B_s mixing and Decays at the Tevatron

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(On behalf of the D∅ and CDF Collaborations)

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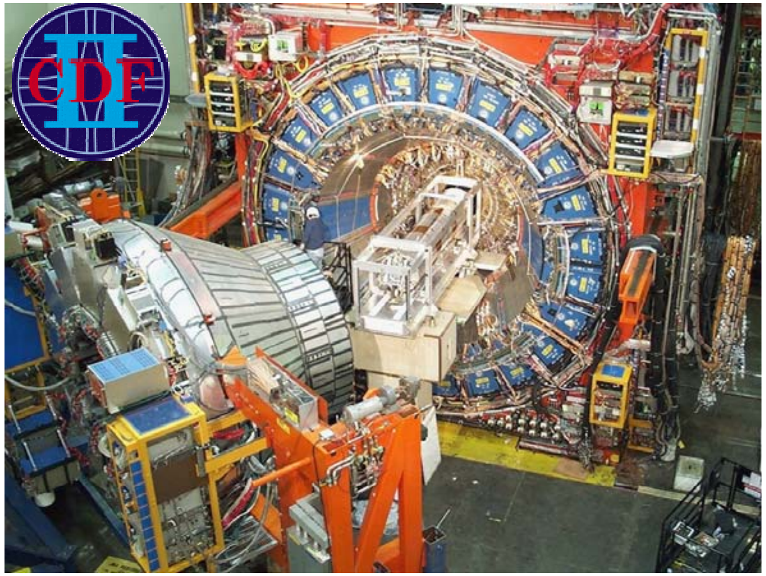
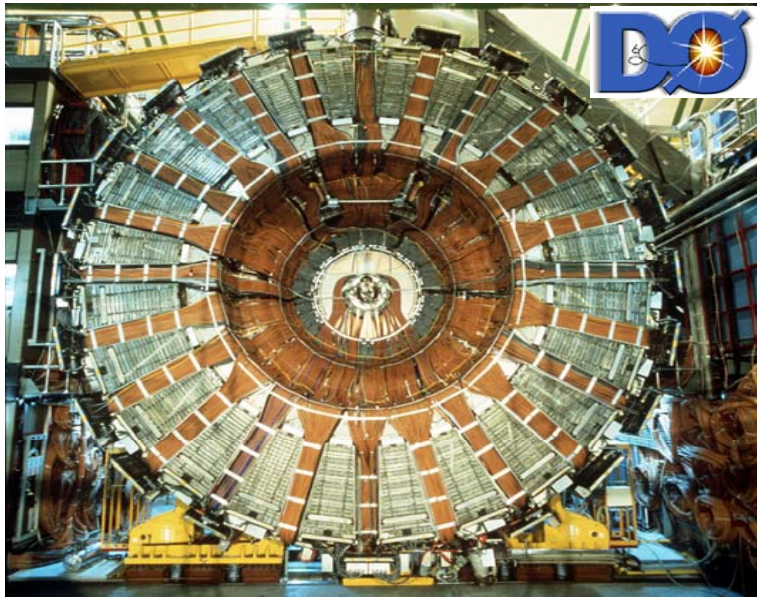
Outline

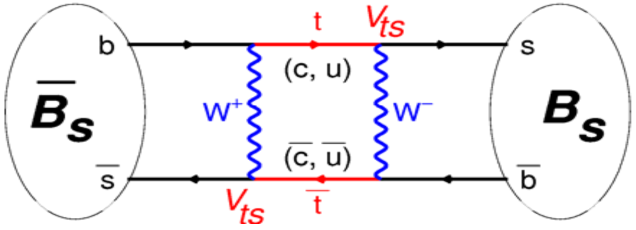
- B_s mixing
  - Ingredients and method
  - D∅ and CDF results
- *b*-Hadrons lifetimes
  - Motivation
  - B_s lifetimes (τ_{Bs} and $\Delta\Gamma_{Bs}$)
  - Λ_b lifetime
- Summary

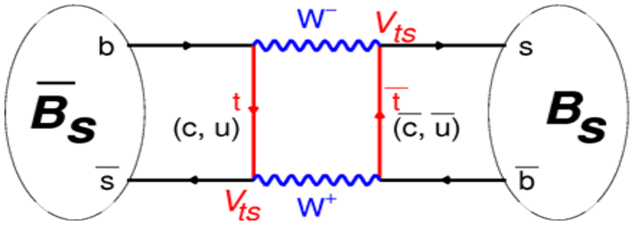




B_s mixing

B_s mixing





B^0 and $\bar{B}^0$ are quantum superposition of two mass eigenstates B_H and B_L:

$B_{L,H} = p|B^0\rangle \pm q|\bar{B}^0\rangle$ and $P(t)_{B^0\to\bar{B}^0} = \frac{e^{-\frac{t}{\tau}}}{2}\left[\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos(\Delta m t)\right]\left|\frac{q}{p}\right|^2$

$\Delta m = m_H - m_L$, $\Delta m_s = \frac{G_F^2}{6\pi^2}\eta_B m_{B_s} f_{B_s}^2 B_{B_s} m_W^2 S(m_t^2/m_W^2)|V_{ts}^\star V_{tb}|^2$

Hadronic uncertainties cancel in ratio:

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}}\xi^2\frac{|V_{ts}|^2}{|V_{td}|^2} , \quad \xi = \sqrt{\frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2}} = 1.21^{+0.047}_{-0.035}$$ (Okamoto, Lattice 05)

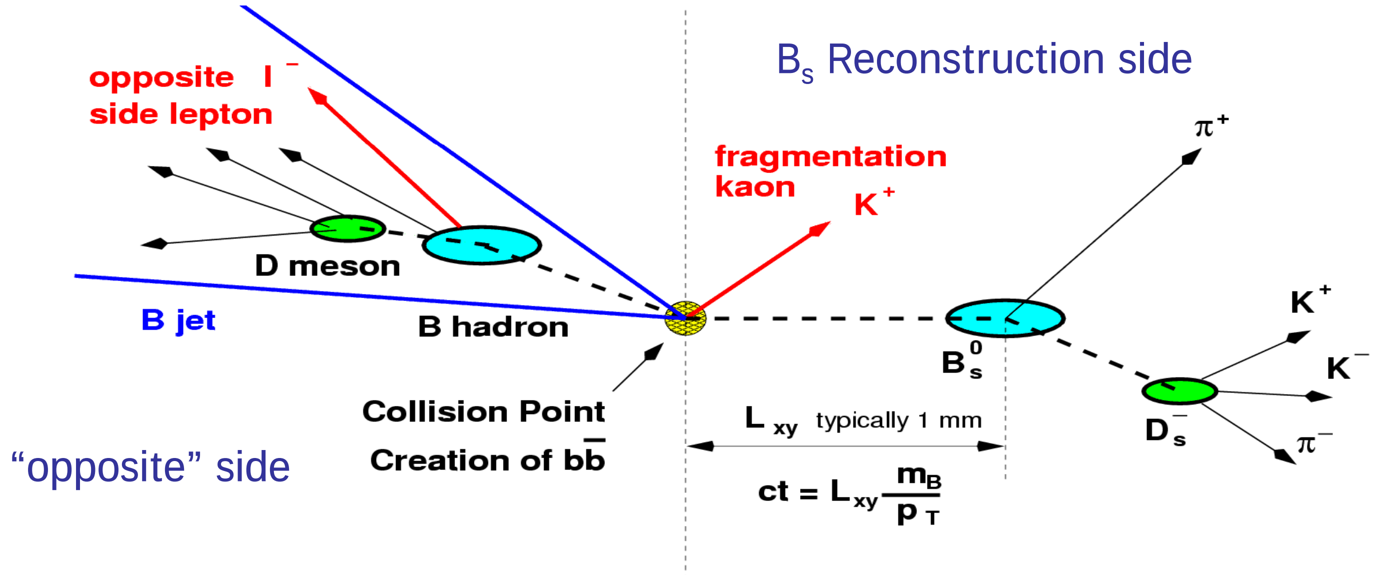
B_d oscillation very well measured (HFAG 2007): $\Delta m_d = 0.507 \pm 0.004\ \mathrm{ps}^{-1}$

From fits of unitarity triangle, assuming Standard Model and using all available

information (CKMFitter group): $\Delta m_s = 18.9^{+5.7}_{-2.8}\ \mathrm{ps}^{-1}$ (Eur. Phys. J. C41)

$$\Rightarrow \quad \Delta m_s > 30 \times \Delta m_d$$

Analysis ingredients



- Analysis ingredients:
  - Reconstruct B_s decays and determine B_s flavor at decay from decay products,
  - Mesure proper time of B_s decays,
  - Determine B_s flavor at production (opposite-side and/or same-side tagging),
  - Mesure Δm_s from an unbinned maximum likelihood fit of mixed and unmixed events:

$$P(t)_{B^0\to B^0,\bar{B}^0} \cong \frac{1}{2\tau}e^{-\frac{t}{\tau}}\left[1 \pm \mathcal{A}\,\mathcal{D}\cos(\Delta m_s t)\right],\quad \text{dilution } \mathcal{D} = 1-2\eta,\quad \eta = \text{mistag prob.}$$

Fit for $\mathcal{A}$ at different Δm_s. For true Δm_s, $\mathcal{A} = 1$, otherwise $\mathcal{A} = 0$.

- Statistical Significance of Δm_s measurement:

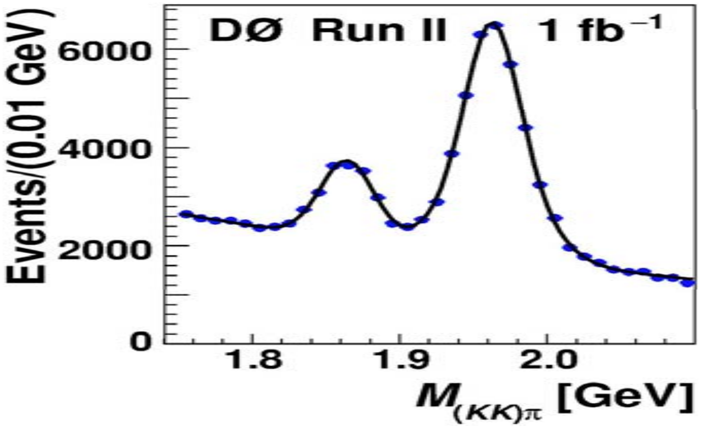
$$\text{Significance} \propto \sqrt{S\,\epsilon\mathcal{D}^2} \times \sqrt{\frac{S}{S+B}} \times e^{-\frac{1}{2}(\sigma_{ct}\Delta m_s)^2}$$

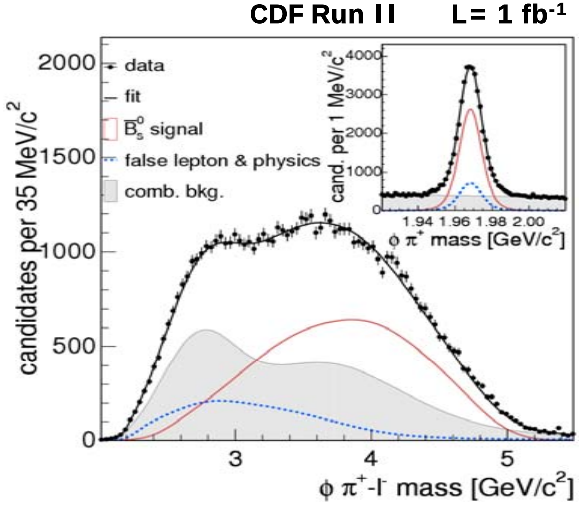
B_s signal reconstruction (1 fb^{-1})

Semileptonic decays: $B_s \to D_s^{+(\star)} \ell^- \bar{\nu}_\ell$

DØ: ~ 42K reconst. events (3 decay modes)

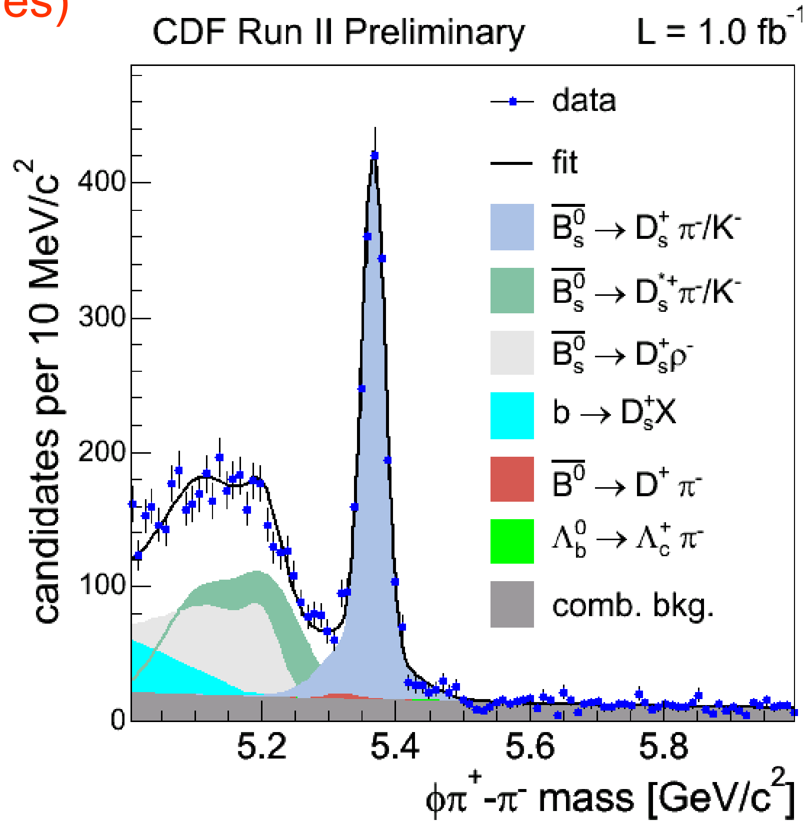
CDF: ~ 61.5K reconst. events (6 decay modes)





Hadronic decays (CDF)

$B_s \to D_s^+\pi^-, D_s^+\pi^-\pi^+\pi^-$



~ 5600 fully reconst. events (6 modes)

~ 3100 partially reconst. events (2 modes)

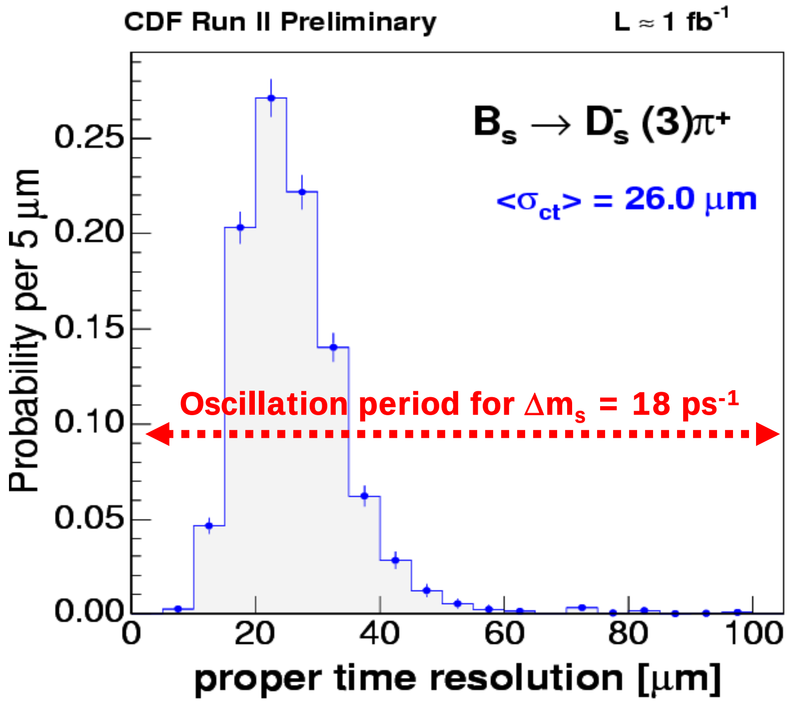
Proper decay time

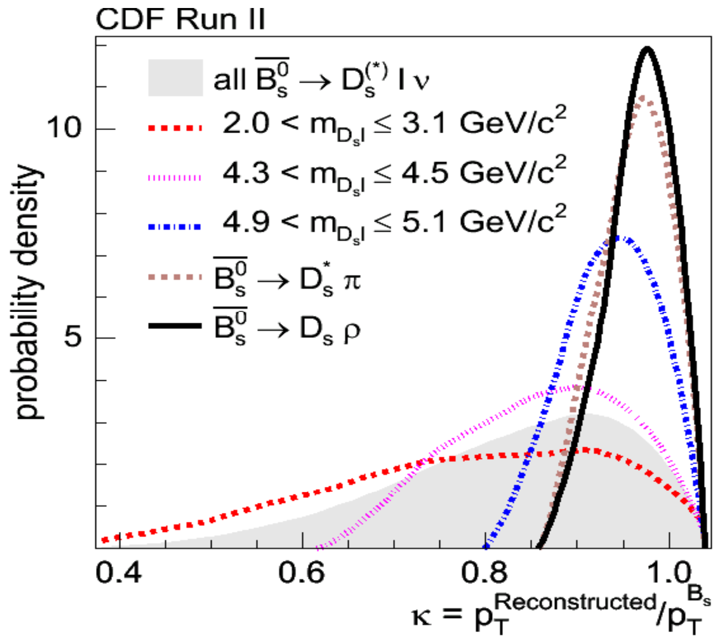
Fully reconstructed hadronic channels:

$$ct = \frac{L_{xy}}{\gamma\beta} = L_{xy}\frac{M(B_s)}{P_T(B_s)}$$

Semileptonic channels: $ct = L_{xy}\frac{M(B_s)}{P_T(D_s\ell)} \times K$

$$K = \frac{P_T(D_s\ell)}{P_T(B_s)}$$ from Monte Carlo





$$\sigma_{ct} = \sqrt{\left(\frac{\sigma_{L_{xy}}}{\gamma\beta}\right)^2 + \left(\frac{\sigma_{\gamma\beta}}{\gamma\beta} \times ct\right)^2}$$

Decay channel	$\langle\sigma_{ct}\rangle$ [μm]
$B_s \to D_s(3)\pi$ (CDF)	26
$B_s \to \ell D_s X$ (CDF)	45
$B_s \to \ell D_s X$ (D0)	50-60

b-flavor tagging @ production

Two methods: opposite-side flavor tagging and same-side flavor tagging:

1. Opposite-side flavor tagging (DØ+CDF):
   - Soft Lepton tagging
   - Jet Charge tagging
   - Secondary vertex charge (DØ)
   - Charge of identified kaon (CDF).

The performance of OST taggers measured in kinematically similar B^+ and B_d samples.

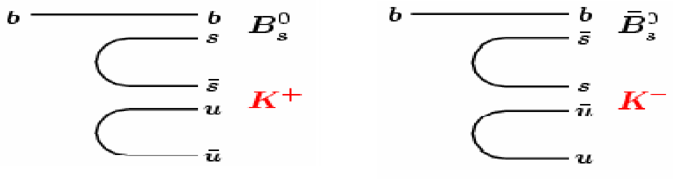
Performances of combined OST taggers

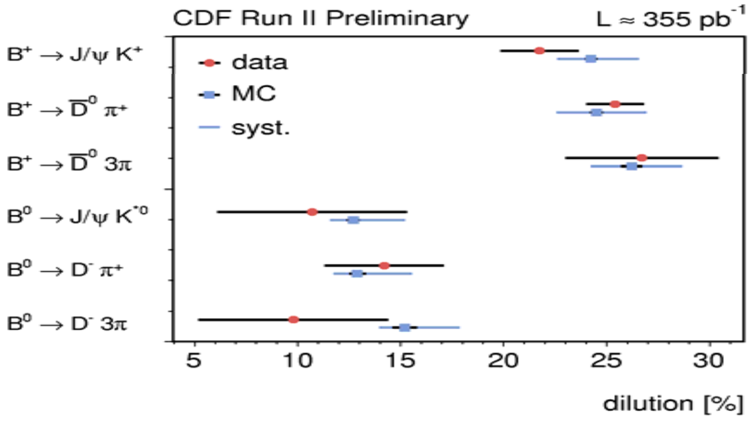
	$\langle \epsilon \mathcal{D}^2 \rangle$ (%)	Δm_d (ps^{-1})
DØ	2.5 ± 0.2	$0.506 \pm 0.020_{\text{stat}} \pm 0.016_{\text{syst}}$
CDF	1.8 ± 0.1	$0.509 \pm 0.010_{\text{stat}} \pm 0.016_{\text{syst}}$
HFAG 2007	-	0.507 ± 0.004

2. Same-side Kaon Tagging (CDF):
   - Use the closest fragmentation track correlated to B_s production flavor.
   - SSKT performance cannot be determined on data (rely on MC).

$$\epsilon \mathcal{D}^2 = 3.7\% \quad \text{(hadronic decays)}$$

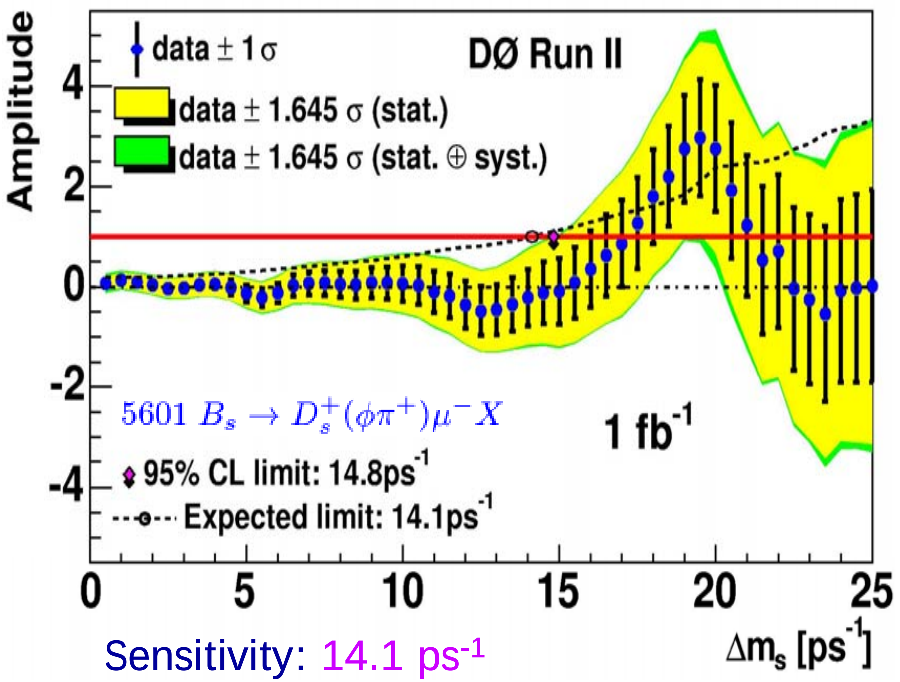
$$\epsilon \mathcal{D}^2 = 4.8\% \quad \text{(semileptonic decays)}$$





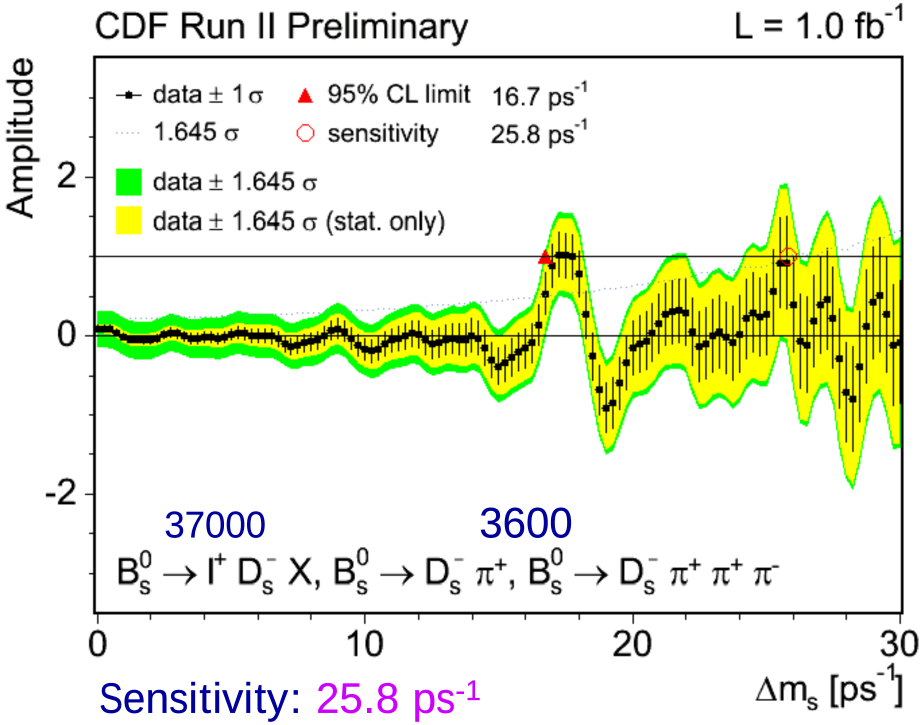
Δm_s measurements (spring 2006)

Phys. Rev Lett. 97 (2006) 021802



Sensitivity: 14.1 ps^{-1}

Phys. Rev Lett. 97 (2006) 062003



Sensitivity: 25.8 ps^{-1}

$$17 < \Delta m_s < 21\ \mathrm{ps}^{-1}\ @\ 90\%\ \mathrm{CL}$$

Most probable value of Δm_s = 19 ps^{-1}

Probability of false result = 5%

$$\Delta m_s = 17.31^{+0.33}_{-0.18}(\mathrm{stat}) \pm 0.07(\mathrm{sys})\ \mathrm{ps}^{-1}$$

3σ signal significance

Probability of random fluctuation = 0.2%

Improved CDF analysis

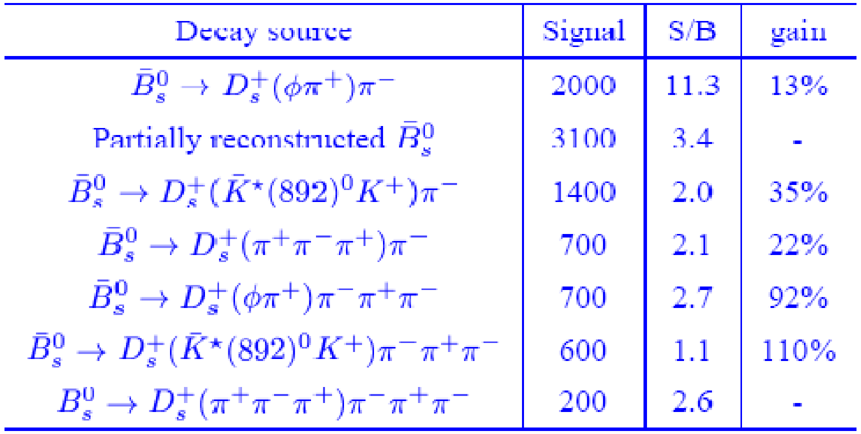
- Increase of B_s signal yield:
  - add partially reconstructed decays:

    $\bar{B}_s^0 \to D_s^{\star +}\pi^-,\ D_s^+ \to D_s^+\gamma/\pi^0$ and $\bar{B}_s^0 \to D_s^+\rho^-,\ \rho^- \to \pi^-\pi^0$ with $D_s^+ \to \phi\pi^+$
  - use particle ID in the selection (Kaon identification)
  - use Neural Net selection for hadronic modes
  - use loose kinematic selection

For $B_s \to D_s^{+(\star)}\ell^-\bar{\nu}_\ell$ gain=66% (37000 → 61500 signal events)

Including hadronic decays ⇒ Effective statistical size increased by a factor of 2.5 !

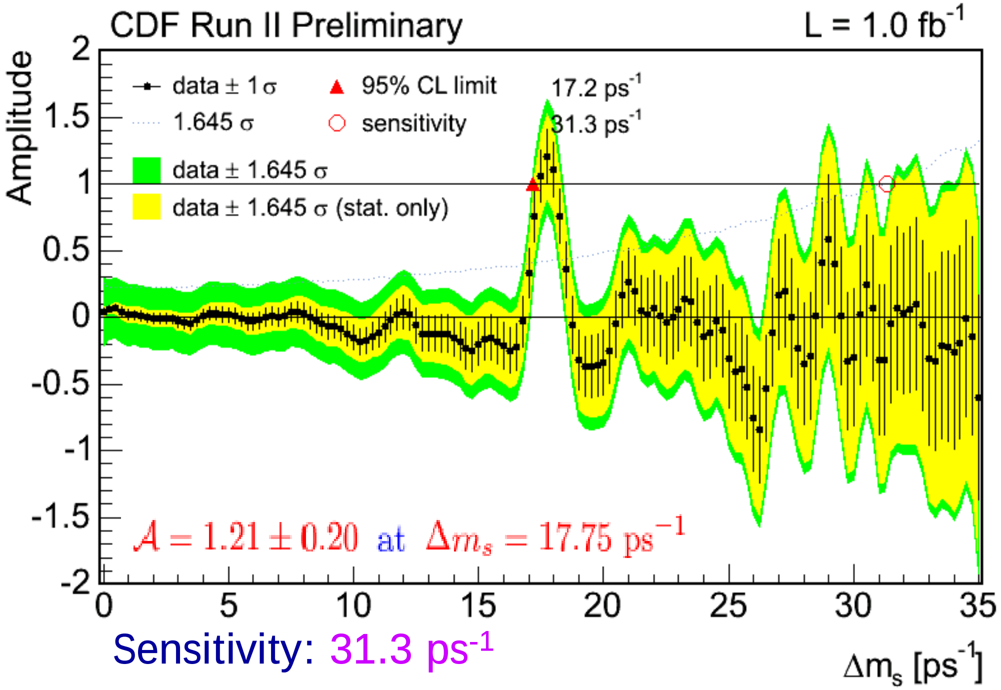
- b-flavor tagging:
  - add opposite side Kaon tag.
  - use NN to combine all opposite side taggers (gain=20%)
  - use NN for same side tagger (gain=10%)



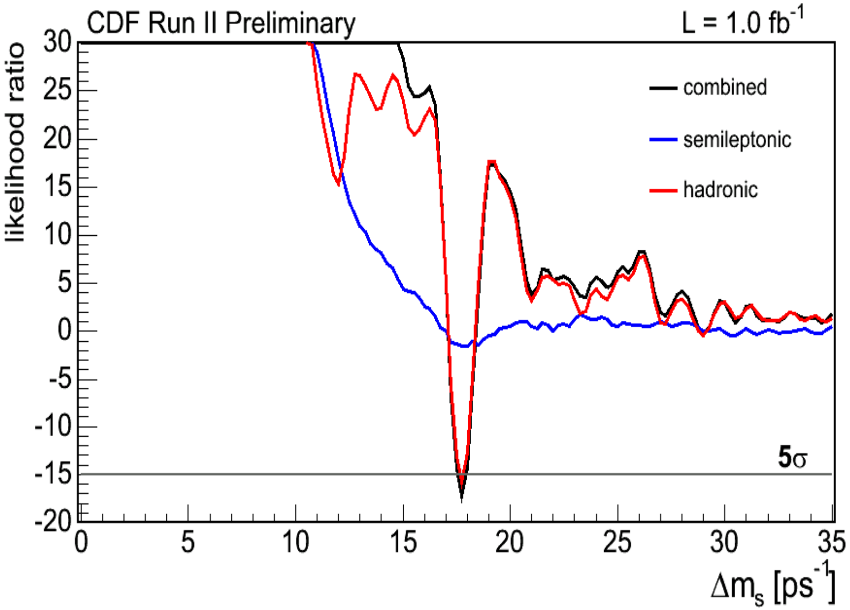
Decay source	Signal	S/B	gain
$\bar{B}_s^0 \to D_s^+(\phi\pi^+)\pi^-$	2000	11.3	13%
Partially reconstructed $\bar{B}_s^0$	3100	3.4	-
$\bar{B}_s^0 \to D_s^+(\bar{K}^\star(892)^0K^+)\pi^-$	1400	2.0	35%
$\bar{B}_s^0 \to D_s^+(\pi^+\pi^-\pi^+)\pi^-$	700	2.1	22%
$\bar{B}_s^0 \to D_s^+(\phi\pi^+)\pi^-\pi^+\pi^-$	700	2.7	92%
$\bar{B}_s^0 \to D_s^+(\bar{K}^\star(892)^0K^+)\pi^-\pi^+\pi^-$	600	1.1	110%
$B_s^0 \to D_s^+(\pi^+\pi^-\pi^+)\pi^-\pi^+\pi^-$	200	2.6	-

Improved CDF analysis

Phys. Rev Lett. 97 (2006) 242003



Sensitivity: 31.3 ps^{-1}



$$\mathcal{A}/\sigma_{\mathcal{A}}(\Delta m_s = 17.75\ \mathrm{ps}^{-1}) \sim 6.05$$

$$\boxed{\Delta m_s = 17.77 \pm 0.1(\mathrm{stat}) \pm 0.07(\mathrm{sys})\ \mathrm{ps}^{-1}}$$

$$\Rightarrow \frac{|V_{td}|}{|V_{ts}|} = \xi\sqrt{\frac{\Delta m_d}{\Delta m_s}\frac{m_{B_s}}{m_{B_d}}} = 0.2062 \pm 0.0011(\mathrm{exp})^{+0.0080}_{-0.0060}(\mathrm{theor})$$

The likelihood ratio minimum at:

$$\Lambda \equiv \log\left[\frac{\mathcal{L}^{\mathcal{A}=0}}{\mathcal{L}^{\mathcal{A}=1}(\Delta m_s)}\right] = -17.26$$

The probability that a randomly tagged data produces a value $\Lambda \leq -17.26$ at any Δm_s value is 8×10^{-8}.

⇒ signal significance >5σ

b-hadrons lifetimes

b-Hadrons lifetimes

- Important test of “non-spectator” effects in heavy hadrons decays:

⇒ “non-spectator” effects give rise to lifetime hierarchy among *b*-hadrons:

$$\tau(B^+) \geq \tau(B_d) \approx \tau(B_s) > \tau(\Lambda_b) \gg \tau(B_c)$$

- Contribution of light quark(s) in *b*-hadron decay width computed in the framework of the Heavy Quark Expansion (expansion in $1/m_b$):

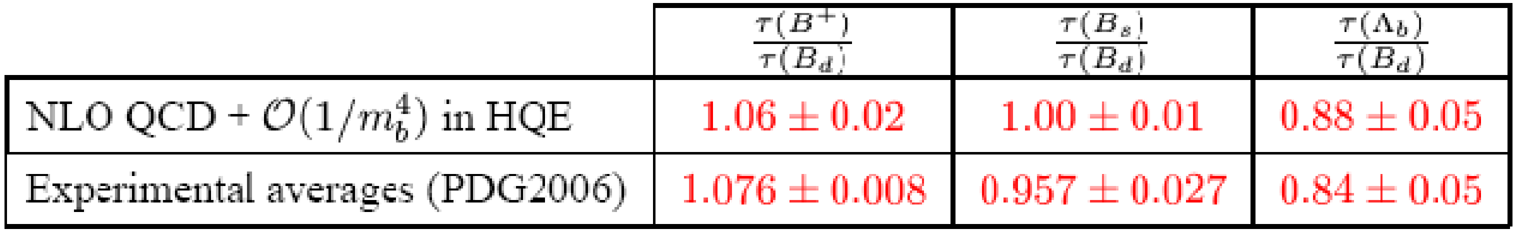
$$\Gamma(H_b \to f) = |\mathrm{CKM}|^2 \sum_n c_n^{(f)} \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^n \langle H_b | O_n | H_b \rangle$$

Non-perturbative corrections arrise only at $\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/m_b^2)$

Difference between meson and baryon lifetimes appears at $\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/m_b^2)$

Splitting of the meson lifetimes occurs at $\mathcal{O}(\Lambda^3_{\mathrm{QCD}}/m_b^3)$

Recent theoretical predictions and experimental averages for the lifetime ratios:

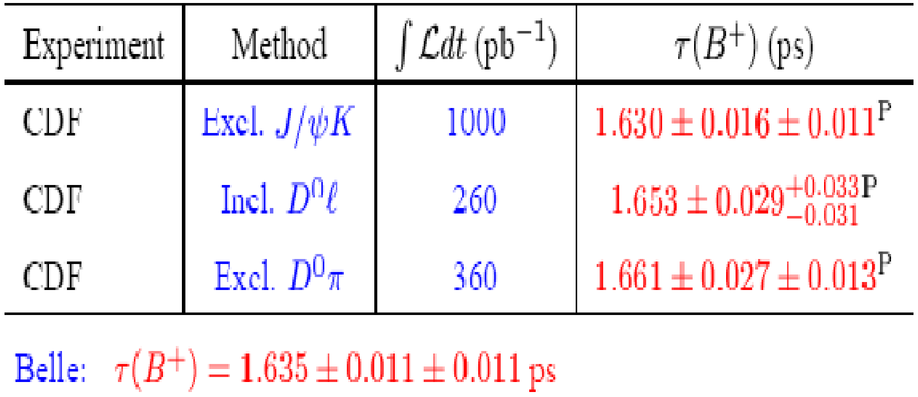


	$\frac{\tau(B^+)}{\tau(B_d)}$	$\frac{\tau(B_s)}{\tau(B_d)}$	$\frac{\tau(\Lambda_b)}{\tau(B_d)}$
NLO QCD + $\mathcal{O}(1/m_b^4)$ in HQE	1.06 ± 0.02	1.00 ± 0.01	0.88 ± 0.05
Experimental averages (PDG2006)	1.076 ± 0.008	0.957 ± 0.027	0.84 ± 0.05

C. Tarantino, Eur. Phys. J. C **33** (2004)

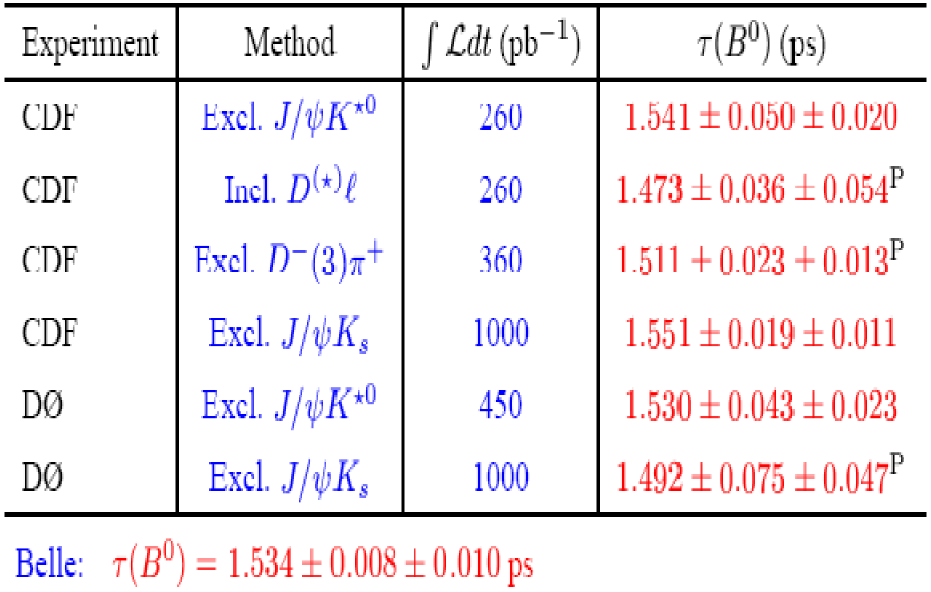
b-Hadrons lifetimes @ Tevatron RunII

Both CDF and D∅ have performed a number of *b*-Hadrons lifetimes measurements for all *b*-Hadron species. For $\tau(B^+)$, $\tau(B^0)$, $\tau(B^+)/\tau(B^0)$ and $\tau(B_c)$ the results are:



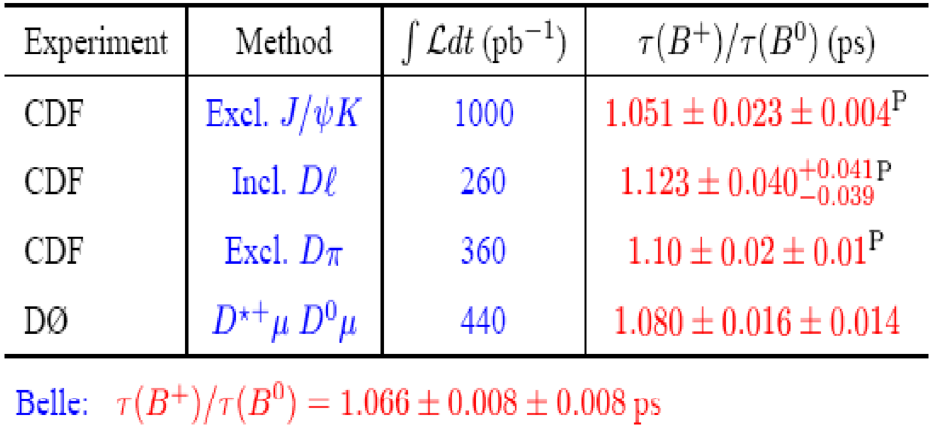
Experiment	Method	$\int \mathcal{L}dt$ (pb^{-1})	$\tau(B^+)$ (ps)
CDF	Excl. $J/\psi K$	1000	$1.630 \pm 0.016 \pm 0.011^{P}$
CDF	Incl. $D^0\ell$	260	$1.653 \pm 0.029^{+0.033}_{-0.031}{}^{P}$
CDF	Excl. $D^0\pi$	360	$1.661 \pm 0.027 \pm 0.013^{P}$

Belle: $\tau(B^+) = 1.635 \pm 0.011 \pm 0.011$ ps



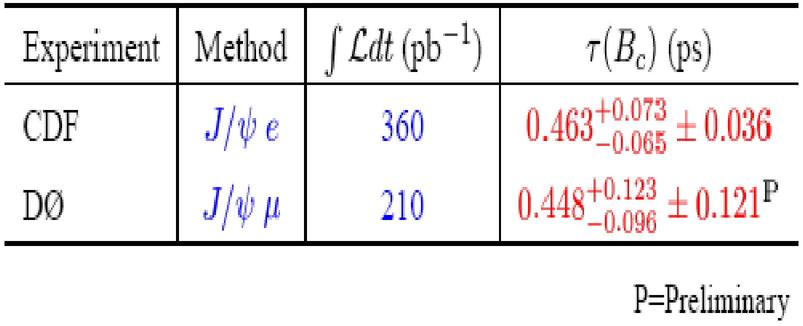
Experiment	Method	$\int \mathcal{L}dt$ (pb^{-1})	$\tau(B^0)$ (ps)
CDF	Excl. $J/\psi K^{*0}$	260	$1.541 \pm 0.050 \pm 0.020$
CDF	Incl. $D^{(*)}\ell$	260	$1.473 \pm 0.036 \pm 0.054^{P}$
CDF	Excl. $D^-(3)\pi^+$	360	$1.511 \pm 0.023 \pm 0.013^{P}$
CDF	Excl. $J/\psi K_s$	1000	$1.551 \pm 0.019 \pm 0.011$
DØ	Excl. $J/\psi K^{*0}$	450	$1.530 \pm 0.043 \pm 0.023$
DØ	Excl. $J/\psi K_s$	1000	$1.492 \pm 0.075 \pm 0.047^{P}$

Belle: $\tau(B^0) = 1.534 \pm 0.008 \pm 0.010$ ps



Experiment	Method	$\int \mathcal{L}dt$ (pb^{-1})	$\tau(B^+)/\tau(B^0)$ (ps)
CDF	Excl. $J/\psi K$	1000	$1.051 \pm 0.023 \pm 0.004^{P}$
CDF	Incl. $D\ell$	260	$1.123 \pm 0.040^{+0.041}_{-0.039}{}^{P}$
CDF	Excl. $D\pi$	360	$1.10 \pm 0.02 \pm 0.01^{P}$
DØ	$D^{*+}\mu$ $D^0\mu$	440	$1.080 \pm 0.016 \pm 0.014$

Belle: $\tau(B^+)/\tau(B^0) = 1.066 \pm 0.008 \pm 0.008$ ps



Experiment	Method	$\int \mathcal{L}dt$ (pb^{-1})	$\tau(B_c)$ (ps)
CDF	$J/\psi\, e$	360	$0.463^{+0.073}_{-0.065} \pm 0.036$
DØ	$J/\psi\, \mu$	210	$0.448^{+0.123}_{-0.096} \pm 0.121^{P}$

P=Preliminary

B_s lifetime measurements

B_s lifetime measurements

In the SM the light and heavy mass eigenstates of the mixed B_s system are expected to have a sizeable decay width difference:

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H = 0.096 \pm 0.039\ \mathrm{ps}^{-1}$$

If CP violation is neglected B_L and B_H are expected to be CP eigenstates:

$$B_\mathrm{L} = \mathrm{CP\ even}\ :\ \text{short lifetime component}\ \tau_\mathrm{L} = 1/\Gamma_\mathrm{L}$$

$$B_\mathrm{H} = \mathrm{CP\ odd}\ :\ \text{long lifetime component}\ \tau_\mathrm{H} = 1/\Gamma_\mathrm{H}$$

Various B_s decay channels have different proportions of B_L and B_H eigenstates:

- Flavor specific decays: $B_s^0 \to D_s^+ \ell^- \bar{\nu}_\ell$ and $B_s^0 \to D_s^+ \pi^-$ have equal fraction of B_L and B_H at t=0.

  Fit to the proper decay lengths distributions with a single signal exponential:
  ⇒ Flavor specific lifetime:

$$\tau(B_s)_\mathrm{fs} = \frac{1}{\Gamma_s} \frac{1 + \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}{1 - \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}, \qquad \Gamma_s = \frac{\Gamma_L + \Gamma_H}{2} = \frac{1}{\bar{\tau}(B_s)}$$

- $B_s^0 \to J/\psi\phi$: contributions from CP even and CP odd states, dominated by CP even. In this decay mode one can measure $\Delta\Gamma_s$ and $\bar{\tau}(B_s) = 1/\Gamma_s$.

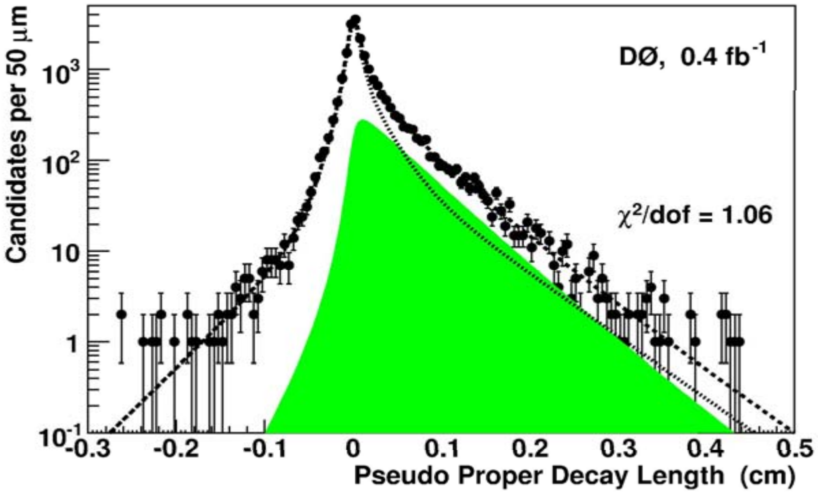
B_s lifetime in flavor specific modes

Both CDF and D∅ have measured B_s lifetime in $B_s^0 \to D_s^+ \ell^- \bar{\nu}_\ell X$

Results based on respectively 360 and 400 pb^{-1} are:

$$\tau(B_s)_{\rm fs} = 1.398 \pm 0.044({\rm stat})^{+0.028}_{-0.025}({\rm sys})\ {\rm ps}\ ({\rm D0})$$

$$\tau(B_s)_{\rm fs} = 1.381 \pm 0.055({\rm stat})^{+0.052}_{-0.046}({\rm sys})\ {\rm ps}\ ({\rm CDF})$$



CDF has also measured B_s lifetime in the fully hadronic modes:

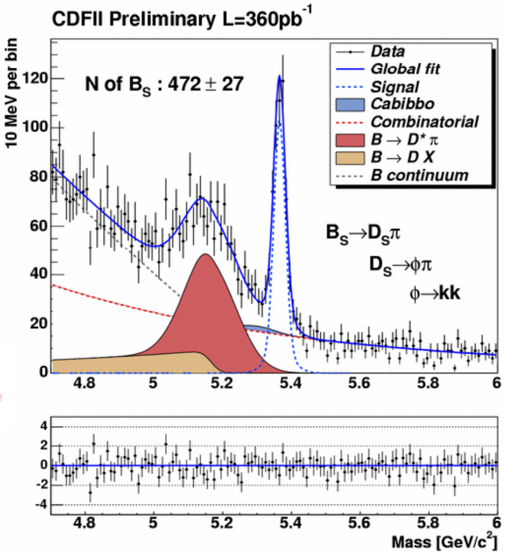
$$B_s^0 \to D_s^+ \pi^- \quad , \quad B_s^0 \to D_s^+ \pi^+ \pi^- \pi^-$$

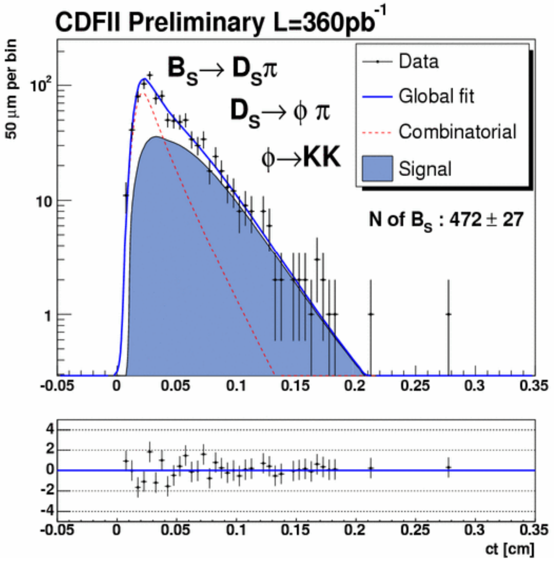
Analysis based on 360 pb^{-1}.

B_s lifetime extracted from a simultaneous fit to the mass and decay length distributions:

$$\tau(B_s)_{\rm fs} = 1.60 \pm 0.10({\rm stat}) \pm 0.02({\rm sys})\ {\rm ps}$$

Will be updated for 1 fb^{-1}.

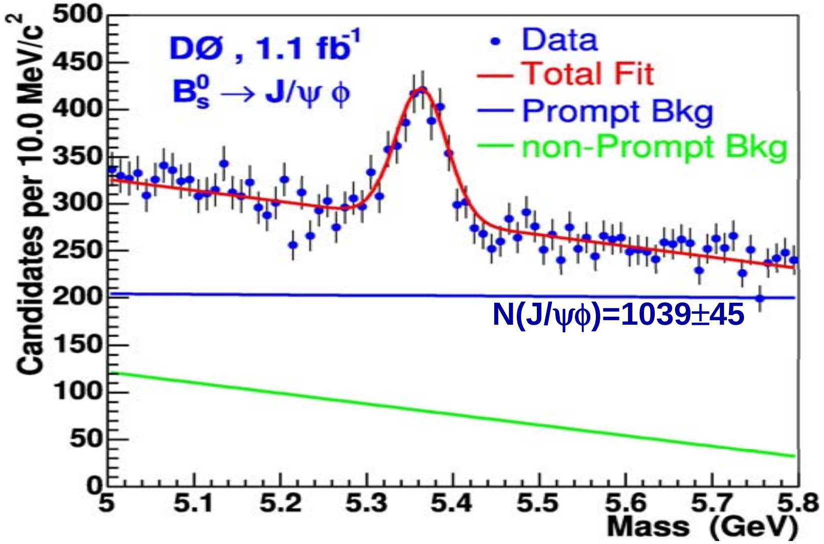




B_s lifetime in $B_s^0 \to J/\psi\phi$

D∅ made a new B_s lifetime measurement in $B_s^0 \to J/\psi\phi$ based on 1.1 fb^{-1}:

D∅ preliminary, 1.1 fb^{-1}



The study of the time dependent angular distribution of the decay products of J/ψ and φ allow to separate the two CP components of the decay. Schematically:

$$\frac{d^3\Gamma(t)}{d\cos\theta\, d\varphi\, d\cos\psi} \propto |A_{\mathrm{even}}(\theta,\varphi,\psi)|^2 + |A_{\mathrm{odd}}(\theta,\varphi,\psi)|^2 + \mathrm{interf.\ term}(\phi_s)$$

ϕ_s ≡ CP violating mixing phase.

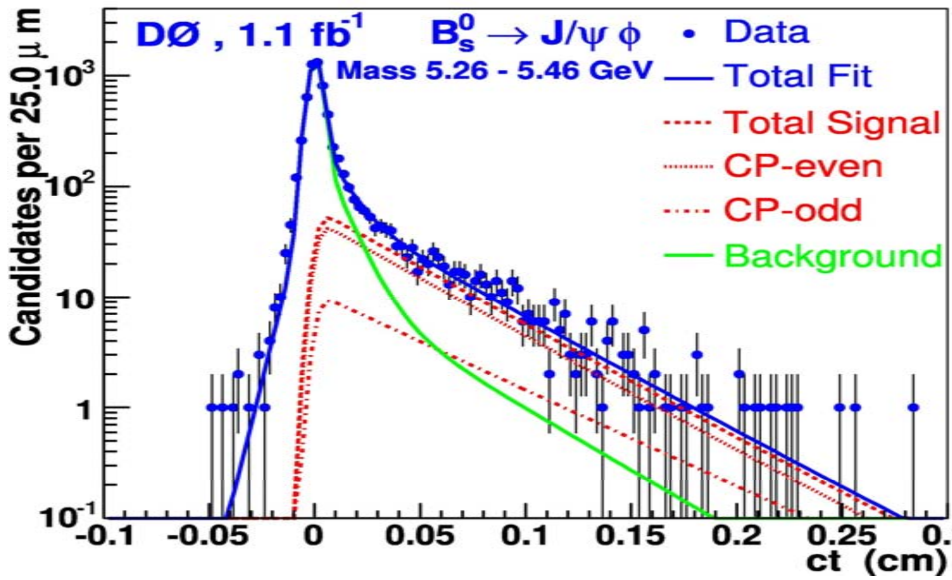
$\Delta\Gamma$ is extracted from a simultaneous unbinned maximum likelihood fit to the proper decay length, the 3 decay angles and the mass.

Assuming no CP violation (ϕ_s=0):

$$\Delta\Gamma = 0.12^{+0.08}_{-0.10}(\mathrm{stat}) \pm 0.02(\mathrm{sys})\ \mathrm{ps}^{-1}$$

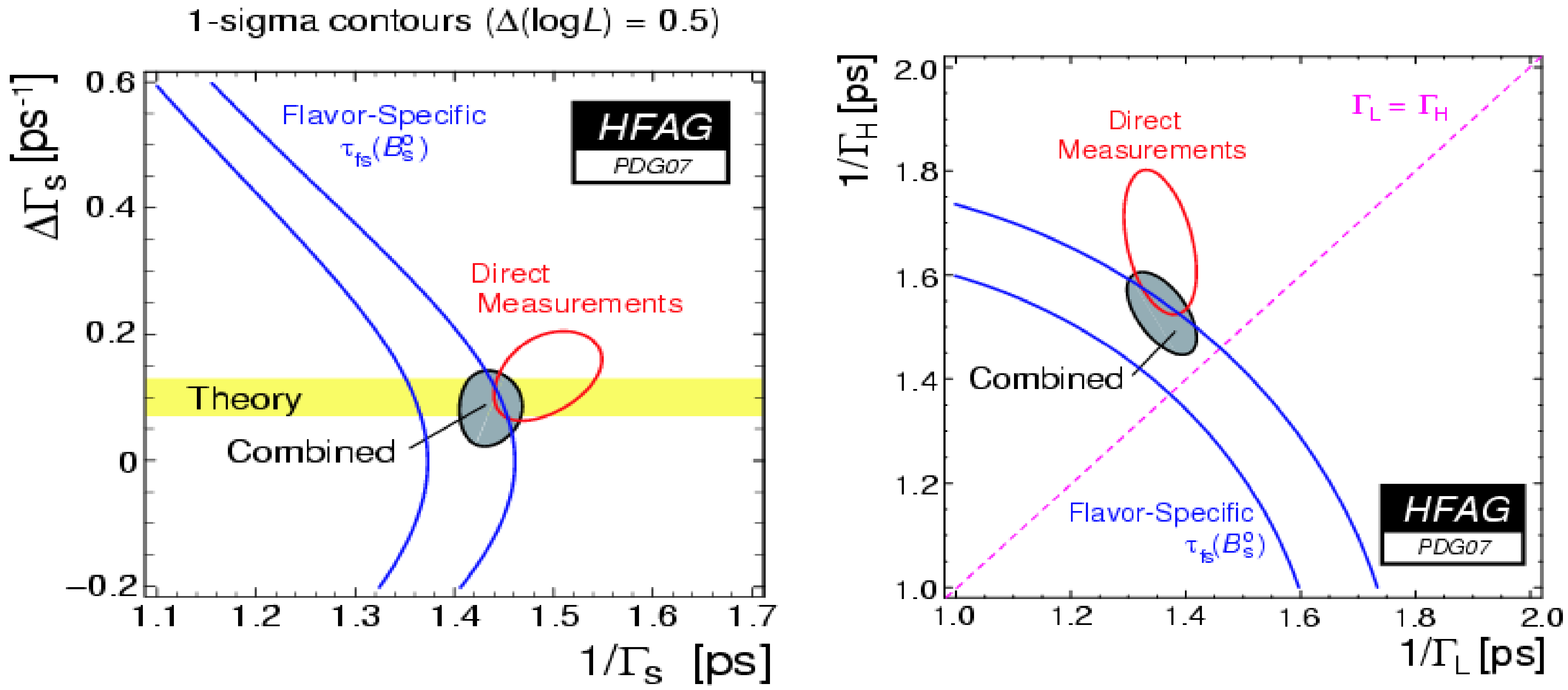
(R. Bernhard and S. Donati talks)

D∅ preliminary, 1.1 fb^{-1}



$$\bar{\tau}(B_s) = 1.52 \pm 0.08(\mathrm{stat})^{+0.01}_{-0.03}(\mathrm{sys})\ \mathrm{ps}$$

B$_s$ lifetime in $B_s^0 \to J/\psi\phi$



Λ_b lifetime measurements

Λ_b lifetime measurements in $\Lambda_b \to J/\psi\Lambda(p\pi^-)$

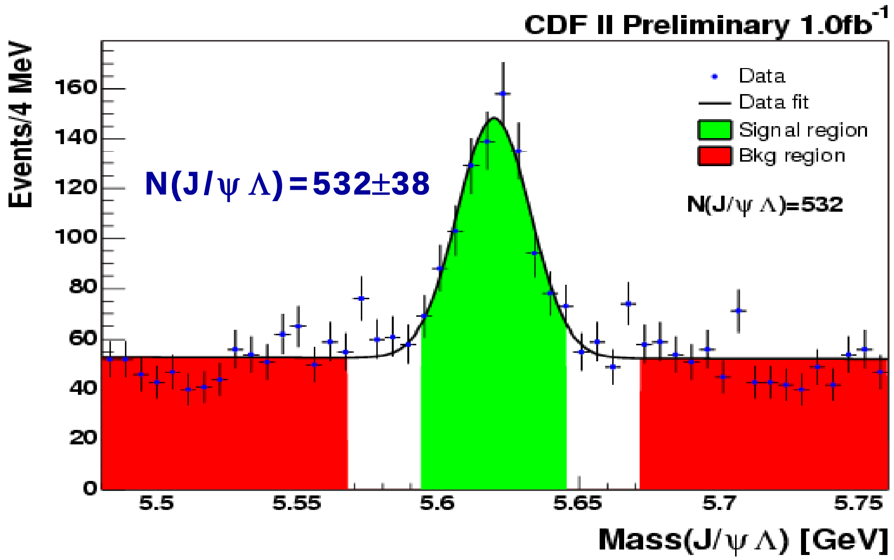
- Both CDF and D∅ have measurements based on respectively 1 fb^{-1} and 1.2 fb^{-1}.
- Similar analysis procedure:

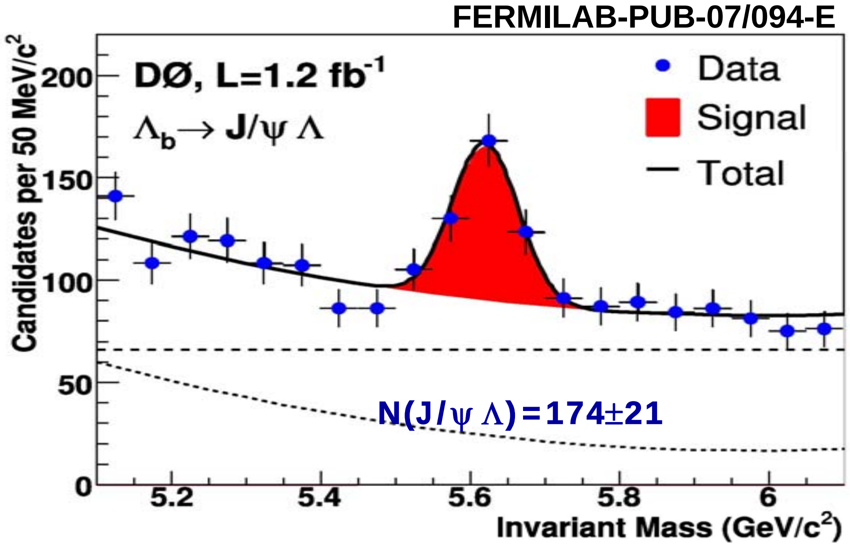
  Λ_b lifetime extracted from an unbinned simultaneous likelihood fit to the mass and proper decay lengths distributions.

$$ct = \frac{L_{xy}}{\gamma\beta} = L_{xy}\frac{M(\Lambda_b)}{P_T(\Lambda_b)} \quad , \quad L_{xy} = (\vec{r}_{J/\psi} - \vec{r}_{PV}) \cdot \vec{u}_{P_T(\Lambda_b)}$$

  Λ_b lifetime cross checked using $B^0 \to J/\psi K_s(\pi^+\pi^-)$

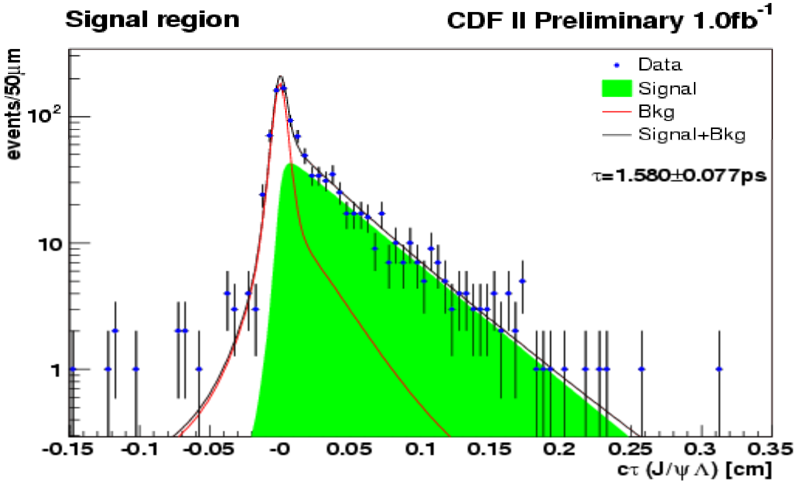
  ⇒ similar signature and kinematics.





CDF tracking and $\Lambda \to p\pi^-$ efficiencies > D∅ ⇒ higher yield

Λ_b lifetime measurements in $\Lambda_b \to J/\psi\Lambda(p\pi^-)$



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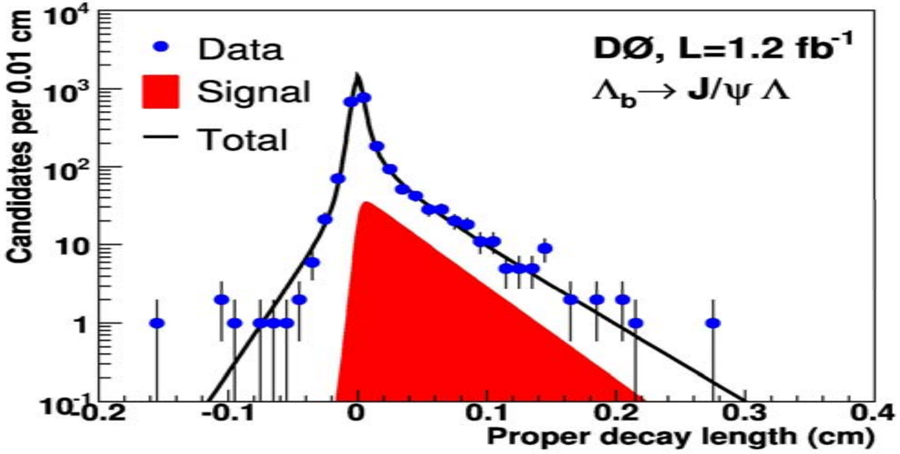


$$\tau(\Lambda_b) = 1.580 \pm 0.077(\mathrm{stat}) \pm 0.012(\mathrm{sys})\ \mathrm{ps}$$

$$\tau(\Lambda_b) = 1.218^{+0.130}_{-0.115}(\mathrm{stat}) \pm 0.042(\mathrm{sys})\ \mathrm{ps}$$

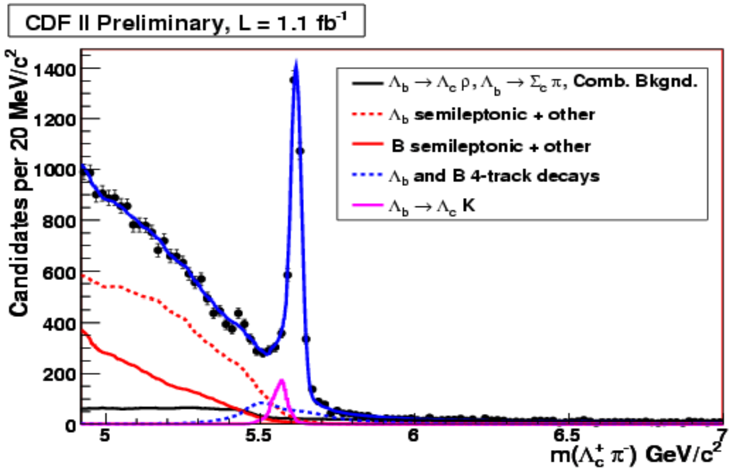
World average (PDG2006): $\tau(\Lambda_b) = 1.230 \pm 0.074$ ps

Cross checks:

CDF : $\tau(B^0 \to J/\psi K_s, K^*) = 1.551 \pm 0.019(\mathrm{stat}) \pm 0.011(\mathrm{sys})$

DØ : $\tau(B^0 \to J/\psi K_s) - 1.501^{+0.078}_{-0.074}(\mathrm{stat}) \pm 0.05(\mathrm{sys})$

D∅ result is consistent with the $\tau(\Lambda_b)$ world average but CDF result is more than 3σ above.



Need more experimental inputs to conclude:
Full hadronic modes ?, CDF has about 3000 reconstructed $\Lambda_b \to \Lambda_c^+(pK^-\pi^+)\pi^-$ more than $\Lambda_b \to J/\psi\Lambda$
⇒ Lifetime measurement in progress.

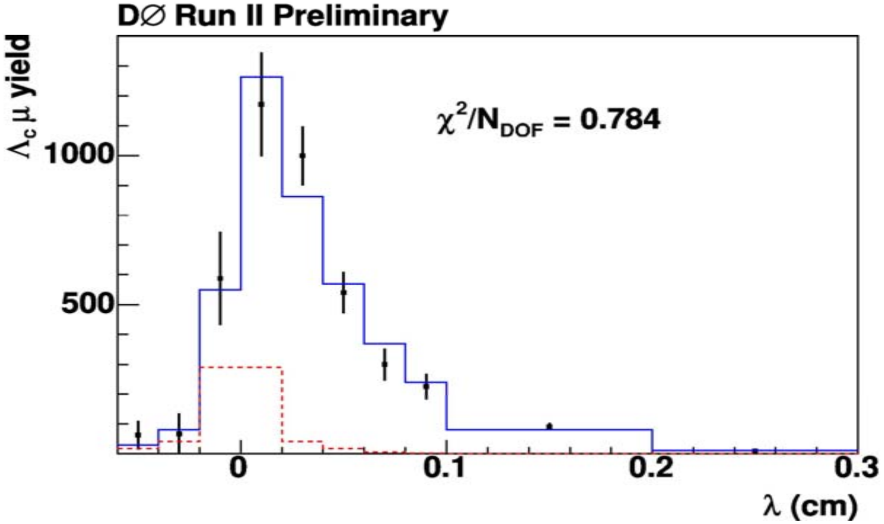
Λ_b lifetime measurements in $\Lambda_b \to \Lambda_c^+(K_s p)\mu^-\bar{\nu}_\mu X$

New measurement by DØ based on 1.3 fb^{-1}.

Partial reconstruction ⇒ $ct = L_{xy}\frac{M(\Lambda_b)}{P_T(\Lambda_c^+\mu^-)} \times K$

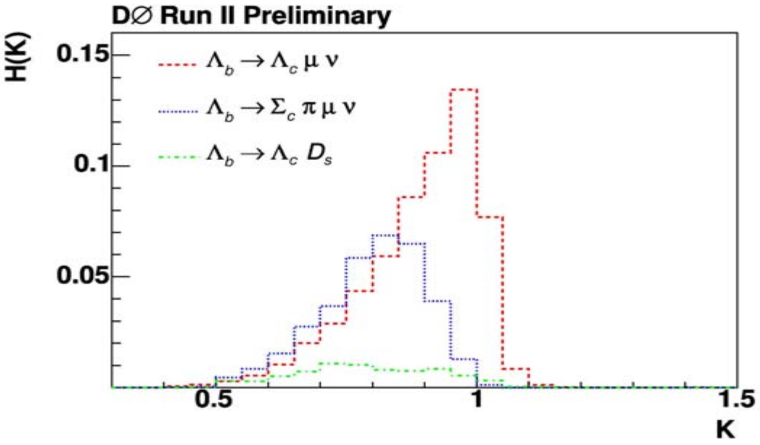
$K = \frac{P_T(\Lambda_c^+\mu^-)}{P_T(\Lambda_b)}$ estimated from Monte Carlo

$\tau(\Lambda_b)$ lifetime extracted from the fit of the number of $K_s p\mu^-$ events versus the visible proper decay length



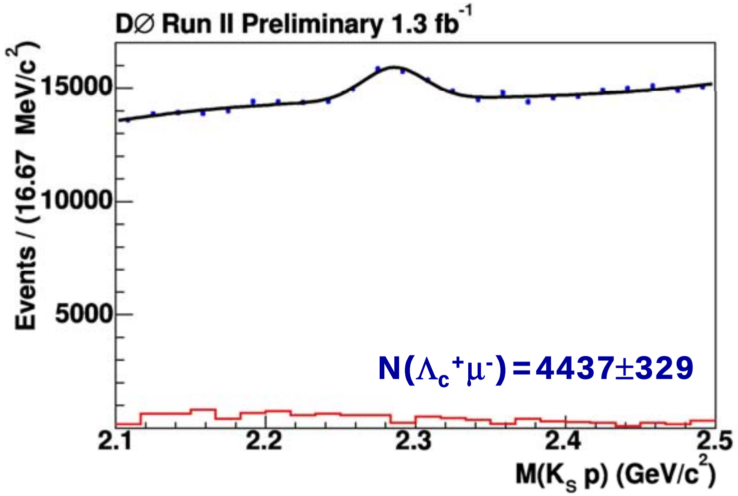
$\tau(\Lambda_b) = 1.28 \pm 0.12(\mathrm{stat}) \pm 0.09(\mathrm{sys})$ ps

Result compatible with $\Lambda_b \to J/\psi\Lambda(p\pi^-)$ and world average.



$\Lambda_b \to \Lambda_c^+\mu^-\bar{\nu}_\mu$ fraction $\sim 0.47^{+0.12}_{-0.10}$

in semileptonic Λ_b decays



m(K_sp) distribution for Λ_b candidates
(3.4 GeV < m($K_s p\mu^-$) < 5.4 GeV)

Summary

- B_s mixing:

  - CDF (improved analysis + additional partially reconstructed B_s hadronic decays):

    $\Delta m_s = 17.77 \pm 0.1(\mathrm{stat}) \pm 0.07(\mathrm{sys})\ \mathrm{ps}^{-1}$ with >5σ signal significance.

- *b*-hadrons lifetimes:

  For B_s lifetimes measurements in the flavor specific modes:

  $\tau(B_s)_{\mathrm{fs}} = 1.398 \pm 0.044(\mathrm{stat})^{+0.028}_{-0.025}(\mathrm{sys})\ \mathrm{ps}\ (\mathrm{D0})$

  $\tau(B_s)_{\mathrm{fs}} = 1.381 \pm 0.055(\mathrm{stat})^{+0.052}_{-0.046}(\mathrm{sys})\ \mathrm{ps}\ (\mathrm{CDF})$

  DØ has new B_s lifetimes measurements in $B_s^0 \to J/\psi\phi$

  - DØ: $\tau(B_s) = 1.52 \pm 0.08(\mathrm{stat})^{+0.01}_{-0.03}(\mathrm{sys})\ \mathrm{ps}$

  CDF and DØ have updated (1 fb-1) their Λ_b lifetimes measurements in $\Lambda_b \to J/\psi\Lambda$:

  - DØ: $\tau(\Lambda_b) = 1.2989 \pm 0.137(\mathrm{stat}) \pm 0.050(\mathrm{sys})\ \mathrm{ps}$

    compatible with world average: $\tau(\Lambda_b) = 1.230 \pm 0.074\ \mathrm{ps}$

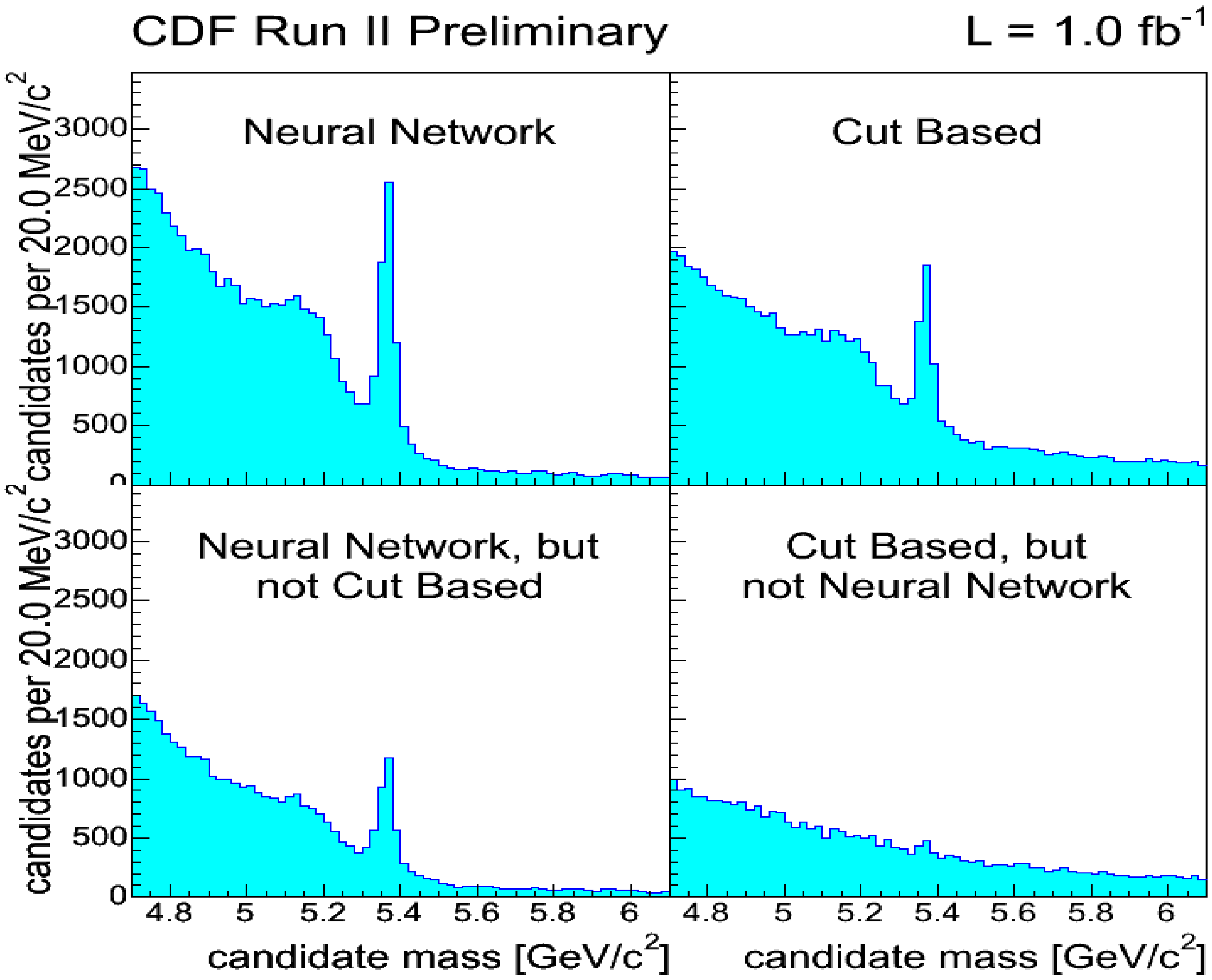
  - CDF: $\tau(\Lambda_b) = 1.580 \pm 0.077(\mathrm{stat}) \pm 0.012(\mathrm{sys})\ \mathrm{ps}$ 3σ above world average.

  DØ has also a new $\tau(\Lambda_b)$ measurement in $\Lambda_b \to \Lambda_c^+(K_s p)\mu^-\bar{\nu}_\mu X$

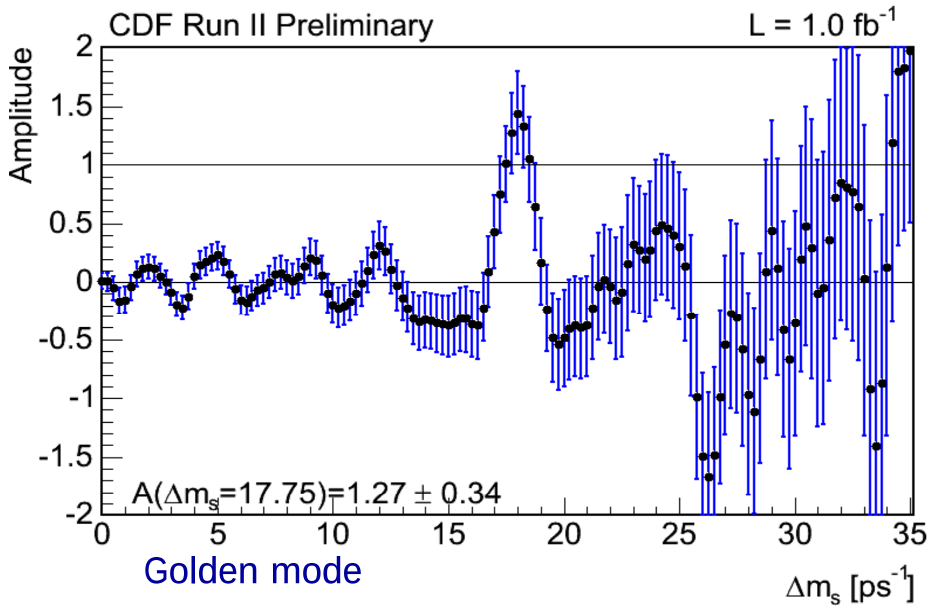
  $\tau(\Lambda_b) = 1.28 \pm 0.12(\mathrm{stat}) \pm 0.09(\mathrm{sys})\ \mathrm{ps}$ compatible with world average.

Backup slides

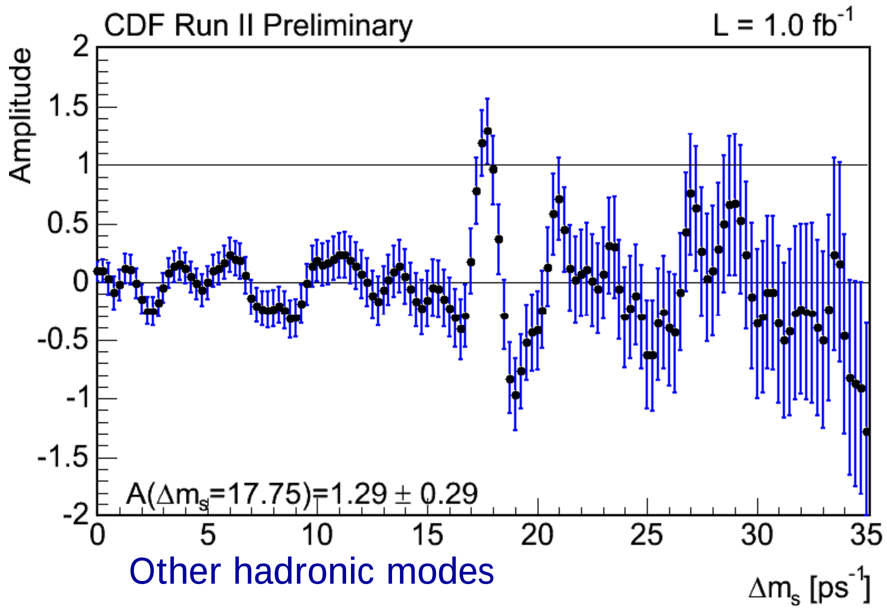
Neural Network selection performance



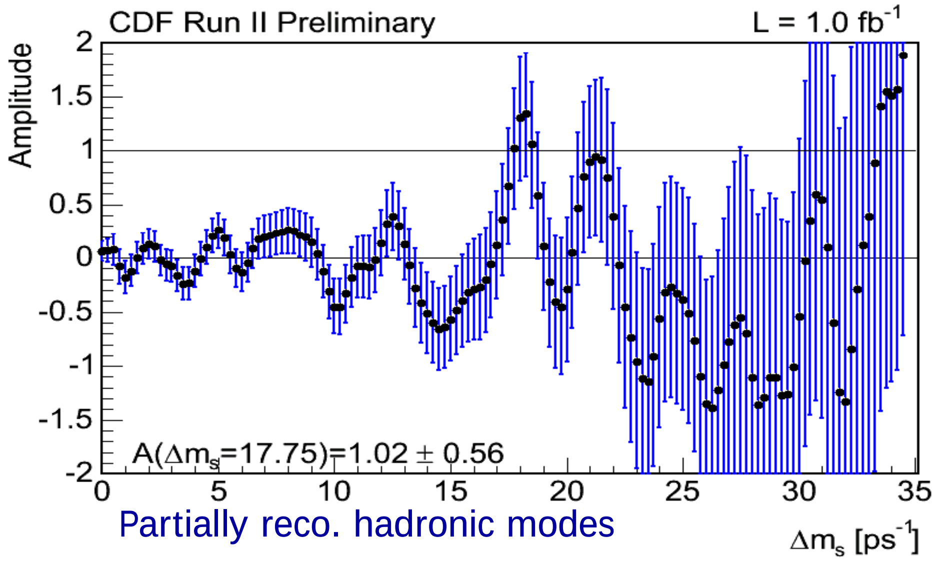
Separate decay modes amplitude scans



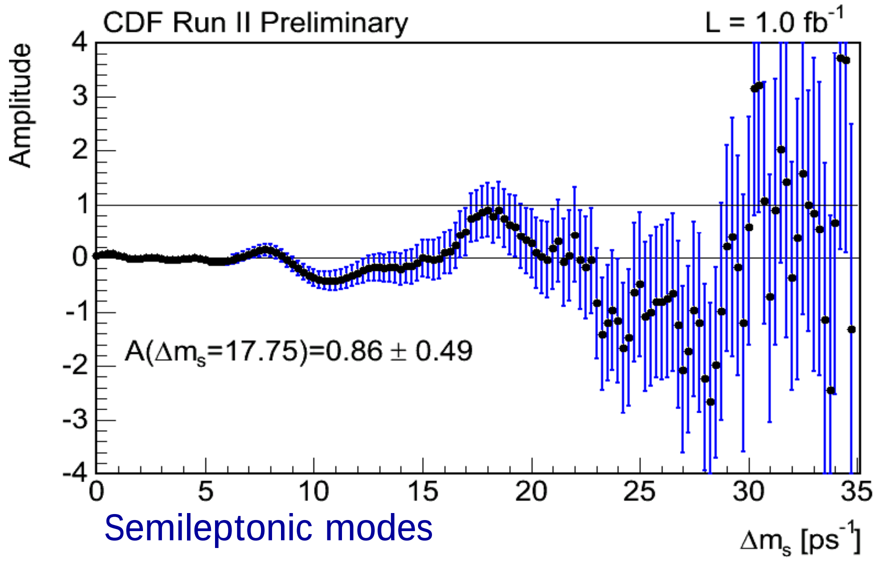
Golden mode



Other hadronic modes

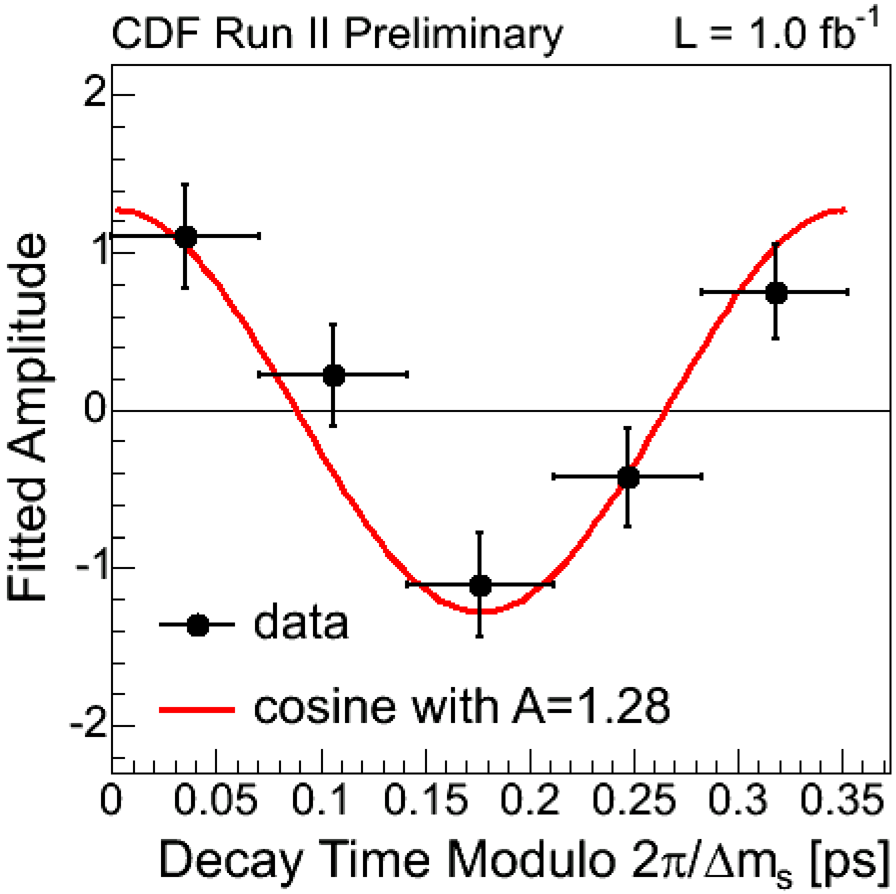


Partially reco. hadronic modes



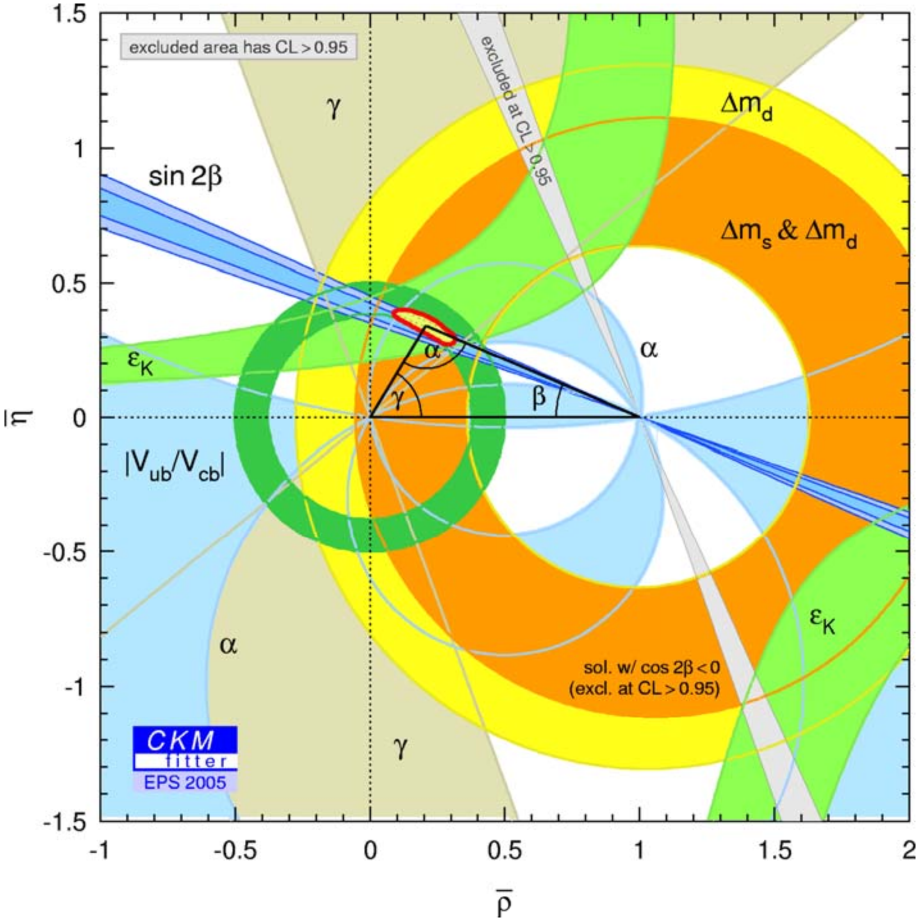
Semileptonic modes

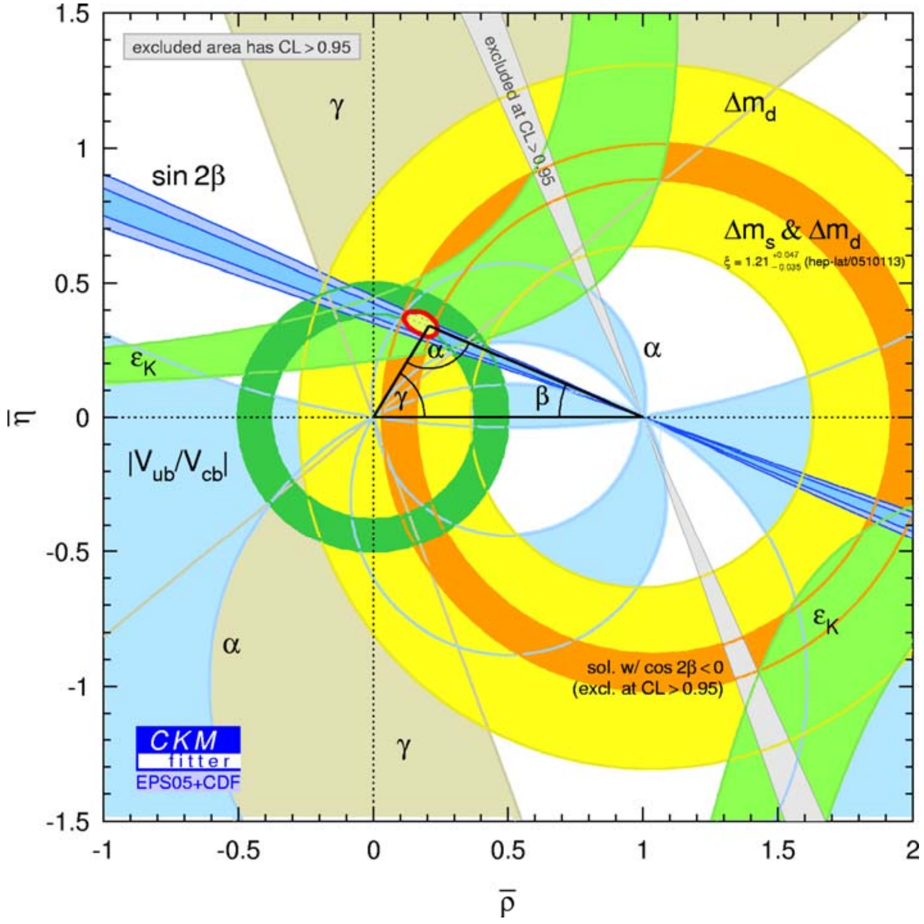
B_s oscillation signal



Impact on the CKM unitarity triangle

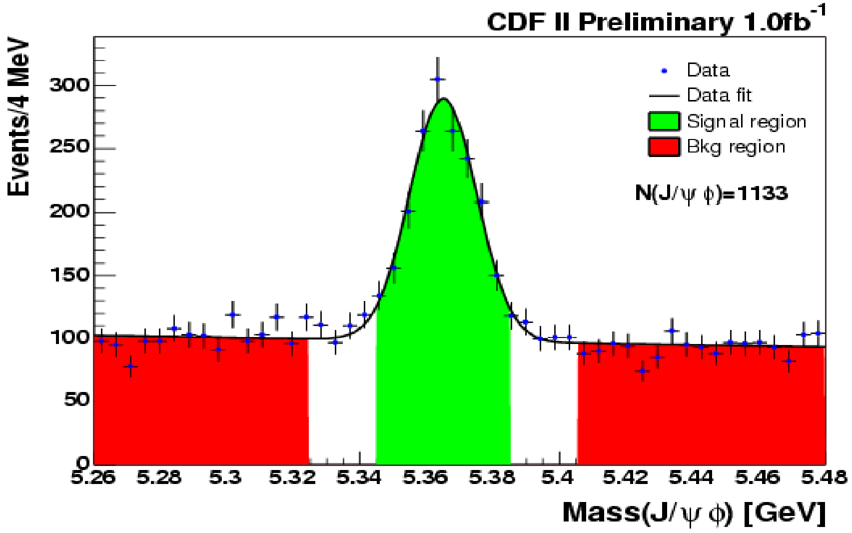
CDF

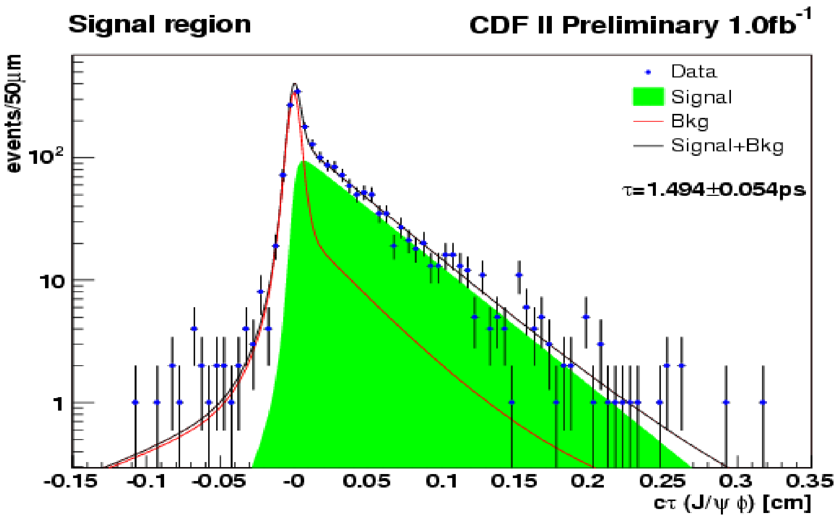




B$_s$ lifetime in $B_s^0 \to J/\psi\phi$

CDF made also a new average B$_s$ lifetime measurement in $B_s^0 \to J/\psi\phi$ from a data sample of 1 fb^{-1}:





$$\hat{\tau}(B_s) = 1.494 \pm 0.054(\text{stat}) \pm 0.009(\text{sys})\ \text{ps}$$

Λ_b lifetime measurements