Beam Transport

Beam transport and matching

Single turn injection

Multi-turn injection

Single turn extraction

Multi-turn extraction

Equations of transverse motion

In previous lectures it was shown that the equations of motion of a charged particle in the paraxial approximation can be reduced to the linear Hill's equation

$$x'' - \left(k(s) - \frac{1}{\rho^2} \right) x = \frac{1}{\rho} \frac{\Delta p}{p_0}$$

$$z'' + k(s) z = 0$$

The solutions of these equations can be written in terms of the optics functions (amplitude and phase)

$$y(s) = \sqrt{\varepsilon \beta(s)} \cos(\phi(s) - \phi_0) + \frac{\Delta p}{p_0} D(s)$$

or equivalently in terms of the principal trajectories

$$y(s) = C(s) y_0 + S(s) y_0' + \frac{\Delta p}{p_0} D(s)$$

No assumption is made about the periodicity of the line.

Principal trajectories (I)

The principal trajectories are two particular solutions of the homogeneous Hill's equation

$$y'' + k(s)y = 0$$

which satisfy the initial conditions

$C(s_0) = 1;$	$C'(s_0) = 0;$	cosine-like solution
$S(s_0) = 0;$	$S'(s_0) = 1;$	sine-like solution

The general solution can be written as a linear combination of the principal trajectories

$$y(s) = y_0 C(s) + y'_0 S(s)$$

We can express amplitude and phase functions in terms of the principal trajectories

$$C(s) = \sqrt{\frac{\beta(s)}{\beta_0}}(\cos\phi(s) + \alpha_0 \sin\phi(s))$$

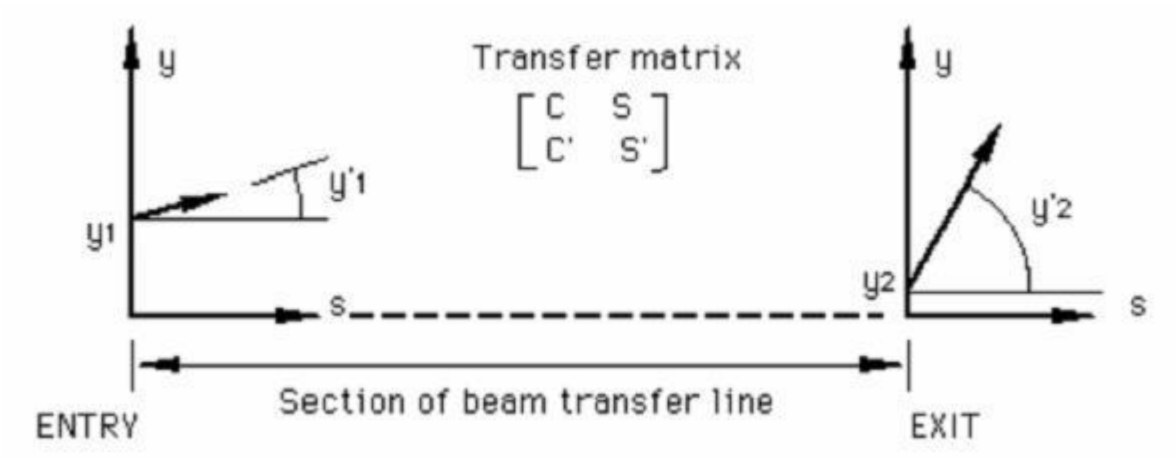
$$S(s) = \sqrt{\beta(s)\beta_0}\,\sin\phi(s)$$

Principal trajectories (II)

As a consequence of the linearity of Hill's equations, we can describe the evolution of the trajectories in a transfer line or in a circular ring by means of linear transformations

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix} \begin{pmatrix} y(s_0) \\ y'(s_0) \end{pmatrix} + \frac{\Delta p}{p_0} \begin{pmatrix} D(s) \\ D'(s) \end{pmatrix}$$

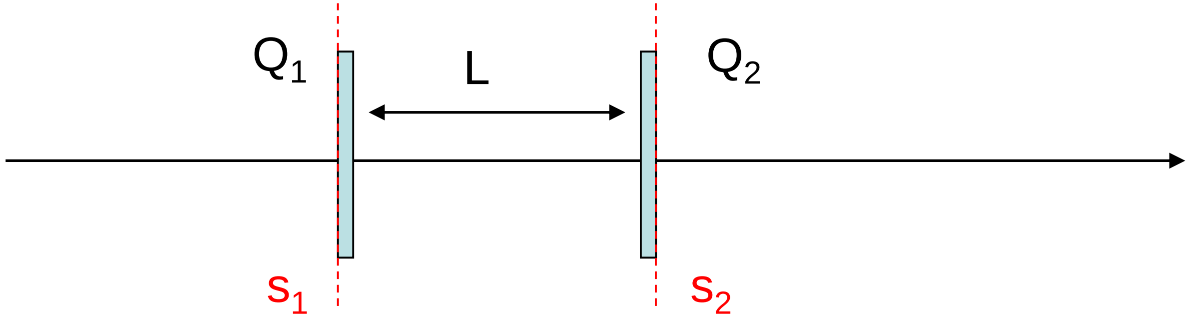
This allows the possibility of using the matrix formalism to describe the evolution of the coordinates of a charged particles in a transfer line, e.g.



$$M_{1\to 2} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix}$$

Matrix formalism for transfer lines (I)

For each element of the transfer line we can compute, once and for all, the corresponding matrix and the propagation along the line will be the piece-wise composition of the propagation through all the various elements



$$M_{1\to 2} = \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} = \begin{pmatrix} 1-L/f_1 & L \\ -1/f^* & 1-L/f_2 \end{pmatrix}$$

$$\frac{1}{f^*} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

Notice that it works equally in the longitudinal plane, e.g.

$$M_{1\to 2} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \qquad \frac{1}{f} = -\frac{qVL\omega \sin\phi_s}{mc^3\gamma_s^3}$$

thin lens quadrupole associate to an RF cavity of voltage V and length L

Matrix formalism for transfer lines (II)

In terms of the amplitude and phase function the transfer matrix will read

$$M_{s_0 \to s} = \begin{pmatrix} C(s) & S(s) \\ -C'(s) & S'(s) \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}}(\cos\Delta\phi + \alpha_0 \sin\Delta\phi) & \sqrt{\beta(s)\beta_0}\sin\Delta\phi \\ -\frac{(\alpha(s)-\alpha_0)\cos\Delta\phi + (1+\alpha(s)\alpha_0)\sin\Delta\phi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}}[\cos\Delta\phi - \alpha(s)\sin\Delta\phi] \end{pmatrix}$$

where β_0 , α_0 and the phase ϕ_0 are computed at the beginning of the segment of transfer line

We still have not assumed any periodicity in the transfer line.

If we consider a periodic machine the transfer matrix over a whole turn reduces to

$$M_{s_0 \to s_0} = \begin{pmatrix} \cos\mu + \alpha_0 \sin\mu & \beta_0 \sin\mu \\ -\sin\mu & \cos\mu - \alpha_0 \sin\mu \end{pmatrix}$$

Optics functions in a transfer line

While in a circular machine the optics functions are uniquely determined by the periodicity conditions, in a transfer line the optics functions are not uniquely given, but depend on their initial value at the entrance of the system.

We can express the optics function in terms of the principal trajectories as

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

This expression allows the computation of the propagation of the optics function along the transfer lines, in terms of the matrices of the transfer line of each single element, i.e. also the optics functions can be propagated piecewise from

$$M_{1\to 2} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix}$$

Examples

In a drift space

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}=\begin{pmatrix}1 & -2s & s^2\\ 0 & 1 & -s\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\beta_0\\ \alpha_0\\ \gamma_0\end{pmatrix} \qquad M=\begin{pmatrix}1 & s\\ 0 & 1\end{pmatrix}$$

The β function evolve like a parabola as a function of the drift length.

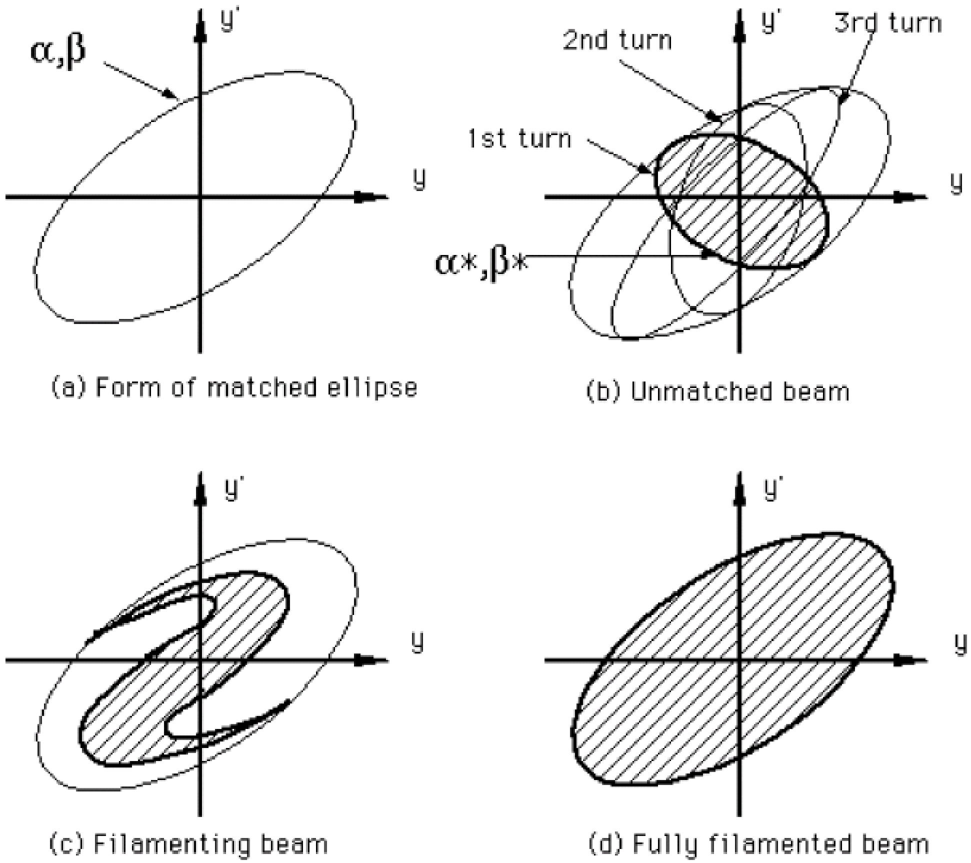
In a thin focussing quadrupole of focal length f = 1/KL

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ KL & 1 & 0\\ (KL)^2 & 2KL & 1\end{pmatrix}\begin{pmatrix}\beta_0\\ \alpha_0\\ \gamma_0\end{pmatrix} \qquad M=\begin{pmatrix}1 & 0\\ KL & 1\end{pmatrix}$$

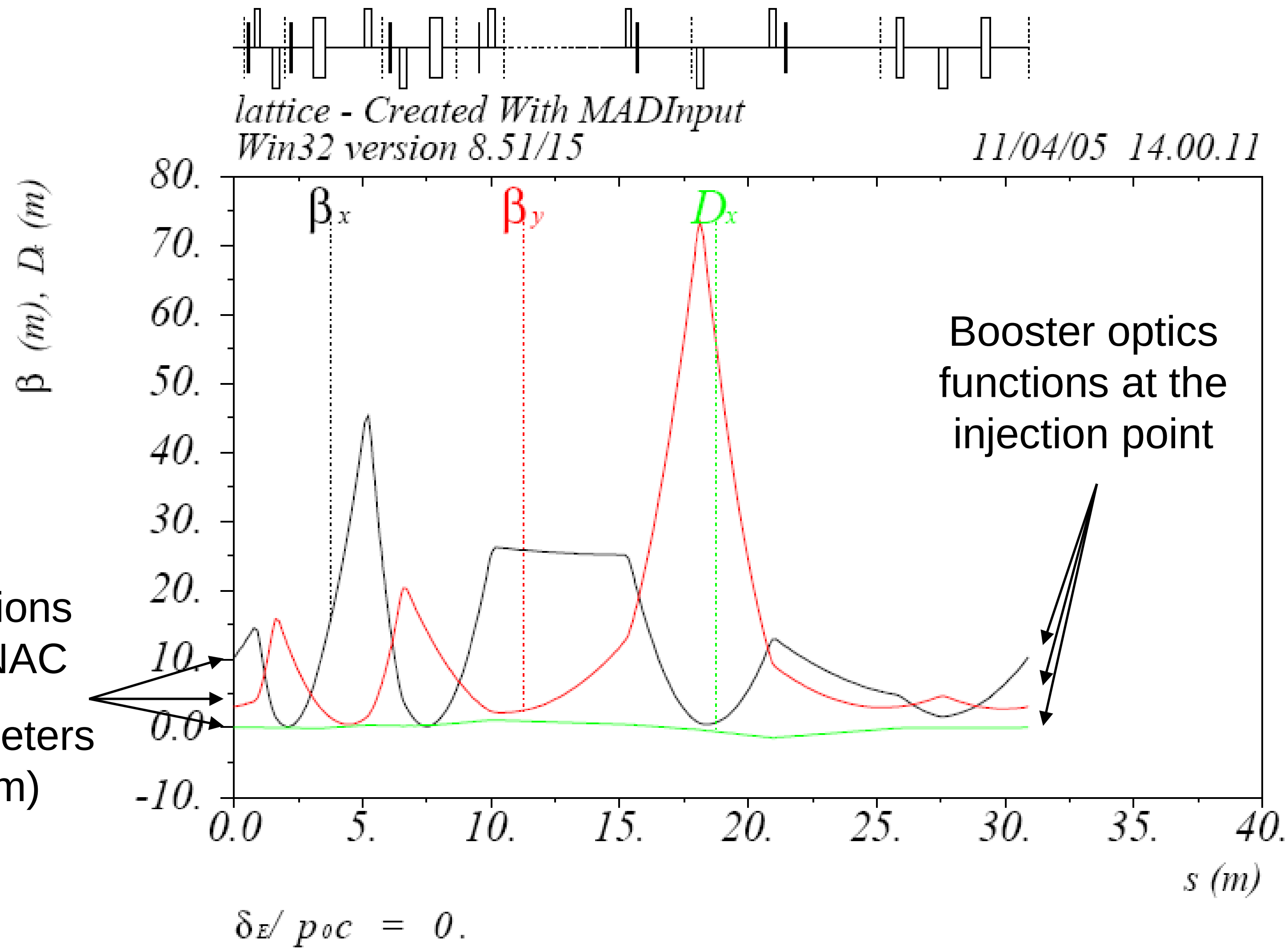
The γ function evolve like a parabola in terms of the inverse of focal length

Matching of optics function in a transfer line

A typical problem in the design of transfer line comes from the requirement of matching the optical function at the end of the transfer line with a set of given optics function, e.g. the optic function of a ring at the injection point



Diamond LINAC to booster transfer line



Optics functions from the LINAC

(Twiss parameters of the beam)

Booster optics functions at the injection point

Matching of optic function in a transfer line

A mad8 example for matching the optics functions to some desired value at the end of the transfer line

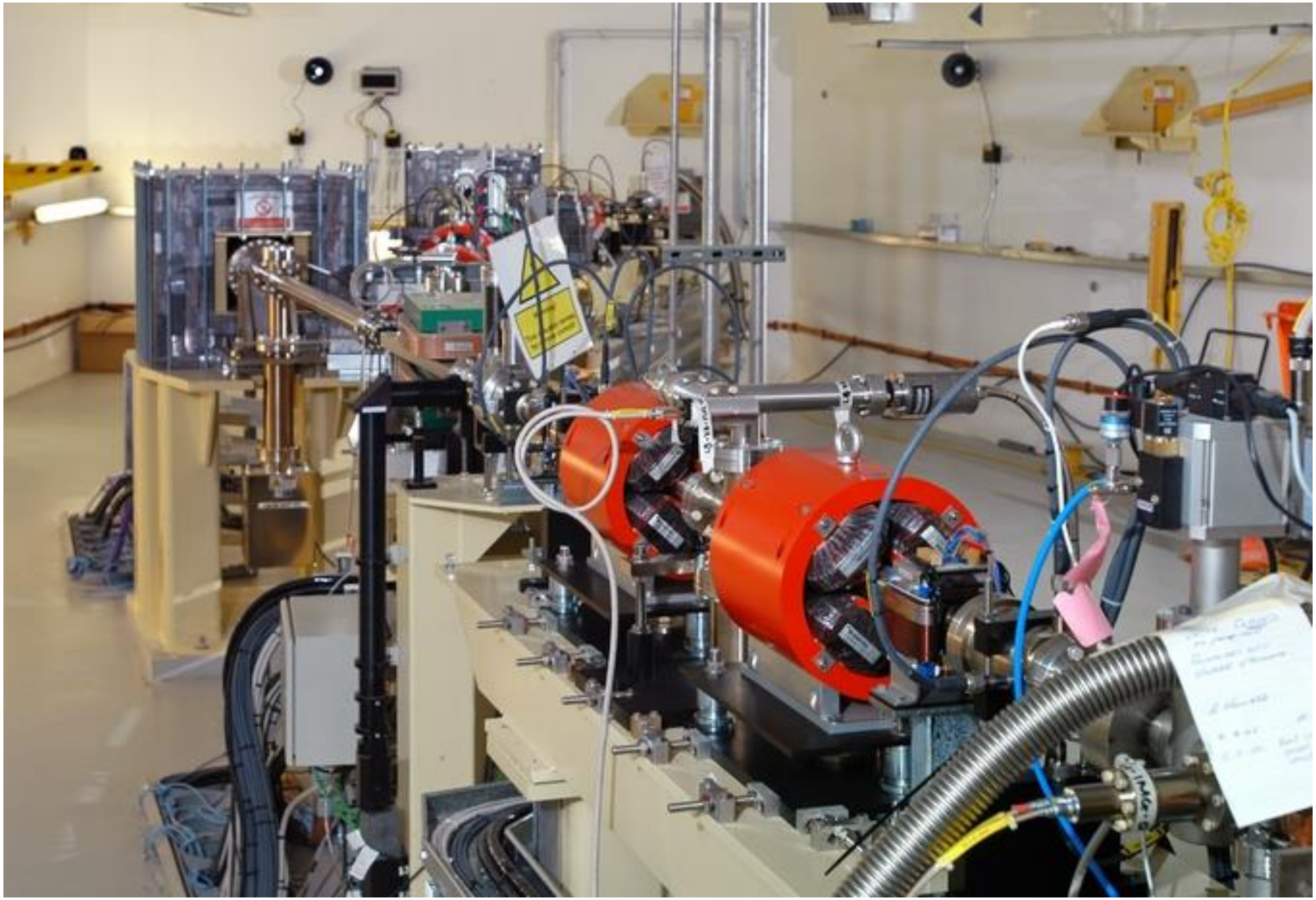
at start

```
Use,newltb
Match,betx=10.0,alfx=-0.5,bety=3.0,alfy=-0.5,dx=0.0,dpx=0.0
Constraint,#E,betx=11.8808,alfx=-2.89418,bety=3.66418,alfy=0.956848
Constraint,#E,dx=0.050281,dpx=0.00605976,dy=0,dpy=0
Constraint,newltb,betx<51,bety<51

Vary,L2BQUAD1[k1],step=0.00001,lower=-6.0000,upper=6.0000
Vary,L2BQUAD2[k1],step=0.00001,lower=-6.0000,upper=6.0000
Vary,L2BQUAD3[k1],step=0.00001,lower=0.0000,upper=6.0000
Vary,L2BQUAD4[k1],step=0.00001,lower=-6.0000,upper=0.0000
Vary,L2BQUAD5[k1],step=0.00001,lower=0.0000,upper=6.0000
Vary,L2BQUAD6[k1],step=0.00001,lower=0.0000,upper=6.0000
Vary,L2BQUAD7[k1],step=0.00001,lower=-6.0000,upper=0.0000
Vary,L2BQUAD8[k1],step=0.00001,lower=0.0000,upper=6.0000

Simplex,calls=500000,tolerance=1E-20
migrad,calls=24000,tolerance=1e-9
EndMatch
```

Transfer line example: Diamond LTB



Achromatic and Isochronous lines

A transfer line is achromatic if the dispersion and its derivative are zero at the end of the line, if they are zero at the beginning of the line

$$D(s) = S(s)\int_{s_0}^{s}\frac{C(t)}{\rho(t)}dt - C(s)\int_{s_0}^{s}\frac{S(t)}{\rho(t)}dt$$

$$D'(s) = S'(s)\int_{s_0}^{s}\frac{C(t)}{\rho(t)}dt - C'(s)\int_{s_0}^{s}\frac{S(t)}{\rho(t)}dt$$

If $D(s_i) = 0$; $D'(s_i)=0$

then $D(s_e) = 0$; $D'(s_e) = 0$

This implies (K. Steffen CAS 85-19) for any trajectory with $dp/p_0 = 0$ that

$$\int_0^s \frac{1}{\rho}\left(x_0 C + x_0' S\right) d\tau = 0$$

A transfer line is isochronous if all trajectories have the same path length, for any x_0, x'_0 and dp/p_0.

$$\Delta\ell = \int_0^s \frac{x}{\rho} d\tau = \int_0^s \frac{1}{\rho}\left(x_0 C + x_0' S + \frac{\Delta p}{p_0} D\right) d\tau = 0$$

If the transfer line is already achromatic, it must satisfy in addition

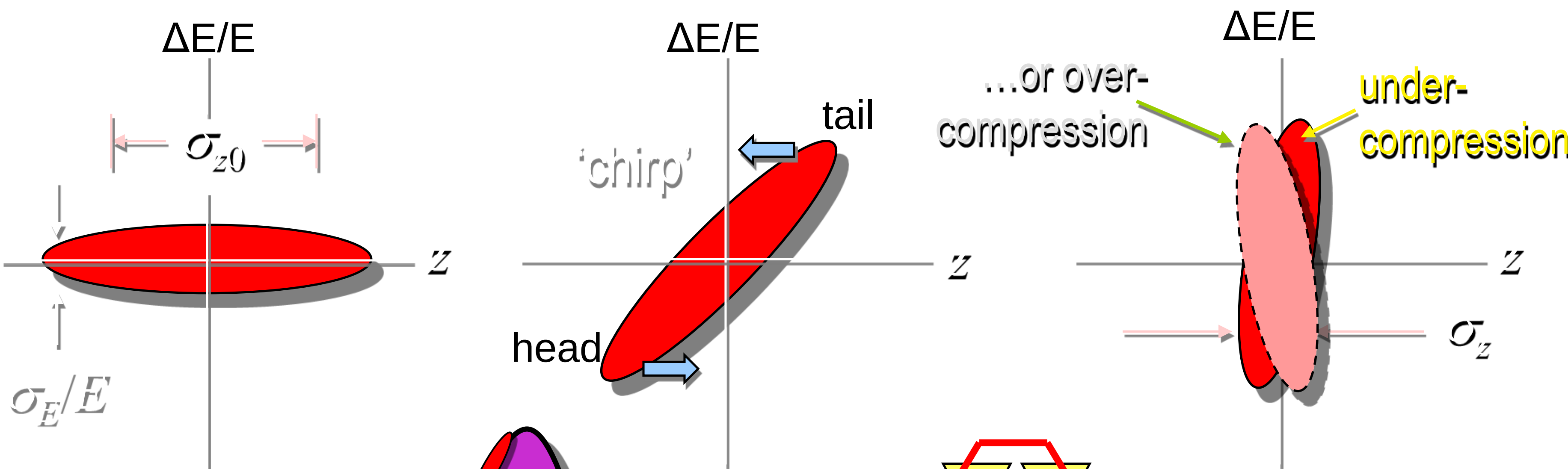
$$\frac{1}{\ell_0}\int_0^s \frac{D}{\rho} d\tau = 0$$

momentum compaction factor (for rings)

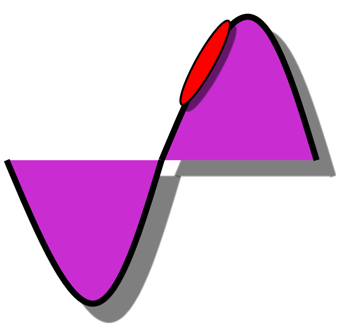
R56 in transfer lines

Example: bunch compressors

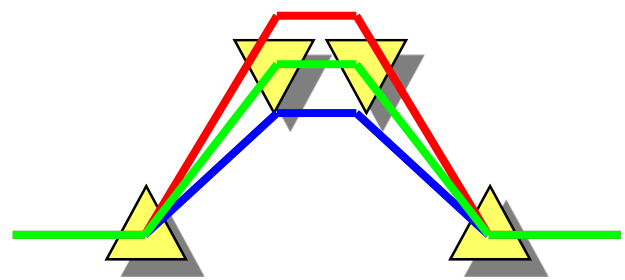
A beam transport line made of four equal dipole with opposite polarity is an example of achromatic transfer line which is non-isochronous. The different time of flight (or path length) for different energies can be used to compress the bunch length



An energy chirp is required for the compression to work



$V = V_0 \sin(\omega t)$



$\Delta z = R_{56}\, \Delta E/E$

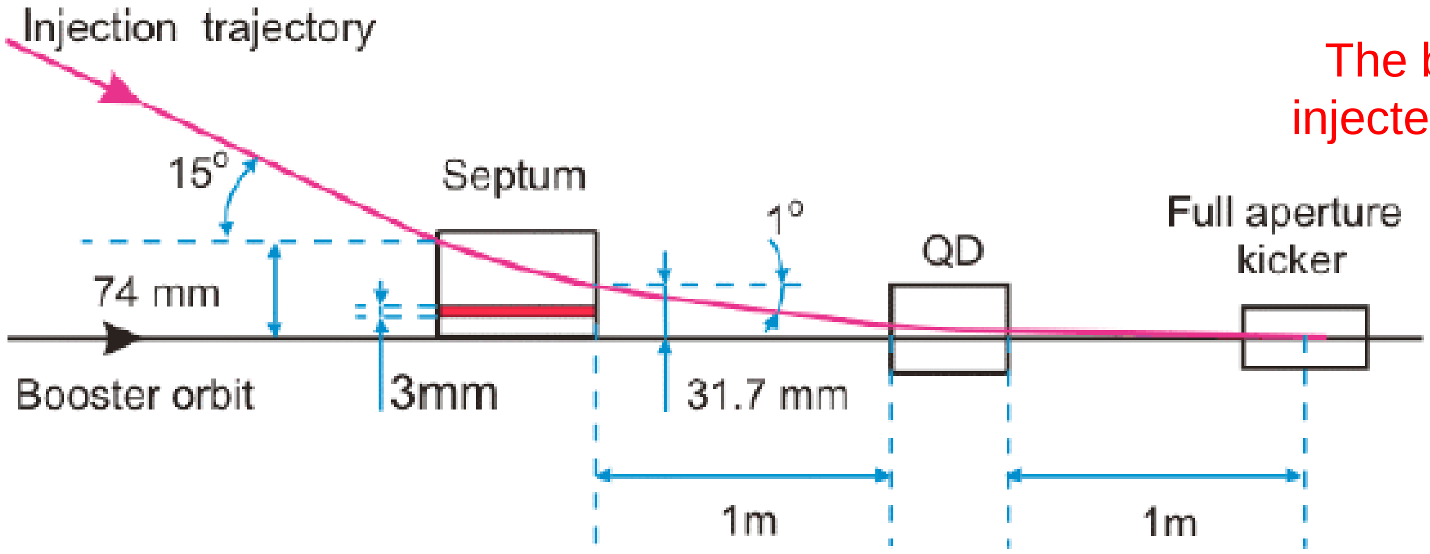
Single turn injection in a circular accelerator

The beam coming from a transfer line is deflected by a septum magnet towards the central orbit of the circular accelerator. This deflection can be many tens of mrads

At the location where the trajectory intersects the central orbit a kicker magnet kicks the beam on axis, removing the residual angle left by the septum (few mrads)

The presence of intermediate magnets (e.g. quadrupoles) between the septum and the kicker has to be taken into account in the definition of the deflection angles

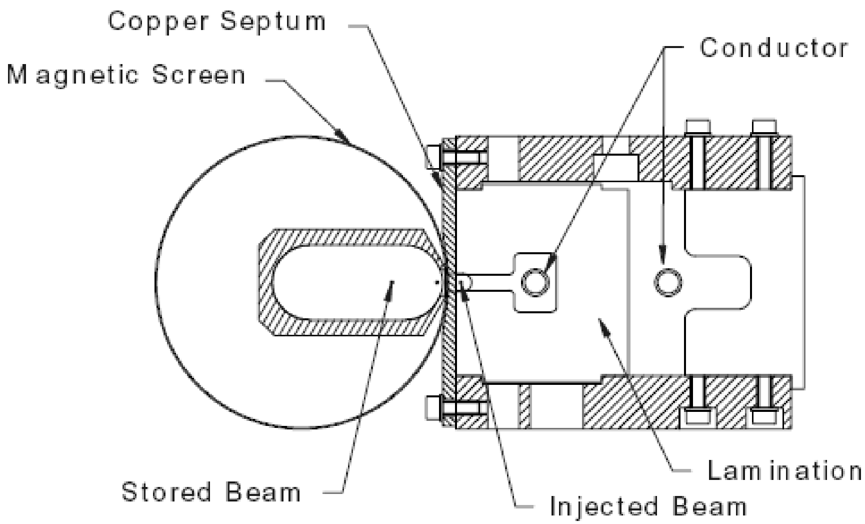
The optics function should be matched;



The beam is injected on axis

Pulsed magnets for single turn injection

The incoming beam is deflected by a septum magnet which is a pulsed magnet. The pulse can be long (many tens of μs) the field should not leak into the aperture where the beam will circulate



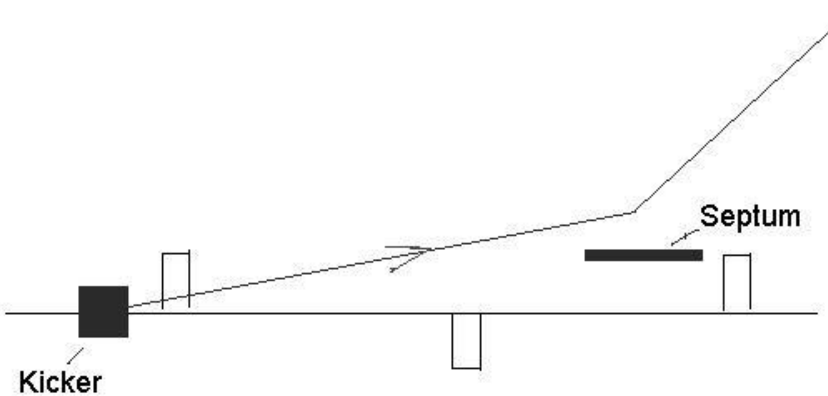
The beam is deflected a pulsed septum magnet and a fast kicker magnet which rises in a time between bunches (50ns)

The kicker pulse should be off when the injected bunch has completed one turn in the ring otherwise it will kick the beam out: the accumulation of current is limited to one turn;

Single turn extraction

The single turn extraction works with a principle very similar to the single turn injection:

A fast kicker deflects the beam from the central orbit. The kicker deflection angle is small and the beam still lies in the aperture of the machine until it enters a magnetic septum which deflects it a large angle beyond the yoke of the next magnet

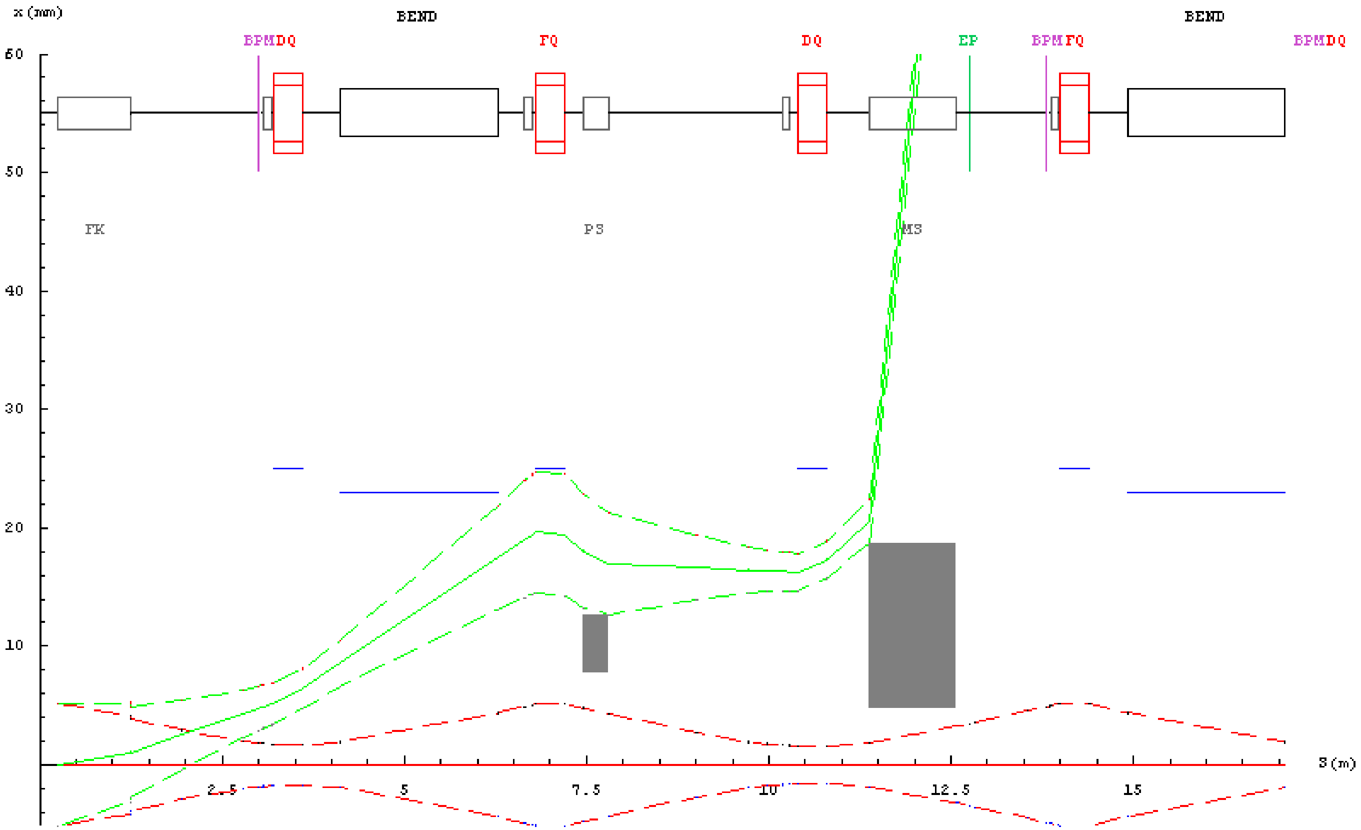


If many elements are present between kicker and septum, the trajectory of the kicked bunch should be computed in detail. The location of the septum should be separated by 90 degrees phase advance to maximise the effect of the kicker.

$$x_{sep} = \theta_{kick} \sqrt{\beta_k \beta_{sep}} \sin(\phi_{sep} - \phi_k)$$

Single turn extraction with pre-septum

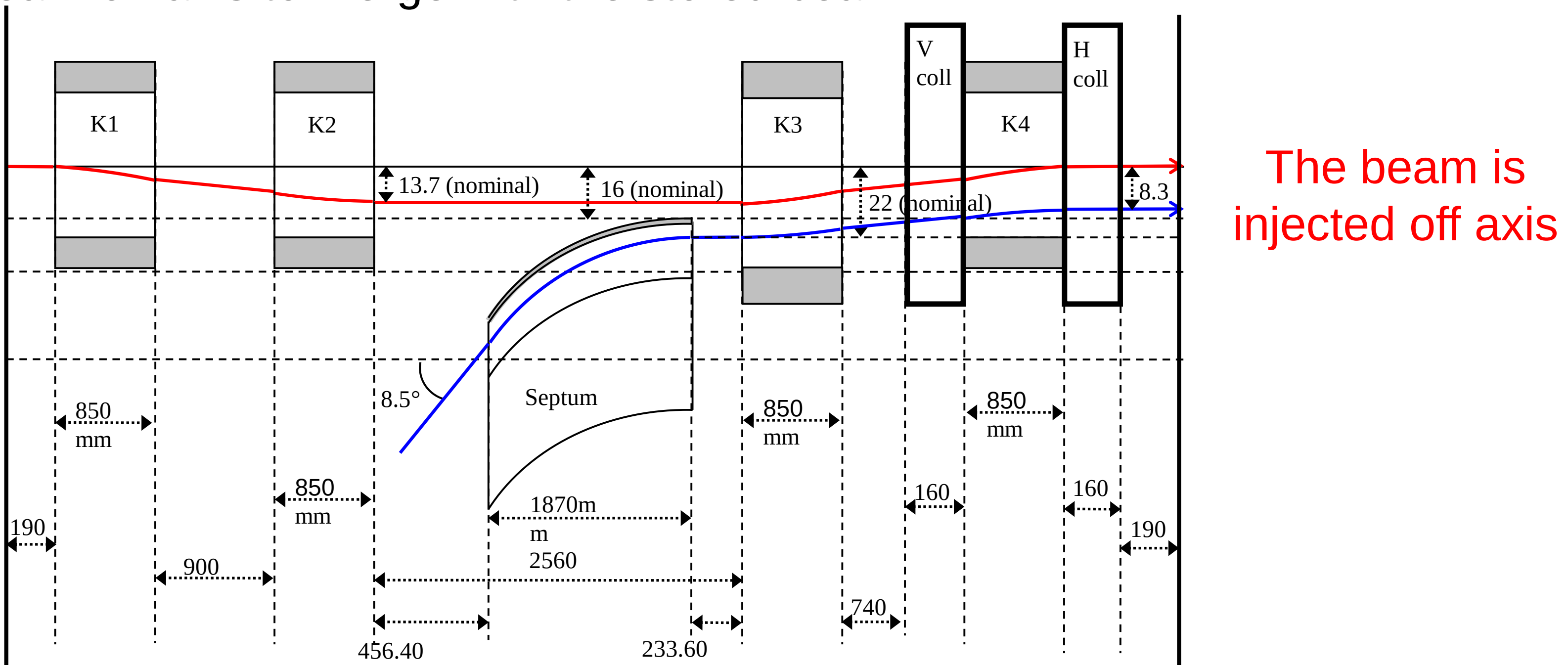
To relax the specifications on the kicker magnet it is possible to envisage the use of more than one septum magnet, e.g. diamond booster which has a single turn extraction system with a kicker, pre-septum and septum



Multi-turn Injection in electron machines

In the multi-turn injection scheme, an electron beam is injected in a circular accelerator with a system made of a septum magnet and four kickers that create a local closed orbit bump.

The bump is created when the injected beam arrives and it is switched off to avoid the injected beam hitting the back of the septum. Radiation damping will put the injected beam on axis to merge with the stored beam

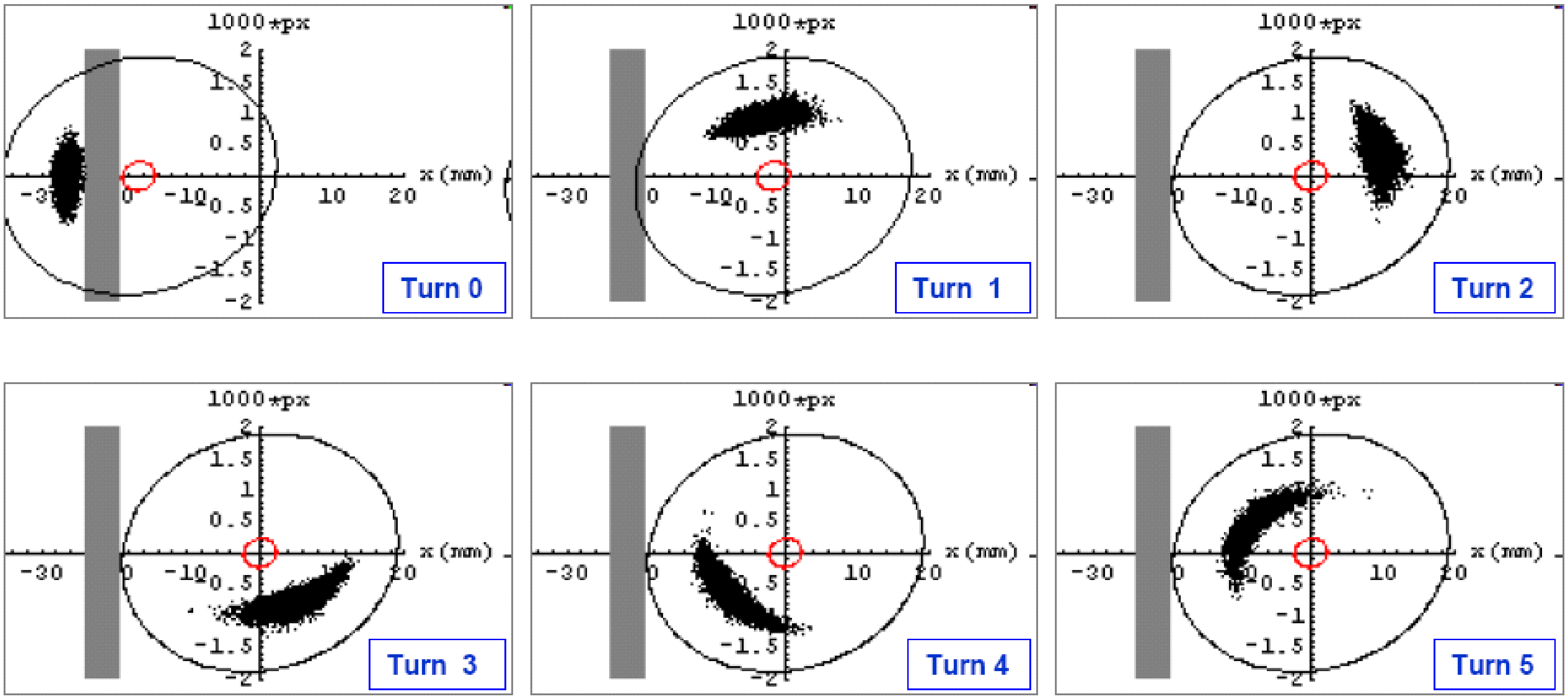


After several damping times, the bump is energised again and a new pulse can be injected in the same way.

The sequence is repeated until the nominal current is reached

Multi-turn Injection at Diamond

The closed orbit bump is completely switched off after two turns. The injected beam clears the septum.

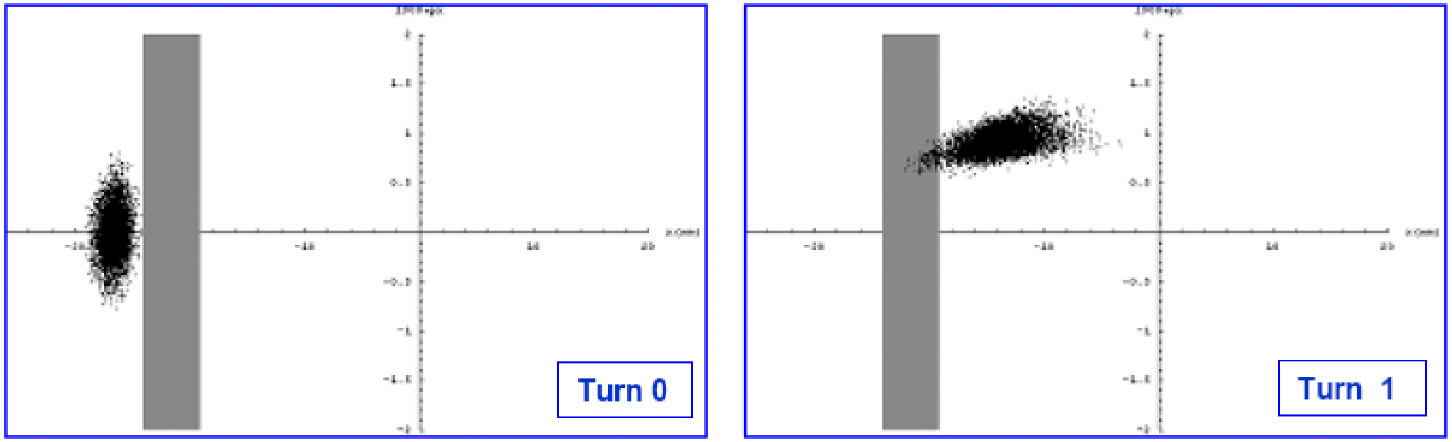


Injection optimisation

The kickers and the septum magnet must be fired simultaneously, synchronously with the incoming bunch.

The pulse length has to be optimised to achieve full injection efficiency

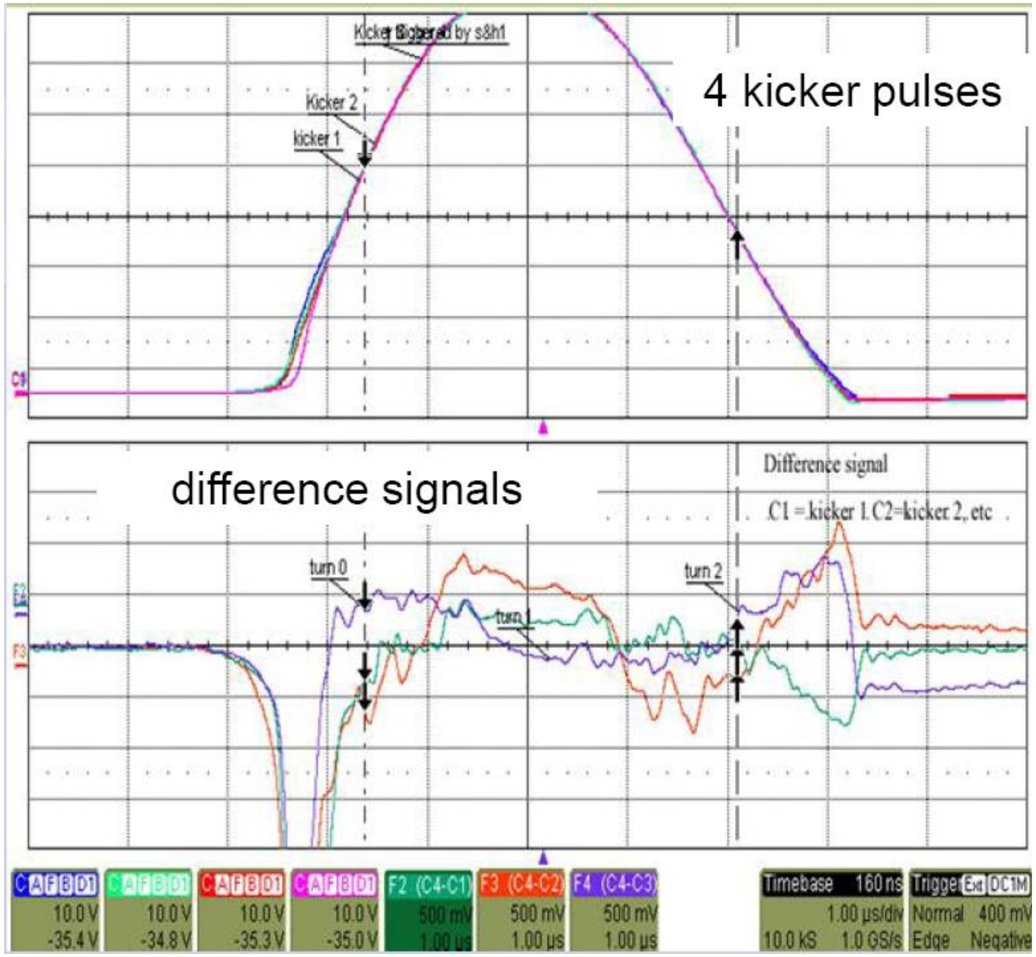
If the kicker pulse is too long the beam might scrape the back of the septum: the optimisation depends on the betatron tune value.



If the pulse is too short we might not kick all the incoming bunch train in the same way, which may result in a poor injection efficiency

Kickers pulse comparison

The orbit bump must be closed, i.e. the kicker's pulse must be the same otherwise we will perturb the stored beam when we are injecting and the injection efficiency can be poor



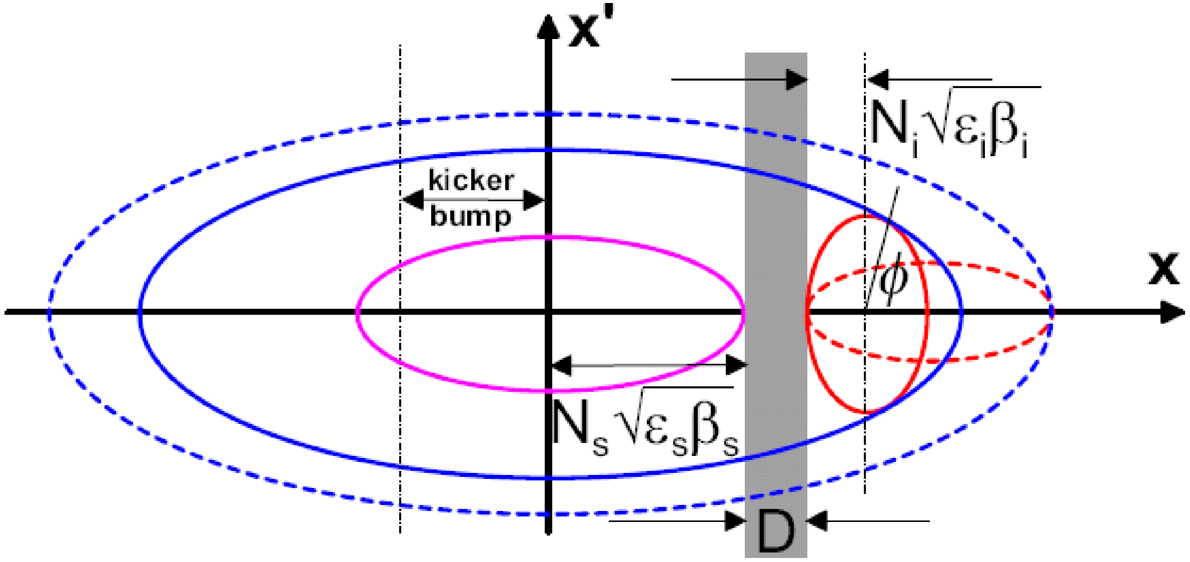
The residual oscillations induced by a non perfectly closed bump must be minimised

The leakage of the septum field is also carefully reduced

Mismatched Injection

To make use of the whole acceptance a mismatched injection scheme is used.

The beta function of the injected beam (at the end of the transfer line) is not matched to the beta function of the ring at the injection point. The beta functions at injection are chosen on the basis of geometrical considerations to fit better the machine acceptance



pink: stored beam displaced; red: injected beam; blue: ring acceptance; grey: septum

Multi-turn Injection for protons

In proton machines the transverse radiation damping time is very long and cannot be used to ease the injection process.

The bump is programmed in order to fill the whole acceptance of the machine (phase space painting)

Some emittance dilution always occurs. Space charge forces limits the injection.

Charge exchange injection is used as an alternative an more effective injection method

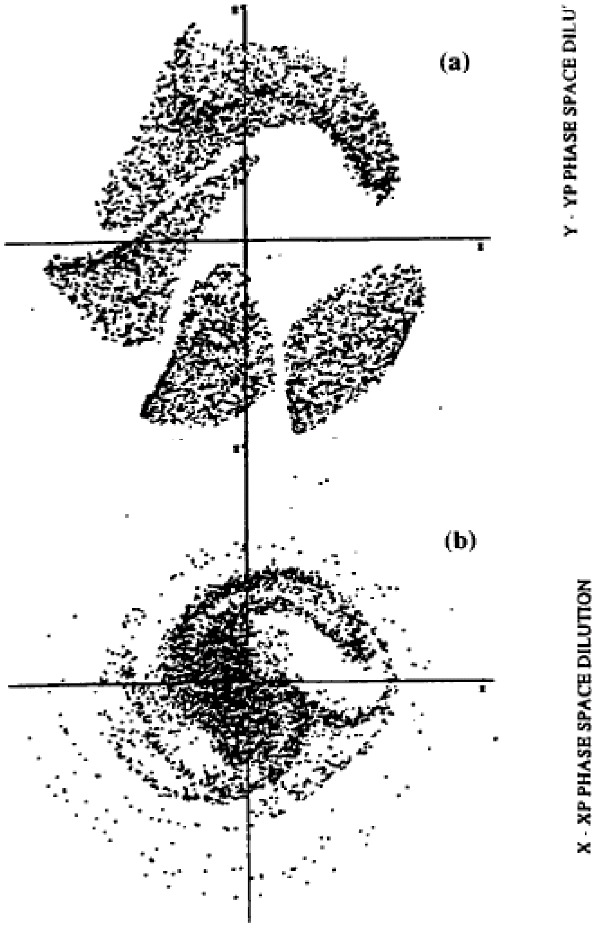
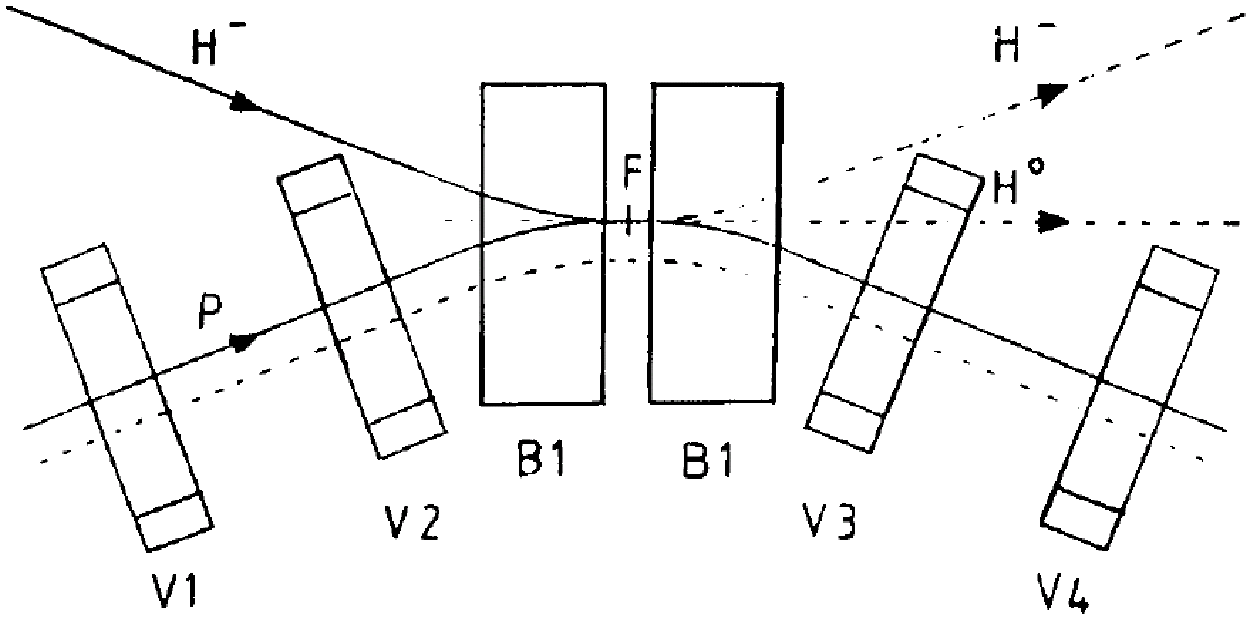


Fig. 3 Distribution after 5-turn injection (a) and 20 turns later (b)

Charge Exchange Injection



H^- and protons are bent in opposite directions by a dipole B1, they travel together in the drift and they both go through a foil F. The foil strips the electron from the H^- and the second bending B1 will select stripped particles with the correct charge to stay in the ring.

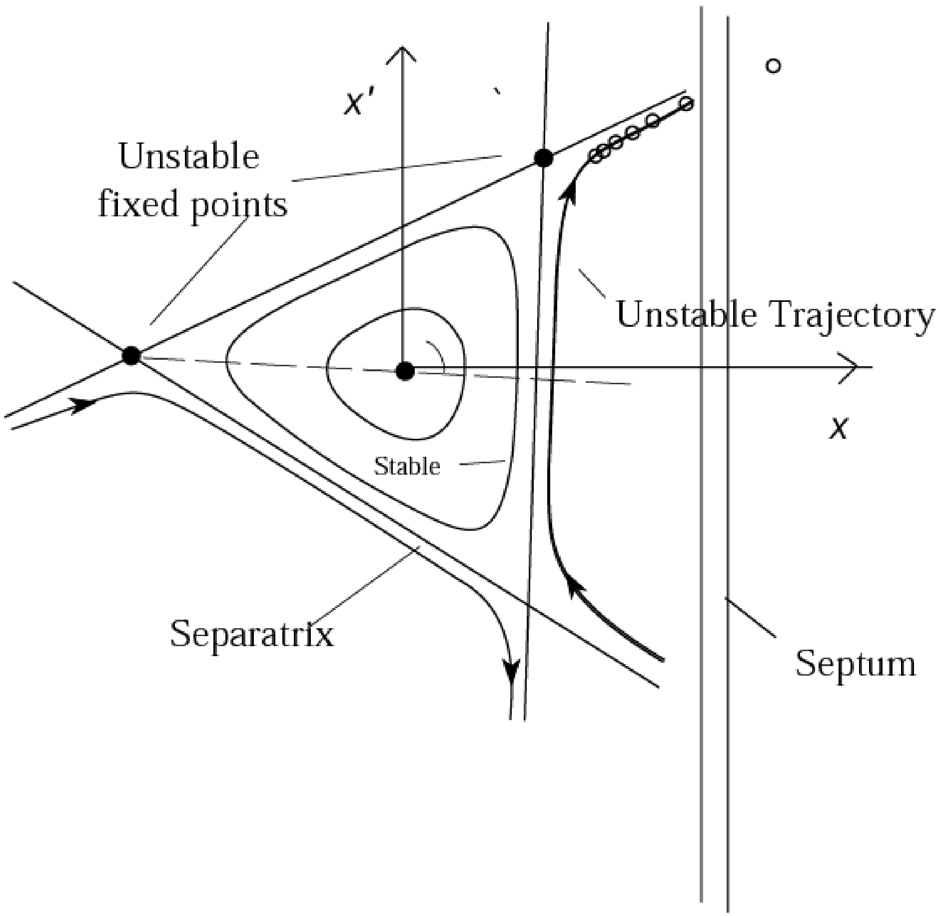
To avoid crossing the foil many times a closed orbit bump is used. The bump is programmed to fill the whole acceptance.

Multi-turn resonant extraction

Placing the tune close to a third order resonance and powering a sextupole we can force the particle to move close to the separatrix of a third order resonance in the horizontal plane.

The amplitude of the particles locked on the resonance will grow, and can cross the septum and be extracted

The strength of the resonance and the speed with which the beam reaches the separatrix can be adjusted so that the extraction can be very slow



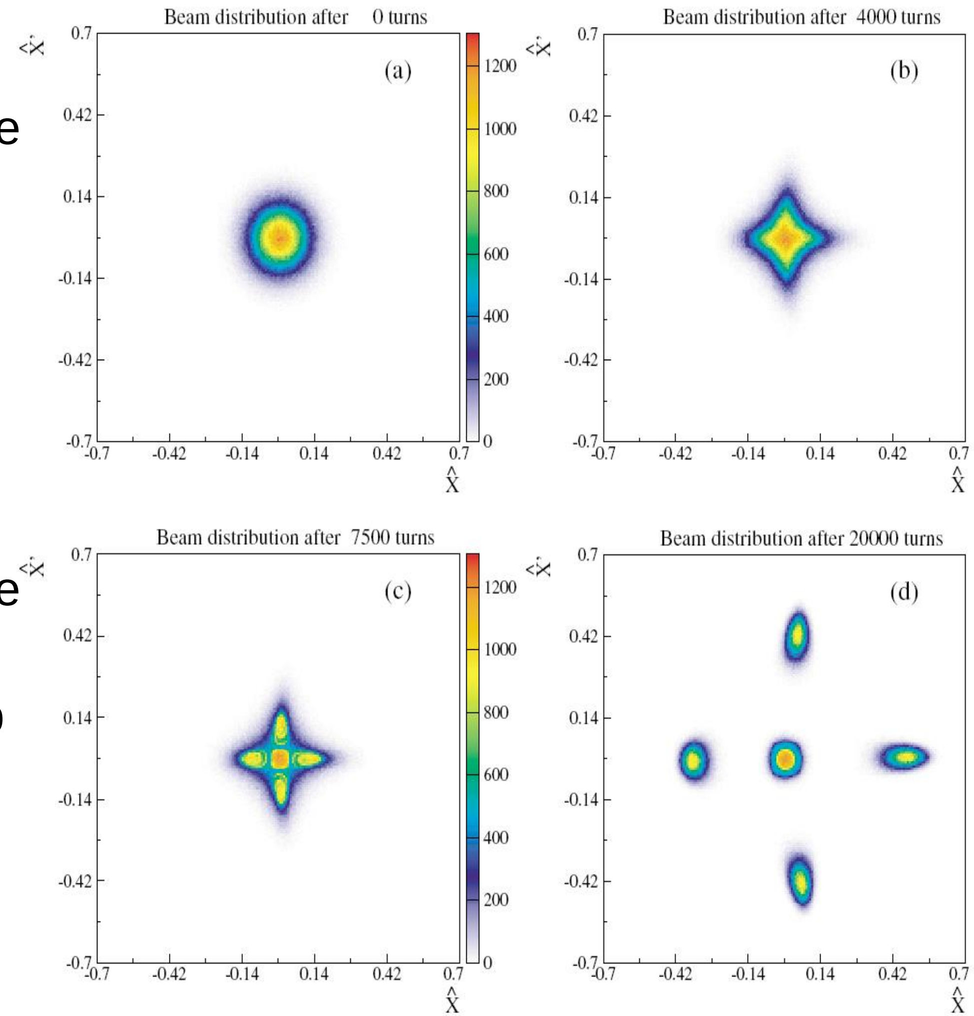
Multi-turn extraction using resonance islands

Powering octupoles creates stable resonance island where particles can be trapped. The bunch can be split into N+1 bunchlets (N is the order of the resonance)

By changing the tune adiabatically (slowly w.r.t. to the beatron motion) the island can be moved in phase space and can be extracted with a N-turn extraction scheme using an orbit bump

Can be done also using 3[rd] order resonance or others

Ref. CERN-SPS studies



Bibliography

CAS 85-19, K. Steffen, Basic course on accelerator Optics

CAS 85-19, G.H Rees, Injection

CAS 85-19, G.H. Rees, Extraction

T. Wilson, Particle Accelerators

H. Schopper, Advances in Accelerator Physics and Technologies