Generalizing the DGLAP Evolution of Fragmentation Functions to the Smallest x-values

Simon Albino[a]

simon@mail.desy.de

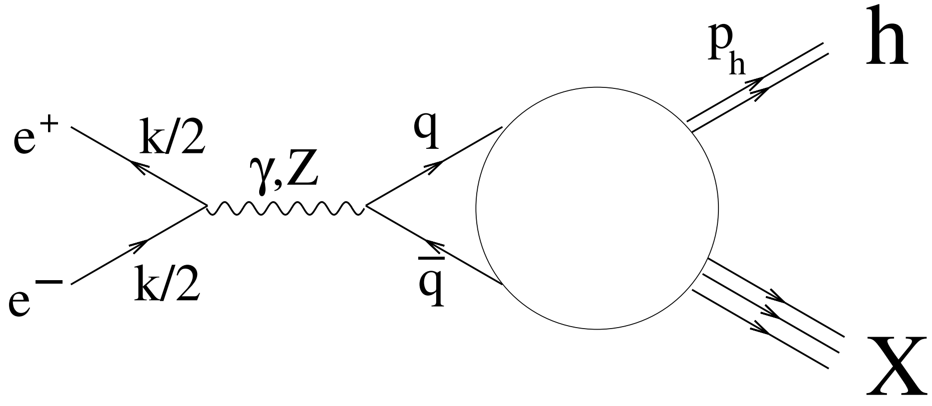
2nd Institute for Theoretical Physics,
University of Hamburg (DESY)

[a] in collaboration with B. A. Kniehl, G. Kramer, W. Ochs (hep-ph/0503170)

Factorization in $e^+e^- \to h + X$

$$\frac{d\sigma^h}{dx_p}(x_p{=}2p_h/\sqrt{s},\ s{=}k^2) =$$

$$(0 \le x_p \le 1)$$



$$\frac{d\sigma^h}{dx_p}(x_p, s) = \sum_i \int_{x_p}^1 \frac{dy}{y} \frac{d\sigma^i}{d(x_p/y)}\left(\frac{x_p}{y}, s, Q^2\right) D_i^h(y, Q^2)$$

$$\mathrm{XS}(h) \ = \mathrm{XS}(\text{parton } i) \ \times \ \mathcal{P}(i \to h,\ \text{mom. frac. } y)$$

$$> Q \qquad\qquad\qquad < Q$$

Q is arbitrary *factorization scale*

Fixed order perturbation theory

- Expand partonic XS $\frac{d\sigma^i}{d(x_p/y)}\left(\frac{x_p}{y}, s, Q^2\right)$ as series in a_s $\left(=\frac{\alpha_s}{2\pi}\right)$
- $\rightarrow$ potentially large $a_s^n \ln^m \frac{Q^2}{s}$ terms
- Solution: choose $Q = O(\sqrt{s})$
- $\rightarrow$ Q dependence of FFs $D_i^h(y, Q^2)$ needed
- Solution at large x_p: DGLAP $\equiv$ pert. Q-evolution

$$\frac{d}{d\ln Q^2} D(x, Q^2) = \int_x^1 \frac{dy}{y} P(y, a_s(Q^2)) D\left(\frac{x}{y}, Q^2\right)$$

$$(D = (D_g, D_d, D_{\bar{d}}, D_u, \ldots))$$

- $\rightarrow$ small x_p: Expansion of P in a_s fails

Applications at large x_p

Fitting procedure (KKP, Kretzer, AKK)

- Theoretical constraints
  $D_i(x, Q_0^2) = N_i x^{\alpha_i}(1-x)^{\beta_i}$, fit $\{N_i,\alpha_i,\beta_i\}$, $\alpha_s(M_Z)$
- Experimental constraints
  $e^+e^- \to (\pi^\pm, K^\pm, p/\bar{p}) + X$ — ALEPH, DELPHI, OPAL, SLD, TPC
  with $x_p > 0.1$ ("large" x_p)

Results

- $\alpha_s(M_Z)$ competitive
- Describe high p_T $pp(\bar{p}) \to (\pi^\pm, K^\pm, p/\bar{p}) + X$ — PHENIX, UA1, STAR

FF and PDF fitting procedures similar

Small x_p vs. small x_B

Type	Timelike (FFs)	Spacelike (PDFs)
$\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}_{\text{NLO term}}$	$\begin{pmatrix} \frac{1}{x} & \frac{\ln^2 x}{x} \\ \frac{1}{x} & \frac{\ln^2 x}{x} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{x} & \frac{1}{x} \\ \frac{1}{x} & \frac{1}{x} \end{pmatrix}$
pert. approach	$x_p > 0.1$	$x_B > 10^{-4}$

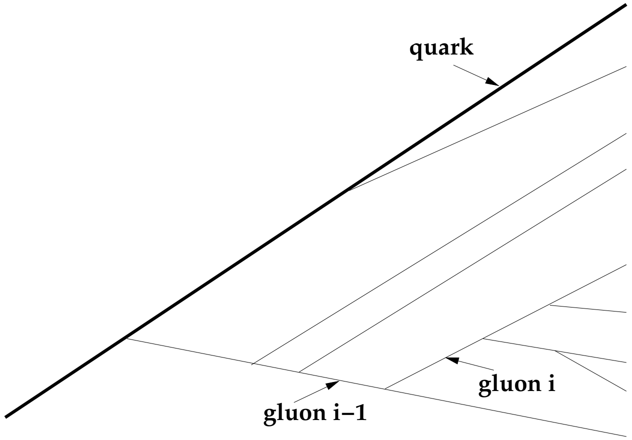
Timelike P contains double logs

$$(1/x)(a_s \ln x)^2 (a_s \ln^2 x)^r, \, r = -1, ..., \infty$$

$\rightarrow$ poor small x_p description when $\ln(1/x_p) = O(a_s^{-1/2})$

Double Logarithmic Approximation

Largest logs (DLs): $q\bar{q} \to q\bar{q} + N$ gluons (tree level)



- $\mathcal{P}(G_{i-1} \to G_i + ...) \propto a_s \frac{d\theta_i}{\theta_i} \frac{dE_i}{E_i} \simeq a_s \ln^2$
- Strong ordering: $(E, \theta)_i \ll (E, \theta)_{i-1}$
- Collinear part $\in$ FFs: $E_T = E_{i-1}\theta_{i-1} > Q$

$$\therefore E_i \theta_i > y Q$$

$$\to \frac{d}{d \ln Q^2} D(x, Q^2) = \int_x^1 \frac{dy}{y} a_s(y^2 Q^2) P^{(0)}(y) D\left(\frac{x}{y}, y^2 Q^2\right)$$

Take small x sings in $P^{(0)}(x)$ only

$$\frac{d}{d \ln Q^2} D(x, Q^2) = \int_x^1 \frac{dy}{y} \frac{2C_A}{y} A \, a_s(y^2 Q^2) D\left(\frac{x}{y}, y^2 Q^2\right)$$

$\to$ DLA equation, contains all DLs

- $A = \begin{pmatrix} 0 & \frac{2C_F}{C_A} \\ 0 & 1 \end{pmatrix}$
- $A_{\text{non-singlet}} = 0$
- $A^2 = A$

DLA vs. DGLAP

Approach	DLA/MLLA	DGLAP
Data studied	small x_p	large x_p
Assumed valid to	$Q_0 = \Lambda_{\rm QCD}$ (limiting spectrum)	$Q_0 \simeq 1$ GeV
Non perturbative input	$D_i(x, \Lambda^2_{\rm QCD}) \propto \delta(1-x)$ (LPHD)	$D_i(x, Q_0^2) = N_i x^{\alpha_i}(1-x)^{\beta_i}$

Our approach:

- keep DGLAP (large x_p)
- improve P at small x_p via DLA

$$a_s P^{(0)}(x) \longrightarrow a_s \overline{P}^{(0)}(x) + P^{\rm DL}(x, a_s)$$

$\uparrow$ LO DL subtracted

DL resummation in P

Insert DGLAP ($P \longrightarrow P^{\rm DL}$) into DLA:

$$2(P^{\rm DL})^2 + \omega P^{\rm DL} - 2C_A a_s A = 0$$

+ boundary condition:

$$a_s P^{{\rm DL}(0)}(\omega, a_s) = \begin{pmatrix} 0 & a_s \frac{4C_F}{\omega} \\ 0 & a_s \frac{2C_A}{\omega} \end{pmatrix} + O(a_s^2)$$

$\longrightarrow$ Solution (agrees with NLO too):

$$P^{\rm DL}(\omega, a_s) = \frac{A}{4}\left(-\omega + \sqrt{\omega^2 + 16 C_A a_s}\right)$$

$$P^{\rm DL}(x, a_s) = \frac{A\sqrt{C_A a_s}}{x \ln \frac{1}{x}} J_1\left(4\sqrt{C_A a_s} \ln \frac{1}{x}\right)$$

As $x \longrightarrow 0$, $P^{\rm DL}(x, a_s) \longrightarrow \frac{1}{x} \ln^{-\frac{3}{2}} \frac{1}{x}$

$\longrightarrow$ Less than LO singularity $\frac{1}{x}$

Results in Mellin space

$$f(\omega) = \int_0^1 dx x^\omega f(x)$$

$\omega \longrightarrow 0$ sings:

$$P^{\rm DL}(\omega, a_s) \ni \frac{a_s}{\omega}\left(\frac{a_s}{\omega^2}\right)^r$$

DGLAP equation:

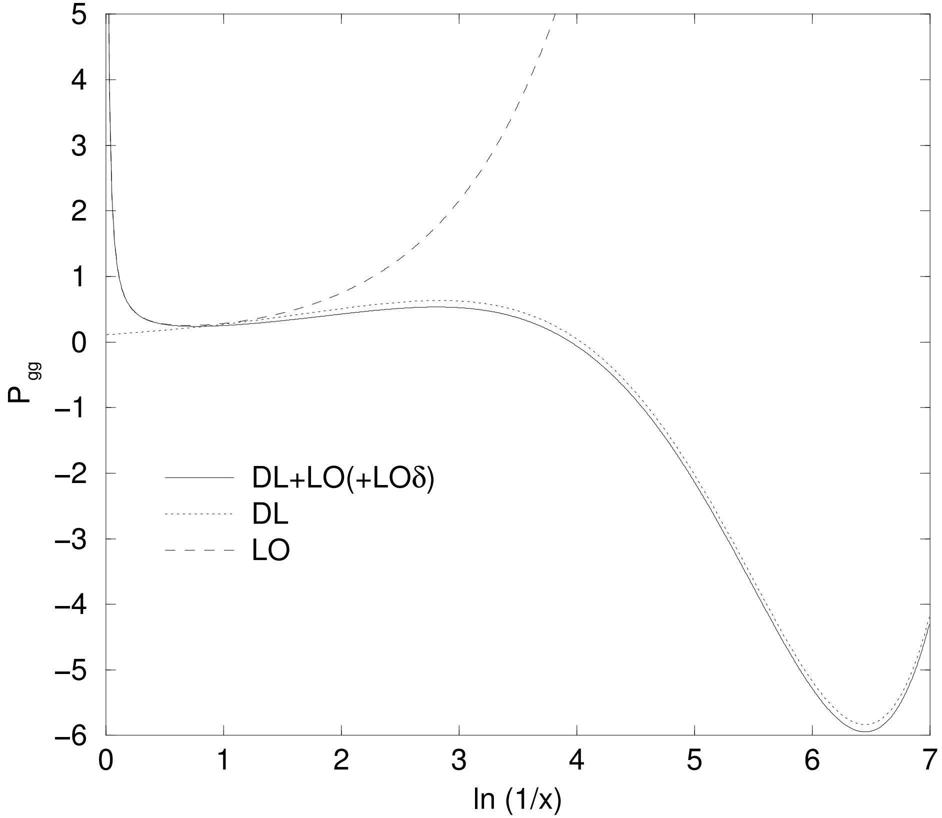
$$\frac{d}{d \ln Q^2} D(\omega, Q^2) = P(\omega, a_s(Q^2)) D(\omega, Q^2)$$

DLA equation:

$$\left(\omega + 2\frac{d}{d\ln Q^2}\right)\frac{d}{d \ln Q^2} D(\omega, Q^2) = 2C_A a_s(Q^2) A D(\omega, Q^2)$$

New P vs. old Ps

E.g. look at P_{gg}



- LO: $P = a_s P^{(0)}$
- DL: $P = P^{\rm DL}$
- DL+LO: $P = a_s \overline{P}^{(0)} + P^{\rm DL}$

Higher logarithms

- Double logs: $\frac{a_s}{\omega}\left(\frac{a_s}{\omega^2}\right)^r$
- General logs: $\left(\frac{a_s}{\omega}\right)^m\left(\frac{a_s}{\omega^2}\right)^r$

Extend approach to any order n, any class of logs m

$$P(\omega, a_s) = \sum_{n=1}^{\infty} a_s^n \overline{P}^{(n)}(\omega) + \sum_{m=1}^{\infty}\left(\frac{a_s}{\omega}\right)^m g_m\left(\frac{a_s}{\omega^2}\right)$$

$a_s^n \overline{P}^{(n)}(\omega)$

- $= a_s^n P^{(n)}(\omega)$, logs subtracted
- Finite as $\omega \to 0$
- Important for large x_p

$\left(\frac{a_s}{\omega}\right)^m g_m\left(\frac{a_s}{\omega^2}\right)$

- Contains all class m logarithms
- $m = 1$: DLs — known
- $m = 2$: SLs — known for P_{gg}
- Finite as $\omega \to 0$?
- Important for small x_p

Relation to MLLA

Approach equivalent to

$$\frac{d}{d\ln Q^2}D(\omega,Q^2)=\left(\omega+2\frac{d}{d\ln Q^2}\right)^{-1}2C_A a_s(Q^2)AD(\omega,Q^2) \quad \longleftarrow \text{DLA eq.}$$

$$+a_s(Q^2)\overline{P}^{(0)}(\omega)D(\omega,Q^2) \quad \longleftarrow \text{Fixed order}$$

MLLA limit:

- Approximations
  - Set $\omega=0$ in $a_s\overline{P}^{(0)}(\omega)$
  - DLA: $D_{q,\overline{q}}\simeq\frac{C_F}{C_A}D_g$
- Take gluon component

$\longrightarrow$ MLLA equation:

$$\left(\omega+2\frac{d}{d\ln Q^2}\right)\frac{d}{d\ln Q^2}D_g(\omega,Q^2)=2C_A a_s(Q^2)D_g(\omega,Q^2)$$

$$-\left(\omega+2\frac{d}{d\ln Q^2}\right)a_s(Q^2)\frac{a}{2}D_g(\omega,Q^2)$$

$\uparrow\ a=\frac{11}{3}C_A+\frac{4}{3}T_R n_f\left(1-\frac{2C_F}{C_A}\right)$

Theory vs. data — method

- Fit to light charged hadron data, $\ln(1/x_p) < \ln\sqrt{s}$
- LO, $Q = \sqrt{s}$, $Q_0 = 14$ GeV, $n_f = 5$

$$\frac{1}{\sigma(s)}\frac{d\sigma}{dx_p}(x_p, s) = \frac{1}{n_f\langle e_q^2(s)\rangle}\sum_q e_q^2(s) D_q^+(x_p, s)$$

Fitted quantities:

- $D_g(x, Q_0^2)$
- $\frac{1}{2}\sum_{\alpha=uc} D_\alpha(x, Q_0^2)$
- $\frac{1}{3}\sum_{\alpha=dsb} D_\alpha(x, Q_0^2)$
- Λ_{QCD}

$D_i(x, Q_0^2) = N\exp[-c\ln^2 x]x^\alpha(1-x)^\beta$

large x: $\rightarrow Nx^\alpha(1-x)^\beta$ (as in DGLAP fits)

small x: $\rightarrow N\exp[-c\ln^2\frac{1}{x} - \alpha\ln\frac{1}{x}]$ (DLA, large Q)

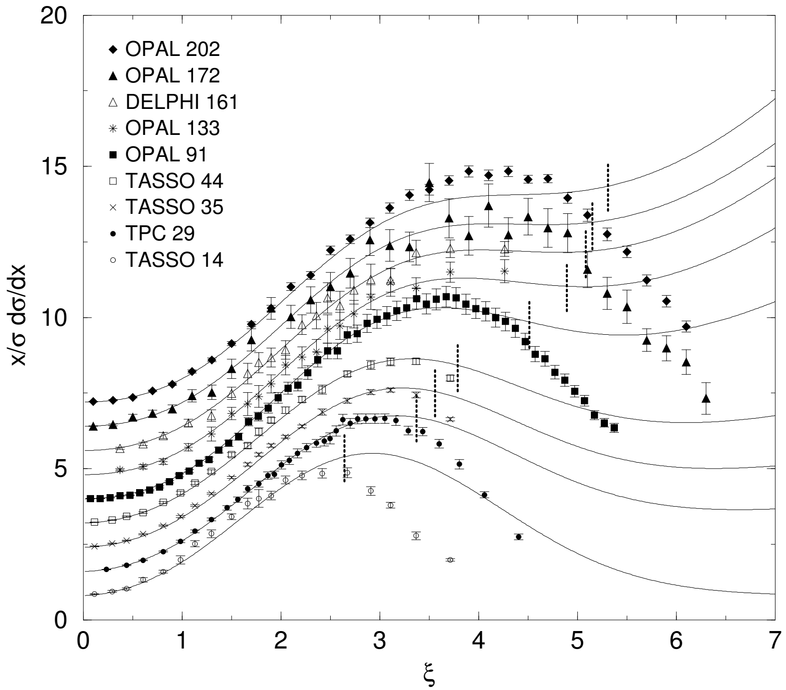
$D_{q,\overline{q}} \simeq \frac{C_F}{C_A} D_g$ (DLA):

$c_{uc} = c_{dsb} = c_g$, $\alpha_{uc} = \alpha_{dsb} = \alpha_g$ (use)

$N_q = \frac{4}{9} N_g$ (ignore)

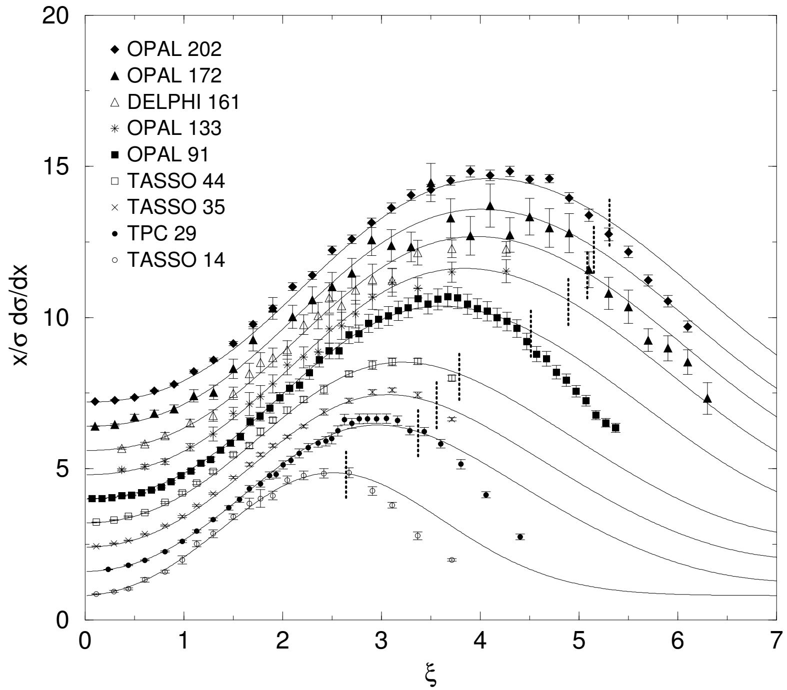
Theory (new and old) vs. data

- $P = a_s P^{(0)}$ (old DGLAP)



$$\chi^2_{\rm DF} = 3.0$$
$$\Lambda_{\rm QCD} = 388 \text{ MeV}$$

- $P = a_s \overline{P}^{(0)} + P^{\rm DL}$ (new DGLAP)



$$\chi^2_{\rm DF} = 2.1$$
$$\Lambda_{\rm QCD} = 801 \text{ MeV}$$

$2\times$ DLA-anticipated N_g/N_q

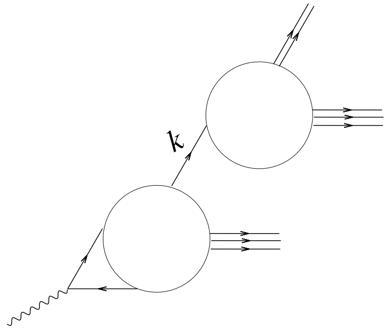
Hadron mass effects at small x_p

Particles confined to 3-axis:

$$(V^+, V^-) = \frac{1}{\sqrt{2}} V^0(+,-)V^3$$

$$\mathbf{V}_T = \mathbf{0}$$

$$p_h = \left(p_h^+, \frac{m_h^2}{2p_h^+} \right)$$



$$q = \left(\frac{p_h^+}{\eta}, \frac{s\eta}{2p_h^+} \right)$$

Variable η:

- Like DIS Nachtmann scaling variable
- 3-axis boost invariant
- true momentum fraction in factorization

Leading twist factorization:

- $k = \left(\frac{p_h^+}{y}, 0 \right)$
- $\frac{d\sigma}{d\eta}(\eta, s) = \int_\eta^1 \frac{dy}{y} \frac{d\sigma}{dy}(y, s, Q^2) D\left(\frac{\eta}{y}, Q^2 \right)$

Relation to experiment:

- $x_p = \frac{2p_h \cdot q}{Q^2} = \eta \left(1 - \frac{m_h^2}{s\eta^2} \right)$
- $\frac{d\sigma}{dx_p}(x_p, s) = \frac{1}{1+\frac{m_h^2}{s\eta^2(x_p)}} \frac{d\sigma}{d\eta}(\eta(x_p), s)$

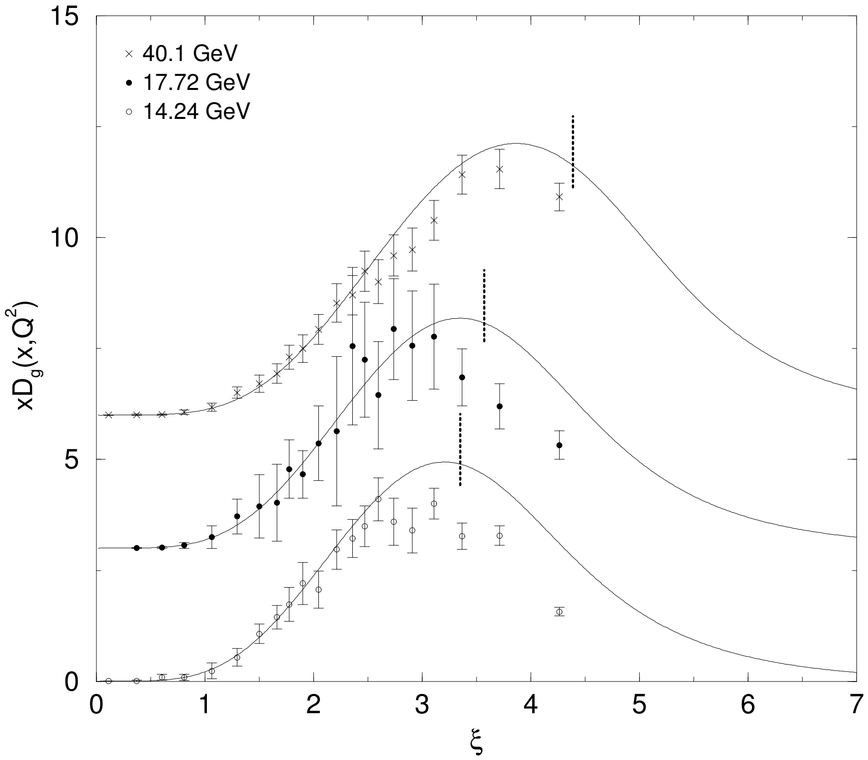
Both fits: $\chi^2_{\rm DF} \simeq 2$, $m_h \simeq 300$ MeV

	DL unresummed	DL resummed
$\Lambda_{\rm QCD}$ (MeV)	1300	400

Resummed fit: $110\% \times$ DLA-anticipated N_g/N_q

OPAL gluon jet data

$E_{\text{jet}} = Q/2 = 14.24,\ 17.72,\ 40.1$ GeV



Hadron mass effects $\rightarrow m_h \simeq 300$ MeV, $\Lambda_{\text{QCD}} \simeq 500$ MeV

Summary

- Resumming DLs extends DGLAP evolution to small x_p (peak region)
- Large x_p description remains good
- Ignore large x_p effects + take DLA limit on FFs $\rightarrow$ MLLA
- Hadron mass effects improve fits, resummation gives better Λ_{QCD}

Further work:

- Extend NLO global fits to smaller x_p (currently $0.1 < x_p < 1$)
- Determine SLs