

Charm Dalitz analyses



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On behalf of BaBar Collaboration

5th Flavor Physics and CP Violation Conference

FPCP 2007

12th - 16th May 2007 - Bled, Slovenia

Outline

- Theoretical motivations.
 - Extract (and exploit) strong interaction dynamics.
- *Dalitz analysis* of multibody hadronic final states
 - Results from B-factories (BaBar)
 - $D^+ \rightarrow \pi^+\pi^-\pi^+$
 - $D^0 \rightarrow K^0K^-\pi^+, K^0K^+K^-, K^0\pi^+\pi^-$.
 - $D_s \rightarrow \pi^+K^+K^-$
 - For Belle results see talk on D-mixing
 - Results from the charm threshold (CLEO-c)
 - $D^+ \rightarrow \pi^+\pi^-\pi^+, D^0 \rightarrow K^0K^-\pi^+$

Latest results available

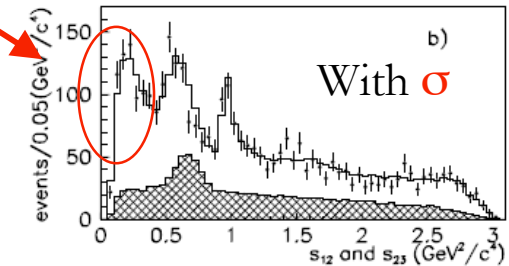
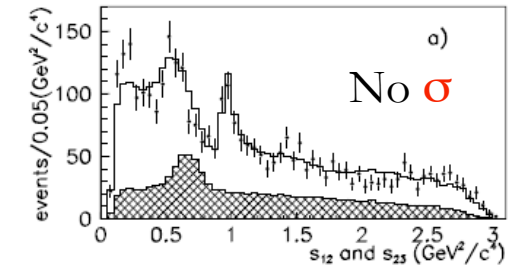
Light meson spectroscopy

- **Scalar mesons**, unknown beasts!

$I = 1$	$I = 1/2$	$I = 0$
		$f_0(600)[?]$
$a_0(980)$	$k(800)[?]$	$f_0(980)$
		$f_0(1370)$
$a_0(1450)$	$K_0^*(1430)$	$f_0(1500)$
		$f_0(1710)$
		$X_0(1550)$

- Too many to fit into a single $q\bar{q}$ nonet
 - » Which nature ? Multiquark ?
 - Meson-meson bound states ?
- Large width, experimentally difficult to resolve

E791 $D^+ \rightarrow \pi^+\pi^-\pi^+$

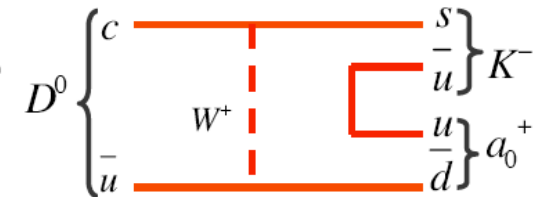
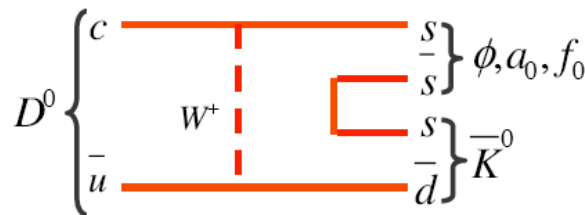
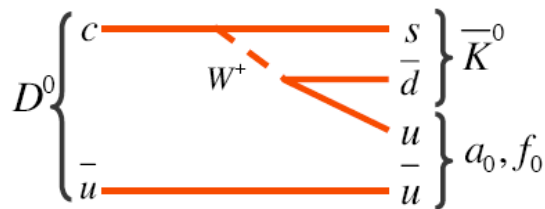


$\sigma \equiv f_0(600)$ or
 low mass $\pi^+\pi^-$ **S-wave**
 is required.

Different parametrization
 FOCUS: K-matrix;
 E791: Rel. Breit-Wigner (BW)

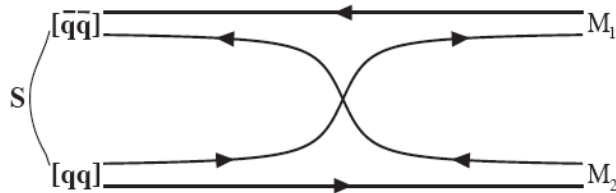
$a_0/f_0(980)$ mesons

- Non-strange and strange content in $f_0(980)$ and $a_0(980)$
- They lie on the KK threshold
 - Resonance shape affected
 - » Flatte's parametrization used:
large errors on the couplings!!
- Analysis of $D^0 \rightarrow K^+K^-\bar{K}^0$ can help!



An example: tetraquarks?

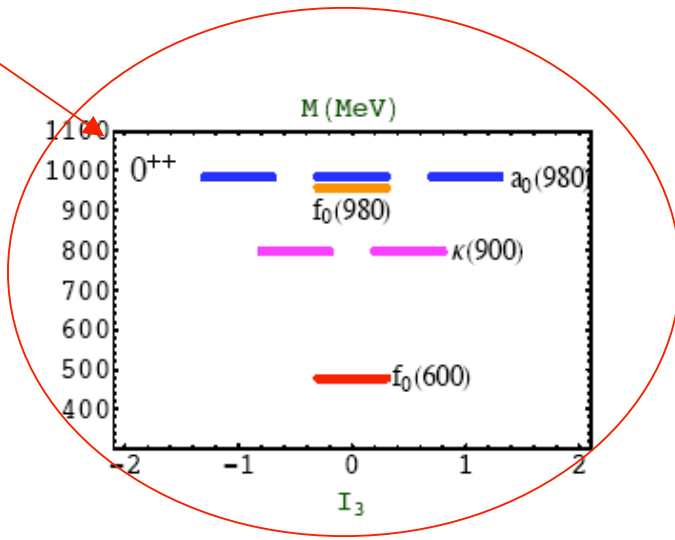
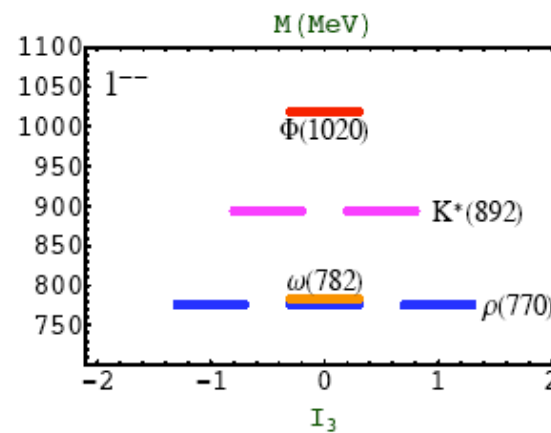
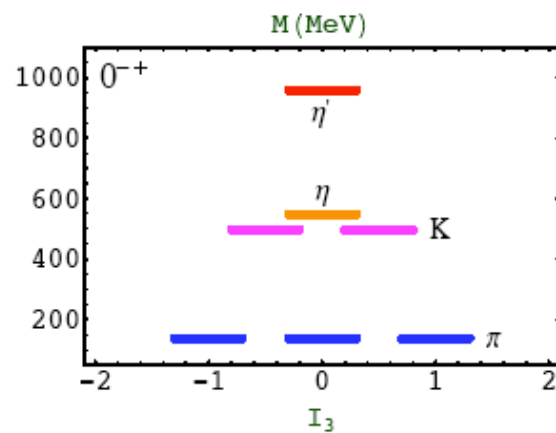
– 2 diquark bound state decay to 2 mesons



S. Nussinov, hep-ph/0307357; M. Karliner and H.J. Lipkin, hep-ph/0307243. R.L. Jaffe and F. Wilczek, Phys. Rev. Lett. **91** (2003) 232003.

L.Maiani et al., Phys.Rev.Lett. 93 (2004) 212002

- **Inverted** mass spectrum predicted!



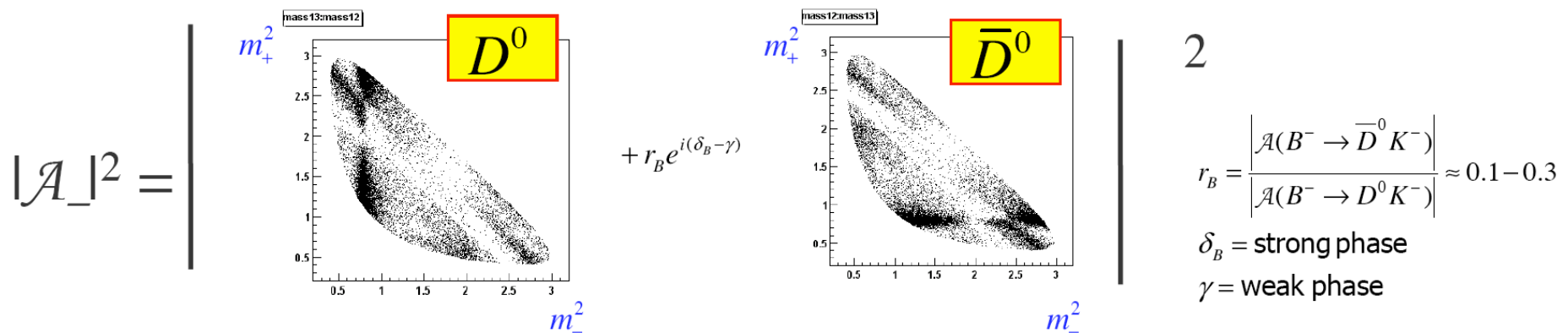
- » Is there a $\kappa(800)$ as well ?
- » What is the $K^{*0}(1430)$?

S-wave and CKM γ extraction

$D^0 \rightarrow K_S \pi^+ \pi^-$ has a rich dynamics

Big $K\pi$ and $\pi\pi$ S-wave

Pictorial view of decay rate of $B^- \rightarrow D^0 K^-$ with $D^0 \rightarrow K_S \pi^+ \pi^-$



D^0 decay amplitude is an *input* for B decay amplitude description

It's crucial to parametrize S-wave correctly!

B-factory and charm factory

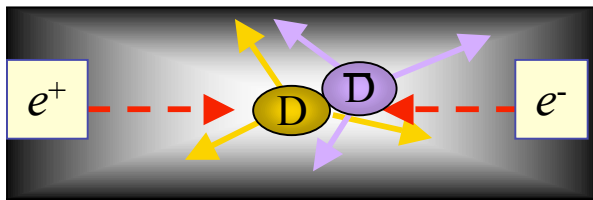
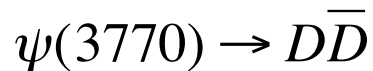
- D meson decays
 - » Large coupling to scalar mesons
 - » Initial state always $\mathbf{J}^P = \mathbf{0}^-$
 - » Isospin and parity violation possible

- B-factories are producing $c\bar{c}$ events too

$$\sigma_{eff}(b\bar{b}) = 1.1 \text{ nb} \quad \sigma_{eff}(c\bar{c}) = 1.3 \text{ nb}$$

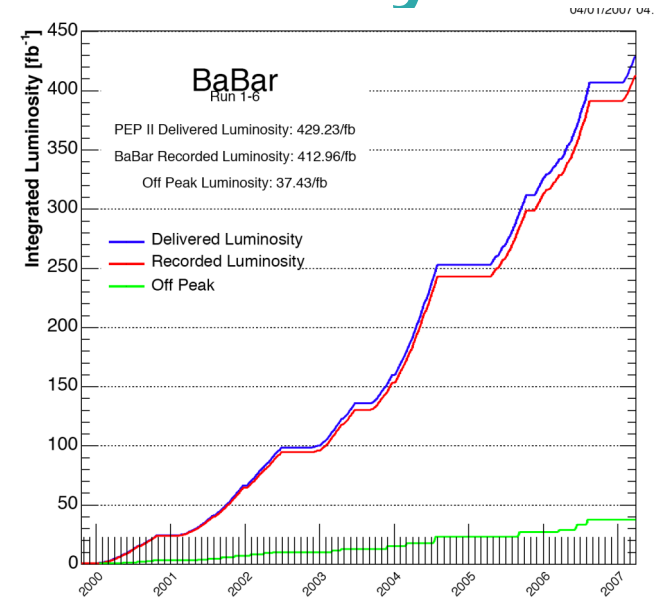
- CLEO-c

- Run on threshold!
- Quantum-coherence



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Run 1-5: more than 500M
cc events

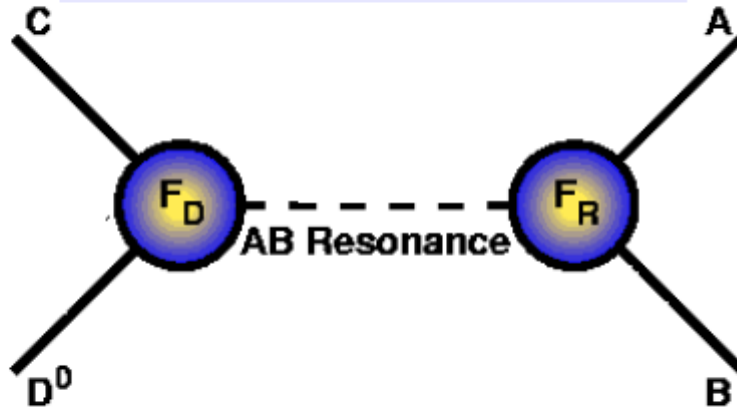
- Pure C = -1
- No additional particles
- Low multiplicity
- Clean ν recon.

281 pb⁻¹
@ $\psi(3770)$

Dalitz amplitudes

D^0 three-body decay $D^0 \rightarrow ABC$ decaying through an $r=[AB]$ resonance

S. Kopp *et al.*, Phys.Rev.D**63**:092001,2001



D^0 three-body amplitude

$$\mathcal{A}_D(s_{12}, s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12}, s_{13})$$

↳ NR term (direct 3 body decay)

$a_0, \delta_0, a_r, \delta_r$: Free parameters of fit

$$\mathcal{A}_r(s_{12}, s_{13}) = F_D^J F_r^J \times M_r^J \times BW_r^J$$

Relativistic Breit-Wigner

$$BW_r^J(s) = \begin{cases} \frac{1}{M_r^2 - s - iM_r \Gamma_r(\sqrt{s})} \\ \frac{1}{M_r^2 - s - iM_r(\rho_1 g_1^2 + \rho_2 g_2^2)} \end{cases} a_0(980)/f_0(980)$$

↳ Angular distribution

↳ D and r Blatt-Weisskopf form factors

$D^+ \rightarrow \pi^+ \pi^- \pi^+$ (CLEO-c)

- $L=281 \text{ pb}^{-1}$ @ $\psi(3770)$
- Untagged analysis

$$m_{BC} = \sqrt{E_{beam}^2 - p_D^2}$$

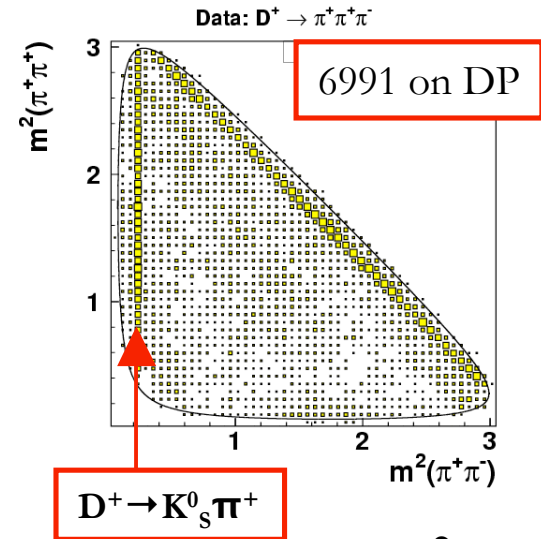
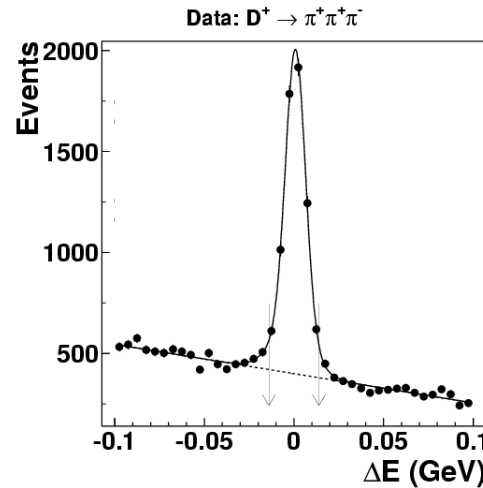
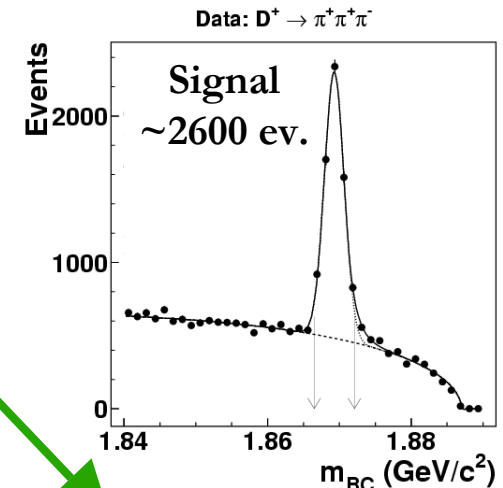
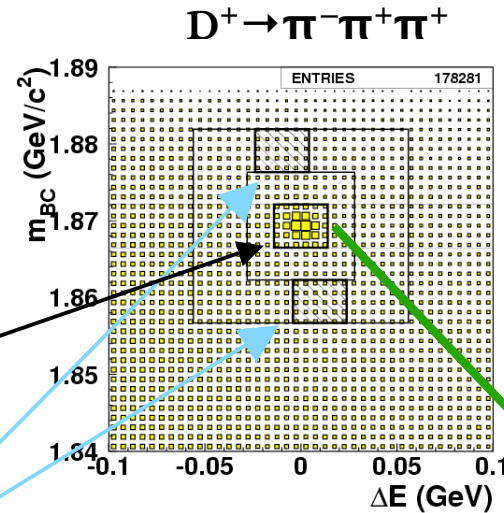
$$\Delta E = E_D - E_{beam}$$

- Signal box for DP
 - $|\Delta E| < 2\sigma$
 - $|m_{BC} - m_D| < 2\sigma$
- Bkg boxes for DP
 - $|\Delta E_{\pm}| < 2\sigma$
 - $5\sigma < |m_{BC} - m_D| < 9\sigma$

DP Statistics:

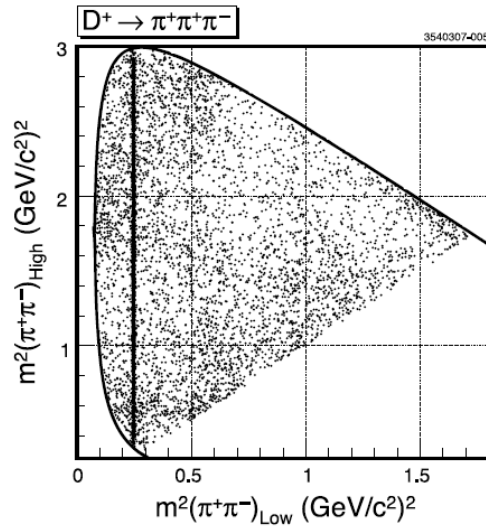
- $N(\pi^- \pi^+ \pi^+) \sim 2600 \text{ ev.}$
- $N(K_s^0 \pi^+) \sim 2240 \text{ ev.}$
- $N_{back} \sim 2150 \text{ ev.}$

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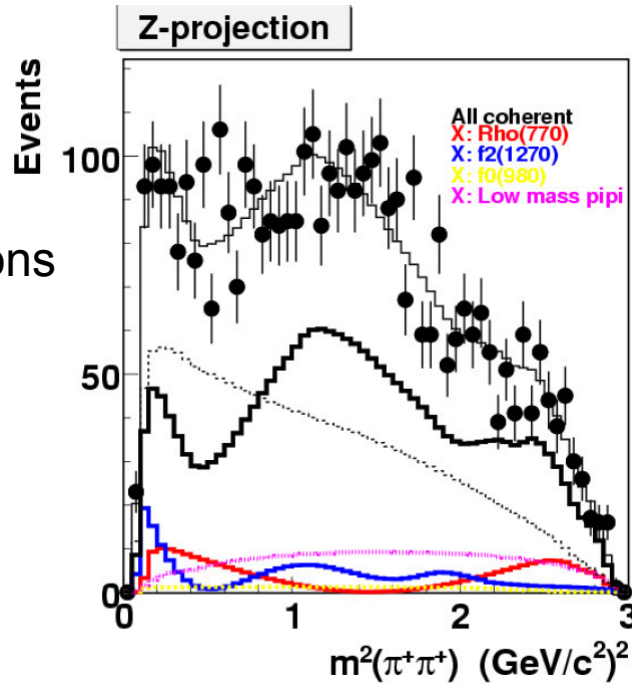
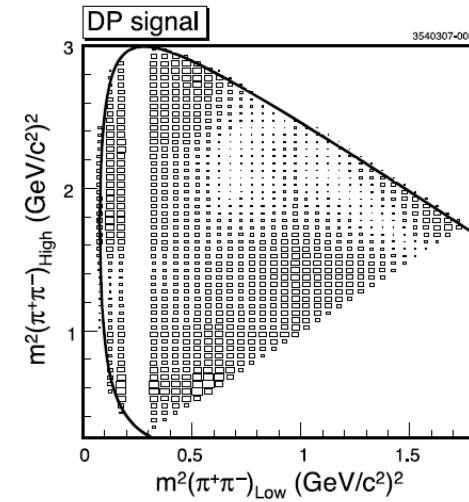
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Nominal fit

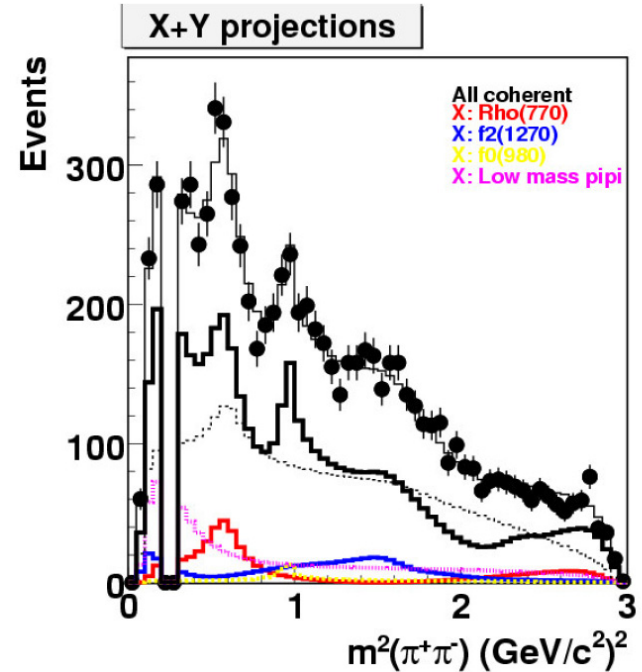


data Dalitz distribution

Signal PDF only
[K_S removed]



Leading fractions shown on projections



CLEO-c $D^+ \rightarrow \pi^+ \pi^- \pi^+$ Results

hep-ex/0704.3954

Mode	Fit Values		
	Relative Amplitude	Phase (degrees)	Fit Fraction (%)
$\rho(770)\pi^+$	1.0	0	$20.0 \pm 2.3 \pm 0.9$
$f_0(980)\pi^+$	$1.4 \pm 0.2 \pm 0.2$	$12 \pm 10 \pm 5$	$4.1 \pm 0.9 \pm 0.3$
$f_2(1270)\pi^+$	$2.1 \pm 0.2 \pm 0.1$	$237 \pm 6 \pm 3$	$18.2 \pm 2.6 \pm 0.7$
$f_0(1370)\pi^+$	$1.3 \pm 0.4 \pm 0.2$	$-21 \pm 15 \pm 14$	$2.6 \pm 1.8 \pm 0.6$
$f_0(1500)\pi^+$	$1.1 \pm 0.3 \pm 0.2$	$-44 \pm 13 \pm 16$	$3.4 \pm 1.0 \pm 0.8$
σ pole	$3.7 \pm 0.3 \pm 0.2$	$-3 \pm 4 \pm 2$	$41.8 \pm 1.4 \pm 2.5$
Limits on Other Contributing Modes			
$\rho(1450)\pi^+$	0.9 ± 0.5	51 ± 22	< 2.4
$f_0(1710)\pi^+$	1.0 ± 1.5	-17 ± 90	< 3.5
$f_0(1790)\pi^+$	1.0 ± 1.1	23 ± 58	< 2.0
Non-resonant	0.17 ± 0.14	-17 ± 90	< 3.5
$I=2 \pi^+ \pi^+$ S-wave	0.17 ± 0.14	23 ± 58	< 3.7

fractions Significant

components on other Upper limits

Consistency with E791

E791 BW σ Fit Fraction
 $(46.3 \pm 9.0 \pm 2.1)\%$

σ pole provides
a good description of the DP

$$Pole_A(s) = \frac{1}{s - s_A},$$

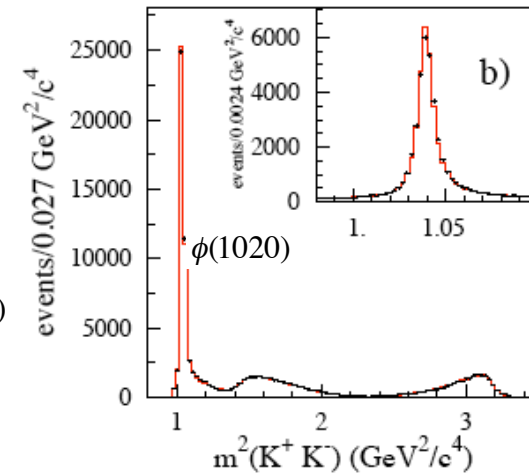
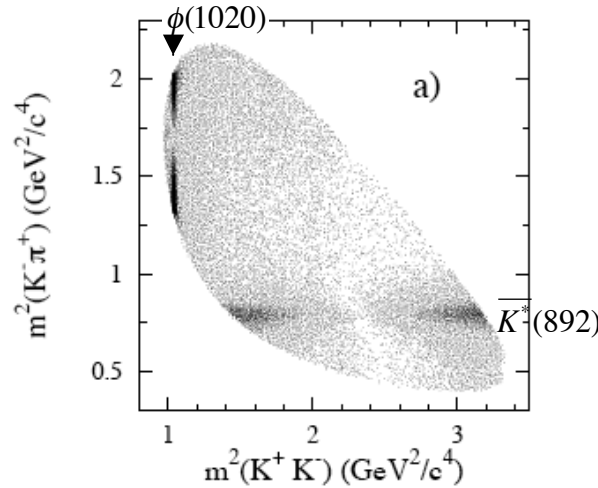
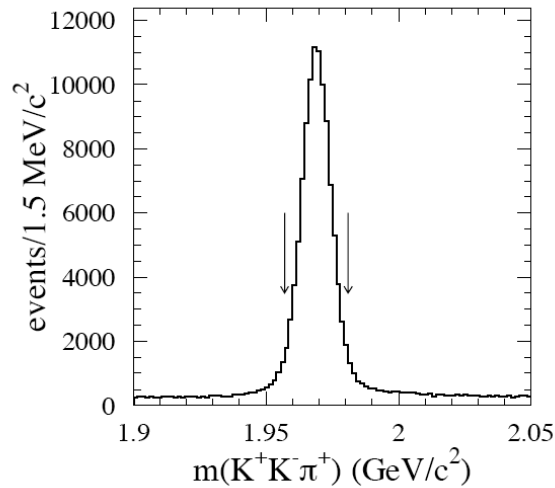
$$s_\sigma = (0.47 - i0.22)^2 GeV^2$$



Dalitz analysis $D_s \rightarrow \pi^+ K^- K^+$

Partial Wave analysis (S-wave extraction)
Measurement of $\phi\pi$ and K^*K fit fraction

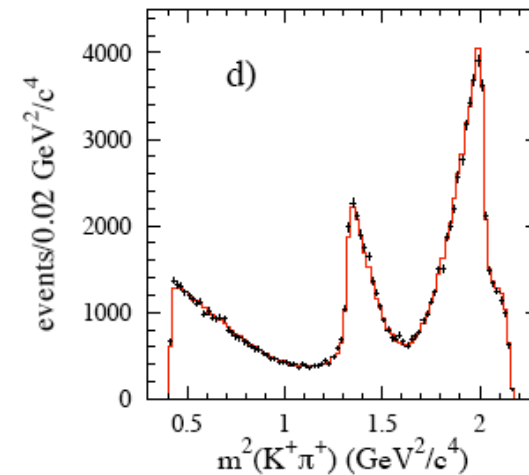
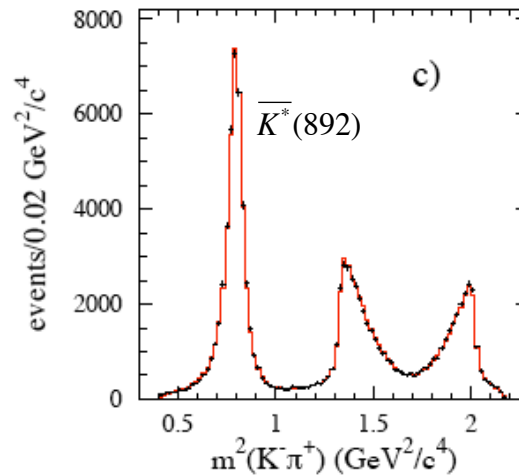
240 fb⁻¹



Vertex separation
and p^*
requirements

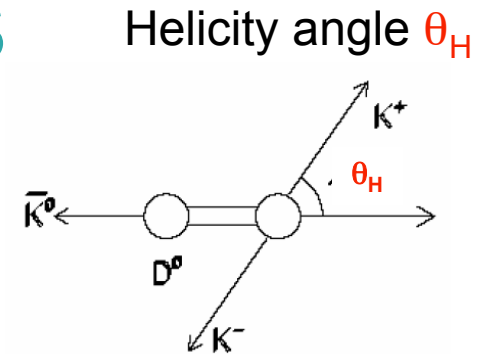
100850 events

95% purity

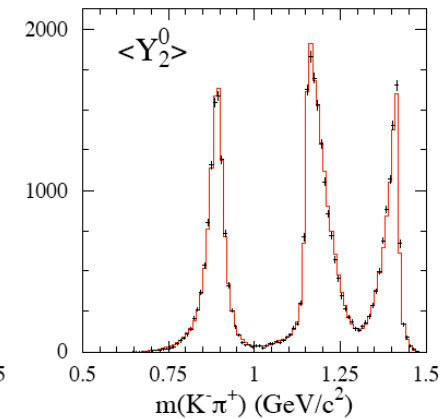
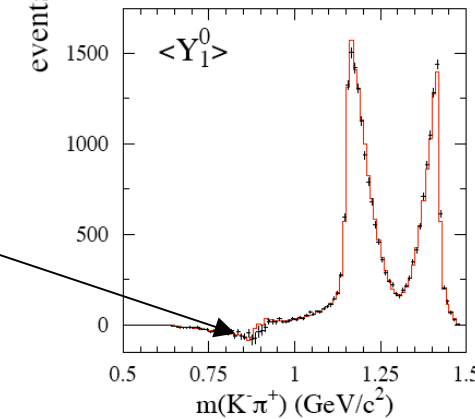
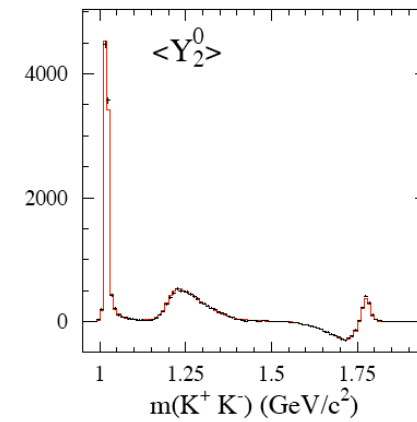
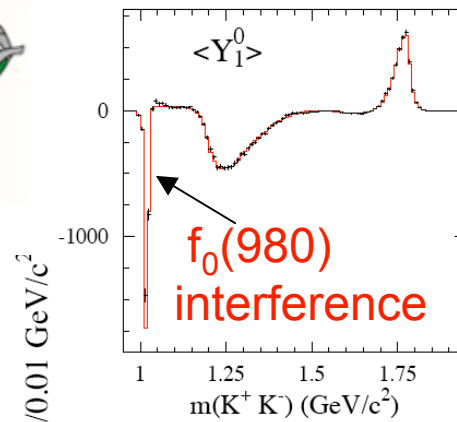


Angular moments

Each event was weighted by the spherical harmonic $Y_L^0(\cos \theta_H)$ ($L=0,1,2,\dots$).



$$\begin{cases} \sqrt{4\pi} \langle Y_0^0 \rangle = S^2 + P^2 \\ \sqrt{4\pi} \langle Y_1^0 \rangle = 2|S||P| \cos \phi_{SP} \\ \sqrt{4\pi} \langle Y_2^0 \rangle = \frac{2}{\sqrt{5}} P^2 \end{cases}$$



Very small
S-P interference.
No $\kappa(800)$?



$D_s \rightarrow \pi^+ K^- K^+$ Dalitz results

BaBar preliminary

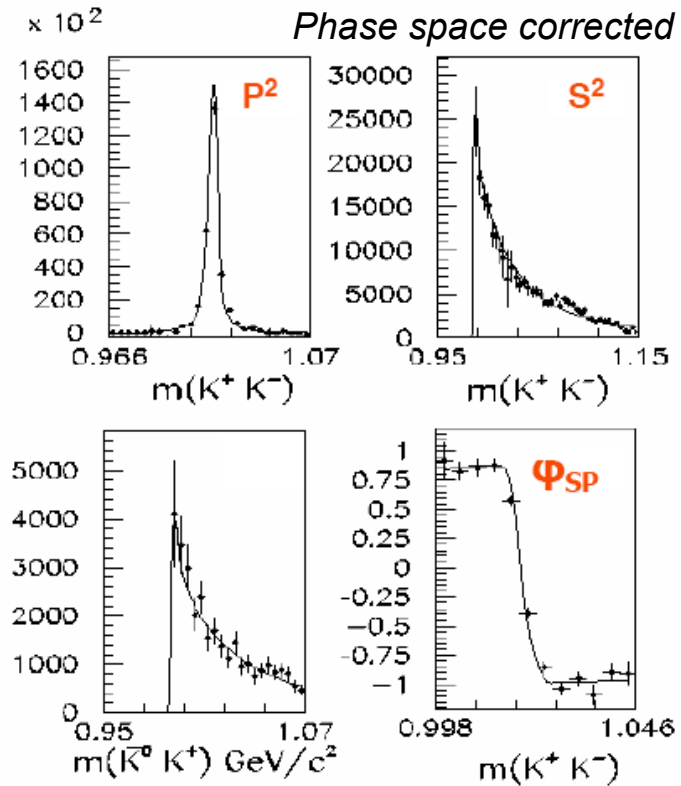
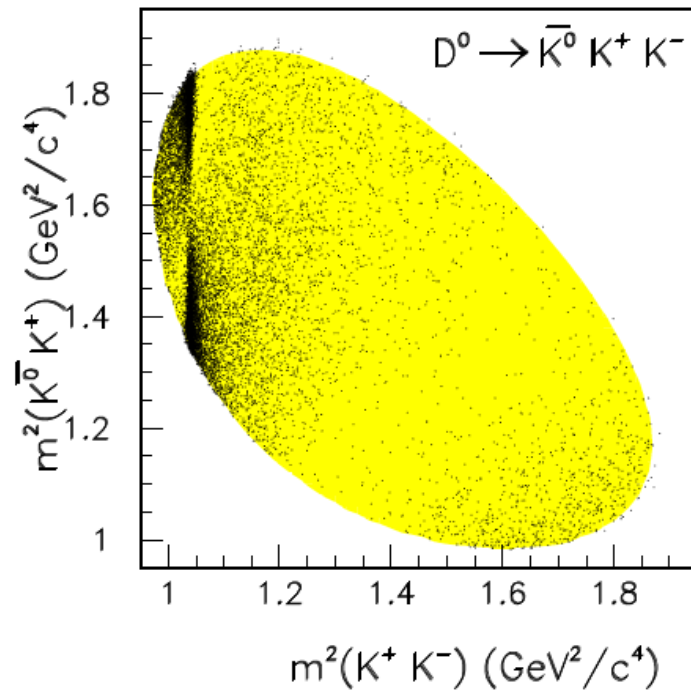
Decay Mode	Decay fraction(%)	Amplitude	Phase(radians)
$\bar{K}^*(892)^0 K^+$	$48.7 \pm 0.2 \pm 1.6$	1.(<i>Fixed</i>)	0.(<i>Fixed</i>)
$\phi(1020)\pi^+$	$37.9 \pm 0.2 \pm 1.8$	$1.081 \pm 0.006 \pm 0.049$	$2.56 \pm 0.02 \pm 0.38$
$f_0(980)\pi^+$	$35 \pm 1 \pm 14$	$4.6 \pm 0.1 \pm 1.6$	$-1.04 \pm 0.04 \pm 0.48$
$K_0^*(1430)^0 K^+$	$2.0 \pm 0.2 \pm 3.3$	$1.07 \pm 0.06 \pm 0.73$	$-1.37 \pm 0.05 \pm 0.81$
$f_0(1710)\pi^+$	$2.0 \pm 0.1 \pm 1.0$	$0.83 \pm 0.02 \pm 0.18$	$-2.11 \pm 0.05 \pm 0.42$
$f_0(1370)\pi^+$	$6.3 \pm 0.6 \pm 4.8$	$1.74 \pm 0.09 \pm 1.05$	$-2.6 \pm 0.1 \pm 1.1$
$\bar{K}_2^*(1430)^0 K^+$	$0.17 \pm 0.05 \pm 0.3$	$0.43 \pm 0.05 \pm 0.34$	$-2.5 \pm 0.1 \pm 0.3$
$f_2(1270)\pi^+$	$0.18 \pm 0.03 \pm 0.4$	$0.40 \pm 0.04 \pm 0.35$	$0.3 \pm 0.2 \pm 0.5$
Sum	$132 \pm 1.2 \pm 15.6$		
χ^2/NDF	1.5		

- Decay dominated by **P-wave**
- Large $f_0(980)$ contribution
 - But big syst. uncertainty (different model used)

Partial Wave Analysis $D^0 \rightarrow \bar{K}^0 K^+ K^-$

Sample = 12540 Events
Purity = 97.3%

@ 91.5 fb⁻¹



– In low K^+K^- mass (no interference with a_0^+)

$a_0(980)$ KK coupling: $g_{kk} = 464 \pm 29(\text{stat.}) (\text{MeV})^{1/2}$



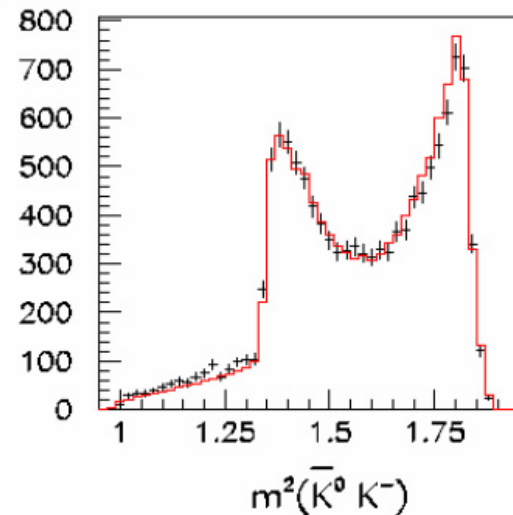
Full Dalitz $D^0 \rightarrow \bar{K}^0 K^+ K^-$ results

Final State	Amplitude	Phase(radians)	Fraction(%)
$\bar{K}^0 a_0(980)^0$	1.(fixed)	0.(fixed)	$66.4 \pm 1.6 \pm 7.0$
$K^0 \phi(1020)$	$0.437 \pm 0.006 \pm 0.060$	$1.91 \pm 0.02 \pm 0.10$	$45.9 \pm 0.7 \pm 0.7$
$K^- a_0(980)^+$	$0.460 \pm 0.017 \pm 0.056$	$3.59 \pm 0.05 \pm 0.20$	$13.4 \pm 1.1 \pm 3.7$
$\bar{K}^0 f_0(1400)$	$0.435 \pm 0.033 \pm 0.162$	$-2.63 \pm 0.10 \pm 0.71$	$3.8 \pm 0.7 \pm 2.3$
Sum			$130.7 \pm 2.2 \pm 8.4$

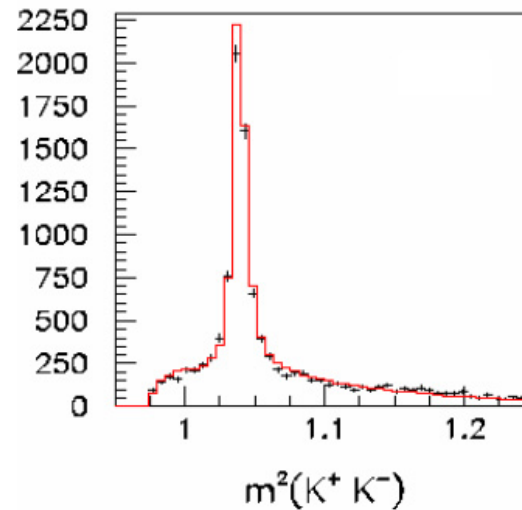
- Big contribution of $a_0(980)$ neutral and charged
- DCS and $f_0(980)$ consistent with zero

$a_0(980)$ KK coupling:

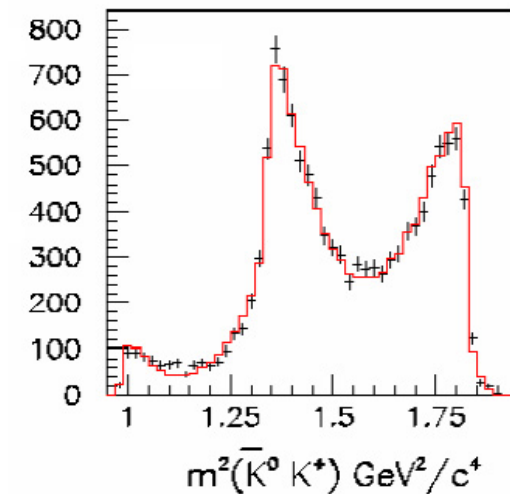
$$g_{kk} = 473 \pm 29(\text{stat.}) \pm 40(\text{syst.}) (\text{MeV})^{1/2}$$



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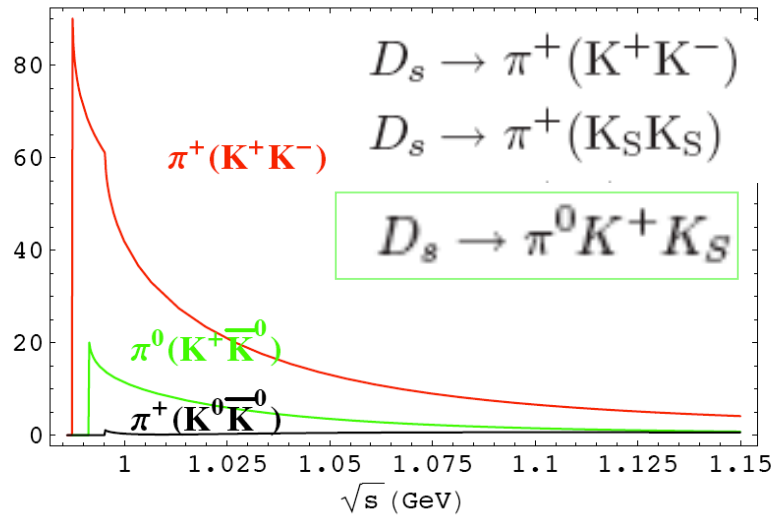
16

$f_0(980)$ parameters \rightarrow M. Ablikim *et al.*, Phys.Lett.B607:243-253,2005

A possible interpretation.

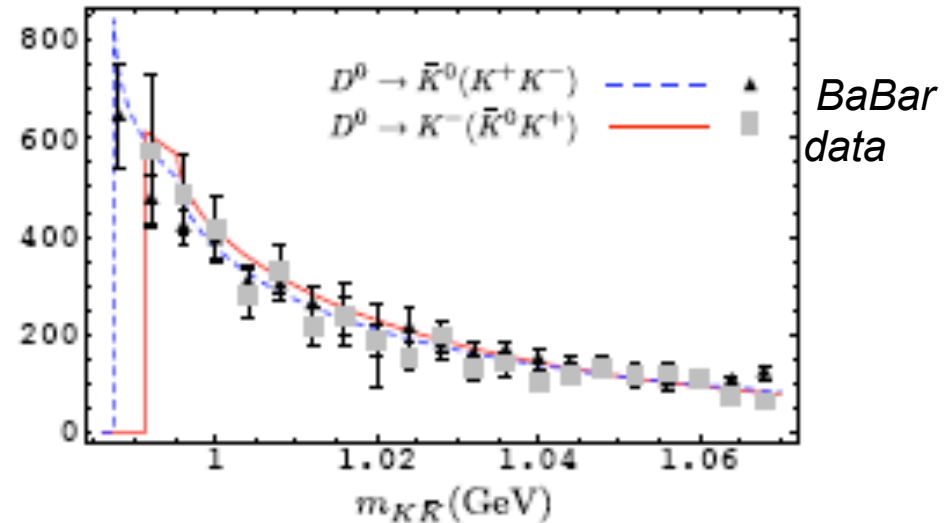
L.Maiani et al. hep-ph/0703272

- $f_0(980)$ as 2 di-quark bound states
 - **Coupling to $K_S K_S$ would vanish!**
 - Decay ratio would be 1/2 for $q\bar{q}$ interpretation



Prediction for rate of
S-wave in $D_s \rightarrow \pi^0 K^+ K_S$

Prediction for rate of
S-wave in $D^0 \rightarrow K_S K_S K_S$



$D^0 \rightarrow K^- K^+ \pi^0$

- 1) Critical for CKM γ extraction
in B decay: ADS method

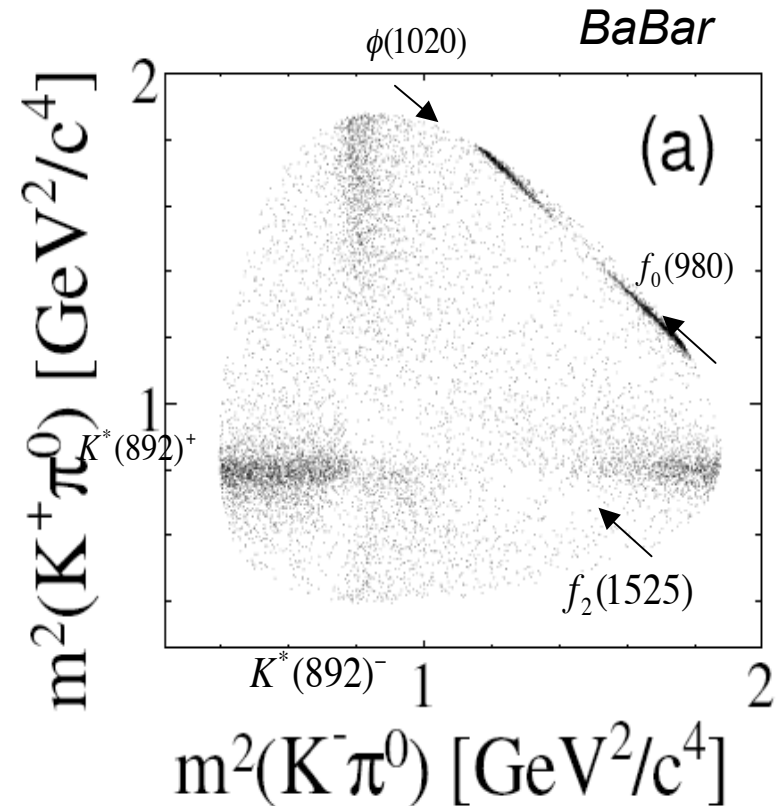
$$A_{\text{ADS}} \equiv \frac{\Gamma[K^+ \pi^-] K^-) - \Gamma([K^- \pi^+] K^+)}{\Gamma[K^+ \pi^-] K^-) + \Gamma([K^- \pi^+] K^+)} = \frac{2 r_B r_D \sin(\delta_B + \delta_D) s}{\mathcal{R}_{\text{ADS}}}$$

$$\frac{A(\overline{D^0} \rightarrow K^{*+} K^-)}{A(D^0 \rightarrow K^{*+} K^-)} = r_D e^{i\delta_D}$$

$$r_D e^{i\delta_D} = \left[\frac{a_{K^* K^+}}{a_{K^* K^-}} \right] e^{[i(\delta_{K^* K^+} - \delta_{K^* K^-})]}$$

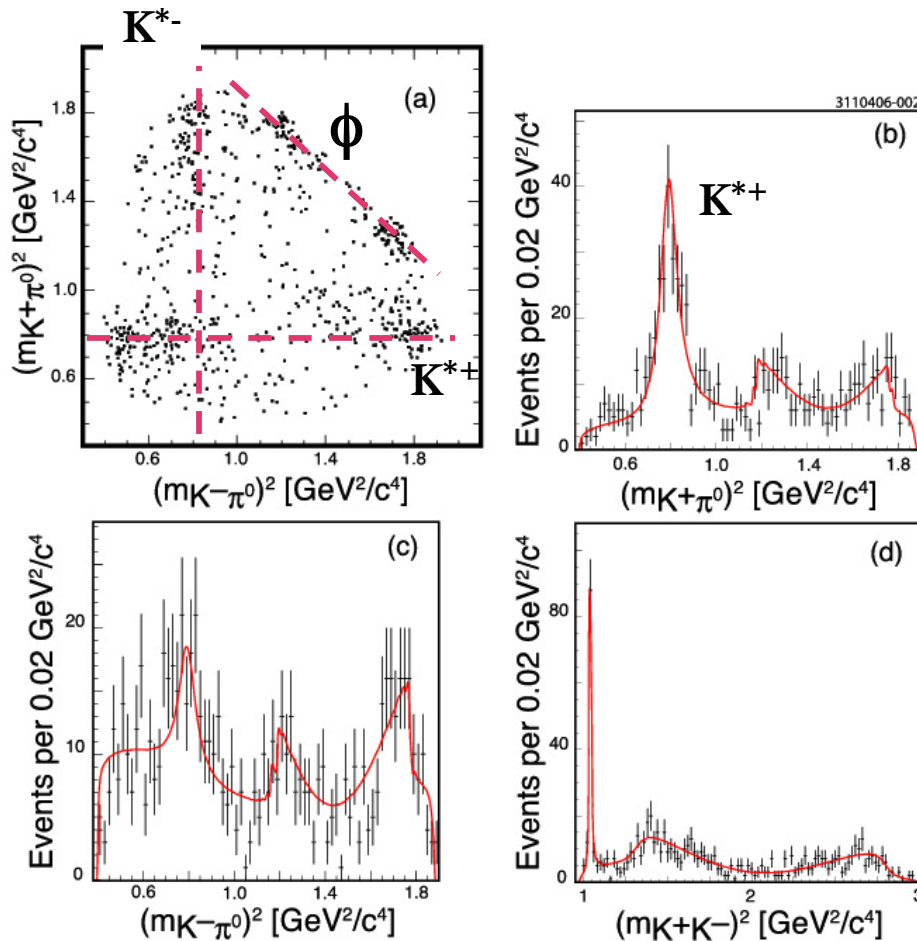
- 2) What's the *nature* of the
 $K\pi$ S-wave below 1.4 GeV ?

$K\pi$ scattering: LASS parametrization Nucl. Phys. B296, 493 (1988); W. Dunwoodie, web notes.



CLEO-c $D^0 \rightarrow K^- K^+ \pi^0$ result

Phys. Rev. D **74**, 031108(R) (2006)



Mode	Fit Values		
	Relative Amplitude	Phase (degrees)	Fit Fraction (%)
$K^{*+}K^-$	1.0	0	46.1 ± 3.1
$K^{*-}K^+$	$0.52 \pm 0.05 \pm 0.04$	$332 \pm 8 \pm 11$	2.3 ± 2.2
$\phi\pi^+$	0.64 ± 0.04	326 ± 9	14.9 ± 1.6
NR	5.62 ± 0.45	220 ± 5	36.0 ± 3.7

Read off the values from the DP fit

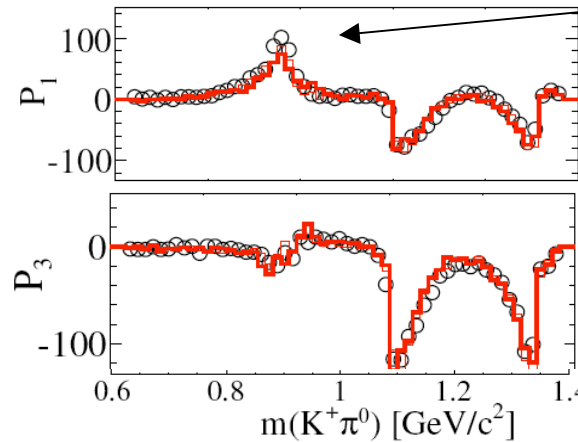
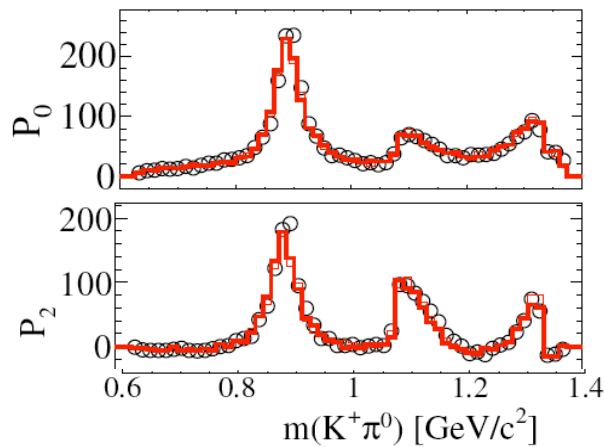
$$r_D = 0.52 \pm 0.05 \pm 0.04$$

$$\delta_D = (332 \pm 8 \pm 11)^\circ$$

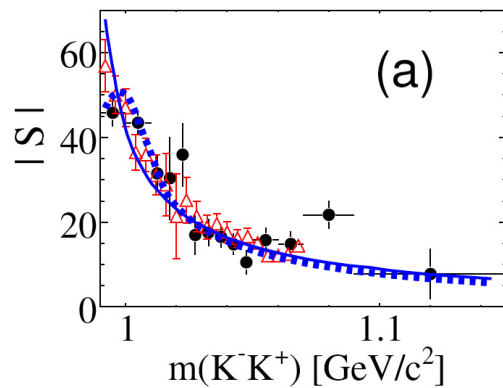
- ❑ First measurement of δ_D .
- ❑ Significant improvement on r_D over previous value using K^*K BF's

Analysis of angular moments

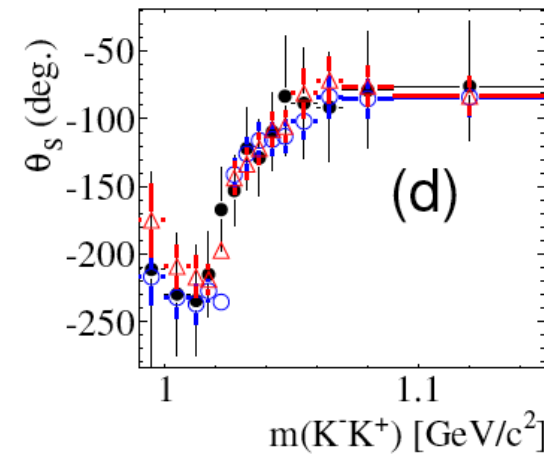
$$P_0 = \frac{|S|^2 + |P|^2}{\sqrt{2}}, \quad P_1 = \sqrt{2}|S||P| \cos \theta_{SP}, \quad P_2 = \sqrt{\frac{2}{5}} |P|^2$$



Significantly large interference between S and P waves.



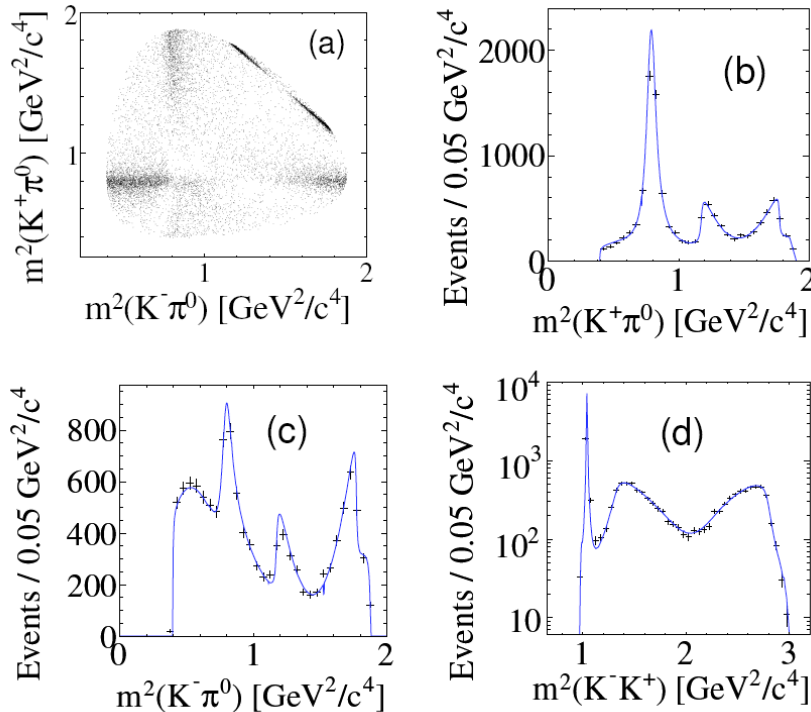
S-wave shape in agreement with $D^0 \rightarrow K^0 K^+ K^-$





BaBar $D^0 \rightarrow K^+ K^- \pi^0$ results

hep-ex/0704.3593 (sub.to PRD)



State	Amplitude, a_r	Phase, ϕ_r ($^\circ$)	Fraction, f_r (%)
$K^*(892)^+$	1.0 (fixed)	0.0 (fixed)	$45.2 \pm 0.8 \pm 0.6$
$K^*(1410)^+$	$2.29 \pm 0.37 \pm 0.20$	$86.7 \pm 12.0 \pm 9.6$	$3.7 \pm 1.1 \pm 1.1$
$K^+ \pi^0 (S)$	$1.76 \pm 0.36 \pm 0.18$	$-179.8 \pm 21.3 \pm 12.3$	$16.3 \pm 3.4 \pm 2.1$
$\phi(1020)$	$0.69 \pm 0.01 \pm 0.02$	$-20.7 \pm 13.6 \pm 9.3$	$19.3 \pm 0.6 \pm 0.4$
$f_0(980)$	$0.51 \pm 0.07 \pm 0.04$	$-177.5 \pm 13.7 \pm 8.6$	$6.7 \pm 1.4 \pm 1.2$
$[a_0(980)^0]$	$[0.48 \pm 0.08 \pm 0.04]$	$[-154.0 \pm 14.1 \pm 8.6]$	$[6.0 \pm 1.8 \pm 1.2]$
$f'_2(1525)$	$1.11 \pm 0.38 \pm 0.28$	$-18.7 \pm 19.3 \pm 13.6$	$0.08 \pm 0.04 \pm 0.05$
$K^*(892)^-$	$0.601 \pm 0.011 \pm 0.011$	$-37.0 \pm 1.9 \pm 2.2$	$16.0 \pm 0.8 \pm 0.6$
$K^*(1410)^-$	$2.63 \pm 0.51 \pm 0.47$	$-172.0 \pm 6.6 \pm 6.2$	$4.8 \pm 1.8 \pm 1.2$
$K^- \pi^0 (S)$	$0.70 \pm 0.27 \pm 0.24$	$133.2 \pm 22.5 \pm 25.2$	$2.7 \pm 1.4 \pm 0.8$

Using LASS parametrization for $K\pi$ S-wave
 $K\pi$ S-wave from E791 as systematics

Model with charged κ

$$m = (870 \pm 30) \text{ MeV}/c^2 \quad \Gamma = (150 \pm 20) \text{ MeV}/c^2$$

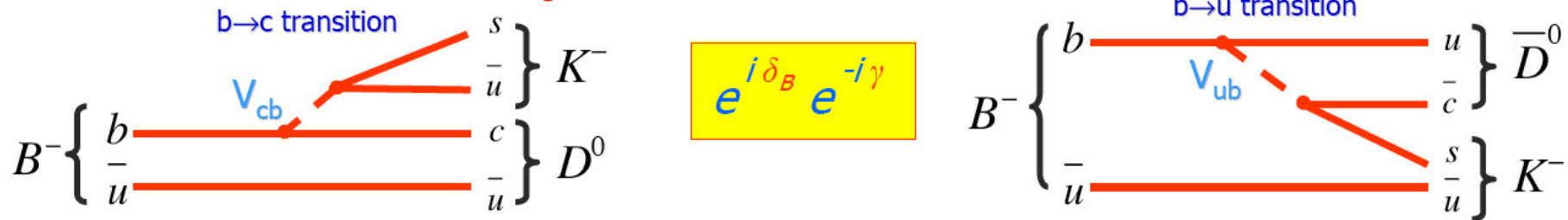
not favoured by data (and parameter different from neutral scalar κ)

$$\delta_D = -35.5^\circ \pm 1.9^\circ \text{ (stat)} \pm 2.2^\circ \text{ (syst)}$$

$$r_D = 0.599 \pm 0.01 \text{ (stat)} \pm 0.011 \text{ (syst)}$$

Towards CKM γ : $D^0 \rightarrow K_S \pi^+ \pi^-$

$B^- \rightarrow D^{0(*)} K^-$ with $D^0 \rightarrow K_S \pi^+ \pi^-$



Interference occurs when some final state is accessible by both D^0 and \bar{D}^0

Giri-Grossman-Soffer-Zupan: PRD68, 054018 (2003): Final state = $K_S^0 \pi^+ \pi^- \Rightarrow$ Dalitz Plot Analysis

$$B^- : \mathcal{A}_-(m_-^2, m_+^2) = |A(B^- \rightarrow D^0 K^-)| \left[f(m_-^2, m_+^2) + r_B e^{i\delta_B} e^{-i\gamma} f(m_+^2, m_-^2) \right]$$

$$B^+ : \mathcal{A}_+(m_-^2, m_+^2) = |A(B^+ \rightarrow \bar{D}^0 K^+)| \left[f(m_+^2, m_-^2) + r_B e^{i\delta_B} e^{+i\gamma} f(m_-^2, m_+^2) \right]$$

$$f(m_+^2, m_-^2)$$

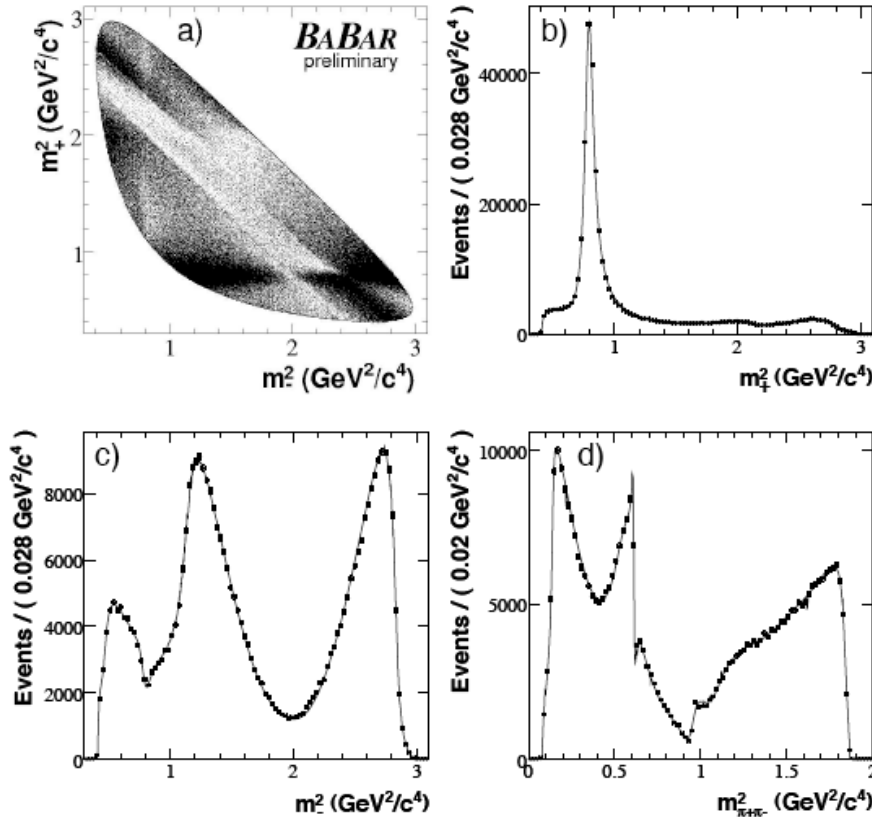
\rightarrow Input from Dalitz analysis

(assumption on the model: **systematic** error on γ)



Dalitz analysis $D^0 \rightarrow K_S \pi^+ \pi^-$

Isobar model resonance + Non resonant term



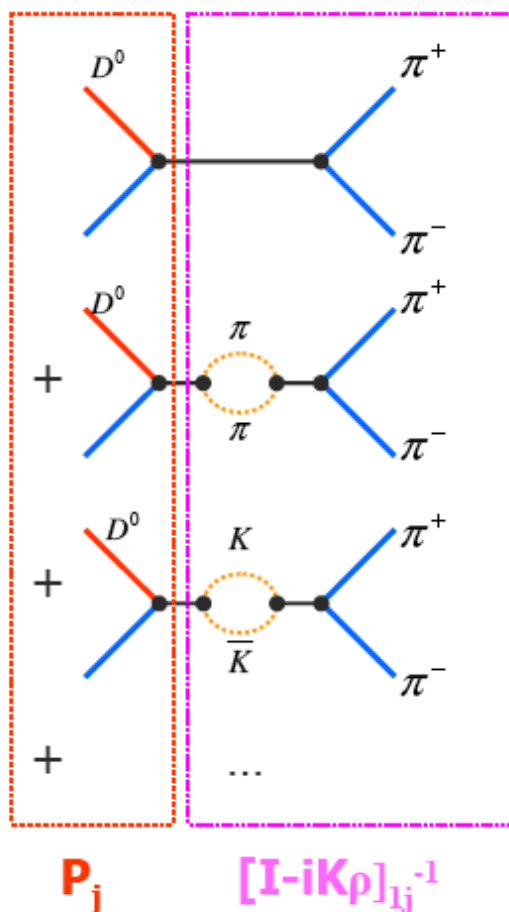
Component	$Re\{a_r e^{i\phi_r}\}$	$Im\{a_r e^{i\phi_r}\}$	Fit fraction (%)
$K^*(892)^-$	-1.223 ± 0.011	1.3461 ± 0.0096	58.1
$K_0^*(1430)^-$	-1.698 ± 0.022	-0.576 ± 0.024	6.7
$K_2^*(1430)^-$	-0.834 ± 0.021	0.931 ± 0.022	3.6
$K^*(1410)^-$	-0.248 ± 0.038	-0.108 ± 0.031	0.1
$K^*(1680)^-$	-1.285 ± 0.014	0.205 ± 0.013	0.6
$K^*(892)^+$	0.0997 ± 0.0036	-0.1271 ± 0.0034	0.5
$K_0^*(1430)^+$	-0.027 ± 0.016	-0.076 ± 0.017	0.0
$K_2^*(1430)^+$	0.019 ± 0.017	0.177 ± 0.018	0.1
$\rho(770)$	1	0	21.6
$\omega(782)$	-0.02194 ± 0.00099	0.03942 ± 0.00066	0.7
$f_2(1270)$	-0.699 ± 0.018	0.387 ± 0.018	2.1
$\rho(1450)$	0.253 ± 0.038	0.036 ± 0.055	0.1
Non-resonant	-0.99 ± 0.19	3.82 ± 0.13	8.5
$f_0(980)$	0.4465 ± 0.0057	0.2572 ± 0.0081	6.4
$f_0(1370)$	0.95 ± 0.11	-1.619 ± 0.011	2.0
σ	1.28 ± 0.02	0.273 ± 0.024	7.6
σ'	0.290 ± 0.010	-0.0655 ± 0.0098	0.9

Total 119.5%

$\pi\pi$ S-wave contains σ and σ'

- σ : $m=(490\pm 6)\text{MeV}/c^2$ $\Gamma=(406\pm 11)\text{MeV}/c^2$
- σ' : $m=(1024\pm 4)\text{MeV}/c^2$ $\Gamma=(89\pm 7)\text{MeV}/c^2$

K-matrix for $\pi\pi$



5 channels: 1= $\pi\pi$ 2=KK 3=multi-meson 4= $\eta\eta$ 5= $\eta\eta'$
 V.V. Anisovitch, A.V Sarantev Eur. Phys. Jour. **A16**, 229 (2003)

K-Matrix formalism overcomes the main limitation of the BW model to parameterize large and overlapping S-wave $\pi\pi$ resonances.

$D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ amplitude

$$\mathcal{A}_D(s_{12}, s_{13}) = \underbrace{F_1}_{\pi\pi \text{ S-wave}} + \underbrace{\sum_r a_r e^{i\delta_r} A_r(s_{12}, s_{13})}_{\substack{\pi\pi \text{ P, D-waves} \\ K\pi \text{ S, P, D-waves}}}$$

$$F_1 = \sum_j [\mathbf{I} - i\mathbf{K}\rho]_{jj}^{-1} \mathbf{P}_j$$

I.J.R. Aitchison, Nucl. Phys. **A189**, 417 (1972)

Initial production vector

Provided by scattering experiment

K-matrix fit

$\pi\pi$ S-wave
with K-matrix
parametrization

BaBar hep-ex/0607104

Cabibbo
Allowed
modes

Doubly Cabibbo
Suppressed
modes

Component	$\text{Re}\{a_r e^{i\phi_r}\}$	$\text{Im}\{a_r e^{i\phi_r}\}$	Fit fraction (%)
$K^*(892)^-$	-1.159 ± 0.022	1.361 ± 0.020	58.9
$K_0^*(1430)^-$	2.482 ± 0.075	-0.653 ± 0.073	9.1
$K_2^*(1430)^-$	0.852 ± 0.042	-0.729 ± 0.051	3.1
$K^*(1410)^-$	-0.402 ± 0.076	0.050 ± 0.072	0.2
$K^*(1680)^-$	-1.00 ± 0.29	1.69 ± 0.28	1.4
$K^*(892)^+$	0.133 ± 0.008	-0.132 ± 0.007	0.7
$K_0^*(1430)^+$	0.375 ± 0.060	-0.143 ± 0.066	0.2
$K_2^*(1430)^+$	0.088 ± 0.037	-0.057 ± 0.038	0.0
$\rho(770)$	1 (fixed)	0 (fixed)	22.3
$\omega(782)$	-0.0182 ± 0.0019	0.0367 ± 0.0014	0.6
$f_2(1270)$	0.787 ± 0.039	-0.397 ± 0.049	2.7
$\rho(1450)$	0.405 ± 0.079	-0.458 ± 0.116	0.3
β_1	-3.78 ± 0.13	1.23 ± 0.16	—
β_2	9.55 ± 0.20	3.43 ± 0.40	—
β_4	12.97 ± 0.67	1.27 ± 0.66	—
f_{11}^{prod}	-10.22 ± 0.32	-6.35 ± 0.39	—
sum of $\pi^+\pi^-$ S-wave			16.2

$\pi\pi$ S-wave



Total fit fraction 116%

Value of χ^2 compatible with nominal model
since it is dominated by the P-wave components,
which are identical between the two models

$B^- \rightarrow D^{0(*)} K^-$ model error on γ

BaBar hep-ex/0607104

Evaluating model uncertainty

- Alternative parametrization for $\pi\pi$ S-wave (BW model and K-matrix model)
- Alternative parametrization for $K\pi$ S-wave (BW E791 and LASS parametrization)



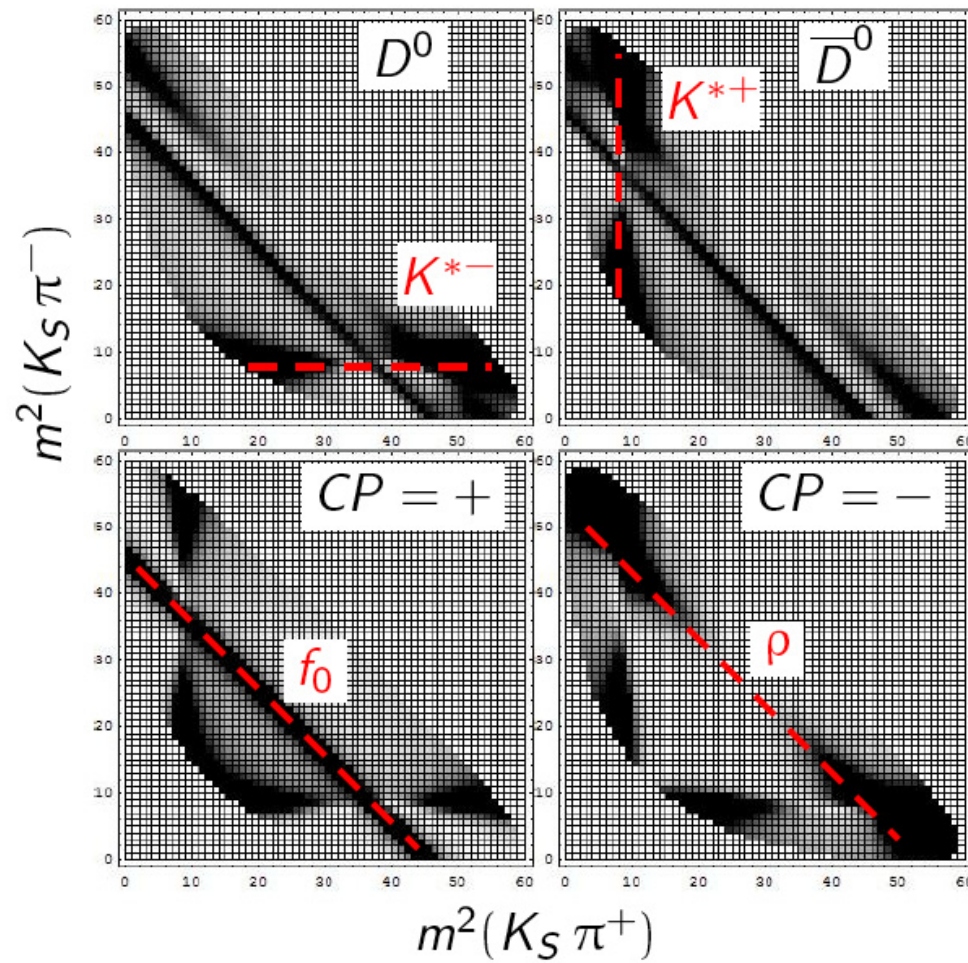
$$\gamma = (92 \pm 41(stat) \pm 11(syst) \pm 12(model))$$

$\pi\pi$ and $K\pi$ S-wave the most critical

CP-tagged Dalitz $D^0 \rightarrow K_S \pi^+ \pi^-$ plots

The $\psi(3770)$ has positive CP

Daughter D^0 mesons have opposite CP (P-wave decay)



Dalitz plots for toy MC with $K^* \pi$, $K_S \rho$, $K_S f_0$ components

Clear difference in flavor-tagged (top) and CP-tagged (bottom) plots

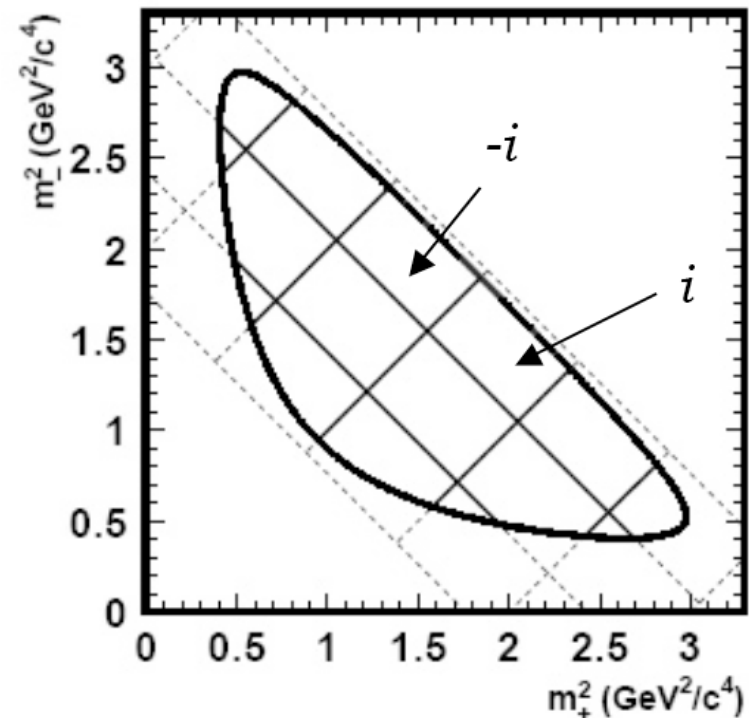
Model independence

Measure c_i to reduce model systematic on γ

$$c_i = \cos \delta_i = \frac{1}{2} \frac{(\bar{M}_i^- - \bar{M}_i^+) (K_i + K_{-i})}{(\bar{M}_i^- + \bar{M}_i^+) \sqrt{K_i K_{-i}}}$$

K_i - flavor tag yield in bin i
 M_{i^\pm} - CP_\pm yield in bin i

CLEO-c will determine c_i by counting yields in bin i for flavor and CP tags



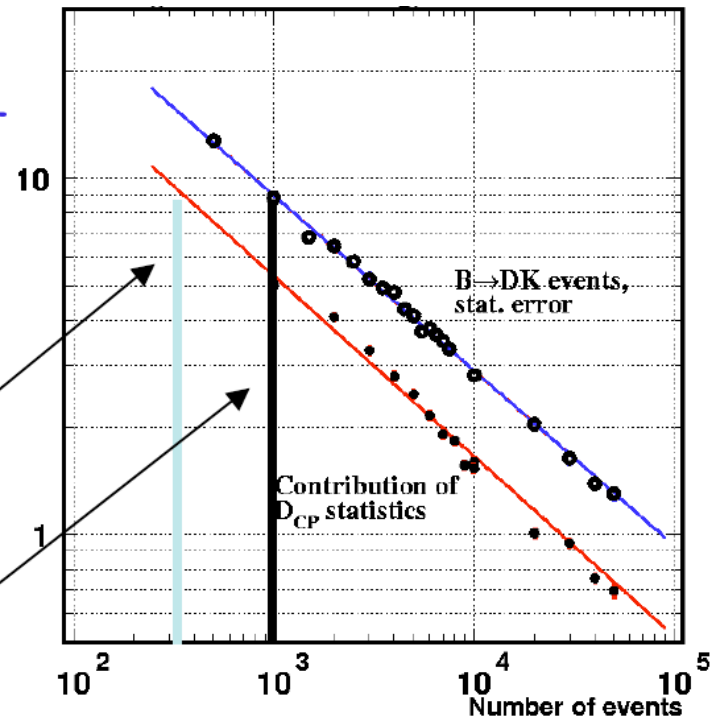
Impact of CLEO-c

D.Asner talk at Flavour at LHC WS

# of events (CLEO-c)	$K_S\pi^+\pi^-$		$K_L\pi^+\pi^-$	
	281 pb ⁻¹	750 pb ⁻¹	281 pb ⁻¹	750 pb ⁻¹
KK	66	175	134	350
$\pi\pi$	27	75	62	150
$K_S\pi^0$	95	250	103	275
$K_L\pi^0$	93	250	-	-
$K_S\pi^+\pi^-$	180	500	457	1200

Sys. Error on γ
Stat. Error on γ

γ error vs. D_{CP} statistics



570 CP tags in 281 pb⁻¹ → 6-7° sys. err.

~1500 CP tags expected in 750 pb⁻¹ → ~4° sys. err.

~1700 "double" Dalitz tags expected in 750 pb⁻¹

of $B^+ \rightarrow DK^+$, $D \rightarrow K_S\pi^+\pi^-$
of CP tagged $D \rightarrow K_S\pi^+\pi^-$

[1] Bondar, Poluektov [hep-ph/0510246]

Conclusion & Outlook

- Charm multi-body decays crucial to determine strong interaction bound states
 - Nature of light mesons resonances still **uncertain**
- More information from comparison of several channels
 - **Multi-channel analyses can be the way to go.**
- Interplay with B physics
 - Determine strong phase variation useful to extract **CKM angle γ**
 - Quantum-coherent states crucial to reduce model error on phase variation
(DD sample at threshold - CP-tagged Dalitz plot)

Current statistical WA error on γ (1ab^{-1} stat.) : $\pm 19^\circ$

SuperB-factory $\rightarrow 25\text{ab}^{-1}$: **stat.** error $\pm 4^\circ$

BES-III $\rightarrow 20\text{fb}^{-1}$: **syst.** error $\pm 1^\circ$

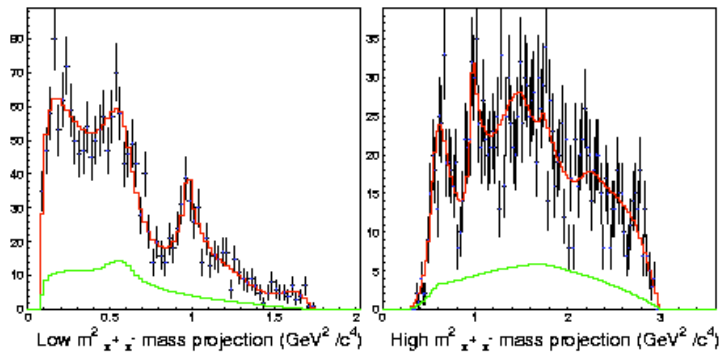
A super-flavour factory the way to go !!!

Back-up

Different parametrizations

FOCUS: K-matrix approach

- No need for a σ , employed a $(\pi\pi)$ S-wave to describe data

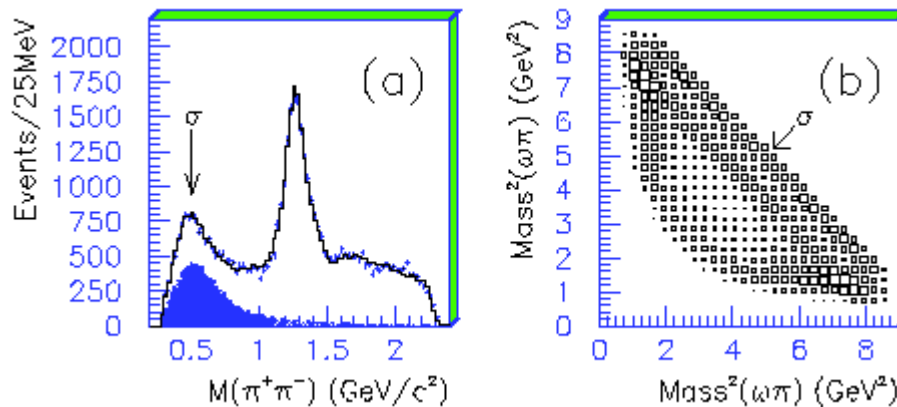


Although not entirely clear this is correct:

“... the K-matrix approach employed in Ref [5] does not meet the chiral requirements of a soft expansion for low energies...”

[J. Oller, PRD71 054030]

BESII- $J/\psi \rightarrow \omega\pi^+\pi^-$

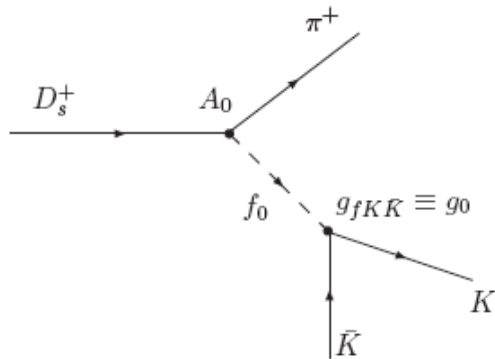


Described via a complex pole

Understand S-wave in (relatively) clear environment
Extend parametrization to more involved decays

Flatte's formula

$$A(D_s \rightarrow \pi^+ K \bar{K}) = \langle K \bar{K}; \text{out} | \left(\sum_{I=0}^1 |S_I\rangle BW_I(s) \langle S_I| \right) |A\rangle$$



$$g_I = \langle K \bar{K}; \text{out} | S_I \rangle$$

$$A_I = \langle S_I | A \rangle$$

$$BW_f = \frac{1}{s - M_f^2 + iM\Gamma_f(s)}$$

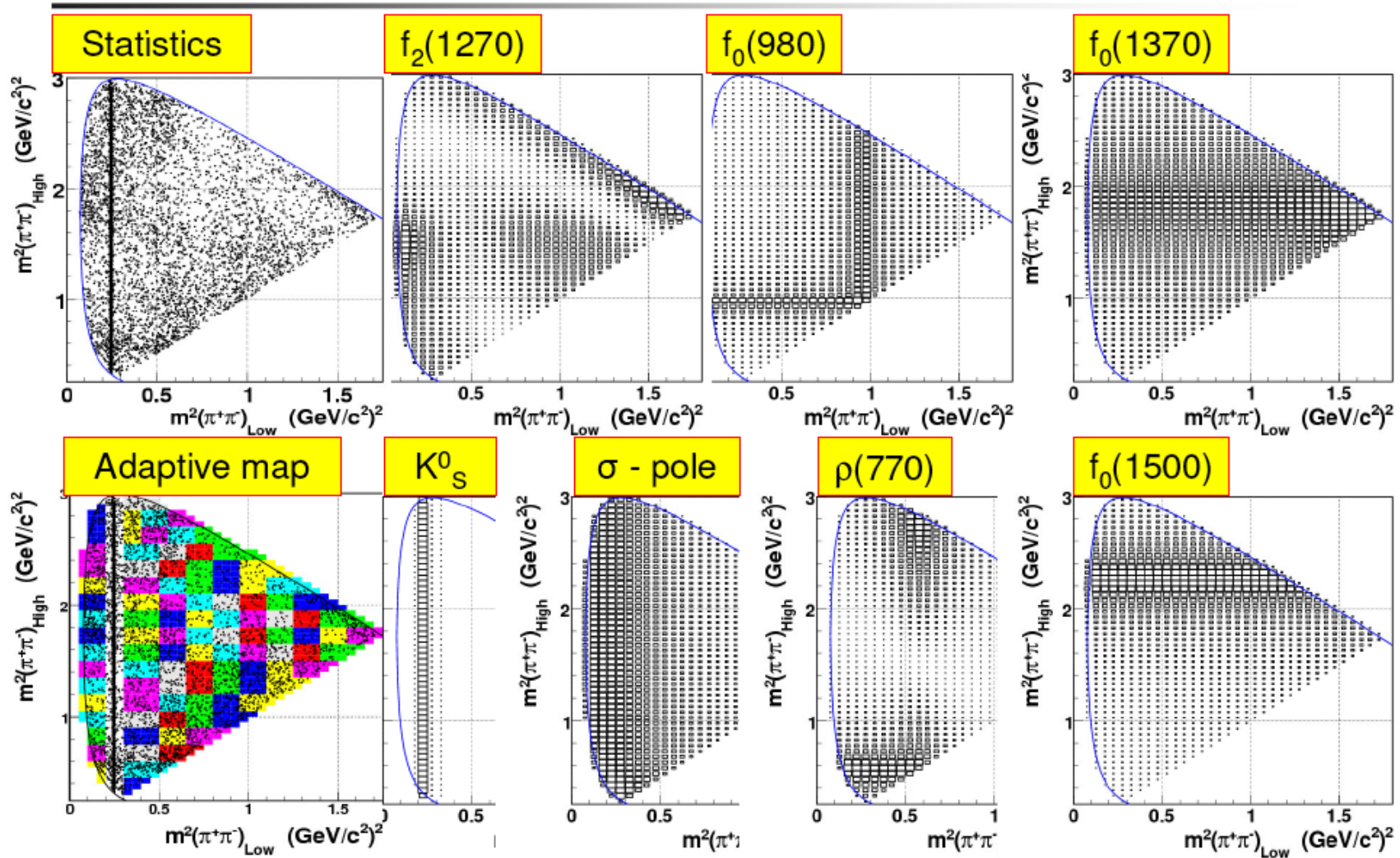
$$\Gamma_{f_0} = \frac{3}{2} g_\pi^2 \frac{2 p_\pi(s)}{\sqrt{s}} + g_0^2 \frac{2}{\sqrt{s}} (p_{\text{ch}} + p_{\text{neu}})$$

$$\Gamma_{a_0^0} = \Gamma_1 + g_1^2 \frac{2}{\sqrt{s}} (p_{\text{ch}} + p_{\text{neu}})$$

$$\Gamma_{a_0^\pm} = \Gamma_1 + 2g_1^2 \frac{2 p_{01}(s)}{\sqrt{s}}$$

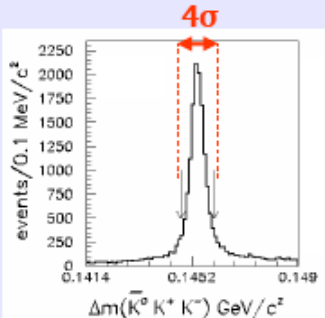
M_a (MeV) [9]	$(g_1)_{CB}$ (MeV) [9]	$(g_2^2/g_1^2)_{CB}$ [9]	$(g_2)_{CB}$ (MeV) [2]
999 ± 2	324 ± 15	1.03 ± 0.14	$473 \pm 29 \pm 40$

Contributing resonances



D⁰ selection and flavour tagging

D* tagging procedure is two-fold useful:



Event selection uses:

$$\Delta m(\bar{K}^0 K^+ K^-) = m(\bar{K}^0 K^+ K^- \pi_s^+) - m(\bar{K}^0 K^+ K^-)$$

1. Selection of high purity sample
2. D⁰ and \bar{K}^0 flavour set by the charge of the slow pion π_s^+

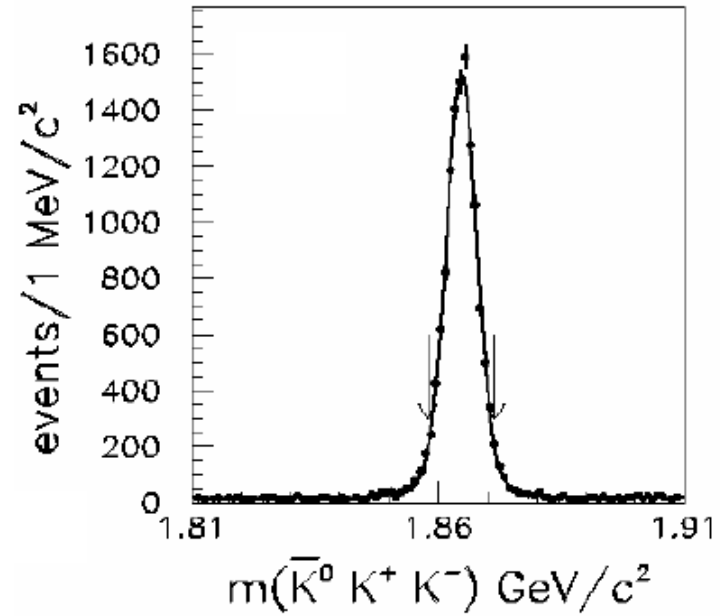
$$e^+ e^- \rightarrow D^{*+} + X$$

$$\quad \quad \quad \searrow \quad \quad \quad D^0 \pi_s^+$$

$$\quad \quad \quad \quad \quad \searrow \quad \quad \quad \bar{K}^0 K^+ K^-$$

$$\bar{K}^0 \rightarrow K_s^0 \rightarrow \pi^+ \pi^-$$

Presence of Doubly Cabibbo Suppressed contribution may lead to misidentification of \bar{K}^0 flavour.



Sample = 12540 Events
Purity = 97.3% @ 91.5 fb⁻¹

$D^0 \rightarrow K^0 K^+ K^-$: fit to PW

- $K^+ K^-$ system: $\Phi(1020)$ and $a_0(980)^0$**

[No evidence of $f_0(980)$ ($I=0$) comparing $\bar{K}^0 K^+$ and $K^+ K^-$ (normalized) S-waves]

$$\underbrace{c_{a_0} BW_{a_0}}_{S\text{-wave}} + \underbrace{c_\phi BW_\phi e^{i\alpha}}_{P\text{-wave}}$$

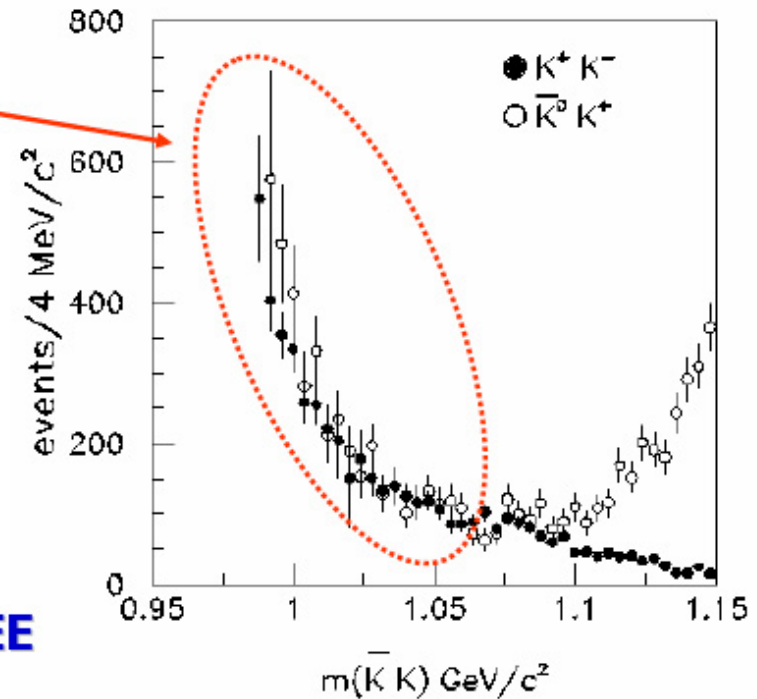
- $\bar{K}^0 K^+$ system: $a_0(980)^+$**

$$BW_\phi = \frac{1}{m_\phi^2 - m_{K^+ K^-}^2 - im_\phi \Gamma_\phi}$$

$$BW_{a_0} = \frac{1}{\underbrace{m_{a_0}^2}_{\text{red}} - m_{KK}^2 - i n_{a_0} (\underbrace{\rho_{\eta\pi} g_{\eta\pi}^2}_{\text{red}} + \underbrace{\rho_{KK} g_{KK}^2}_{\text{blue}})}$$

FREE

Fixed to Crystal Barrel measurement
A. Abele et al., Phys. Rev. **D57**, 3860(1998)





$I=1/2$ $K\pi$ S-wave

$K\pi$ S-wave in mass range $0.6\text{--}1.4$ GeV/c^2 is not well-understood. A possible κ state ~ 800 MeV/c^2 has been conjectured, but this has only been reported in the neutral state.

- For the $K^+\pi^0$ and $K^-\pi^0$ S-wave amplitudes, we try three models:

- Amplitude obtained from LASS $K^-\pi^+\rightarrow K^-\pi^+$ scattering.

Nucl. Phys. B296, 493 (1988); W. Dunwoodie, web notes.

- $K^-\pi^+$ amplitude extracted from a model-independent partial-wave analysis of $D^+\rightarrow K^-\pi^+\pi^+$ decay by the E791 collaboration. Phys. Rev. D73, 032004 (2006)

- [coherent sum of $\kappa(800)$ + uniform NR + $K^*_0(1430)$]. { No evidence in $K\pi$ elastic scattering. }

Normalized to arbitrary scale for $m(K\pi)>1.1$ GeV/c^2 for easy comparison.

