds\, e^{-\eta'} n \cdot A_s(e^{-\eta'} ns) \right] = Y_n(0) \\
\mathcal{E}_T(\eta) &\to \mathcal{E}_T(\eta + \eta')
\end{aligned}$$

Boost Invariance in Soft Sector

- Implies $\left\langle \mathcal{E}_T(\eta) \right\rangle = \left\langle \mathcal{E}_T(\eta + \eta') \right\rangle$ for any η'
  - choose $\left\langle \mathcal{E}_T(0) \right\rangle$
- Can perform integral over rapidity:

$$\begin{aligned} \langle e \rangle &= \langle e \rangle_{\mathrm{PT}} + \frac{1}{Q} \int d\eta\, f_e(\eta) \langle \mathcal{E}_T(0) \rangle \\ &= \langle e \rangle_{\mathrm{PT}} + \frac{1}{Q} c_e \mathcal{A}, \end{aligned}$$

where:

$$c_e = \int_{-\infty}^{\infty} d\eta\, f_e(\eta) \qquad \mathcal{A} = \left\langle \mathcal{E}_T(0) \right\rangle$$

- Reproduces usual prediction for shift in mean value
  - provides field theory interpretation of universal quantity

$\mathcal{A}$

Not Enough Boost Invariance

- Does not give simple shift of full distribution:

$$S(e) = \left\langle \delta\left(e - \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, \mathcal{E}_T(\eta) f_e(\eta)\right)\right\rangle$$
$$\neq \delta\left(e - \frac{c_e \mathcal{A}}{Q}\right)$$

  - e.g. next term in series is:

$$\frac{1}{2}\delta''(e)\frac{1}{Q^2}\int d\eta_1 \int d\eta_2\, f_e(\eta_1) f_e(\eta_2)\left\langle \mathcal{E}_T(\eta_1)\mathcal{E}_T(\eta_2)\right\rangle$$
$$= \frac{1}{2}\delta''(e)\frac{1}{Q^2}\int d\eta_1 \int d\eta_2\, f_e(\eta_1) f_e(\eta_2)\left\langle \mathcal{E}_T(0)\mathcal{E}_T(\eta_2 - \eta_1)\right\rangle$$

  can perform one rapidity integral, but not both

$\Rightarrow$

- Correlations in soft radiation spoil simple shift

Soft Function for Angularities

- Recall: $\tau_a = \frac{1}{Q}\sum_i |\mathbf{k}_i^{\perp}| e^{-(1-a)|\eta_i|}$

$$S(\tau_a) = \left\langle \delta\left(\tau_a - \frac{1}{Q}\int d\eta\, \mathcal{E}_T(\eta) e^{-(1-a)|\eta|}\right)\right\rangle$$

- Useful to look at Laplace transform:

$$\tilde{S}_a(\nu) = \int d\tau_a\, e^{-\nu\tau_a} S(\tau_a)$$

$$\tilde{S}_a(\nu) = \left\langle \exp\left[-\frac{\nu}{Q}\int d\eta\, e^{-(1-a)|\eta|}\mathcal{E}_T(\eta)\right]\right\rangle$$

- Re-expressed as an expansion in *cumulants*

$$\tilde{S}_a(\nu) = \exp\left[\sum_{n=1}^{\infty}\left(-\frac{\nu}{Q}\right)^n \left\langle\!\!\left\langle \left[\int d\eta\, e^{-(1-a)|\eta|}\mathcal{E}_T(\eta)\right]^n \right\rangle\!\!\right\rangle\right]$$

$$\langle\!\langle \hat{X} \rangle\!\rangle = \langle X \rangle,\ \langle\!\langle \hat{X}^2 \rangle\!\rangle = \langle X^2 \rangle - \langle X \rangle^2$$

Scaling Rule for Angularities

- Resummed pQCD and single gluon approximation predict the simple dependence on *a*:

$$\tilde{S}_a(\nu) = \exp\left[\frac{1}{1-a}\sum_{n=1}^{\infty}\lambda_n\left(-\frac{\nu}{Q}\right)^n\right]$$

Berger, Sterman (2003); Berger, Magnea (2004)

λ_n independent of *a*

- *n=1*: $\left\langle\!\!\left\langle \int d\eta\, e^{-(1-a)|\eta|}\mathcal{E}_T(\eta)\right\rangle\!\!\right\rangle = \frac{1}{1-a}\mathcal{A}$
- *n=2*: $\left\langle\!\!\left\langle \int d\eta_1\, d\eta_2\, e^{-(1-a)(|\eta_1|+|\eta_2|)}\mathcal{E}_T(\eta_1)\mathcal{E}_T(\eta_2)\right\rangle\!\!\right\rangle$

$$= \int d\eta\left(\frac{1}{1-a}+|\eta|\right)e^{-(1-a)|\eta|}\langle\!\langle\mathcal{E}_T(0)\mathcal{E}_T(\eta)\rangle\!\rangle$$

$$= \frac{1}{1-a}\int d\eta\left\{1-\frac{1}{2}[(1-a)|\eta|]^2+\cdots\right\}\langle\!\langle\mathcal{E}_T(0)\mathcal{E}_T(\eta)\rangle\!\rangle$$

absence of correlations in $\eta \Rightarrow \frac{1}{1-a}$ scaling

Scaling Violations = Energy Flow Correlations

- Equation for Laplace-transformed correlator:

$$\mathcal{C}_2(a) - \frac{\partial}{\partial \ln(1-a)}\mathcal{C}_2(a) = \lambda_2(a)$$

$$\mathcal{C}_2(a) = \frac{1}{2}\int d\eta\, e^{-(1-a)|\eta|}\langle\!\langle \mathcal{E}_T(0)\mathcal{E}_T(\eta)\rangle\!\rangle \quad \tilde{S}_a(\nu) = \exp\left[\frac{1}{1-a}\sum_{n=1}^{\infty}\lambda_n(a)\left(-\frac{\nu}{Q}\right)^n\right]$$

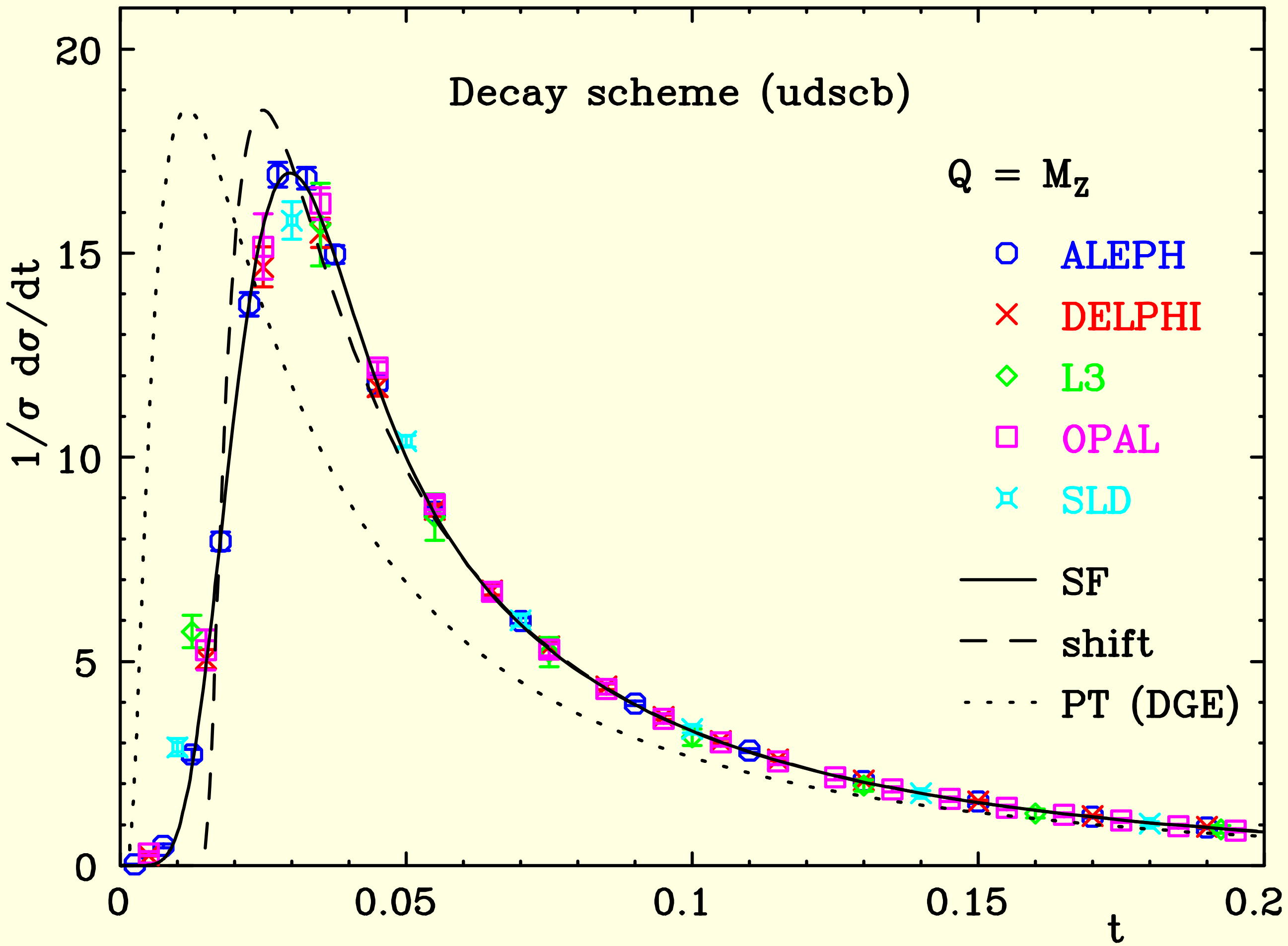
- Extract $\lambda_2(a)$ from scaling violation in *1/Q*^{2} corrections to event shape distributions
- Solve for momentum correlations in soft radiation:

$$\mathcal{C}_2(a) = (1-a)\int_{-\infty}^{a}\frac{da'}{(1-a)^2}\lambda_2(a')$$

Summary

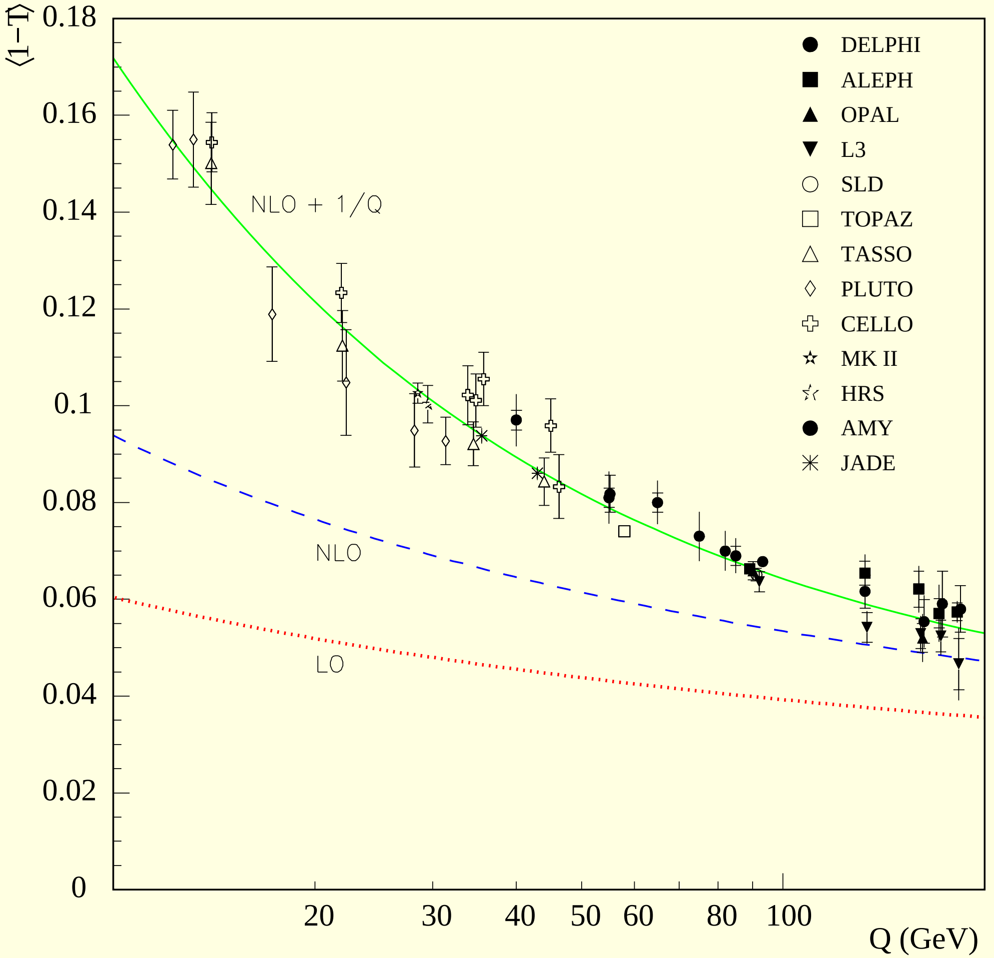
- Decoupling of soft from collinear modes in leading-order SCET Lagrangian facilitates proof of factorization.
- Nonperturbative shifts in mean values of event shapes controlled by single universal parameter
  - Due to boost invariance of soft radiation in leading-order of SCET expansion
  - NP corrections to full event shape distributions not found to be universal
- Violation of *a* scaling in angularity distributions yields information about correlations in soft radiation

Thrust Distribution in *Z*-to-Hadrons



From
E. Gardi
J. Rathsman
(2002)

Average Thrust: 1/Q corrections



Shifts from perturbative predictions well fit by 1/Q power corrections…
are these universal amongst different event shapes?

From
M. Dasgupta
G. Salam (2003)

(Shifts in peaks of angularity distributions)

From
C. Berger
G. Sterman
(2003)