Theoretical Tool II
factorization approaches

Hsiang-nan Li
Academia Sinica, Taiwan

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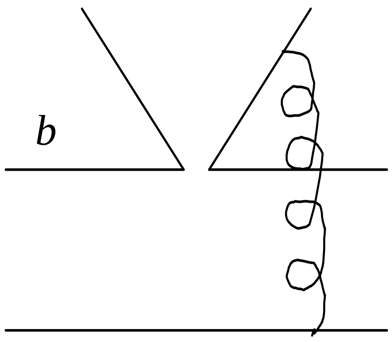
Outlines

- Introduction (Iain's talk)
- Collinear vs k_T factorization
- B decays in QCDF, $SCET_O$, PQCD
- Recent results
- Comments

Factorization assumption vs factorization theorem

• FA	FT
• of process	of dynamics
• $A(B\to M_1M_2) \propto f_{M2}F^{BM1}$	$\phi_B\otimes\phi_{M1}\otimes\phi_{M2}\otimes H$
• Vac$\to M_2$, $B\to M_1$	nonpert pert

- Factorizable: in the above form
- Nonfactorizable: not in the above form
- This nonfactorizable amplitude is factorizable.



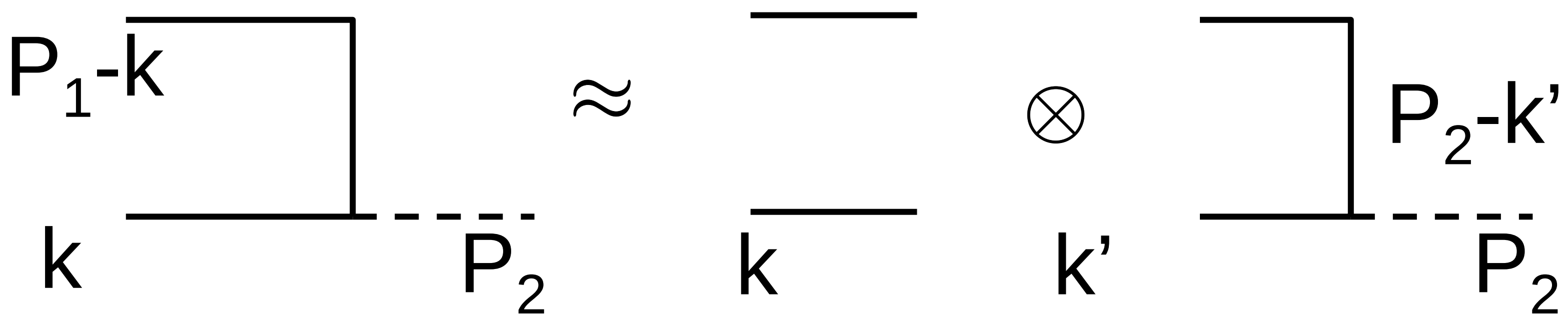
Collinear vs k_T factorization

Collinear: QCDF, SCET

K_T: PQCD

Simplest example

- $\pi^0(P_1)\gamma^* \to \gamma(P_2)$
- At leading order (LO)



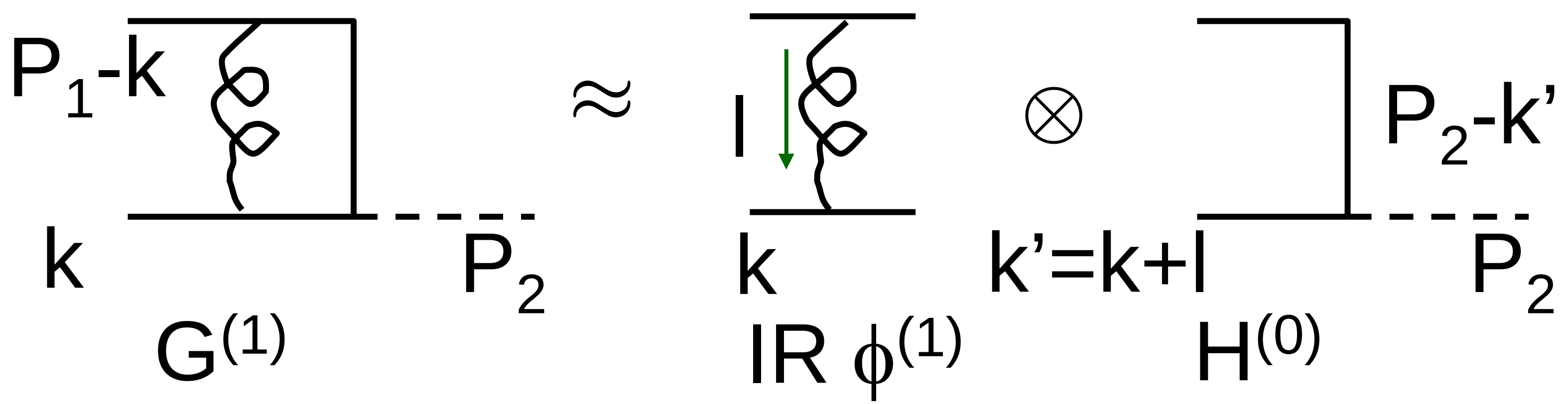
Full QCD $G^{(0)}$ | Effective theory Wave function $\phi^{(0)}$ | Wilson coeff Hard kernel $H^{(0)}$

- Initial parton $k=(xP_1^+,0,0_T)$, $Q^2=2P_1\cdot P_2$

LO factorization

- Collinear factorization, $k'=(x'P_1^+,0,0_T)$
- $G^{(0)}(x,Q^2)=\int dx'\phi^{(0)}(x;x')H^{(0)}(x',Q^2)$
- No gluon exchange, $\phi^{(0)}=\delta(x-x')$,
- $H^{(0)}\propto 1/(P_2-k')^2 \propto 1/(x'Q^2)$
- k_T factorization, $k'=(x'P_1^+,0,k'_T)$
- $G^{(0)}(x,Q^2)=\int dx'dk'_T\phi^{(0)}(x;x',k'_T)H^{(0)}(x',k'_T,Q^2)$
- No gluon exchange, $\phi^{(0)}=\delta(x-x')\delta(k'_T)$,
- $H^{(0)} \propto 1/(x'Q^2+k'_T{}^2) \rightarrow 1/(x'Q^2)$
- $H^{(0)}$ does not acquire k_T dependence

NLO collinear factorization



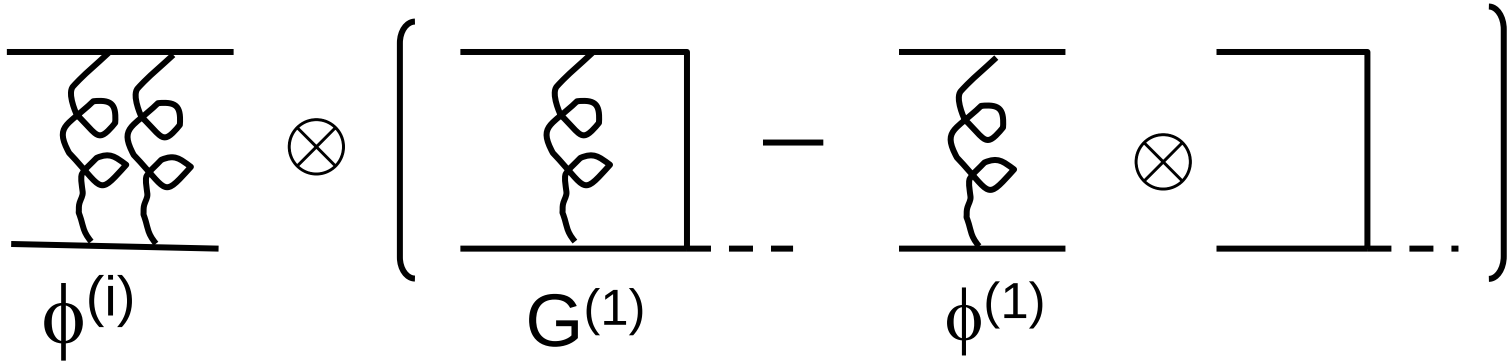
- Collinear factorization: $k'=(k^+ + l^+, 0, 0_T)$ IR finite
- $G^{(1)}(x,Q^2)=\int dx'\phi^{(1)}(x;x')H^{(0)}(x',Q^2)+H^{(1)}(x,Q^2)$
- One gluon exchange, $\phi^{(1)} \propto \delta(x-x'+l^+/P_1^+)$,
- Collinear gluon exchange modifies longitudinal parton momentum in $H^{(0)}$ (transverse momentum set to zero).

NLO k_T factorization

- k_T factorization: $k' = (k^+ + l^+, 0, l_T)$
- $G^{(1)}(x,Q^2) = \int dx' dk'_T \phi^{(1)}(x;x',k'_T) H^{(0)}(x',k'_T,Q^2)$
- $+H^{(1)}(x,Q^2)$
- One gluon exchange, $\phi^{(0)} \propto \delta(x - x' + l^+/P_1^+)\delta(k'_T - l_T)$,
- Nontrivial k_T dependence in $H^{(0)} \propto 1/(x'Q^2 + k'^2_T)$ at this order
- Collinear gluon exchange modifies both parton longitudinal and transverse momenta in $H^{(0)}$

$H^{(1)}$ in k_T factorization

- At NLO, partons in $H^{(1)}$ are on shell
- Beyond NLO, $H^{(1)}$ acquires nontrivial k_T dependence (Nandi, Li 07)



Initial parton $k=(xP_1^+,0,k_T)$ in $G^{(1)}$ and $\phi^{(1)}$

- $H^{(1)}(x,k_T,Q^2)=G^{(1)}(x,k_T,Q^2)$
  $-\int dx'dk'_T\phi^{(1)}(x,k_T;x',k'_T)H^{(0)}(x',k'_T,Q^2)$
- IR divergences cancel between $G^{(1)}$ and $\phi^{(1)}$

Gauge invariance

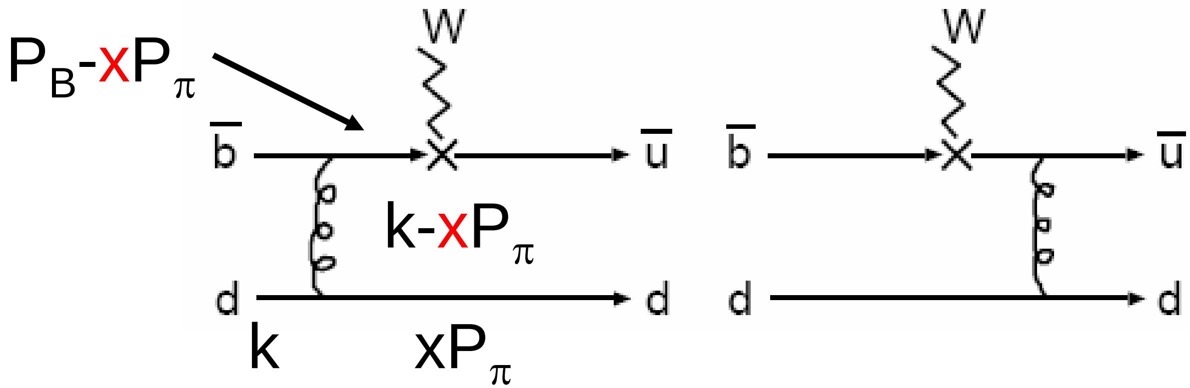
- Partons are off-shell by k_T^2. Both quark diagrams (full QCD) and effective diagrams (wave function) depend on gauge.
- Gauge dependences in $G^{(1)}$ and $\phi^{(1)}$ cancel, and $H^{(1)}$ is gauge-invariant.
- Hard kernel is infrared-safe, gauge-invariant, so are predictions from k_T factorization.

B decays in QCDF, $SCET_O$, PQCD

See 0705.1624 by Silvestrini

End-point singularity

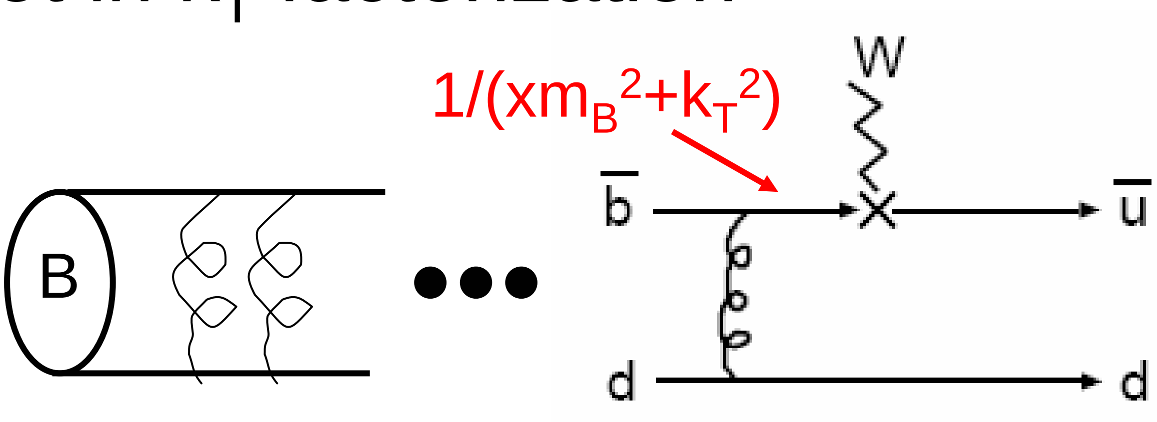
- End-point singularity in collinear factorization for $B \to \pi$ form factor



$$\int_0^1 dx \frac{\phi_\pi(x)}{x^2} \to \infty,$$

$$\phi_\pi(x) \propto x(1-x)$$

- But not in k_T factorization



Collinear gluons generate k_T

$B\to\pi$ form factor

- In QCDF and SCET (collinear factorization)

  Form factor $F^{B\pi}=\zeta^{B\pi}$ (with singularity) $+\zeta_J^{B\pi}$

  $\zeta^{B\pi}$ nonfactorizable, $\zeta_J^{B\pi}$ factorizable

  different orders in α_s: α_s^0 and α_s

- In PQCD (k_T factorization) and $SCET_O$ (Manohar, Stewart 06)

  Both $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ are factorizable

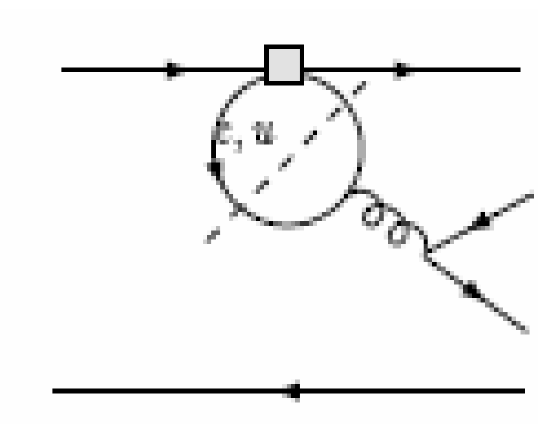
  same order in α_s

- Regularization of singularity, log of cutoff at every order, $\ln^2(m_b/k_T)$ in PQCD and $\ln\mu_\pm$ in $SCET_{O,}$, need to be resummed.

Annihilation in QCDF, $SCET_O$, PQCD

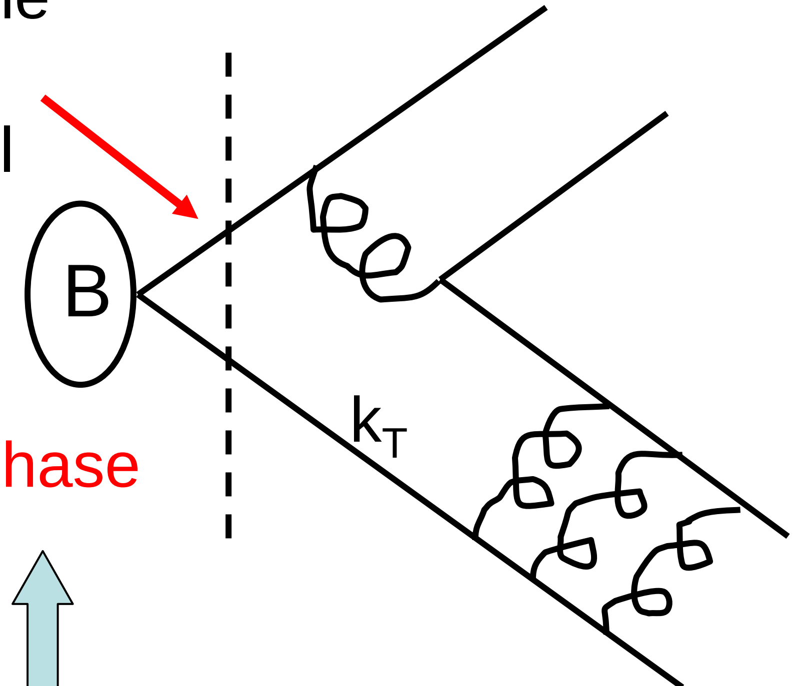
- Applied to charmless nonleptonic decays, scalar penguin annihilation is treated as:
- Parameter in QCDF due to end-point singularity (nonfactorizable), $X_A = \ln(m_B/\Lambda)[1+\rho_A \exp(i\phi_A)]$
- What mechanism generates ϕ_A?
- Factorizable in $SCET_O$, but real (ALRS 06). Strong phase appears at $\alpha_s^2\Lambda/m_b$.
- Phase generated by nonfactorizable contribution at $\alpha_s m_0 \Lambda/m_b^2$ (Chay, Li, Mishima in preparation)?

Imaginary in PQCD



Loop line
can go
on-shell

Strong phase



Bander-Silverman-Soni Mechanism for strong phase

$$\frac{1}{xm_B^2 - k_T^2 + i\epsilon} = \frac{P}{xm_B^2 - k_T^2} - i\pi\delta(xm_B^2 - k_T^2).$$

Imaginary annihilation! (Li, Mishima 06)

- Whether scalar penguin annihilation carries strong phase can be tested by comparing

	$A_{CP}(K^+\pi^0)$	$A_{CP}(K^+\rho^0)$
Transition	$B \to P$	$B \to V$
Emission penguin	a_4+a_6	a_4-a_6
Annihilation is	less,	more important

- Real annihilation, $A_{CP}(K^+\pi^0) \approx A_{CP}(K^+\rho^0)$
- Imaginary annihilation, small large phase

$$A_{CP}(K^+\pi^0) << A_{CP}(K^+\rho^0)$$

- Data (HFAG) 0.047 ± 0.026 $0.31^{+0.11}_{-0.10}$

Recent results

ΔS puzzle, Zupan's talk

$B(\pi^0\pi^0)$ puzzle?

- Large $B(\pi^0\pi^0) \approx 10^{-6}$ not understood.
- Complete $O(\alpha_s^2)$ T^{II} (spectator scattering) in QCDF/SCET enhances C (Beneke, Yang 05; Beneke, Jager 05; 06).
- $BR(\pi^0\pi^0)$ can not be enhanced too much due to bound from data $BR(\rho^0\rho^0) \approx 1.07\times 10^{-6}$ (Li, Mishima 06).
- PQCD matches data $B(\rho^0\rho^0)$, but gives $B(\pi^0\pi^0)=(0.3^{+0.5}_{-0.2})\times 10^{-6}$

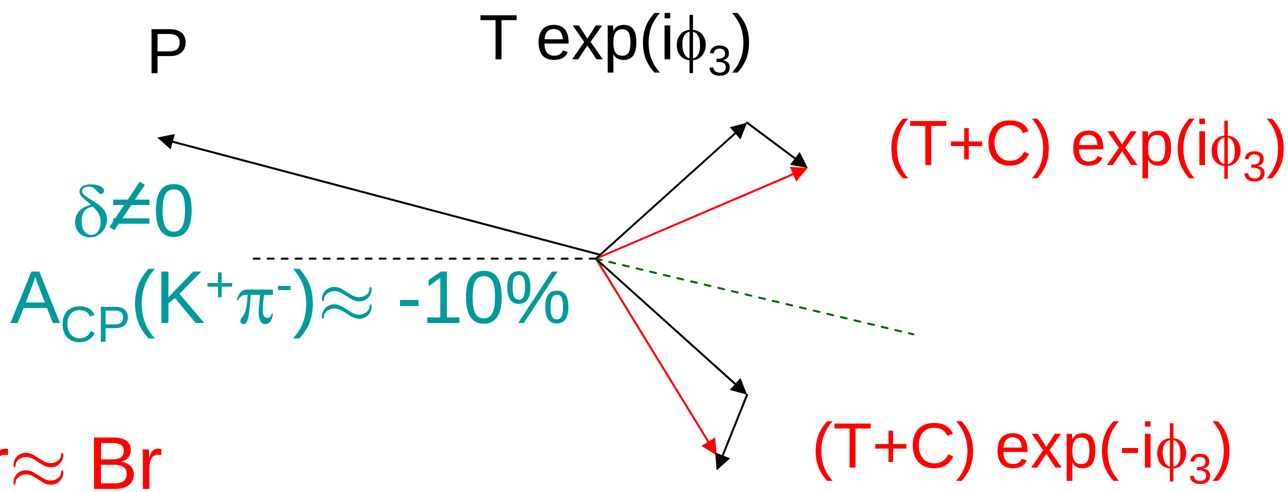
$f_L(\phi K^*)$ puzzle? (Gritsan's talk)

- No "satisfactory" SM explanation, but…
- With annihilation (chirally enhanced, Chen, Keum, Li 02; Kagan 04) and small form factor $A_0^{BK^*}$ (Li 04), it is possible to accommodate data.
- $f_L(K^{*+}\rho^0) \neq f_L(K^{*0}\rho^+)$ dramatically may be a puzzle (tree not dominant) (Lu et al 06).

Mode	PDG2004 Avg.	BABAR	Belle
$K^{*+}\rho^0$	$0.96^{+0.04}_{-0.15} \pm 0.04$	$0.96^{+0.04}_{-0.15} \pm 0.05$	
$K^{*0}\rho^+$	$0.43 \pm 0.11^{+0.05}_{-0.02}$	$0.52 \pm 0.10 \pm 0.04$	$0.43 \pm 0.11^{+0.05}_{-0.02}$
ϕK^{*+}	0.50 ± 0.07	$0.46 \pm 0.12 \pm 0.03$	$0.52 \pm 0.08 \pm 0.03$
$\rho^+\rho^0$	0.96 ± 0.06	$0.905 \pm 0.042^{+0.023}_{-0.027}$	$0.95 \pm 0.11 \pm 0.02$
$\omega\rho^+$	$0.88^{+0.12}_{-0.15} \pm 0.03$	$0.82 \pm 0.11 \pm 0.02$	

$A_{CP}(K\pi)$ puzzle? (Gronau's talk)

- Vertex correction in NLO PQCD gives large imaginary C, rotating T in $K^+\pi^0$, $C/T \approx 0.3\ \exp(-80^o)$(Li, Mishima, Sanda 05)
- Alleviated, but not gone away completely



$\overline{Br} \approx Br$

$A_{CP}(K^+\pi^0) \approx -8\% \rightarrow (-1^{+3}_{-5})\%$

LO NLO

Hadronic B_s decays

- Test SU(3) or U-spin symmetry
- Framework basically the same as of $B_{u,d}$ decays: QCDF (Beneke, Neubert 03), SCET (Williamson, Zupan 06), PQCD (Ali et al 07; Xiao et al)
- BR^{PQCD} similar to BR^{QCDF}, A_{CP}^{PQCD} opposite to A_{CP}^{QCDF} in sign
- PQCD predictions consistent with existing data, except:
- $B(B_s \to K^+\pi^-)$ twice of data, like $B(B_d \to \pi^+\pi^-)$
- $B(B_s \to \phi\phi) \approx 44\times10^{-6}$ too large.

Comments

- Factorization approaches are systematic tools, better not to be used for data fitting (S1,… S4, G in QCDF, charming penguin in SCET, BPRS 04)
- End-point singularities in emission and annihilation were not treated in a consistent way in QCDF.
- SCET_O is encouraging, counting rules consistent with PQCD. But need to deal with arbitrary ln $\mu_{\pm}$. ($\ln^2 k_T$ resummed in PQCD).
- NLO, $1/m_B$ corrections not yet fully studied, crucial for identifying puzzles as new physics signals.

Back-up slides

k_T expansion

- Parton momentum $k=(k_+,0,k_T)$
- Large x ~ O(1), k_T negligible

twist-3 contribution

$$\frac{\not{P}-\not{k}}{(P-k)^2} \not\approx \frac{\not{P}-\not{k}^+}{(P-k^+)^2} + \frac{\not{P}-\not{k}^+}{(P-k^+)^2}\not{k}_T\frac{\not{P}-\not{k}^+}{(P-k^+)^2}$$

- Small x, $xQ^2 \approx k_T^2$, different k_T expansion

$$\frac{P-k}{(P-k)^2} \approx \frac{\not{P}-\not{k}^+}{(P-k^+)^2-k_T^2} + \frac{\not{P}-\not{k}^+}{(P-k^+)^2-k_T^2}\not{k}_T\frac{\not{P}-\not{k}^+}{(P-k^+)^2-k_T^2}$$

- Setting k_T to zero in cases with end-point singularity will be a too strong approximation.