Progress in lattice QCD: leptonic decays and neutral meson mixing

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Fermilab

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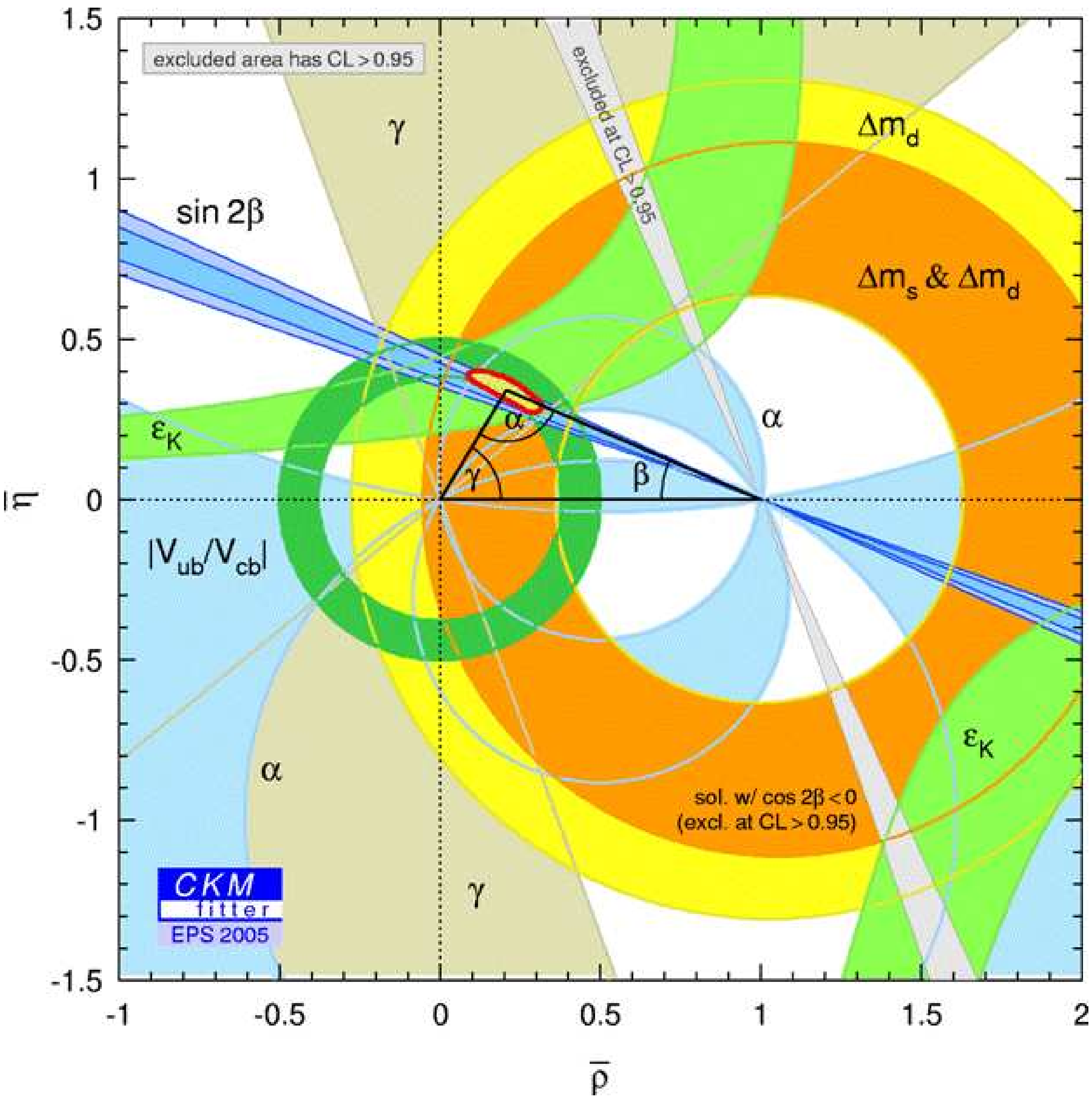
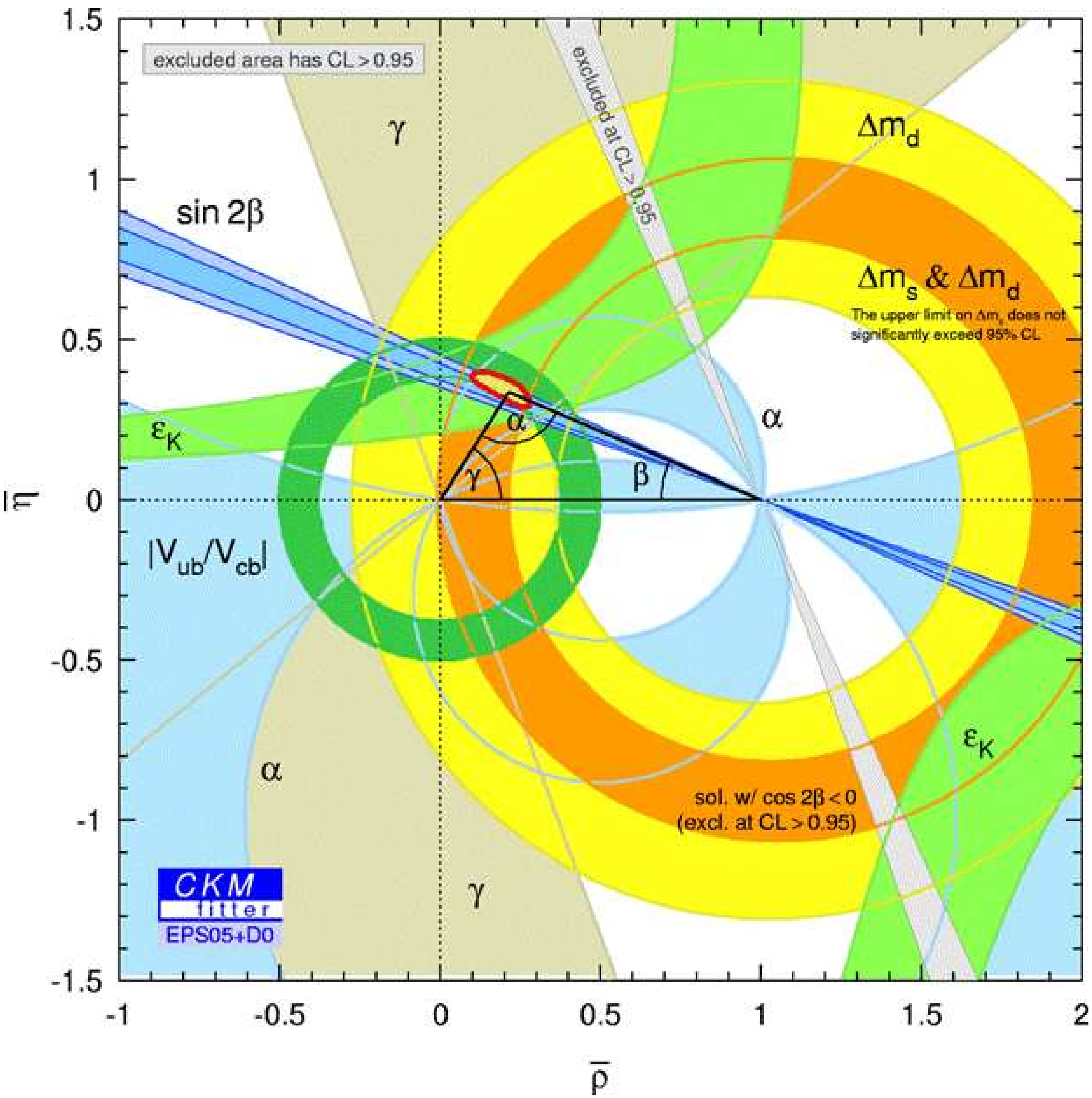
Constraining the Unitarity Triangle



Constraining the Unitarity Triangle



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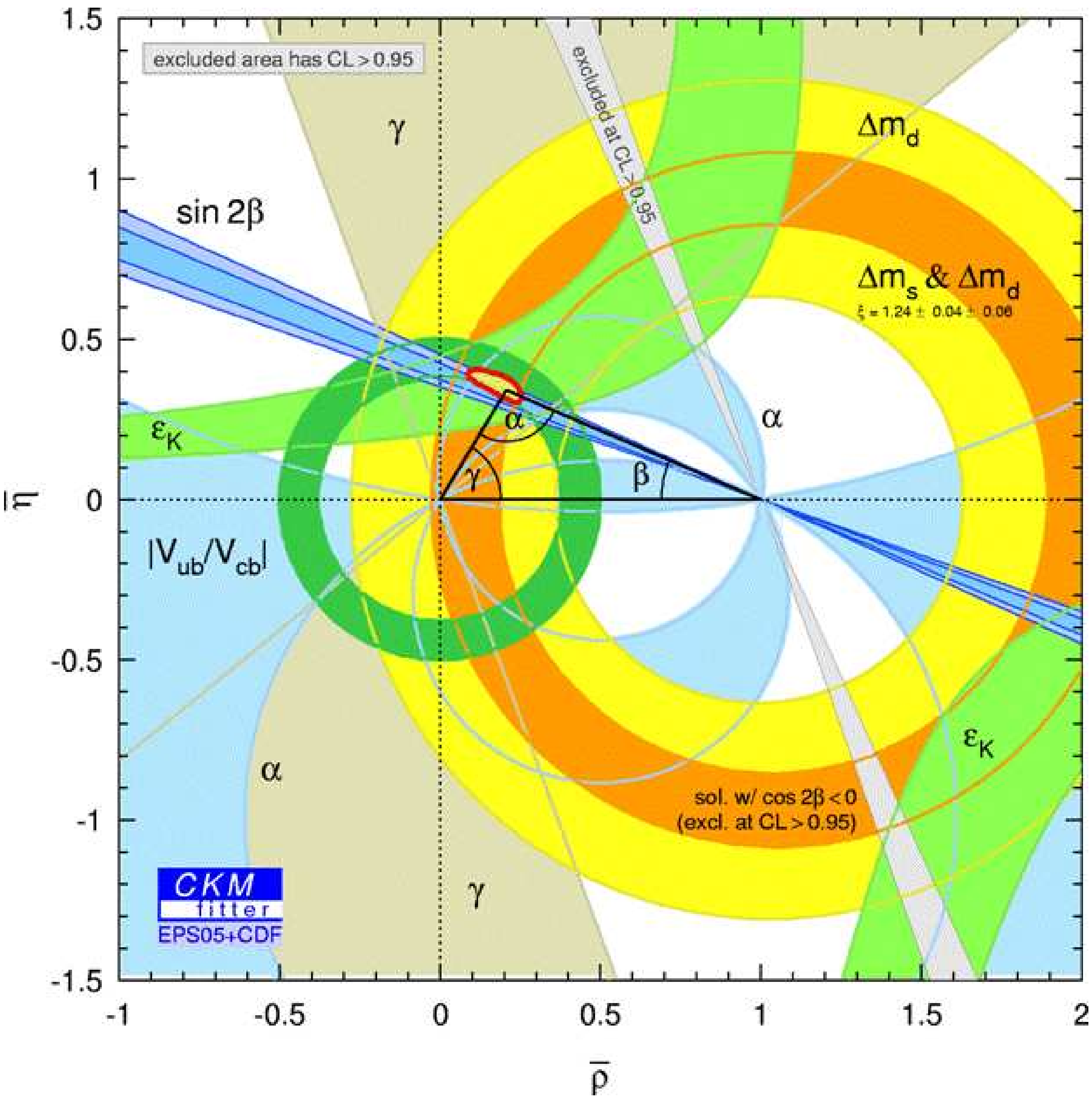


Introduction

I. Review of quantities $f_{B_{d,s}}$ and $f_{B_{d,s}}\sqrt{B_{B_{d,s}}}$ needed to extract CKM parameters from $\Delta M_{d,s}$

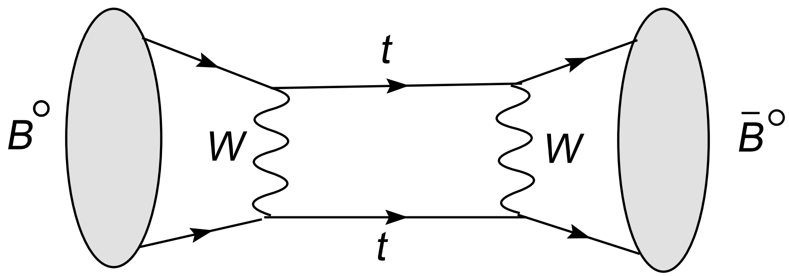
II. Lattice QCD and discussion of errors for non-experts

III. Review of recent unquenched f_B calculations

IV. Review of latest on $f_{B_{d,s}}\sqrt{B_{B_{d,s}}}$

V. Conclusion

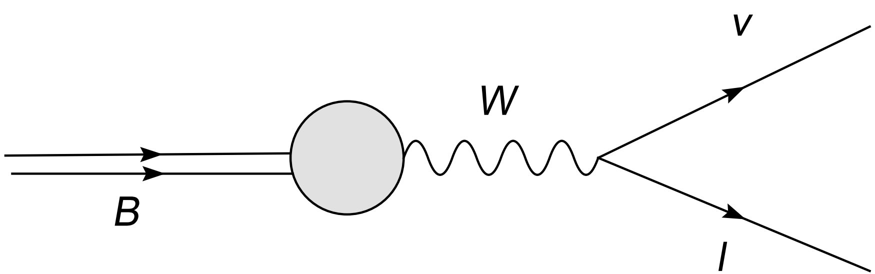
B_B



$$\langle \overline{B}^0 | (\bar{b}d)_{V-A} (\bar{b}d)_{V-A} | B^0 \rangle \equiv \frac{8}{3} m_B^2 f_B^2 B_B, \qquad (1)$$

$$\Delta M_s = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \widehat{B}_{B_s} \qquad (2)$$

f_B



$$\mathcal{B}(B \to \tau \overline{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \qquad (3)$$

Lattice QCD

- Allows non-perturbative calculations from first principles
- Can be simulated on a computer using Monte Carlo methods
- Simulations require a finite-sized grid with lattice spacing a and size L
- Even with today's computers this is still a difficult task!

Quenched Approximation

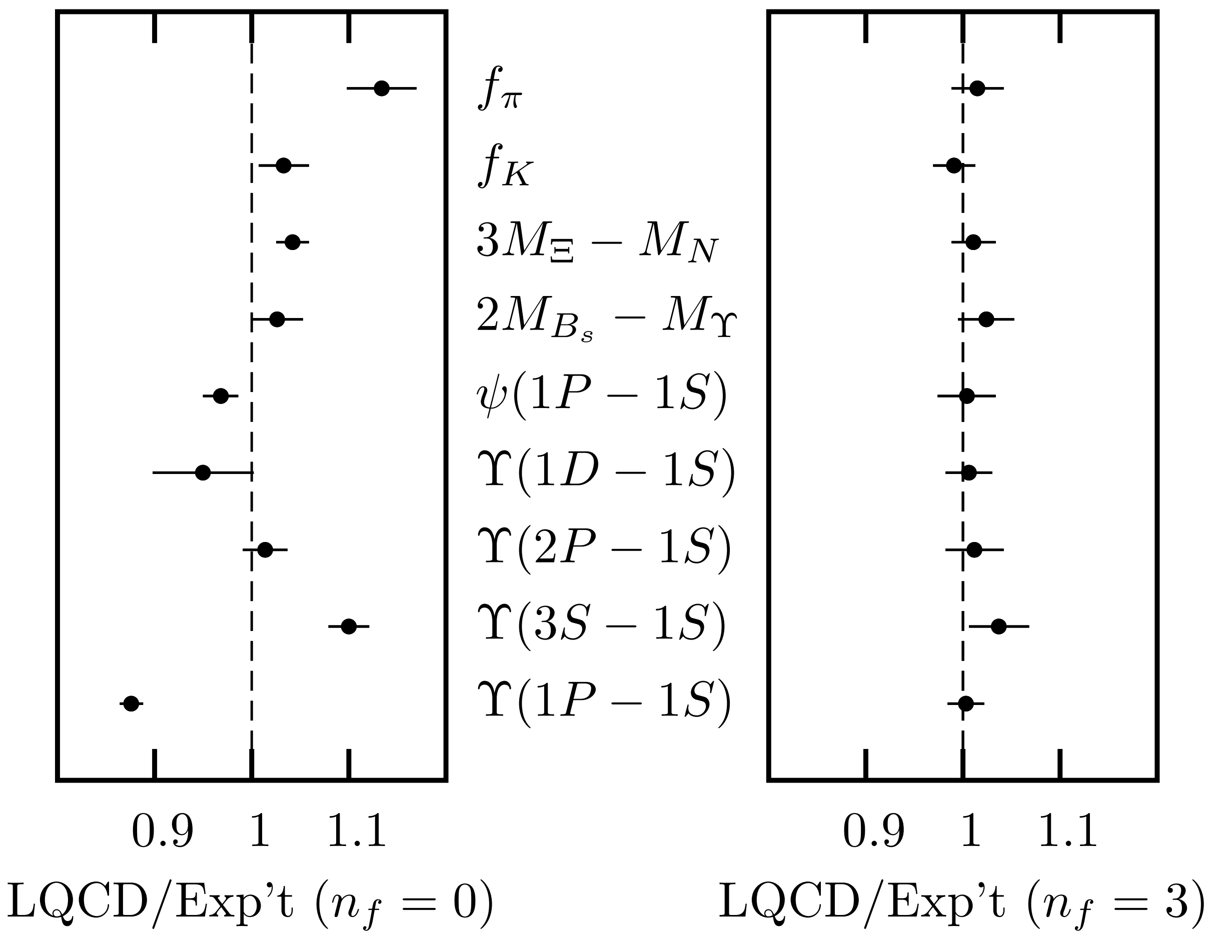
Configurations are generated with a weighting given by the gauge field and fermion determinant. Expensive.

“Unquenched” calculations, where the fermion determinant is not ignored are just now starting to be the norm.

The quenched approximation ignores fermion-antifermion vacuum bubbles. This is an uncontrolled systematic error.

“Partial quenching” treats the valence quarks (which appear in operators) differently from the sea quarks (which weight the gauge configurations). This isn’t necessarily a bad thing if one has the right number of flavors in the two sectors.

Unquenching Lattice QCD



- Hadron spectroscopy – masses and decay constants (hep-lat/0304004)
- *Good agreement for simple quantities!*

Systematic Errors

Because QCD with physical quark masses is a nonlinear multiscale problem ($\Lambda_{QCD} \approx 100 - 200$ MeV, $m_{u,d} \approx 2 - 6$ MeV, $m_b \approx 4.3$ GeV), it is very expensive to simulate at the physical quark masses.

1.) Statistics and fitting

2.) Tuning lattice spacing, a, and quark masses

3.) Matching lattice gauge theory to continuum QCD

4.) Extrapolation to continuum

5.) Chiral extrapolation to physical up, down quark masses

Chiral perturbation theory

Chiral perturbation theory (ChPT) is an expansion about small quark masses and momenta.

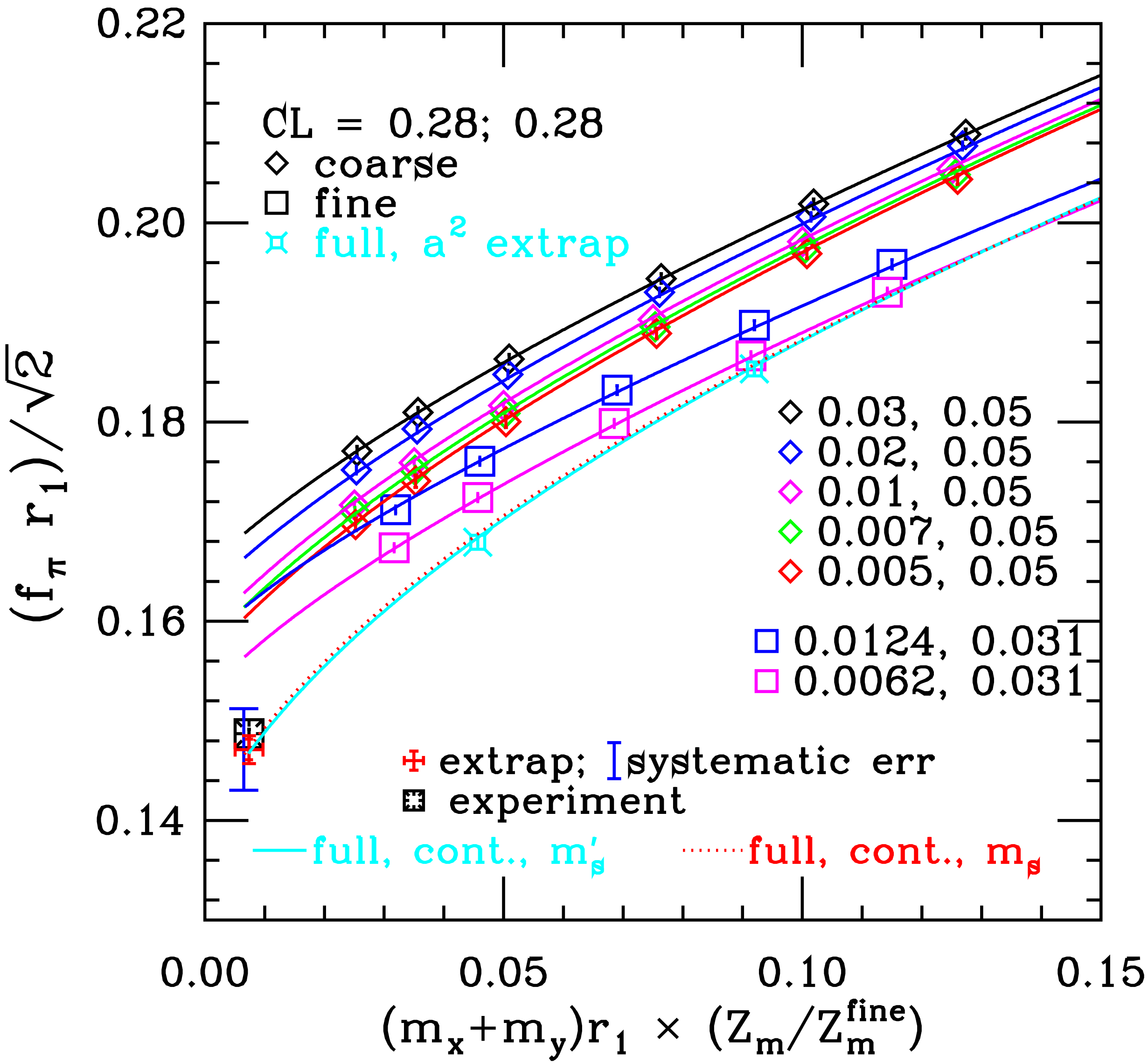
At each order new terms must be introduced to cancel the renormalization scale dependence. These terms are not determined within ChPT.

When combined with lattice calculations, these constants can be determined. In fact, varying valence and sea quarks separately gives a better handle on the fits to various quantities.

Finally, it is possible to account for lattice artifacts in the ChPT by introducing the appropriate symmetry breaking terms in the chiral lagrangian (finite lattice spacing) or restricting the Feynman integrals to finite volume.

Chiral perturbation theory is not a model!

Chiral Extrapolation



(MILC, hep-lat/0407028)

Heavy quarks on the lattice

The lattice cut-off is smaller than the heavy quark masses for realistic lattices.
The solution(s): heavy quark effective theory(HQET) or nonrelativistic QCD

Fermilab Method:

Continuum QCD $\longrightarrow$ Lattice gauge theory
(using HQET)

nonrelativistic QCD method:

Continuum QCD $\longrightarrow$ Nonrelativistic QCD $\longrightarrow$ Lattice gauge theory

- Both methods require tuning parameters of the lattice action
- The currents and 4-quark operators must be matched as well. Typically this is done in lattice perturbation theory.

Other Approaches

The extrapolation method (Becirevic, et al, hep-lat/0002025; QCDSF, hep-lat/0701015):

In this case one simulates at masses around the charm quark and extrapolates to bottom with fit functions determined from HQET.

The step-scaling method (Guazzini, Sommer, and Tantalo, hep-lat/0609065):

One starts with a small volume where the b quark can be computed directly, where the finite size effects can be eliminated through step scaling functions which give the change of the observables when L is changed to $2L$.

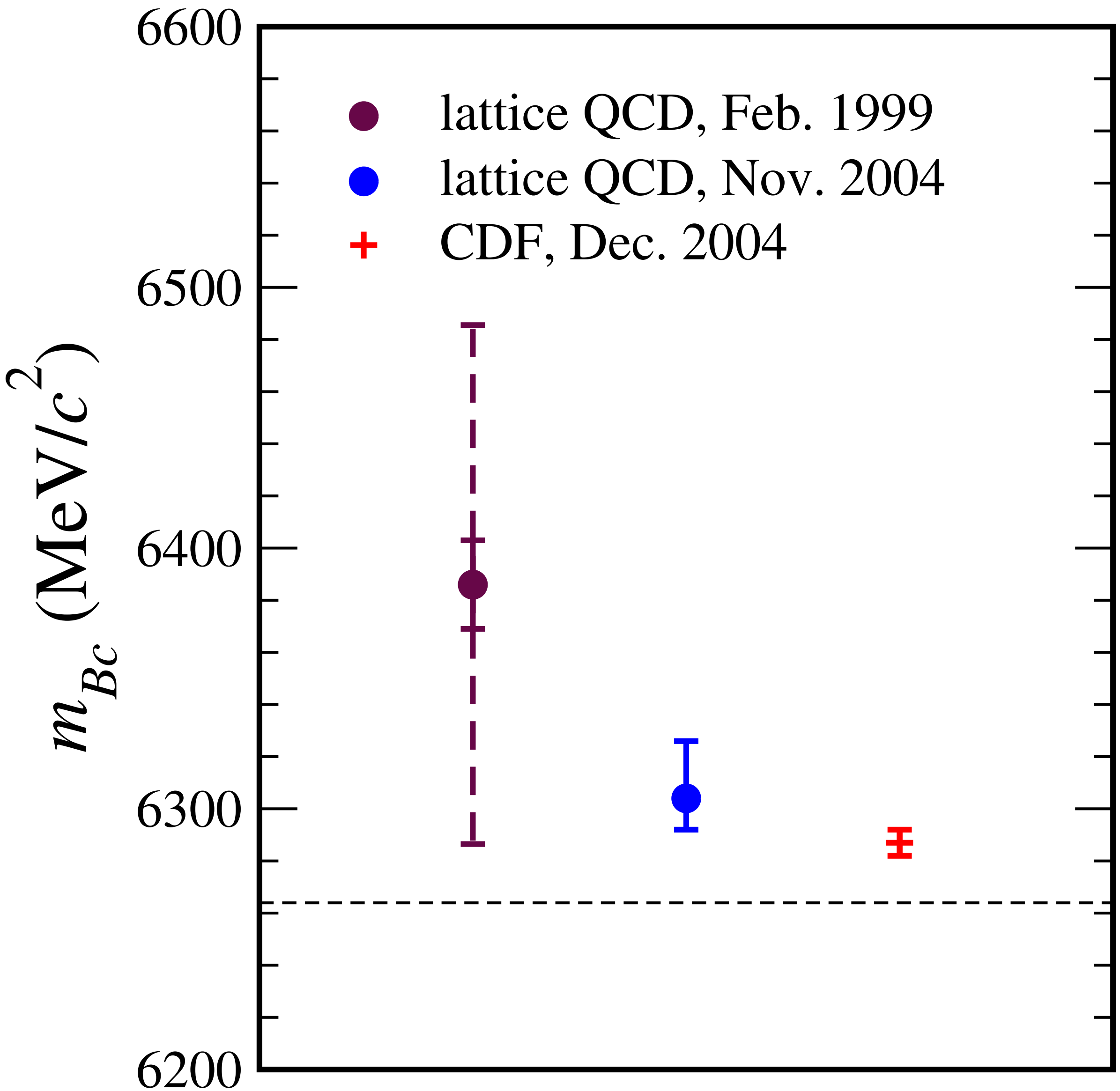
Matching Errors

One must estimate errors due to inexact matching of the lattice to the continuum.

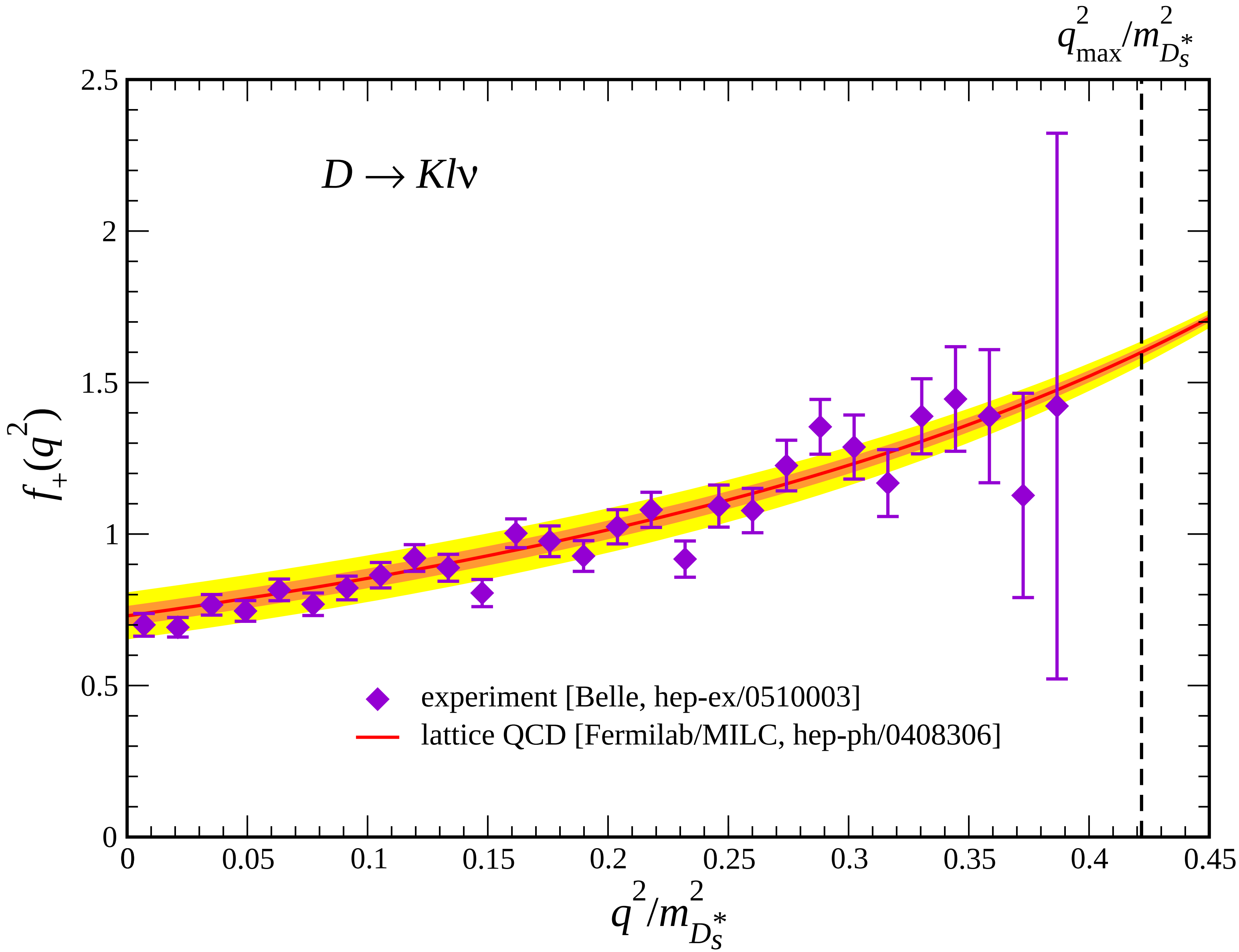
In the Fermilab method, all errors associated with discretizing the action are combined. These errors are then estimated using knowledge of HQET power counting.

In the nonrelativistic QCD method, there are “relativistic errors” associated with using NRQCD [$O(\alpha_s \Lambda_{QCD}/m_Q)$, $O(\Lambda_{QCD}^2/m_Q^2)$], and “perturbation theory errors” associated with matching NRQCD to the lattice [$O(\alpha_s^2)$].

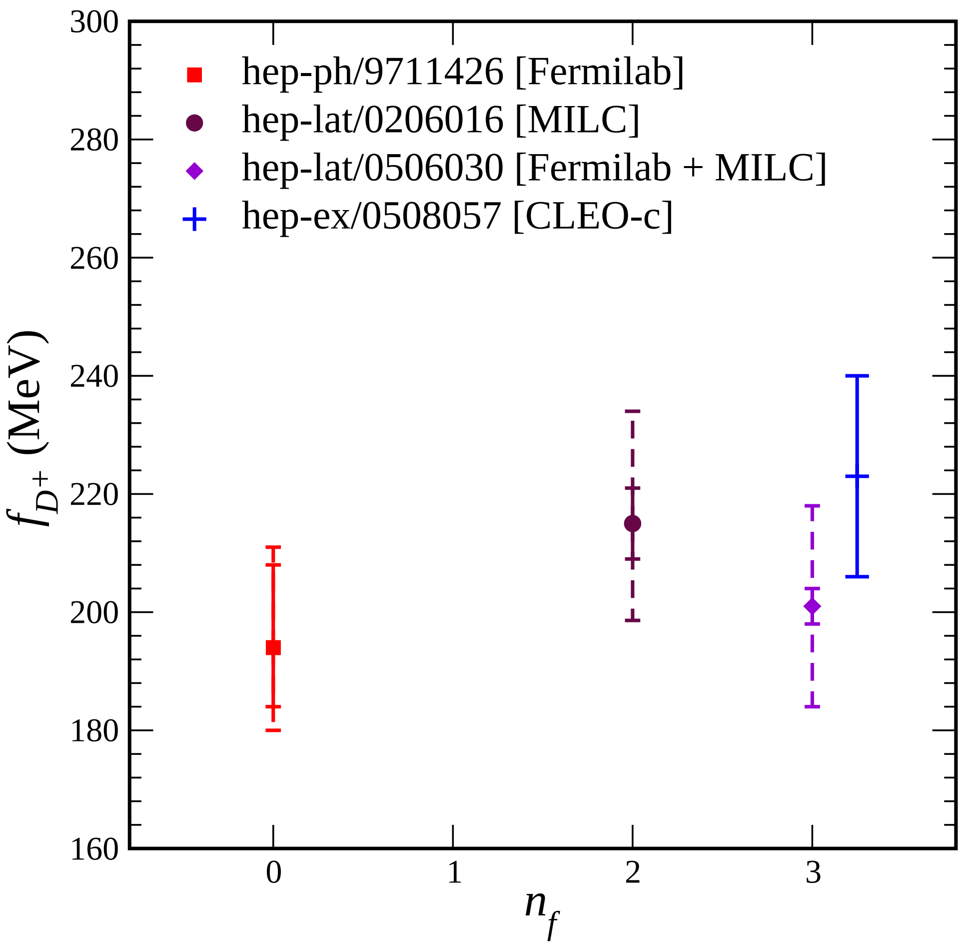
Prediction of the B_c mass



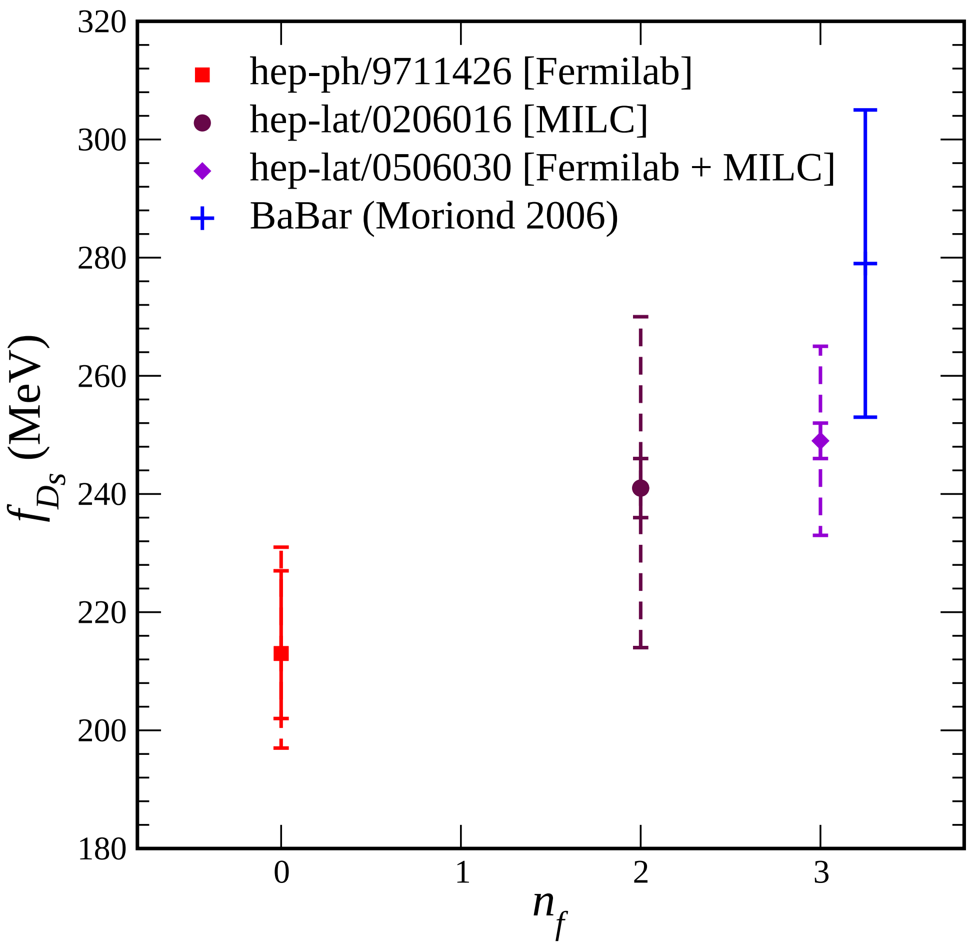
Prediction of form factor shape



Prediction of f_D



Prediction of f_{D_s}

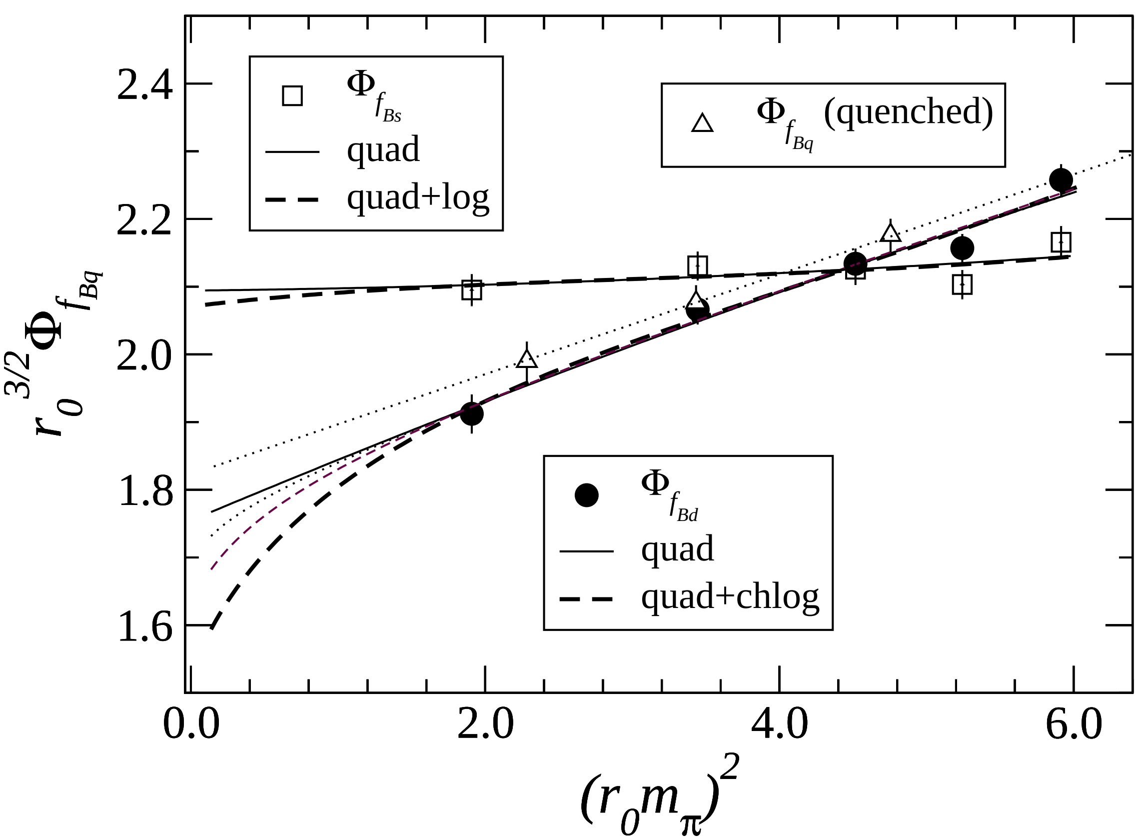


Recent lattice calculations of f_B

I will focus on three flavor unquenched lattice calculations of f_B, since these are now available. (For mixing this is not yet the case. For that part of the review I will also mention some quenched calculations.)

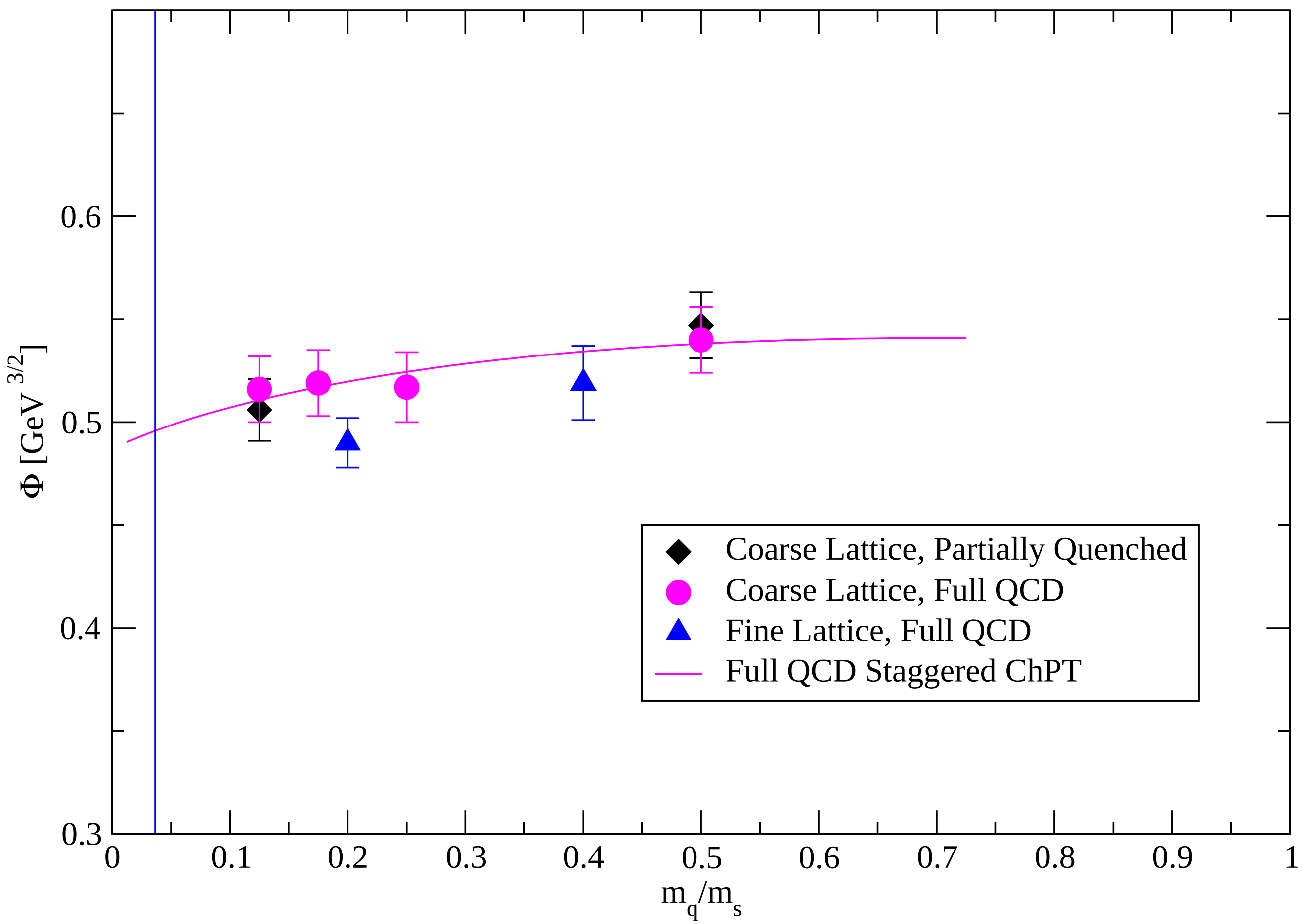
- There are currently two groups working on heavy light physics with three flavors of dynamical quarks: Fermilab/MILC and HPQCD
- Both groups use the (publicly available) “2+1 flavor” MILC lattices with three flavors of improved staggered quarks. HPQCD has a published result for f_B.
- These calculations have two light quark masses and one heavy around m_s
- The light quark masses range between m_s and $m_s/10$.

Old chiral extrapolation



JLQCD (hep-ph/0307039)

New chiral extrapolation



HPQCD (hep-lat/0507015)

HPQCD result for f_B

This was presented in hep-lat/0409040.

$f_{B_d} = 0.216(9)(19)(4)(6)$ GeV

4% statistics, scale, chiral extrapolation

9% higher order matching

2% generic discretization errors

3% Tuning m_b and relativistic corrections

Total: 10%

$f_{B_s}/f_{B_d} = 1.20(3)(1)$

The first error comes from statistics and the chiral extrapolation; the second comes from discretization errors, relativistic corrections and higher order matching.

Recent lattice calculations of $f_B\sqrt{B_B}$

- For $f^2_{B_s}B_{B_s}$, HPQCD has published a result (hep-lat/0610104). They are working on $f^2_{B_d}B_{B_d}$, as well as other matrix elements needed to compute the partial decay widths. Fermilab is also at work on this.

- For now the state of the art on $f^2_{B_d}B_{B_d}$ is a two flavor calculation with the light quark around m_s. For the other matrix elements relevant for the widths, the most recent results are still quenched.

HPQCD result for $B_s^0 - \overline{B}_s^0$ mixing

Presented in hep-lat/0610104

$f_{B_s}\sqrt{\widehat{B}_s} = 0.281(21)$ GeV

Error in $f_B^2 \widehat{B}_s$:

9% statistics and fitting

9% higher order matching

4% discretization errors

3% relativistic corrections

5% lattice scale uncertainty (a^{-3})

Total: 15%

Using $|V_{ts}^* V_{tb}| \approx |V_{cs}^* V_{cb}| \approx 4.1 \times 10^{-2}$ one gets $\Delta M_s(theory) = 20.3(3.0)(0.8) ps^{-1}$.
Conversely, $|V_{ts}^* V_{tb}| = 3.8(3)(1) \times 10^{-2}$.

$B_d^0 - \overline{B}_d^0$ mixing

JLQCD (hep-lat/03070139) quotes

$$f_{B_d}\sqrt{\widehat{B}_{B_d}} = 0.215(11)(^{+0}_{-23})(15)$$

where the errors are

5% statistical

11% chiral extrapolation error

7% lattice matching

Total: 14% (28% for the square)

Width Splittings

$$\Delta_s = 0.097^{+0.041}_{-0.042}\text{ps}^{-1} \longrightarrow \left(\frac{\Delta\Gamma}{\Gamma}\right) \approx 0.15 \pm 0.06$$

from last year's FPCP Unofficial World Average.

Using the NLO formula of Lenz and Nierste [Lenz, LHC Workshop, June 2006], HPQCD quoted the following at Lattice 06

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{245MeV}\right)^2 [0.170 B_{B_s} + 0.059 \overline{B}_s - 0.044]$$

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} (thoery) = 0.16(3)(2)$$

Conclusion

f_{B_d} has approximately 10% errors

f_{B_s} has about the same, around 10%

$f_{B_s}^2 B_{B_s}$ has a 15% error

$f_{B_d}^2 B_{B_d}$ has close to a 30% error

Ratios have considerably lower errors.

Many MILC lattice ensembles at finer lattice spacings and larger volumes exist and are being analyzed now by both Fermilab/MILC and HPQCD. These should make a significant dent in the error budgets for these important quantities.