

# Instabilities I & II

JAI Graduate Course

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# Introduction

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# Instabilities

*So you've carefully tuned the machine to produce the best possible performance at the highest intensity and you've invited the lab director to the control room to observe, for the first time, the machine working at design intensity. The intensity comes up, the pulse goes up and just when you're about to get there... there's a sudden loss of beam. You try again, and it repeats. This is characteristic of an instability.*

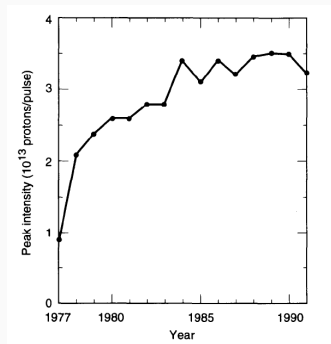
*- E. J. N. Wilson, CERN.*



Ted Wilson in the SPS control room in 1977, Image ©CERN

# Instabilities

Instabilities are one of the main factors that limit performance.



Peak beam intensity in the CERN PS over time, Image from [2] pp. 2 and Jacques Gareyte, 1991.

# Types of Instabilities

When pushed in terms of performance, accelerators tend to reach an intensity limit. With analysis, understanding and (hopefully) mitigation, a new limit emerges. The same pattern can be seen with many high intensity and high energy accelerators.

## Why does this happen?

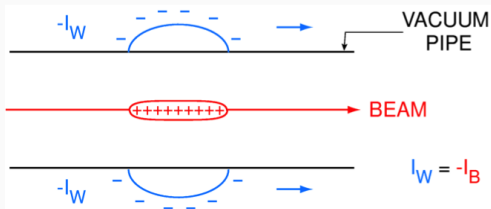
Electromagnetic interactions with the environment can affect both individual particles and the collective motion of the whole bunch.

We can have both **transverse** and **longitudinal** instabilities. There are also both **single** and **multi-bunch** instabilities.

# Impedance of the wall

- There is a wall current  $I_W$  due to the circulating bunch
- Vacuum pipe is not smooth, so  $I_W$  sees an *impedance*.

$$Z = Z_r + iZ_i \quad (1)$$



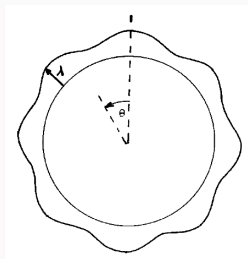
- The induced voltage is  $V \approx I_W Z = -I_B Z$  which acts back on the beam.

Instabilities are intensity dependent.

## Test of an Example Instability

From an initial small perturbation, we can test if the perturbation is:

- Increased, thus INSTABILITY
- Decreased, thus STABILITY



Example: perturbation in the local line density of charge around a synchrotron.

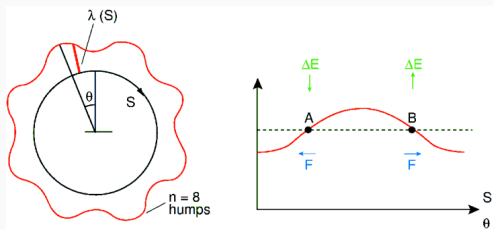
If the forces set up by a pattern of perturbation reinforce the shape, it is sure to grow exponentially.



# Negative Mass Instability

No longer a problem, but helps us understand the mechanisms of instabilities.

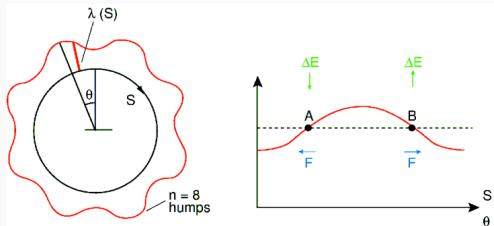
Imagine a ring with a modulation in the line density  $\lambda(s)$ , around the ring.



What is the result?  $E = - \left[ \frac{q}{4\pi\epsilon_0\gamma^2} \right] \frac{\partial\lambda}{\partial s}$ .

- Particle B finds itself with a larger charge density behind it than in front of it, pushing it forward.
- Conversely particle A will be decelerated by the mountain of charge in front of it.

# Negative Mass Instability



$$E = - \left[ \frac{q}{4\pi\epsilon_0\gamma^2} \right] \frac{\partial\lambda}{\partial s}.$$

So is this stable or unstable? Depends on  $\gamma_t$ .

## Stability

If  $\gamma < \gamma_t$ : if energy is gained, revolution frequency **increases**, and A and B move away from the 'hump' of charge **STABLE**

If  $\gamma > \gamma_t$ : if energy is gained, revolution frequency **decreases**, and A and B move toward the 'hump' of charge. **UNSTABLE**

# Impedance

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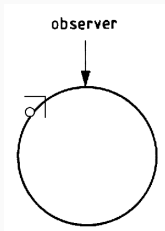
# Impedance and Instabilities

- In general, impedances are complex, and are functions of the frequency  $Z(\omega) = Z(\omega)_{real} + iZ(\omega)_{imag}$
- Strong coupling between beam and vacuum chamber if the impedance and particle beam have a significant component at the same frequency
- Impedance depends on each piece of vacuum chamber including cavities, or changes in beam pipe diameter, material, shape etc...

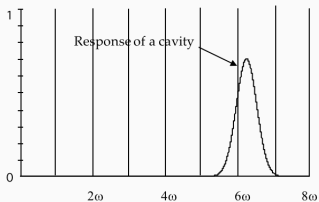
Impedances for a particular component can be narrow band quality factor  $Q \gg 1$  as in an accelerating cavity

OR they can be broadband with  $Q \approx 1$  due to change in vacuum chamber cross section.

# Driving terms



Fourier analysis of a circulating delta function bunch of charge passing an observer.

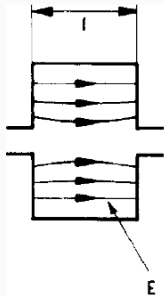


Spectrum from a bunch showing response of an r.f. cavity

$$I = \sum I_n e^{in\omega_0 t} \quad (2)$$

Produces a fundamental at the revolution frequency plus all higher harmonics in equal strength

## Impedance in a cavity



The voltage experienced in local enlargement in the beam pipe (which acts like a cavity) has the form:

$$I = \hat{I}e^{-i\omega t}, V = \hat{V}e^{-i\omega t} \quad (3)$$

We can relate force on particles to the Fourier component of the beam current which excites the force.

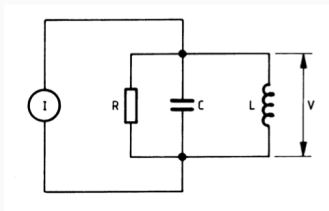
The impedance is a complex quantity.  $V(\omega) = -Z(\omega)I(\omega)$

- REAL if voltage and current are in phase
- IMAGINARY if 90 degrees or  $i$  between voltage and current.  
(Inductive = +, Capacitive = -)
- Differs from RF wave by 90 degrees

The resistive part of the impedance can lead to a shift in the betatron oscillation frequency of the particles while the reactive or imaginary part may cause damping or anti-damping.

# RLC Circuit Impedance

A cavity can be modelled as an AC resonant circuit:



$$\omega_r = \frac{1}{\sqrt{LC}} \quad (4)$$

Where the quality factor  $Q = R\sqrt{C/L} = R/L\omega_r = RC\omega_r$

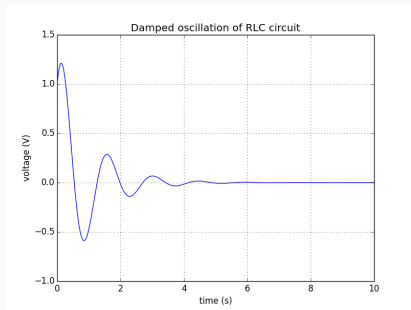
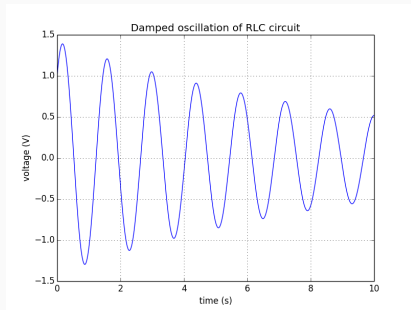
And a differential equation can be written down for voltage and current:

$$\ddot{V} + \frac{\omega_r}{Q} \dot{V} + \omega_r^2 V = \omega_r \frac{R}{Q} i \quad (5)$$

# RLC Circuit Impedance

The solution is a damped resonant circuit, damping rate  $\alpha = \omega_r/2Q$

$$V = V_0 e^{-\alpha t} \sin \left[ \omega_r \sqrt{1 - \frac{1}{4Q^2}} t \right] + \phi \quad (6)$$



<http://www.amanogawa.com/archive/CircuitsA.html>



## RLC Circuit Impedance

If the current in the circuit is  $I = \hat{I}e^{i\omega t}$  it can be shown [4] that the impedance seen is:

$$Z(\omega) = Z_r(\omega) + iZ_i(\omega) = R \left[ \frac{1 - iQ \left( \frac{\omega^2 - \omega_r^2}{\omega\omega_r} \right)}{1 + Q^2 \left( \frac{\omega^2 - \omega_r^2}{\omega\omega_r} \right)^2} \right] \quad (7)$$

- When  $\omega$  is below resonant frequency, the reactive component is inductive or positive
- When the driving frequency is above the resonant frequency, it becomes negative and capacitive

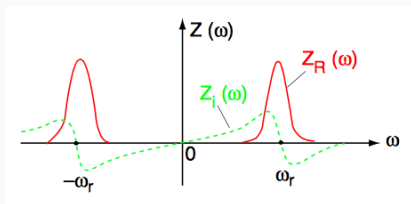
# Impedance effects

- For a narrow-band impedance, high Q factor and low damping rate  $\alpha$ . Thus signal will oscillate for many turns and produce *multi-bunch effects*
- For a broad-band cavity, Q is low,  $\alpha$  is large, the fields collapse rapidly and don't affect subsequent bunches. May produce *single bunch effects*

# RLC Circuit Impedance

For a high Q cavity (narrow band resonator) this can be simplified near the resonance frequency with  $\Delta\omega = \omega - \omega_r$  to:

$$Z(\omega) \approx R_s \frac{1 - i2Q \frac{\Delta\omega}{\omega_r}}{1 + \left(2Q \frac{\Delta\omega}{\omega_r}\right)^2} \quad (8)$$



# **General Method for Studying Instabilities**

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# Our Approach

- Propose a physical concept by which a perturbation to the beam might arise
- Try to determine whether this can lead to an instability
- Figure out under which conditions it is unstable

## Negative Mass Instability again

We can describe the beam line density  $\lambda(\theta)$  and the corresponding instantaneous current  $I(\theta)$  as DC with a small AC component:

$$I = I_0 + I_1 e^{i(n\theta - \Omega t)} \quad (9)$$

Where  $n$  describes the 'humps' ( $n = 8$  in previous slides),  $\Omega = n\omega_0$  is the angular frequency.

This induces a voltage per turn due to the longitudinal impedance:

$$U_s = -I_1 e^{i(n\theta - \Omega t)} Z(\Omega) \quad (10)$$

# Frequency Shift

We postulate that this produces a *complex frequency shift*  $\Delta\Omega$  which modifies the pattern:

$$\Omega = n\omega_0 + \Delta\Omega \quad (11)$$

Now we take a little short-cut... recalling that the motion in an accelerating cavity with voltage  $V_0$  and frequency  $hf_0$  with a phase angle  $\phi_s = 0$  (stationary bucket), we get an equation of motion in  $\phi$ :

$$\left[ \frac{E_0\beta^2\gamma}{2\pi\eta hf_0^2 e} \right] \ddot{\phi} + V_0\phi = 0 \quad (12)$$

And the small amplitude synchrotron oscillation frequency is:

$$\omega_s^2 = \left[ \frac{e\eta h V_0 \omega_0^2}{2\pi E_0 \beta^2 \gamma} \right] \quad (13)$$

## Frequency Shift

$$\omega_s^2 = \left[ \frac{e\eta h V_0 \omega_0^2}{2\pi E_0 \beta^2 \gamma} \right] \quad (14)$$

From this, we replace the term describing voltage and harmonic number with:  $V_0 h \rightarrow -inZl_0$

Note that the  $i$  is due to the fact that unlike the RF, the voltage induced by a resistive impedance passes zero 90 degrees after the particle passes.

Which happens to be the correct result, and gives us the frequency shift:

$$(\Delta\Omega)^2 = (\Omega - n\omega_0)^2 = -i \left[ \frac{\eta\omega_0^2 n l_0}{2\pi\beta_2 E} \right] (Z_r + iZ_i) \quad (15)$$



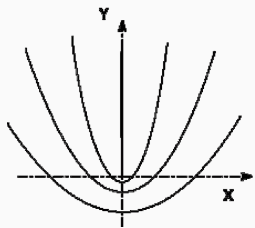
## Growth, Damping and Frequency Shift

If we put the complex frequency shift back into the equation for instantaneous current, we get:

$$I(t, \theta) = I_0 + I_1 e^{\Delta\Omega_i t} e^{i(n\theta - (n\omega_0 + \Delta\Omega_r)t)} \quad (16)$$

- the  $\Delta\Omega_i$  term describes the growth or damping of the mode
- the  $\Delta\Omega_r$  term is the real frequency shift of the rotating pattern

## Stability diagram



Take Equation 15 and lump all the beam parameters into a constant  $\xi$ , to get:

$$(\Delta\Omega)^2 = \xi(Z_r - iZ_i) = (\Delta\Omega_r + i\Delta\Omega_i)^2 \quad (17)$$

Equate the real and imaginary parts, to get parabolic contours for  $\Delta\Omega_i = \text{const}$ :

$$Z_r = 2\Delta\Omega_i \sqrt{Z_i/\xi + \Delta\Omega_i^2/\xi^2} \quad (18)$$

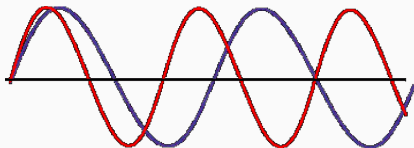
Relates the imaginary part of  $\Delta\Omega$  the growth (or damping) rate, to the complex impedance  $Z$  as a plot of contours of constant growth rate in the  $Z_r, Z_i$  plane.

# Landau Damping

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## Landau Damping - the idea

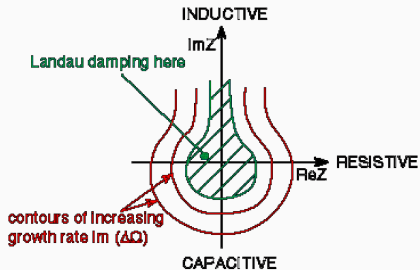
In a real machine, not all particles in the beam have the same frequency. The coherent motion from an instability therefore de-coheres over time, potentially damping the instability.



Two oscillators excited together become incoherent and give zero centre of charge motion after a number of turns comparable to the reciprocal of their frequency difference.

## Landau Damping - stability diagram

Landau damping applies not just to longitudinal but also transverse, single and multi-bunch instabilities. Along with active feedback systems, it is a powerful way to overcome coherent beam instabilities



The line defining zero growth rate leads us to a handy approximation for the stability limit of unbunched beams, the 'Keil-Schnell Stability Criterion':

$$\left| \frac{Z}{n} \right| \leq \frac{F m_0 c^2 \beta^2 \gamma \eta}{l_0} \left( \frac{\Delta p}{p} \right)_{FWHM}^2 \quad (19)$$

# Types of Instabilities

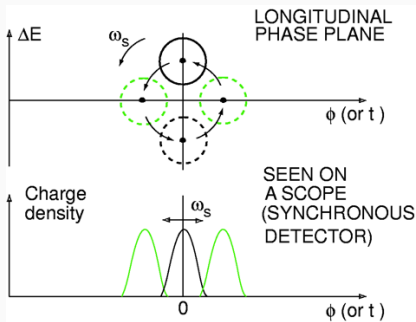
**Table 1:** A non-exhaustive list of instabilities

	Transverse	Longitudinal
Single bunch	Rigid bunch instability	Negative mass instability Head tail instability Robinson instability Longitudinal microwave instability
Multi-bunch	Coupled bunch modes Resistive wall instability	Coupled bunch modes

For some more detailed discussion on these, [1] and [3] are useful references. Useful books include Wiedemann [5] and Chao [2].

# Robinson Instability

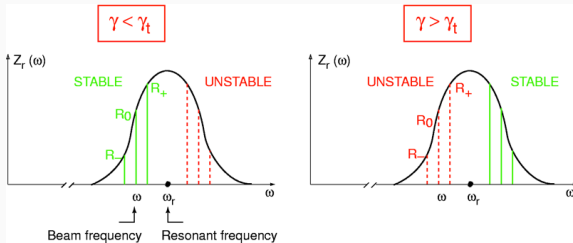
A single bunch and a resonator over multiple turns.



The single bunch in 'dipole' or 'rigid bunch' mode rotates in longitudinal phase plane with  $\omega_s$ , the phase  $\phi$  and energy  $\Delta E$  also vary with  $\omega_s$ .

# Robinson Instability

The bunch sees a resonator impedance at  $\omega_r \approx \omega_0$



Whenever  $\Delta E > 0$ , for  $\omega < \omega_r$ :

- $\omega$  increases
- sees a larger real impedance  $R_+$
- more energy taken from the beam
- STABLE

The opposite is true for  $\omega > \omega_r$ .

Whenever  $\Delta E > 0$ , for  $\omega > \omega_r$ :

- $\omega$  decreases (above transition)
- sees a smaller  $R_+$
- less energy taken from the beam
- UNSTABLE



# Robinson Instability

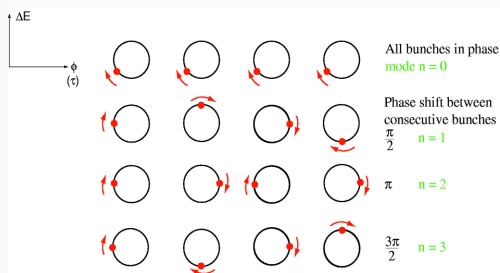
This instability used to be removed just by fine tuning the cavity's resonant frequency  $\omega_r$  slightly away from the beam frequency  $\omega = n\omega_0$

Nowadays a feedback system on the cavity tune is an efficient way of removing it for increased performance.

# Multi-bunch coupling instability

In this instability, the fields induced in the resonator hang around long enough to influence subsequent bunches.

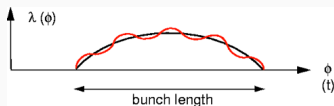
If there are  $M = 4$  bunches, they can couple together in 4 ways:



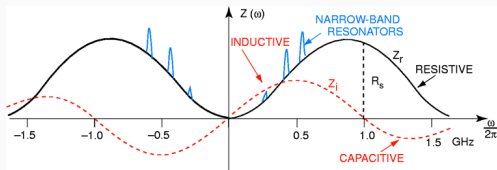
With four possible phase shifts between the four bunches, above transition,  $n=1$  is UNSTABLE.

# Longitudinal Microwave instability

This is a single bunch effect, driven by a broad-band impedance, which is caused by discontinuities in the beam pipe.



Typically results in a high-frequency density modulation superimposed on the bunch shape. Has fast growth rates and also affects lepton machines.

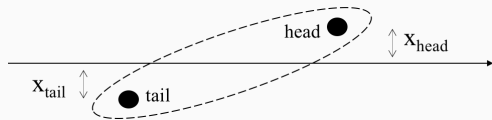


In older machines, the impedance was as much as 20 – 50 $\Omega$ , whereas it is now  $< 1\Omega$  in a modern synchrotron.

# Head Tail Instability

- Single bunch effect of transverse wakefields generated by head of the bunch on it's own tail.
- Occurs for broad-band impedances, which act very quickly and decay quickly, so only affects a single bunch.
- In a linear accelerators, can lead to beam break-up, as they have many cavities.

We can represent the head and tail as a two macro-particle model:



This produces a single bunch current  $I_b = qNl_b f_{rev}$  limit of:

$$I_b \leq \frac{4\pi q\gamma\omega_0\nu_\beta\nu_s}{r_c\beta c (Z_\perp/n)_{imag}} \quad (20)$$

Severe limitation on single-bunch currents in storage rings - special care must be taken to minimize transverse impedance of the vacuum chamber

# Summary

- Broad-band impedances are mainly responsible for single-bunch beam instabilities.
- Narrow-band impedances can cause multibunch instabilities but usually don't affect single bunch intensity limits.
- Both can cause longitudinal or transverse instabilities.

**Questions?**



Us particle accelerator school.

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